# Dynamic Asset Liability Management:
# A Profit Testing Model for Swiss Pension Funds

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**Abstract**

Profit Testing, an established concept in the life insurance industry, is developed and applied to self-governed (self-insured) DC pension schemes in Switzerland. The dynamics of the pension scheme's population are modelled by Markov chains. This allows for accurately capturing the impacts of different personnel policies such as reduction or increase of staff, promotion of early retirement etc. Profit profiles, cash flows, funding ratios and other key characteristics are calculated for DC schemes taking into account the Swiss regulatory framework. It is shown, that the close to reality modelling of the pension scheme's dynamics within the Profit Testing Model provides an appropriate and reliable basis for asset-liability management and helps to determine the optimal investment strategy.

**Keywords**

Risk Control, Profit Testing, Asset Liability Management, Dynamic Pension Fund Modelling

# 1 Introduction

## 1.1 General Information about Swiss Pension Funds

The principle of occupational provisions for retirement, death and disability in Switzerland has been put through its paces by the time the so-called “Three Pillar System” was added to the Swiss Constitution in 1972. The Swiss Federal Old Age and Surviving Dependants’ Insurance (AHV), in conjuncture with the disability insurance (IV), forms the “First Pillar”. Together with the “Second Pillar” (mandatory occupational provisions in force since 1985), it is supposed to provide adequate coverage for the assured and his family.

Coverage is considered adequate when benefits provided in accordance with the Swiss Federal Law on Occupational Retirements, Survivors’ and Disability Pension Plans (BVG), together with AHV, allow maintaining the accustomed standard of living. This condition is considered satisfied when the benefits provided by the two “pillars” total around 60% of the former salary assuming all required contributions have been paid.

According to the BVG, every employer has to provide all his employees with a guaranteed minimum pension scheme in a fully funded plan. The majority of pension schemes provide more generous benefits than required under BVG that mainly serves as a benchmark. However, BVG regulations and minimum benefits must be included in the company plan and are guaranteed.

The schemes generally provide the following benefits:

- Retirement pensions (the benefit formula is often 60% to 70% of final salary after 40 years’ service). Although retirement pensions are not legally or formally indexed, pension schemes usually grant increases on an ad hoc basis;
- Pre- and post-retirement spouses’ pensions of 60% and orphans’ pensions of 10% to 20%, based on the member’s projected/actual pension. A death benefit of one or two times salary is sometimes paid where there are no survivors’ pensions;
- Long-term disability (temporary and lifetime disability) pensions equal to service projected retirement pensions;
- The vested amount or vested benefit for leaving employees. For defined contribution (DC) schemes the vested amount will be the total of the employee’s and employer’s contributions, plus interest credited until the day of leaving. For defined benefit (DB) schemes it will be the value of the accrued benefits based on the ratio of completed pensionable service to projected pensionable service.
- The possibility to pledge or draw money from the company’s pension scheme to invest in property used by an employee and his or her family. The amount, which can be pledged or withdrawn, depends on the vested benefit.

Top-hat pension plans and special arrangements exist for senior executives very often in form of profit sharing, stock option plans, equity linked saving accounts and benefits for a salary higher than the specified value.

Larger plans are self-insured or partially covered through insurance of risk benefits. Law does not restrict actuarial methods. Swiss pension and risk premiums are tariff rated, but profit-sharing bonuses are paid. Pension monies are required by law to be deposited in a foundation, kept completely separately from the company and its balance sheet.

In this paper we focus on self-governed (or self-insured) pension schemes set up as foundations, i.e. legal entities in their own right, governed by a board of trustees.

The aim of this paper is to create a stochastic Markov based model for Swiss pension schemes that captures the complexity of pension schemes and thus goes beyond using simply Monte Carlo simulations. This allows investigating the influence of different economical factors on the funding ratio and contribution rate.

## 1.2 Profit Tests in Life Insurance and for Pension Schemes

For more than thirty years, profit testing has been recognized as a major tool available to actuaries involved in product development. Many papers have been written on the use of profit tests in the calculation of gross premiums. The algebra for developing profit profiles is well documented in papers such as Smart [1977], Darbellay/Veraguth [1998] or Koller [2000].

