NATURAL HEDGING OF LIFE AND ANNUITY MORTALITY RISKS

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ABSTRACT. The values of life insurance and annuity liabilities move in opposite directions in response to a change in the underlying mortality. Natural hedging utilizes this to stabilize aggregate liability cash flows. Our study shows empirical evidence that insurers who utilize natural hedging also charge lower premiums than otherwise similar insurers. This indicates that insurers who are able to utilize natural hedging have a competitive advantage. In addition, we show how a mortality swap might be used to provide the benefits of natural hedging to a firm that writes only one of the lines of business.

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1. INTRODUCTION

If future mortality improves relative to expectations, the life insurer liabilities decrease because death benefit payments will be later than expected initially. However, the annuity insurers have a loss relative to current expectations because they have to pay annuity benefits longer than expected initially. If the mortality deteriorates, the situation is reversed: life insurers have losses and annuity writers have gains.

The purpose of this paper is to study natural hedging of mortality risks and to propose mortality swaps as a risk management tool. Natural hedging utilizes the interaction of life insurance and annuities to a change in mortality to stabilize aggregate cash outflows. The same mortality change has opposite impacts on life insurance and annuities.

Few researchers investigate the issue of natural hedging. Most of the prior research explores the impact of mortality changes on life insurance and annuities separately, or investigates a simple combination of life and pure endowment life contracts (Frees et al., 1996; Marceau and Gaillardetz, 1999; Milevsky and Promislow, 2001; Cairns et al., 2004). Studies on the impact of mortality changes on life insurance focus on "bad" shocks while those on annuities focus on "good" shocks.

Wang et al. (2003) analyze the impact of the changes of underlined factors guiding the process of the mortality hazard rates and propose an immunization model to calculate the optimal level of product mix between annuity and life insurance to hedge longevity risks based on the mortality experience in Taiwan. However, they do not use separate mortality tables to explore life insurance and annuity mortality experience. In practice, life insurance and annuity mortality experience can be very different, so there is "basis risk" involved in using annuities to hedge life insurance mortality risk. Their model cannot pick up this basis risk.

Marceau and Gaillardetz (1999) examine the calculation of the reserves in a stochastic mortality and interest rates environment for a general portfolio of life insurance policies. In their numerical examples, they use portfolios of term life insurance contracts and pure endowment polices, like Milevsky and Promislow (2001). They focus on convergence of simulation results. There is a hedging effect in their results, but they do not pursue the issue.

Froot and Stein (1998) develop a framework for analyzing the capital allocation and capital structure decisions facing financial institutions. Their model suggests that the hurdle rate of an investment opportunity consists two parts, the standard market-risk factor and the unhedgeable risk factor. Froot and O'Connel (1997) have documented the very high average hurdle rate of the catastrophe reinsurance business. On average, over the period

1980-1994, the price is on the order of four times the actuarial value. Since the risks being insured are essentially uncorrelated with the market portfolio and a classical model would imply prices roughly equal to actuarial values, this type of pattern suggests striking markup of unhedgeable catastrophe risks. Until now, no attention has been paid to the risk premium of unhedgeable mortality risks. Our hypothesis is that the insurance price is positively related to unhedgable mortality risks after we control for each company's size and reserve. Our results support the hypothesis that the insurance price is inversely related to the degree of natural hedging.

Although it is common for an insurer to write both life insurance and annuities, its mix of life and annuity mortality risks is not likely to provide an optimal mortality hedge. It may make sense to create a swap with another company to acquire the missing line of business and improve the natural hedge. We propose and price a mortality swap between a life insurer and an annuity insurer. It works like natural hedging within a company. Our research investigates the overall impact of a mortality swap on a life insurer's reserves and, therefore, contributes to the solution of the insurer's asset-liability management problem. If an insurer can successfully hedge its mortality risk, the mortality risk premium in its products will be reduced, and thus its prices will be lower. It will improve its competitiveness in the market.

The paper proceeds as follows: In Section 2, we use an example to illustrate the idea of natural hedging by simulation; In Section 3, using market quotes of single–premium immediate annuities (SPIA) from A. M. Best, we find empirical support for natural hedging. That is, insurers who naturally hedge mortality risks have a competitive advantages over otherwise similar insurers. In Section 4, we propose and price a mortality swap between life insurers and annuity insurers. Section 5 is the conclusion and summary.

2. INTRODUCTORY EXAMPLE

This example illustrates the idea of a natural hedge. Consider a portfolio of life contingent liabilities consisting of whole life insurance policies written on lives age (35) and immediate life annuities written on lives age (65). If mortality improves, what happens to the insurer's total liability? We know that on average, the insurer will have a loss on the annuity business and a gain on the life insurance business. And if mortality declines, the effects are interchanged. This example shows what can happen if mortality risk increases as a result of a common shock. Here are our assumptions:

(1) Mortality for (35) is based on the 1990-95 SOA Male Basic Table and the table for (65) is based on the 1996 US Individual Annuity Mortality Male Basic Table.

(2) The annuity has an annual benefit of 510 and it is issued as an immediate annuity at age 65.
(3) The face amount of life insurance on (35) is $100,000$ and the life insurance is issued at age 35. For this amount of insurance, the present value of liabilities under the life insurance and under the annuity are about equal.
(4) Premiums and annuity benefits are paid annually. Death benefits are paid at the end of the year of death.
(5) The initial number of life insured ℓ_{35} is 10,000 which is the same as that of annuitants ℓ_{65}.
(6) The mortality shock ϵ is expressed as a percentage of the force of mortality μ_{x+t}, so it ranges from -1 to 1, that is, $-1 \leq \epsilon \leq 1$ with probability 1. Without the shock, the survival probability for a life age (x) at year t is $p_{x+t} = \exp(-\mu_{x+t})$. With the shock, the new survival probability $p^{'}_{x+t}$ can be expressed as:

$$p^{'}_{x+t} = (e^{-\mu_{x+t}})^{1-\epsilon} = (p_{x+t})^{1-\epsilon}.$$

If $0 < \epsilon \leq 1$, mortality experience improves. If $-1 \leq \epsilon < 0$, mortality experience deteriorates.
(7) The term structure of interest rates is flat; there is a single interest rate $i = 0.06$.