A profit test uses projection mathematics to establish the prospective profit profile of a policy on a given set of assumptions. The resulting profit profile is discounted at a risk rate to give the present value of future profits (PVFP). A major strength of profit testing over conventional gross premium formulae is its ability to incorporate the statutory reserving basis independent of the assumed rate of return on one the hand and to use an empirical basis such as event probabilities on the other hand.

The profit profile derived from a profit test is the stream of profits, which flow from the policy over its lifetime. Profit or surplus is defined as the excess of income over expenditures during any period, where expenditures include the necessary increase in valuation reserves. The shape of the profit profile varies greatly from product to product and will depend upon the assumptions incorporated into the profit test. In a life insurance company parts of the profit are distributed to shareholders and/or policyholders, in a pension scheme, however, it could be only distributed to its members as a bonus or used for contribution holidays.

There are two important aspects of risk for pension funds: the probability of having a funding ratio below one (shortfall risk) and the absolute amount of being underfunded. To estimate the behaviour of these important characteristics we use the idea of Profit Testing and adopt the standpoint of the pension fund expert. Thus, we calculate the expected statutory profit or loss for each year of the projection horizon in the same manner as he or she does for the actuarial balance of a pension scheme.

## 1.3 Investment Objectives for Pension Schemes and Profit Testing

The BVG formulates restrictions and objectives for Swiss pension schemes. First of all, it shall be guaranteed that the pension obligations could be met permanently. This objective is subdivided into a short-term, a mid-term and a long-term component:

- In every year of a planning-period the fund needs to have enough liquid assets to meet the pension payments already due,
- Vested benefits due need to be paid,
- Assured benefits need to be covered by the assets (sufficient funding ratio).

The pension scheme has to achieve the lowest possible and stable level of contributions. All these objectives have to be treated for the actual structure and the expected evolution of the population of the pension scheme. Therefore, the investment principle should be to maximize the rate of asset return but considering the financial risk capacity of the pension scheme and the regulatory (statutory) restrictions. The financial risk capacity of the pension scheme is determined by liquidity requirements, and development and commitment of capital is induced by pension obligations (liabilities) and free parts of the risk reserves. Profit testing is a very powerful vehicle to make sure the investment strategy suits such objectives.

# 2 Dynamic Management of Swiss Pension Funds

## 2.1 The Markov Model

The Markov Model is an extremely useful instrument to model life insurances and annuities since it provides a level of generality suitable for our purpose. In life insurance applications of the Markov model, one has to determine actuarial values of payments that are contingent upon the stay in certain states or upon transitions between the states. Transitions between states are governed by the adopted transition probabilities. They satisfy the so-called Markov property, which implies that the future course of the stochastic process does not depend on earlier visited states.

The evolution of the population of a pension fund is modelled with an inhomogeneous discrete time Markov chain, since transition probabilities largely depend on the age of the insured. To this respect our paper uses the idea of Ruhdorfer [1988].

To follow the dynamic evolution of a pension fund it is necessary to calculate the value of incomes and expenditures of the fund. Furthermore, it is necessary to compute the present value of all financial flows that are involved. For the purpose of discounting and accumulation we will use the usual compounded interest rate model.

There are five groups of participants in a self-governed pension scheme in Switzerland:

- employees (state A),
- disability pensionists (state I),
- early and elderly retired pensionists (states EP and P),
- widow's and widower's pensions (state W),
- orphan's pensions and pensions for children of retirees and disabled persons.

We do not model the later type of pensions. They are paid as a lump sum equal to the present value of such a pension. There are two absorbing states in the model: the states Death and Resignation. They are grouped together on the state diagram and used as a reserve to hire new employees according to the employer's personnel policy (introduced through dismissal or resignation rates and the new generation density). Figure 1 shows the transition diagram for the Markov model of a Swiss pension scheme, each arrow denotes a possible transition from one state to another. The dashed arrow from the state Death to the state W represents new widows and widowers (hired in the State W) in case of death of pension scheme members such as employees, disabled persons or retirees. There are no benefits in case of death of widows and widowers.

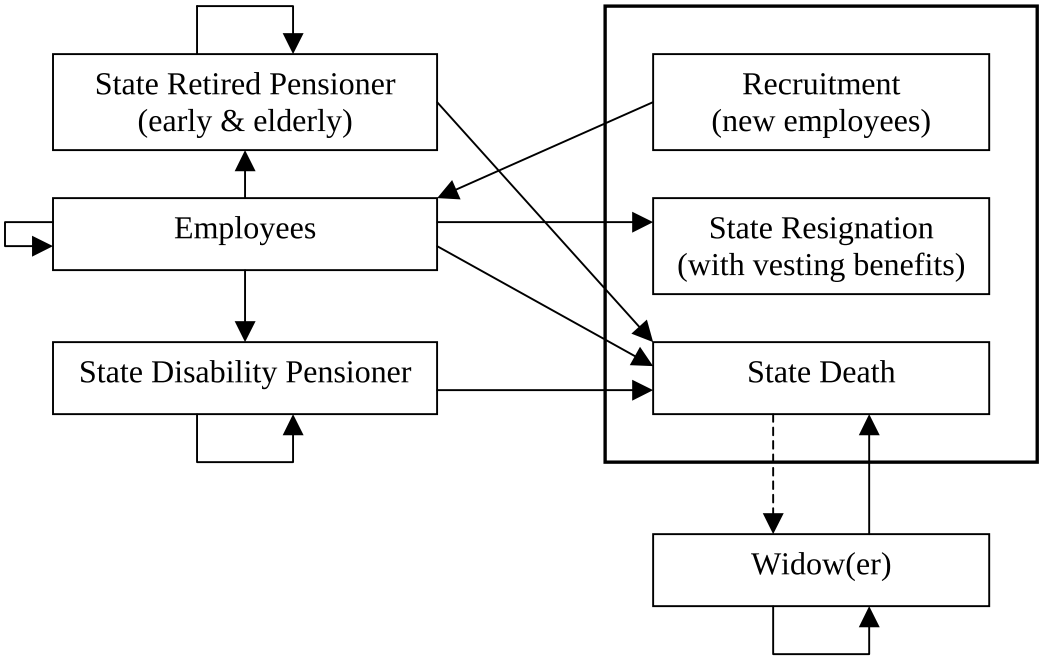


Figure 1. Markov model state diagram for a pension scheme.

Arrows returning to the same state denote the possibility to stay in the same state, e.g. to be an employee and not to get disabled or retired for a period of time. Recruitment is shown as a separate state to stress the fact that only new employees can enter the pension scheme from outside. The states I (disabled), EP and P (retired) can be reached only through the state A (employees). We do not model special cases such as a merger between two firms or an integration of a portfolio with disabled pensioners and retirees into the pension scheme. On the other hand, retirees and disabled can not cancel their policies and leave alive the pension scheme, but only via the state Death. As we have already mentioned earlier, new widows and widowers enter the scheme from outside, but their age and gender depend on the dead insured person and their number can not be varied by personnel policy. The probability to be re-married for a widow(er) is zero, so she or he can not leave the pension scheme alive. A more detailed description of state spaces for Markov models relevant to life policies can be found in Wolthuis [1994]. Benefit calculations with Markov chains in life insurance are described in Koller [2000].

### 2.2 Liability Scenarios

In general, there are three possible scenarios for the number of employees (in state A) in the pension scheme:

- Closed scheme (no new employees): the number of persons in the state A decreases depending on transition probabilities, especially dismissal and early retirement rates.
- Open balanced scheme: the number of persons in state A is constant (growth rate 0%) or grows with a small annual growth rate of, say, 1% - 2%.
- Reduction of the number of employees due to a certain personnel policy (for example, retired employees are not or only partially substituted by new employees).

The impact of each scenario on cash flow profiles will be analyzed separately. We will not work with a combination of different scenarios.