2.1. **Life insurance.** For the life insurance, the present value of 1 paid at the end of the year of death is v^{k+1} and the expected present value is

$$A_x = \sum_{k=0}^{\infty} v^{k+1}{}_kp_x q_{x+k}$$

where x is the age when the policy issued ($x = 35$ in this example). For a benefit of F the expected present value is FA_x.

The present value of 1 per year, paid at the beginning of the year until the year of death, is

$$\ddot{a}_{\overline{K(x)+1}|} = \frac{1 - v^{K(x)+1}}{d}.$$

The expected present value

$$\ddot{a}_x = \mathrm{E}\left[\ddot{a}_{\overline{K(x)+1}|}\right] = \sum_{k=0}^{\infty} v^k{}_kp_x.$$

The net annual premium rate for 1 unit of benefit is determined so that the present value of net premiums is equal to the present value of benefits. This means

$$P_x\ddot{a}_x = A_x$$

and for a benefit of F the annual premium is

$$FP_x = FA_x/\ddot{a}_x.$$

If the insured dies at $K(x) = t$, then the insurer's net loss is the present value of the payment, less the present value premiums. For a unit benefit, the loss is

$$L = v^{K(x)+1} - P_x \ddot{a}_{\overline{K(x)+1}|} = v^{K(x)+1} - P_x \frac{1 - v^{K(x)+1}}{d}.$$

It follows from the definition of the net premium P_x that the expected loss is zero. For a benefit of F, the loss is FL. Of course, the loss can be negative in which case the result turned out in the insurer's favor. On average, the loss is zero.

2.2. **Annuities.** For an annuitant (y), the present value of 1 per year paid at the beginning of the year is

$$\ddot{a}_{\overline{K(y)+1}|} = \frac{1 - v^{K(y)+1}}{d}.$$

The expected present value

$$\ddot{a}_y = \mathrm{E}\left[\ddot{a}_{\overline{K(y)+1}|}\right] = \sum_{k=0}^{\infty} v^k {}_kp_y.$$

The policy is purchased with a single payment of $\ddot{a}_y$. In our example $y = 65$ and the mortality table is based on annuity experience. For an annual benefit of b, the net single premium is $b\ddot{a}_y$. The company's loss per unit of benefit is

$$\ddot{a}_{\overline{K(y)+1}|} - \ddot{a}_y = 1/d - \ddot{a}_y - v^{K(y)+1}/d.$$

The expected loss is zero.

2.3. **Portfolio.** The portfolio has a life insurance liability to pay a benefit of F at the end of the year of the death of (x) and a liability to pay a benefit of b at the beginning of each year as long as (y) is alive. The total liability is

$$Fv^{K(x)+1} + b\ddot{a}_{\overline{K(y)+1}|}.$$

To offset the liability the company has

$$FP_x\ddot{a}_{\overline{K(x)+1}|} + b\ddot{a}_y.$$

The difference is the total loss:

$$L = Fv^{K(x)+1} + b\ddot{a}_{\overline{K(y)+1}|} - FP_x\ddot{a}_{\overline{K(x)+1}|} - b\ddot{a}_y.$$

The expected loss is zero. However, this expectation is calculated under the assumption that the mortality follows the tables assumed in setting the premiums. If we replace the before–shock lifetimes with the after shock lifetimes, what happens to the loss?

2.4. **Calculation results.** Table 1 presents the results of the present value of life insurance cash flows and annuity cash flows at time $t = 0$, separately and aggregate. It shows the percentage deviation of the present value of benefits from the life insurance premiums and that of annuity payments from the total annuity premium collected at time $t = 0$. We also show the present value of the sum of both life insurance and annuity payments and the percentage of deviation from the present value of total premiums collected. Each result includes a shock improvement or deterioration relative to the table mortality, modelled by multiplying the force of mortality by a factor $1 - \epsilon$ in each year. With a small mortality improvement shock $\epsilon = 0.05$ (Table 1), the present value of the total annuity payments increases from 54,054,326 without shock to 54,702,000. In this scenario, annuity insurers will lose 1.2% $[= (54,702,000 - 54,054,326)/54,054,326]$ of their expected total payments. In this scenario, life insurers will gain 2.3% of their expected total payments. If the above life insurance and annuity are written by the same insurer, the shock has a much smaller effect on its business (a 0.6% gain). With a small mortality bad shock $\epsilon = -0.05$ (Table 1), annuity insurers will gain 1.2% $[= (53,432,000 - 54,054,326)/54,054,326]$ of their expected total payments. In this scenario, life insurers will lose 2.3% of their expected total payments. If the above life insurance and annuity are sold by the same insurer, a bad shock has little effect on its business (a 0.6% loss). When there is a big good shock $\epsilon = 0.50$, the present value of total annuity payments will increase by 15.0% and the life insurer will gain 27.0% of their total expected payments on average. The overall effects will be 6% gain on a big good shock. Writing both life and annuity business reduces the impact of a big bad shock $\epsilon = -0.50$ to a 5.6% loss.

3. Empirical Support For Natural Hedging

Life insurance and annuities have become commodity-like goods, meaning that the price variable is a primary source of competition among insurance industry participants. Through various marketing campaigns, consumers are well aware whether a price offered by an insurer is attractive or not. Life insurance and annuity pricing elements include the probability of the insured event occurring, the time value of money, the benefits promised and loadings to cover expenses, taxes, profits, and contingencies (Black and Skipper, 2000). Natural hedging provides an efficient internal financial market for an insurer and reduce its external hedging costs to handle contingencies. Froot and Stein (1998) and Froot and O'Connel (1997) suggest striking markup of unhedgeable catastrophe risks. Until now, no

TABLE 1. Results for 5%, 10%, 25% and 50% mortality improvement or deterioration relative to life and annuity mortality tables (The present values are in thousands).