Transition probabilities from state A to the states I and P are given in the actuarial bases. Dismissal rates and early retirement probabilities can vary to a great extend according to the personnel policy of the employer and are strongly age dependent.

The number of new employees entering the pension scheme at the end of each year is calculated with respect to a chosen scenario. The new generation density determines the age and gender of each new employee. This density is also defined as a part of the personnel policy. As a rule in Switzerland, each new employee transfers his or her vesting benefit from the former employer to the new pension scheme.

We investigate a typical defined contribution (DC) scheme. Its benefits are somewhat higher than the BVG requires. Contributions are age dependent and there is a guaranteed (minimum) interest rate of 3.25% per annum. Pension payments and salaries are indexed to inflation. There are no contribution holidays or bonus payments.

First, we focus on cash flows. They are an important characteristic of a pension scheme. They consist of inflows (contributions according to the benefit plan and vesting benefits from former employers brought by new employees) minus outflows. Outflows are different lump sum benefits and pensions due to be paid during a year, and also vesting benefits for resigned or dismissed employees. The whole value of outflows depends on the age structure of the pension scheme, dismissal or resignation rates and the number of retirees, spouses and disabled persons. We will calculate cash flow profiles for the open balanced pension scheme with a growth rate of 0% using four different new generation densities and the same resignation rates. The early retirement rates used are statistically observed and representative data for Swiss pension schemes. The results for cash flows are shown in Figure 2 and the applied densities in Figure 3. This example shows the sensitivity of the Markov model for different types of personnel policy.

**Cash Flow Profiles for Different New Generation Densities**

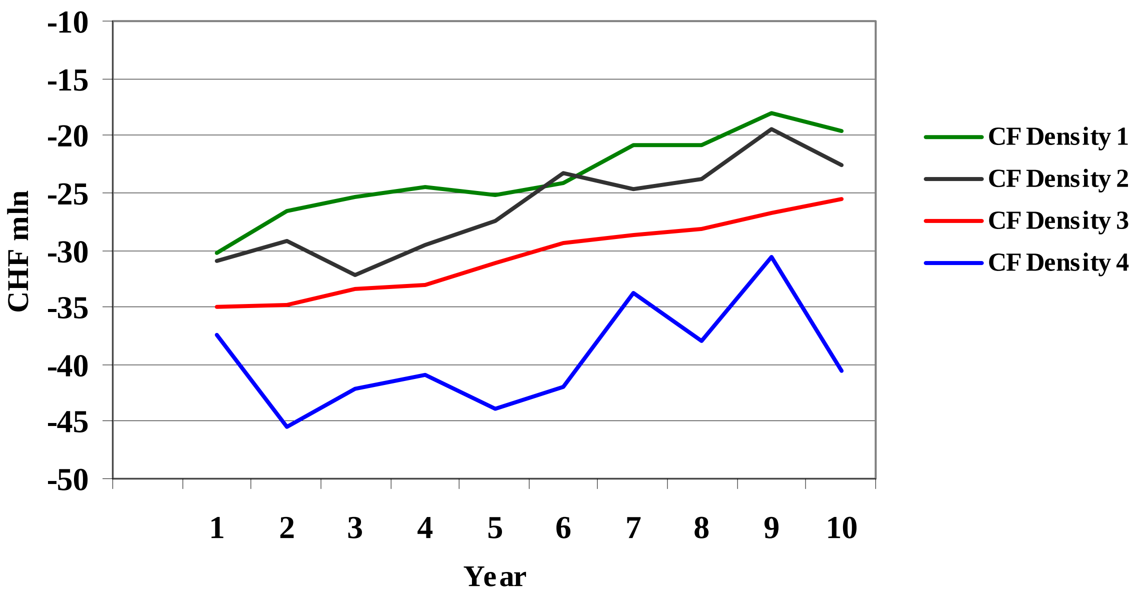


Figure 2: Cash flow profiles for different new generation densities.