Improvement level aged (65)/Improvemnent level aged (35)=1

Present Value Life Benefits Payments	% Life	Present Value Annuity Payments	% Annuity	Total Present Value	% Total
$\epsilon = 0.05$					
52,787	-2.3	54,702	1.2	107,489	-0.6
$\epsilon = 0.10$					
51,488	-4.7	55,376	2.4	106,864	-1.2
$\epsilon = 0.25$					
47,373	-12.4	57,589	6.5	104,963	-2.9
$\epsilon = 0.50$					
39,486	-27.0	62,144	15.0	101,630	-6.0

Deterioration level aged (65)/Deterioration level aged (35)=1

Present Value Life Benefits Payments	% Life	Present Value Annuity Payments	% Annuity	Total Present Value	% Total
$\epsilon = -0.05$					
55,293	2.3	53,432	-1.2	108,725	0.6
$\epsilon = -0.10$					
56,506	4.5	52,833	-2.3	109,338	1.1
$\epsilon = -0.25$					
60,003	11.0	51,158	-5.4	111,161	2.8
$\epsilon = -0.50$					
65,448	21.1	48,704	-9.9	114,152	5.6

attention has been paid to the risk premium of unhedgeable mortality risks. Our hypothesis is that natural hedging is inversely related to the mortality risk premium and thus insurance price. Natural hedging may provide a competitive advantage to an insurer in a super-competitive landscape.

3.1. **Pricing of Unhedgeable risks.** Froot and Stein (1998) investigate the pricing of risks that cannot be hedged. Their model includes two periods, defined by Time 0,1 and 2. The initial portfolio of exposure will result in a Time 2 random payoff of $Z_p = \mu_p + \varepsilon_p$, where μ_p is the mean and ε_p is a mean-zero disturbance term. In our case, the initial portfolio of exposure is the original business composition of life insurance and annuities. The firm invests in a new investment at Time 1, e.g. selling new annuity business. The new investment offers a random payoff of Z_N at Time 2, which can be written as $Z_N = \mu_N + \varepsilon_N$, where μ_N is the mean and ε_N is a mean-zero disturbance term. The risks can be classified into two categories: (i) perfectly tradeable exposures, which can be unloaded frictionlessly on fair-market terms, and (ii) completely non-tradeable exposures, which must be retained by the financial intermediaries no matter what. The disturbance terms, that is, the pre-existing and new risks, ε_p and ε_N, can be decomposed as:

$$\varepsilon_p = \varepsilon_p^T + \varepsilon_p^N,$$

$$\varepsilon_N = \varepsilon_N^T + \varepsilon_N^N,$$

where ε_p^T is the tradeable component of ε_p, ε_p^N is the non-tradeable component, and so forth. The intermediary's realized internal wealth at Time 2 is denoted by w. The investment at Time 2 requires a cash input of I, which can be funded out of internal sources w, or can be raised externally in an amount e. Thus $I = w + e$. The investment yields a gross return of $F(I)$. And the convex costs to raising external finance e are given by $C(e)$. The solution to this intermediary's Time-2 problem can be denoted by $P(w)$, as follows:

$$P(w) = \text{max } F(I) - I - C(e), \text{subject to } I = w + e.$$

Froot et al. (1993) demonstrate that $P(w)$ is, in general, an increasing concave function, so that $P_w > 0$, and $P_{ww} < 0$. If the bank must make a decision to either accept or reject an investment opportunity of small fixed size, the hurdle rate μ_N^* is given by

$$\mu_N^* = \gamma \text{cov}(\varepsilon_N^T, M) + G\text{cov}(\varepsilon_N^N, \varepsilon_p^N). \tag{1}$$

where $G \equiv -EP_{ww}/EP_w$ is a measure of the firm's effective risk aversion and also the unit price of non-tradeable risk. When $G > 0$ and investment requires the assumption of non-tradeable risk, Eq.(1) is a two-factor pricing model. γ is the per unit price of market systematic risks. M in the first factor is the tradeable market portfolio. If the correlation between tradeable part of the new investment ε_N^T and the market portfolio is 0, e.g. catastrophe bonds, the first term in Eq.(1) will be 0. The second factor in Eq.(1) shows that μ_N^* is an increasing function of the correlation between pre-existing and new unhedgeable risks $\text{cov}(\varepsilon_N^N, \varepsilon_p^N)$. If a life insurer is able to realize the natural hedging, $\text{cov}(\varepsilon_N^N, \varepsilon_p^N)$ will be lower. Thus its required μ_N^* will decrease and the price of its life insurance products will be lower than otherwise similar insurers which cannot naturally hedge their liabilities.

Eq.(1) applies to insurance markets. Compared with other financial markets, insurance markets normally are incomplete. The "novel twist" in the addition of the second factor in Eq.(1) reflects the risk premium required by the insurers taking unhedgeable risks. Froot and O'Connel (1997) estimate the hurdle rate of catastrophe reinsurance by subtracting the value of the expected loss from the market price charged by the reinsurer for a policy. They conclude that on average, over the period 1980–1994, price is on the order of four times the actuarial value. Froot and O'Connel (1997) focus on the role of capital, that is, G. Our paper, on the other hand, will explore the impact of the correlation between different liabilities, that is, cov$(\varepsilon_N^N, \varepsilon_p^N)$ on the hurdle rate μ_N^*.

3.2. **Data, Measures and Methodologies.**

Data and Measures. Part of the data used in this analysis were monthly payouts of non–qualified life–only option single–premium immediate annuities (SPIAs) for a 65–year–old male from 1995 to 1998 (Kiczek, 1995, 1996, 1997; A.M.Best, 1998). Each year A.M. Best Company surveyed about 100 companies on monthly payments for a 65–year–old male with \$100,000 to invest. The lifetime-only option provides the highest monthly payment to the annuitant until death. That's because benefits are not payable thereafter to any beneficiary. SPIAs guarantee the annuitant a steady flow of lifetime income. The difference between qualified and non-qualified is that the principal of qualified plans is taxable upon withdrawal. Non-qualified plans are not taxable at withdrawal because they were taxed prior to the investment. Since non–qualified plans reflect the competitive market behavior better than qualified plans, and they are not affected by the governmental subsidy, we use only non–qualified plans.

Investors interested in purchasing a single–premium immediate annuity may compare products offered by competing companies, including monthly payouts and all options and enhancements. Table 2 Panel A shows that monthly payments for a 65–year–old male (Male 65) with \$100,000 to invest in the lifetime only option ranges from \$653 to \$992. The higher the monthly payouts, the lower the annuity price. The annuity price of each company is equal to \$100,000 divided by its monthly payments. We divide the mean of the monthly payments (\$765.24) by each company's monthly payouts to rescale the annuity price of each company. We code it "$PRICE$". The mean of the rescaled annuity price is close to 1.