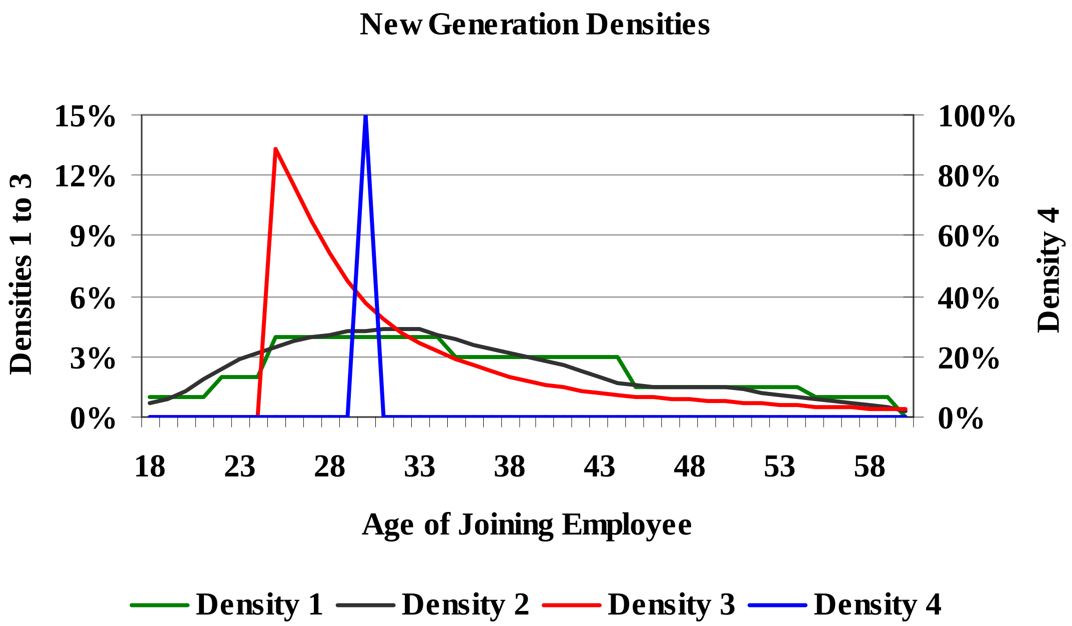


Figure 3. New generation densities used in Figure 2.

The densities 1 and 2 are typical for pension schemes for civil servants (e.g.), the density 3 is more general: the probability to enter the pension scheme is very small for employees older than 40. The density 4 is 1 for the age of 30 and zero otherwise. One concludes that the possible profile of the new generation density should be selected carefully since its impact is considerable.

### 2.3 Profiles of Cash Flows and Statutory Reserves of Pension Funds

Now we turn to the impact of different dismissal personnel policies on the cash flows and statutory reserves profiles. New employees will be hired according to density 1, its profiles for males and females are shown in Figure 4 together with the applied dismissal rates.