Table 2: Descriptive Statistics for the Annuity Insurer Sample from 1995 to 1998

Panel A

Variable	Description	All Firms ($N = 322$)				
		Mean	Std. Dev.	Min	Median	Max
Male 65	Monthly annuity payments to male aged 65[a]	765.24	44.42	653.00	767.00	992.00
PRICE	Mean of Male 65[d]/Male 65	1.0036	0.0584	0.7716	0.9979	1.1721
resann	Annuity reserve[c]	3,189,334	6,510,098	41	952,398	43,011,379
reslife	Life insurance reserve[c]	3,025,688	7,664,022	0	734,909	45,725,773
LNresann	Log(resann)	20.4927	1.8851	10.6315	20.6745	24.4847
LNreslife	Log(reslife)	19.9610	3.1666	0.0000	20.4152	24.5459
RATIO	Log((reslife+1)/resann)	-0.5317	3.4088	-23.8619	0.0108	10.3162
Tasset	Total assets[c]	9,336,635	19,464	33,351	3,032,935	127,097,380
LNtasset	Log(Tasset)	28.7591	1.4700	24.2304	28.7405	32.4760
lrate	lag of A. M. Best rating[b]	1.5901	0.6506	1.0000	2.0000	4.0000
COMEXP_NPW	% of commisson and expense to net premium	23.7643	13.6719	2.5000	21.4000	102.2000
LNcomexp	Log(COMEXP_NPW)	3.0207	0.5522	0.9163	3.0634	4.6269
pw_A	Annuity written premium[c]	215,535	408,850	1	40,808	2,656,478
pw_L	Life insurance written premium[c]	580,890	1,469,252	0	114,226	9,225,301

Panel B

Variable	Description	resann/(resann+reslife)>0.05 ($N = 311$)				
		Mean	Std. Dev.	Min	Median	Max
Male 65	Monthly annuity payments to male aged 65[a]	765.70	44.46	653.00	767.00	992.00
PRICE	Mean of Male 65[d]/Male 65	1.0030	0.0584	0.7716	0.9979	1.1721
resann	Annuity reserve[c]	3,293,352	6,599,341	3,953	985,393	43,011,379
reslife	Life insurance reserve[c]	2,944,875	7,403,864	0	718,453	44,670,882
LNresann	Log(resann)	27.5273	1.7262	22.0978	27.6163	31.3925
LNreslife	Log(reslife)	26.7193	3.9331	0.0000	27.3004	31.4303
RATIO	Log((reslife+1)/resann)	-0.8080	4.0844	-30.7696	-0.0765	2.9166
Tasset	Total assets[c]	9,361,727	19,484,156	33,351	3,142,893	127,097,380
LNtasset	Log(Tasset)	28.7630	1.4759	24.2304	28.7762	32.4760
lrate	lag of A.M. Best rating[b]	1.5884	0.6558	1.0000	2.0000	4.0000
COMEXP_NPW	% of commisson and expense to net premium	23.2791	13.4370	2.5000	21.2000	102.2000
LNcomexp	Log(COMEXP_NPW)	3.0017	0.5478	0.9163	3.0540	4.6269
pw_A	Annuity written premium[c]	220,054	412,744	1	42,419	2,656,478
pw_L	Life insurance written premium[c]	574,379	1,454,724	0	113,317	9,225,301

Panel C

Variable	Description	resann/(resann+reslife)>0.10 ($N = 299$)				
		Mean	Std. Dev.	Min	Median	Max
Male 65	Monthly annuity payments to male aged 65[a]	765.17	44.63	653.00	766.75	992.00
PRICE	Mean of Male 65[d]/Male 65	1.0037	0.0587	0.7716	0.9982	1.1721
resann	Annuity reserve[c]	3,397,424	6,706,626	3,953	1,042,774	43,011,379
reslife	Life insurance reserve[c]	2,640,877	6,663,400	0	696,900	44,670,882
LNresann	Log(resann)	27.5966	1.6868	22.0978	27.6729	31.3925
LNreslife	Log(reslife)	26.6526	3.9786	0.0000	27.2699	31.4303
RATIO	Log((reslife+1)/resann)	-0.9441	4.1075	-30.7696	-0.1253	2.0547
Tasset	Total assets[c]	9,069,208	19,262,442	33,351	2,996,357	127,097,380
LNtasset	Log(Tasset)	28.7374	1.4725	24.2304	28.7284	32.4760
lrate	lag of A. M. Best rating[b]	1.5853	0.6619	1.0000	2.0000	4.0000
COMEXP_NPW	% of commisson and expense to net premium	23.2104	13.5495	2.5000	21.2000	102.2000
LNcomexp	Log(COMEXP_NPW)	2.9962	0.5524	0.9163	3.0540	4.6269
pw_A	Annuity written premium[c]	220,358	415,248	1	45,496	2,656,478
pw_L	Life insurance written premium[c]	537,185	1,400,053	0	111,911	9,225,301

Panel D

Variable	Description	resann/(resann+reslife)>0.25 ($N = 243$)				
		Mean	Std. Dev.	Min	Median	Max
Male 65	Monthly annuity payments to male aged 65[a]	764.97	45.03	653.00	767.00	992.00
PRICE	Mean of Male 65[d]/Male 65	1.0040	0.0592	0.7716	0.9979	1.1721
resann	Annuity reserve[c]	3,971,020	7,286,773	3,953	1,324,643	43,011,379
reslife	Life insurance reserve[c]	2,325,016	6,608,182	0	599,076	44,670,882
LNresann	Log(resann)	27.8763	1.5893	22.0978	27.9122	31.3925
LNreslife	Log(reslife)	26.3613	4.3045	0.0000	27.1187	31.4303
RATIO	Log((reslife+1)/resann)	-1.5150	4.3606	-30.7696	-0.5512	1.0981
Tasset	Total assets[c]	9,079,913	20,055,796	33,351	3,256,612	127,097,380
LNtasset	Log(Tasset)	28.7424	1.4687	24.2304	28.8117	32.4760
lrate	lag of A. M. Best rating[b]	1.5802	0.6466	1.0000	2.0000	4.0000
COMEXP_NPW	% of commisson and expense to net premium	21.9519	12.1816	2.5000	20.3000	93.5000
LNcomexp	Log(COMEXP_NPW)	2.9515	0.5341	0.9163	3.0106	4.5380
pw_A	Annuity written premium[c]	239,695	440,024	1	52,730	2,656,478
pw_L	Life insurance written premium[c]	481,784	1,392,476	0	96,443	9,225,301

Note:
[a] Lifetime-only option: $100,000 Single Premium Non-Qualified Annuities
[b]if ‘A++’ or ‘A+’ then lrate=1;
if ‘A’ or ‘A-’ then lrate=2;
if ‘B++’ or ‘B+’ then lrate=3;
if ‘B’ or ‘B-’ then lrate=4;
if ‘C++’ or ‘C+’ then lrate=5;
if ‘C’ or ‘C-’ then lrate=6;
if ‘D’ then lrate=7;
if ‘E’ or ‘F’ or ‘S’ then lrate=8.
[c]in Millions
[d]mean of total 322 observations

The data with respect to life insurers characteristics are obtained from the National Association of Insurance Commissioners (NAIC). The ratio of life insurance reserve to annuity reserve reflects the level of natural hedging provided by life insurance business to annuity business. The higher this ratio, the less longevity risks the annuity business may face. We use the logarithm of this ratio to adjust for extreme values[1]. The natural hedging indicator is named "$RATIO$".

In an informationally efficient, competitive insurance market, the price of insurance will be inversely related to firm default risk (Phillips et al., 1998; Cummins, 1988; Merton, 1973). We use the A. M. Best rating[2] to control for the default risk of annuity insurers. Our hypothesis is that the higher the default risks, the lower the annuity prices. Other factors which may affect the annuity prices are also included in our regression model. We use the logarithm of total asset ($LNtasset$) to control for the size of insurance companies. The logarithm of the percentage of the commission and expenses to the total net premium written ($LNcomexp$) are also included. The higher expenses are expected to relate to higher annuity prices. Since A. M. Best Company does not report ratings for all of the companies, our final sample includes only 322 observations of A. M. Best rated companies from 1995 to 1998.

Methodologies. The relation between annuity price and natural hedging is estimated using the pooled OLS technique controlling for year effects. The model is as follows:

$$PRICE = \alpha + \beta RATIO + \gamma X + \delta D,$$

where X is a vector of control variables and D is a vector of year dummies. $PRICE$ is the normalized price for a male aged 65 life–option only SPIA. $RATIO$ measures the weight of life insurance business of an insurer. The expected sign of β is negative which means natural hedging lowers the annuity price. We run the regression with four specifications. The first regression includes all firms in our sample. We use the proportion of annuity reserves to the sum of annuity reserves and life insurance reserves to measure the longevity risk exposure of an annuity insurer. The second specification regresses on a sample where the proportion of annuity reserve to the sum of annuity reserve and life insurance reserve is more than 5%. This proportion in the third regression is 10% and 25% for the fourth specification. Table 2 Panel B, C and D show the summary statistics for Specification 2, 3 and 4. There are 311 observations for Specification 2 out

[1]We add 1 to the life insurance reserve to avoid no definition of log of zero. Our sensitivity test shows that it does not affect the final results.

[2]lag of one period.

of total 322 observations. And only 243 observations are in Specification 4. White statistics are used to test the heteroscedasticity.

3.3. **Findings and Implications.** White statistics fail to reject the homoscedasticity hypothesis for all four regressions which means that the OLS model is appropriate. The coefficient estimates and their standard errors are presented in Table 3. The signs of all the coefficients are consistent with the predicted effects. In addition, each estimated coefficient is large relative to its standard error.

Table 3: Pooled OLS Regression—Relationship between Annuity Price and Natural Hedging

Variable	$PRICE$			
	Regression 1[a]	Regression 2[b]	Regression 3[c]	Regression 4[d]
Intercept	0.9071*** (0.0789)	0.9185*** (0.0797)	0.9146*** (0.0803)	0.9844*** (0.0837)
$RATIO$	-0.0012 (0.0008)	-0.0012 (0.0008)	-0.0015* (0.0008)	-0.0017** (0.0008)
$LNresann$	-0.0043 (0.0027)	-0.0035 (0.0034)	-0.0089** (0.0039)	-0.0222*** (0.0054)
$LNtasset$	0.0060* (0.0033)	0.0048 (0.0038)	0.0100** (0.0043)	0.0206*** (0.0057)
$lrate$	-0.0102** (0.0048)	-0.0099** (0.0049)	-0.0084** (0.0049)	-0.0103** (0.0053)
$LNcomexp$	0.0070 (0.0060)	0.0070 (0.0061)	0.0068 (0.0061)	0.0081 (0.0067)
Year Dummy 1998	0.0788*** (0.0079)	0.0800*** (0.0081)	0.0801*** (0.0083)	0.0818*** (0.0089)
Year Dummy 1997	0.0288*** (0.0078)	0.0292*** (0.0079)	0.0304*** (0.0081)	0.0292*** (0.0086)
Year Dummy 1996	0.0434*** (0.0081)	0.0436*** (0.0082)	0.0445*** (0.0083)	0.0453*** (0.0090)
N	322	311	299	243
White test p-value	0.1626	0.2483	0.1843	0.2950
R^2	0.2737	0.2719	0.2815	0.3292
Adj R^2	0.2551	0.2526	0.2617	0.3062

Note: Standard errors are presented below the estimated coefficients;

[a] all firms;

[b] resann/(resann+reslife) > 0.05;

[c] resann/(resann+reslife) > 0.10;

[d] resann/(resann+reslife) > 0.25;

*** Significant at 1% level;

** Significant at 5% level;

* Significant at 10% level.

The results support our hypothesis that natural hedging lowers annuity prices. Increased life insurance business is related to lower annuity prices because the signs of coefficient of $RATIO$ are negative in all four regressions. In the first and the second regressions, the coefficient of $RATIO$ is not significant although it is negative. One possible explanation for this result is that when the annuity business only accounts for a very small proportion of an life insurer's business, the marginal effects of natural hedging are negligible. When the annuity business increases relative to the life insurance business, the need for longevity risk hedging increases. Our results coincides with the above theoretical analysis. When we focus on those observations with the proportion of annuity reserve to the sum of annuity reserve and life insurance reserve higher than 10%, the coefficient of $RATIO$ becomes significant at the level of 7%. If the proportion of annuity reserve to the sum of annuity reserve and life insurance reserve is higher than 25%, the coefficient of $RATIO$ is negative at the significant level of 3% and its magnitude (-0.0017) is higher than other regressions. This suggests that when an insurer sells relatively more annuities, the increase in the life insurance has a higher marginal effect in lowering the annuity price.