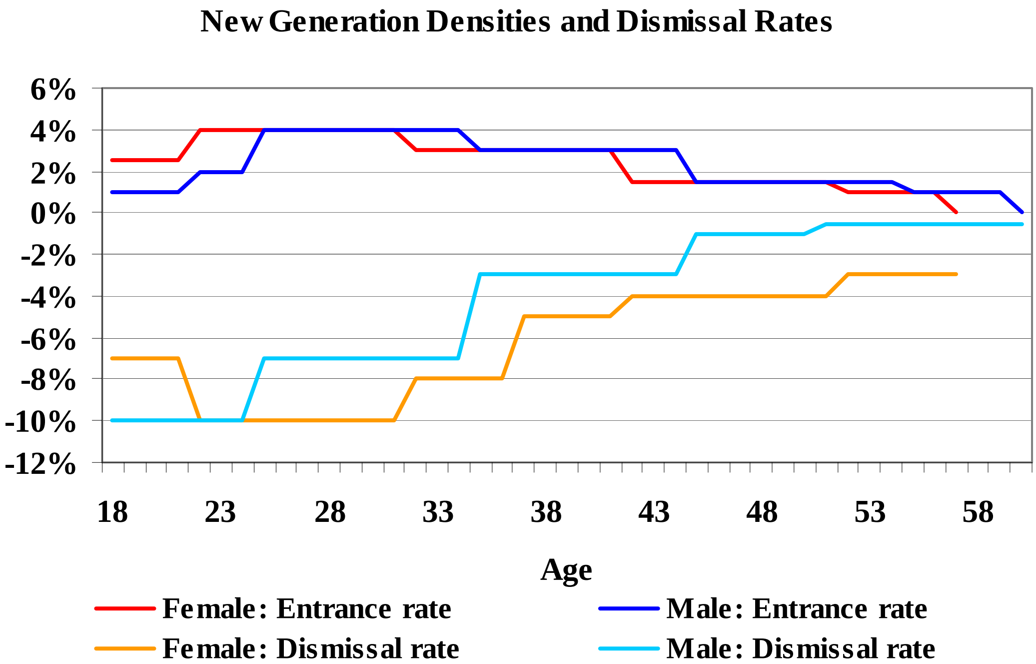


Figure 4: New generation densities (using density 1) for males and females and dismissal rates.

The macroeconomic parameters that where used are as follows: the annual inflation rate is 1%, the annual wage growth rate is 1%, and the total return from investments is 3% p.a.. There are four scenarios for the evolution of the pension scheme population:

1. The pension scheme is open and the growth rate for the number of active members is 0%.
2. The pension scheme is closed, i.e. no new employees at all.
3. The pension scheme is open, but retired and early retired employees are not substituted with new employees, so the number of active members is decreasing during the projection period.
4. The pension scheme is open and the number of active members is increasing linearly at 2% per year.

Table 1 summarizes the values for different insured groups in the pension scheme. The third scenario is not shown in Table 1, as its values are exactly between those of scenario 1 and 2, but Figures 5 and 6 show all values.

| | | Growth Rate 2% | | Growth Rate 0% | | Closed Scheme | |
|---|---|---|---|---|---|---|---|
| | T=0 | T=5 | T=10 | T=5 | T=10 | T=5 | T=10 |
| Actives | 4467 | 4914 | 5360 | 4467 | 4467 | 3256 | 2304 |
| Disabled | 199 | 236 | 251 | 234 | 240 | 227 | 210 |
| Retirees | 1609 | 1692 | 1902 | 1691 | 1885 | 1687 | 1837 |
| Spouses | 560 | 591 | 680 | 591 | 677 | 589 | 670 |
| Total | 6835 | 7433 | 8194 | 6982 | 7269 | 5760 | 5022 |
| Total Reserves CHF mln | 1304 | 1414 | 1723 | 1371 | 1604 | 1269 | 1345 |

Table 1. Evolution of pension scheme population during projection horizon.

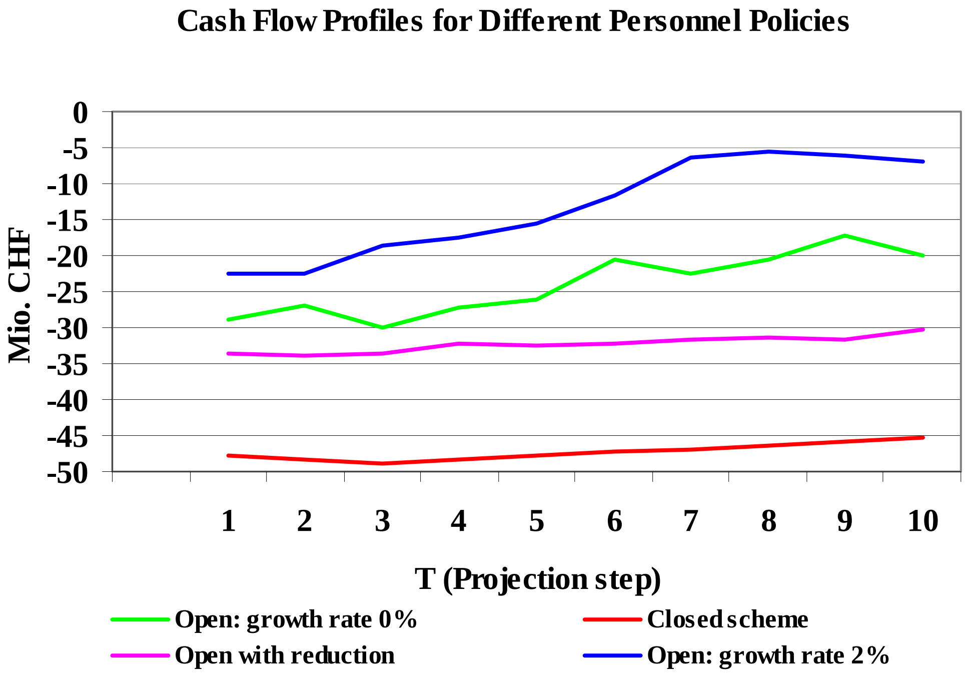


Figure 5: Cash flow profiles for different scenarios.

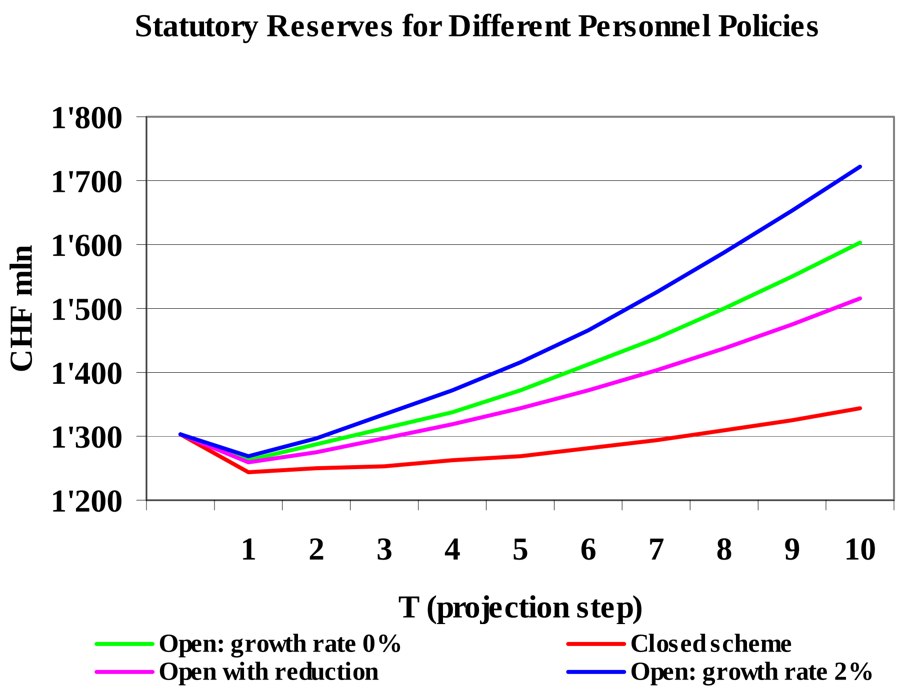


Figure 6: Evolution of statutory reserves for different scenarios.

### 2.4 Profit Testing in ALM

An important application of profit testing is the simultaneous dynamic Asset and Liability Management (ALM). Pension schemes with profit sharing or ad hoc contribution holidays can be modelled only by ALM with profit testing to ensure that in each macroeconomic scenario there are enough actuarial provisions and reserves to guarantee statutory benefits in each year during the projection horizon.

The formulation of the objectives and their transformation in an investment strategy are subject to several restrictions (e.g. a frame for the funding ratio, solvability directives, restrictions for certain asset classes). The realisation of investment concepts depends on the investment opportunities offered by the financial markets. The pension obligations restrict the objectives and the strategy to the extent that they determine the development and the commitment of capital, the contribution stream and the flow of pension payments.

## 3 Final Results and Conclusions

The presented Markov based profit testing model allows to manage a wide variety of Swiss pension schemes. If the transition probabilities and parameters for the liability part of the model (e.g. dynamics of new entrants, salaries etc.) are estimated correctly, the simulated results of the liquidity requirements and liabilities will be satisfying.