Since the annuity price in our model is a normalized price, how do we interpret the coefficient of $RATIO$ (-0.0017)? Suppose an life insurer has 5% of its business in life insurance and 95% of its business in annuities. It sells life-only SPIAs to males aged 65 at the market average monthly payouts \$765. If it can realize full natural hedging, that is, 50% of business in life insurance and 50% business in annuities, its SPIA monthly payouts will increase by \$24, that is, from \$765 to \$789. It will make its SPIAs more attractive than other similar competitors.

The signs of other control variables are all consistent with our expectation. The log–annuity reserve is negative and significant in Regression 3 and 4. Insurance relies on the law of large numbers to minimize the speculative element and reduce volatile fluctuations in year-to-year losses (Black and Skipper, 2000). Unsystematic risk and uncertainty diminish as the number of exposure units increases. When a life insurer writes more annuities, it may reduce its annuity prices. The size of an insurer is positively related to the price of its SPIA which may reflect the market power of bigger firms. This is consistent with prior research (Sommer, 1996; Froot and O'Connel, 1997). Some previous evidence to support the hypothesis that insurance prices reflect firm default risk can be found in (Berger et al., 1992; Sommer, 1996). Our default risk proxy $lrate$ is negative and significant in all four specifications. Our results support the hypothesis that insolvency risk of life insurers is reflected in the prices they receive for their annuities. The sign of our expenses proxy $LNcomexp$ is positive but not significant.

4. Mortality Swaps

In section 3, we conclude that natural hedging helps an insurer lower its price all else equal. However, in reality it may be too expensive and not realistic for an insurer to realize fully natural hedging by changing its business composition. First, there exists technical difficulties for an insurer specialized in annuities or life insurance to switch to another type of business. Second, natural hedging is not a static process. Dynamic natural hedging is required for new life or annuity business. Dynamic natural hedging by adjusting life and annuity sales may be expensive. Third, even if an insurer is keenly interested in the dynamic natural hedging, whether it can sell adequate life or annuity business required by full natural hedging is an open question. Innovation has become an absolute necessity to survive and perform well in almost every industry (Hitt et al., 2003). If an insurer is able to take advantage of natural hedging at a low cost by financial innovation, it can gain competitive advantage in the market by selling insurance products at lower prices. We propose a mortality swap to accomplish this goal.

4.1. **Basic Ideas.** Although it is not realistic for an life and/or annuity insurer to realize fully natural hedging, it can find a counter-parties in the industry to swap its life or annuities business with the counter-party's annuities or life business. Dowd et al. (2004) propose the possible uses of survivor swaps as instruments for managing, hedging and trading mortality-dependent risks. Their proposed survivor swap involves transferring a mortality risk relating to a specific population with another population, that is, one specific longevity risk for another specific longevity risk. Their mortality swap can be used to diversify the mortality risks. However, if a good shock or a bad shock strikes every population, the survivor swap can not efficiently eliminate the mortality risks. Our idea of mortality swap is motivated by the annuity insurer's desire to pay variable-level payments on the counter-party's life insurance for a series of variable-level payments on its annuity business. Swapping mortality risks between life insurer and annuity insurer may be more efficient than swapping between two annuity insurers because life insurance and annuity handle different tail of mortality risks. Life insurance can reduce or eliminate the systematic risks of annuities and vise versa. Without any collateral, the swap payments are subject to counter–party risk. Assuming there is no counter–party risk, our proposed mortality swaps contracts are described as follows.

Suppose it is not an off-market mortality swap contract which refers to one where there is no payment at initiation to either party. Then each year, the annuity insurer pays floating to the life insurer (or a swap originator) based on the actual number of death and the face value F per person and gets a floating annuity benefit based on the actual number of survivors and the annual payout b per annuitant. There are no other payments. This is a floating for floating swap from each insurer's perspective. So long as there is no counter–party risk, the insurer can get essentially the same reinsurance benefit from a swap. Figure 1 shows how a mortality swap works. An insurer can swap its annuity directly with another insurer's life insurance or indirectly with a special purpose company (SPC). The mortality swap transactions between these two insurers are similar to cash inflows and outflows between two sections in a parent company. Although these two insurers do not actually merge together, the mortality swap realizes the benefits of a merge and stabilize the cash flows of these two insurance companies. Since the life insurance or annuity annual payments are in the same currency, there is no need for both counter-parties to actually transfer the cash. The difference between the floating life insurance payment and the variable annuity payment is calculated and paid to the appropriate counter-party. Net payment is paid by the one who owes it. Paying net payments rather than gross ameliorates counter-party credit risks. Transactions through a fully collateralized special purpose company can eliminate counter-party credit risk entirely. The mortality swap is a zero-sum game. What one party gains, the other party loses.

4.2. **Mortality Swap Pricing.** Cairns et al. (2004) discuss a theoretical framework for pricing mortality derivatives and valuing liabilities which incorporate mortality guarantees. Their stochastic mortality models require certain "reasonable" criteria in terms of their potential future dynamics and mortality curve shapes. Rogers (2002) shows that mortality operates within a complex framework and is influenced by socioeconomic factors, biological variables, government policies, environmental influences, health conditions and health behaviors. Not all of these factors improve with time. There are different opinions on future mortality trends: improvement (Buettner, 2002), life table entropy (Hayflick, 2002) and deterioration (Goss et al., 1998; Rogers, 2002). How to estimate the parameters in such a stochastic mortality model seems to be an open question at this point.