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## 5 APPENDIX: State Space and Formulae for the Markov Chain for Pension Scheme Population

The sequence of random variables $(X(t))_{t \in T}$ with $T \subseteq N_0$ defined on the same probability space with the finite discrete state space Z : $(Z_1, Z_2, \ldots Z_h)$ is called Markov chain of the first order with the state space Z and the initial distribution p(0), if and only if

$$P\{X(0) = j\} = p_j(0), \forall j \in Z$$

$$P\{X(t+1) = j_{t+1} \mid X(t) = j, \ldots, X(0) = j_0\} = P\{X(t+1) = j_{t+1} \mid X(t) = j_t\}, \qquad \text{(A.1)}$$
$$\forall t \in T, \forall j_0, j_1, \ldots, j_{t+1} \in Z$$
$$\forall i, j \in Z, \forall t \in T$$

The transition probability from state i at time t to state j at time t+1 varies in time and the Markov chain is not homogeneous any more, as is the case in most life insurance applications:

$$\pi_{ij}(t) := P\{X(t+1) = j \mid X(t)\}, \forall i, j \in Z, \forall t \in T \qquad \text{(A.2)}$$

$\Pi(t) := (\pi_{ij}(t))_{1 \le i, j \le h}$ , h is the number of all possible states in the Markov chain.

The h x h matrix of transition probabilities is denoted by Π(t) is a stochastic matrix, it means that

$$\begin{aligned} &\pi_{ij}(t) \ge 0, \forall i, j \in Z, \\ &\sum_{j \in Z} \pi_{ij}(t) = 1, \forall i \in Z \end{aligned} \qquad \text{(A.3)}$$

$$\Pi(t) = \begin{pmatrix} \Pi_{11} & \Pi_{12} & (\tilde{\pi}_{iS})_{i \in ZA} & (\tilde{\pi}_{iT})_{i \in ZA} \\ 0 & \Pi_{22} & 0 & (\pi_{iT})_{i \in ZB} \\ \tilde{\pi}'_{ZS}\, x_S(t) & 0 & 1 - x_S(t) & 0 \\ \tilde{\pi}'_{ZT}\, x_T(t) & 0 & 0 & 1 - x_T(t) \end{pmatrix}, \qquad \text{(A.4)}$$

where ZA denotes the State A ( Active), ZB is the State I (disabled) together with the States EP and P (retirees). Disabled and retirees are combined together in this sub-matrix to show the stochastic matrix in its compact form. S stands for the State with dismissed or resigned employees, and T stands for the State Death.

$\tilde{\pi}'_{ZS}$ is the new generation density at time t as a vector with values for the whole age interval of the State A as depicted in Figures 3 and 4 as a replacement for resigned employees,

$\tilde{\pi}'_{ZT}$ is like $\tilde{\pi}'_{ZS}$, but as a replacement for dead employees during the year. $x_S(t)$ and $x_T(t)$ are the quotes to calculate for each year, the values are in [0,1). $\Pi_{11}$ is a sub-matrix for all states in ZA (to be an active and to stay as an active), $\Pi_{12}$ is a sub-matrix for transitions between ZA and ZB (to be an active and to get disabled or to get retired if the age is at least 58-60, with the age of 65 all actives become retired), $\Pi_{22}$ describes transitions within ZB (to be disabled at first, to die as a disabled or/and then to get retired at the age of 65, for persons older than 65: to stay as retiree or to die).

Having the initial distribution p(0) (simply the age distribution in each state) in the pension scheme we can calculate the development of the Markov chain in each point of time t:

$$p'(t+1) = p'(t)\Pi(t), \forall t \in T \qquad \text{(A.5)}$$

The state space of our Markov chain is shown on the Figure 1. The States I, EP and P are grouped to ZB in the above description. Active members, disabled members or retirees can not visit the State W after their death, but a new spouse could be "hired" from outside in the pensions scheme with a different gender and age dependent on that of the defunct. On the other hand spouses can not visit the States A, I, EP or P, therefore we do not model the situation, when a married couple is employed at the same firm and the wife, for example, continues to work after the death of her husband and is an active and a widow at the same time in the pension scheme.