Wang (1996, 2000, 2001) has developed a method of pricing risks that unifies financial and insurance pricing theories. We are going to apply this method to price mortality swaps. Let $\Phi(x)$ be the standard normal cumulative distribution function with a probability density function

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

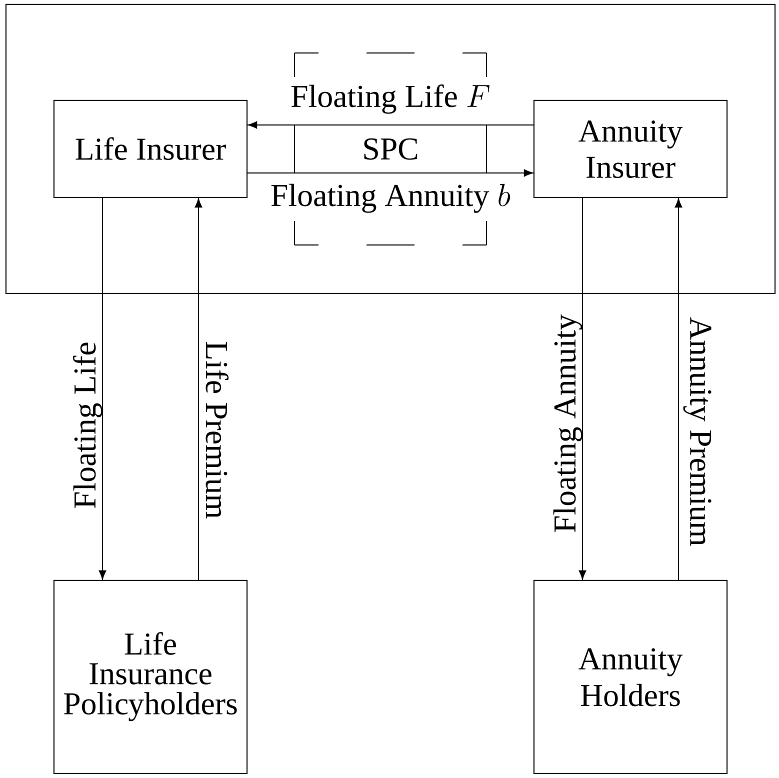

FIGURE 1. Mortality Swap Diagram.

for all x. Wang defines the distortion operator as

$$g_\lambda(u) = \Phi[\Phi^{-1}(u) - \lambda] \tag{2}$$

for $0 < u < 1$ and a parameter λ. Now, given a distribution with cumulative density function $F(t)$, a "distorted" distribution $F^*(t)$ is determined by λ according to the equation

$$F^*(t) = g_\lambda(F)(x) = \Phi[\Phi^{-1}(F(x)) - \lambda]. \tag{3}$$

Consider an insurer's liability X over a time horizon $[0, T]$. The value or fair price of the liability is the discounted expected value under the distribution obtained from the distortion operator. Omitting the discount for now, we have the formula for the price:

$$H(X, \lambda) = \mathrm{E}^*(X) = \int x dF^*(x). \tag{4}$$

where the parameter λ is called the market price of risk, reflecting the level of systematic risk. Thus, for an insurer's given liability X with cumulative density function F, the Wang transform will produce a "risk–adjusted" density function F^*. The mean value under F^*, denoted by $\mathrm{E}^*[X]$, will define a risk–adjusted "fair-value" of X at time T, which can be further discounted to time zero, using the risk-free rate. Wang's paper describes the utility of this approach. It turns out to be very general and a generalization of well known techniques in finance and actuarial science. Our idea is to use observed annuity prices to estimate the market price of risk for annuity mortality, then use the same distribution to price mortality bonds.

The Wang transform is based on the idea that the annuity market price takes into account the uncertainty in the mortality table, as well as the uncertainty in the lifetime of an annuitant once the table is given. The market price of risk does not, and need not, reflect the risk in interest rates because we are assuming that mortality and interest rate risks are independent. Moreover, we are assuming that investors accept the same transformed distribution, and independence assumption for pricing mortality swaps.

Market price of risk. First we estimate the market price of risk λ. We defined our transformed distribution F^* as:

(5) $$F_i^*(t) = g_\lambda(F_i)(t) = \Phi[\Phi^{-1}({}_tq_x) - \lambda_i],$$

where $i = l, a$. l stands for life insurance and a is for annuity. For the distribution function $F_a(t) = {}_tq_{65}$, we use the 1996 IAM 2000 Basic Table for a male life age sixty–five and, separately, for a female life age sixty–five. Then assuming an expense factor equal to 4%, we use the 1996 market quotes of qualified immediate annuities (Kiczek, 1996) and the US Treasury yield curve on December 30, 1996 to get the market price of risk λ_a by solving the following equations numerically:

(6) $$128.40 = 7.48a_{65}^{(12)} \qquad \text{for males},$$

(7) $$138.39 = 6.94a_{65}^{(12)} \qquad \text{for females}.$$

The market price of risk for males and females respectively is shown in Table 4 and Figure 2. The market price of risk is 0.2134 for male annuitants and 0.2800 for female annuitants. Figure 2 shows that the market prices of the annuities are higher than the mortality experience of the 1996 IAM 2000 Basic Table and the market curve lies above the 1996 IAM 2000 Basic mortality experience curve. We think of the 1996 IAM 2000 Basic Table as the actual or physical distribution, which requires a distortion to obtain market prices. That is, a risk premium is required for pricing annuities.

Similarly, for the distribution function $F_l(t) = {}_tq_{35}$, we use the 1995 US SOA Basic Age Last Table for a male life age thirty–five and, separately, for a female life age thirty–five. A.M.Best (1996) reports market quotes for both preferred non-smokers and standard smokers. The 1995 US SOA Basic Age Last Table is created based on a mixture of smokers and non-smokers. We assume the 1995 US SOA Basic Age Last Table reflects the

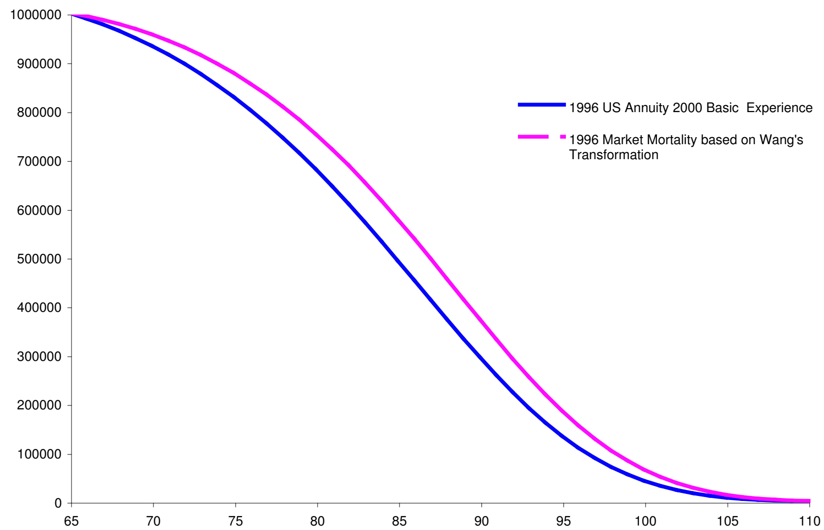

FIGURE 2. The result of applying the Wang transform to the survival distribution based on 1996 IAM experience for males (65) and prices from Best's Review, 1996.

TABLE 4. The market price of risk λ_a, determined by the 1996 IAM 2000 Basic Table, the US Treasury constant maturity interest rate term structure for December 30, 1996, and annuity market prices from Best's Review (1996) net of our assumed expense factor 4%. The payment rate is the dollars per month of life annuity per $1,000 of annuity premium at the issue age. The market value is the price (net of annuity expenses) for $1 per month of life annuity.

	Payment Rate	Market Value	Market price of risk λ_a
Male (65)	7.48	128.40	0.2134
Female (65)	6.94	138.39	0.2800

US population smoking percentage reported by the Center for the Disease

Control (CDC) [3] in 1995. Assuming an expense factor equal to 10%, we use the 1996 market quotes of ten–year level $250,000 term policy (A.M.Best, 1996) based on 97 companies and the US Treasury yield curve on December 30, 1996 to get the market price of risk λ_l by solving the following equations:

$$456.73 = 250,000 \sum_{k=0}^{9} v^{k+1} {}_k p_{35}^{lm} q_{35+k}^{lm} \qquad \text{for males,} \tag{8}$$

$$341.50 = 250,000 \sum_{k=0}^{9} v^{k+1} {}_k p_{35}^{lf} q_{35+k}^{lf} \qquad \text{for females.} \tag{9}$$

where $x = 35$ is the age when the policy issued and the benefit is $F = 250,000$.

The market price of risk λ_l for male and female life insureds aged 35 respectively is shown in Table 5.

TABLE 5. The market price of risk λ_l, determined by the 1995 US SOA Basic Age Last Table, the US Treasury constant maturity interest rate term structure for December 30, 1996, and ten–year level $250,000 term policy market prices from Best's Review (1996) net of our assumed expense factor 10%.

	Net Total Premium	Market price of risk λ_l
Male (35)	456.73	0.1933
Female (35)	341.50	0.0971

Mortality Swap Pricing. A swap can be regarded as a series of forward contracts, and hence they can be priced using the concept of forwards. Different from other swaps, pricing mortality swaps between life insurers and annuity insurers should take into account differences between these two kinds of mortality experiences caused by adverse selection problems. Good-health persons are more likely to purchase annuities and bad-health ones are more willing to buy life insurance. The probability of death for each age in the annuity mortality tables is normally lower than that for the life insurance. We use different transformed mortality tables to price a 10–year mortality swap between life insurers and annuity insurers. We obtain the transformed annuity mortality rates from Equation 6 for male and Equation 7 for female. The transformed life mortality rates are calculated from

[3]Source: www.cdc.gov. In 1996, male current smokers account for about 27%, former smoker 27.5% and non-smoker 45.5%. For female, these percentages are about 22.6%, 19.5% and 57.9% respectively.

Equation 8 for male and Equation 9 for female. One insurer swaps an annuity paying b annually if the annuitant survives with a life insurance sold by the other insurer which pays F if the insured dies in ten years. The mortality swap can be priced by Equation 10:

$$0 = FN \sum_{k=0}^{9} v^{k+1} {}_k p_{35}^{lj} q_{35+k}^{lj} - bN \sum_{k=0}^{9} v^{k+1} {}_{k+1} p_{65}^{aj}. \tag{10}$$

where $j = m, f$. The superscript m stands for male and f is for female. q_x^{lj} is the transformed life insurance mortality rate and q_x^{aj} is the transformed annuity mortality rate. N is the number of life insureds or annuitants specified in the mortality swap.

The results for 10-year mortality swaps are shown in Table 6. When the life insurer swaps $F = 1,000,000$ face–value life insurance on a male insured aged (35) with the annuity insurer, it needs to pay the annuity insurer $b = 267.33$ annually if a male annuitant aged (65) survives at the end of the year. In practice, insurers may trade a mortality swap on N_1 life insureds and N_2 annuitants where $N_1 \neq N_2$. The pricing principle is the same as Equation 10.

TABLE 6. Mortality Swap Pricing

	Life Face Value F	Annual Annuity Payment b
Male	1,000,000	267.33
Female	1,000,000	195.03

5. CONCLUSIONS AND DISCUSSION

Mortality risk has long been a major issue for insurance companies. Natural hedging utilizes the interaction of life insurance and annuities to a change in this mortality to stabilize aggregate cash outflows. Untradeable mortality risks are one kind of unhedgeable risks. There are residual risks that can not be diversified by selling a large pool of life or annuity business or by incomplete natural hedging. As for unhedgeable risks, the insurer behaves like an risk-averse individual. For a certain level of capital it holds, it will requires a higher risk premium as unhedgeable mortality risks increase, all else equal. Although it is prudent to charge a higher risk premium, the insurer is likely to lose its market shares because the consumers may switch to other similar insurers with a lower price. The other insurers are able to charge a lower price based on their more balanced life and annuity business. Our empirical evidences suggest natural hedging is an important factor contributing to annuity price differences after we control for other variables.

These differences become more significant for those insurers selling relatively more annuity business. We expect that life insurers may reach the same conclusion.

Most insurance companies still have considerable net exposures to mortality risks even if they reduce their exposure by pooling individual mortality risk and by balancing their annuity positions against their life positions (Dowd et al., 2004). Natural hedging is good, but it is not without cost. In Section 4, we point out that it will be too expensive to balance the life position and annuity position internally. First, there exists technical difficulties for an insurer specialized in annuities or life insurance to switch to another type of business. Second, natural hedging is not a static process. Dynamic natural hedging is required for new life insurance or annuity business. Dynamic natural hedging is expensive. Third, even if an insurer is keenly interested in the dynamic natural hedging, whether it can sell adequate life or annuity business required for full natural hedging is an open question.

Mortality derivatives have a great deal of potential as instruments for managing mortality-related risks. We propose and price a mortality swap between life insurers and annuity insurers to realize natural hedging at a relatively low costs. Compared with other derivatives, such as mortality bonds, mortality swaps can be arranged at lower costs and in a more flexible way to suit diverse circumstances. Thus, there are good reasons to anticipate a healthy market for mortality swaps between life insurers and annuity insurers. The mortality swap counter-parties can achieve "mutual gains" by swapping two-tail mortality risks with each other at a relatively low cost.

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