About the Risk Quantification of Technical Systems

by

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Abstract

The risk quantification of complex technical systems (for instance technical devices or processes) is often very demanding. One reason is the lack of large, homogenous portfolios and therefore the absence of reliable statistics. On the other hand it is very important for insurance companies to calculate a risk adequate price for technical systems and to evaluate the risk of technical portfolios in the context of risk management.
This paper shows how a standard engineering method, Fault Tree Analysis (FTA), and actuarial techniques can be combined to evaluate the risks of technical systems. For this purpose engineering judgement and experience is combined with actuarial knowledge in a consistent way. The result is a new method to generate the distribution of the total claim size, the basis for pricing (especially high severity claims) and risk management of technical systems. In this way all modern risk measures as for instance VaR or expected shortfall are accessible. The new method is demonstrated by applying it in a case study.

Keywords:
Risk management of technical systems, engineering risk analysis,
fault tree analysis, technical insurance, actuarial risk calculation, Monte Carlo simulation.

1. Introduction

The risk evaluation of technical systems and processes is often a very difficult task for insurers and underwriters. This is due to several reasons: The special knowledge of technicians and engineers is needed, the high probability for risk increase via positively correlated interaction, the differences in safety / quality control and risk culture as well as the fact that statistical data are missing. The last point is due to the fact that the insurer is far away from writing huge portfolios of homogeneous risks in that segment. But instead of this, high insured sums of different individual risks are written. On the other hand, it is of course, very important for the insurer to have a reliable risk evaluation and risk management, in order to calculate economically sound as well as competitive prices, to make reasonable reinsurance decisions and to meet the right financial and technical risk management strategies.

In the engineering sciences, there are well known methods of technical risk analysis as for instance the Fault Tree Analysis (FTA) or the Event Tree Analysis (ETA) [1]. These methods are designed to detect all possible sources of faults of technical systems. Their aim is technical progress and minimisation of risk. In special cases they also provide quantitative results as the probability of a claim event. The application of all technical systems, however, provides some unavoidable risks that have to be accepted. Here begins the task of insurance mathematics and quantitative risk management [3]: The risks have to be evaluated to calculate the price of the risk or to decide how the financial risks can be dealt with. In insurance practice the lack of large, homogenous portfolios makes quantitative risk analysis of technical portfolios often impossible. To receive a risk evaluation going beyond the crude estimation of the claim expectation, one needs the distribution of the claim sizes.

The construction of claim size distributions for technical systems is the aim of the work in hand: The above mentioned engineering risk analysis method, FTA, is the basis of the method. It delivers a model for the claim numbers. Via the FTA all possible claims can be detected and classified. Additionally the probability for the claim events can be estimated. This is the scope of this standard engineering method. Once the FTA is done, one is left with clearly defined and specified single claim events. Due to the clear definition of the single claim event, it is easy for experts (technicians, engineers) to give information from which actuaries can extract plausible single claim distributions. Using these ingredients (FTA-tree and single claim distribution) the total claim distribution of the technical system can be generated by methods of quantitative risk analysis and Monte Carlo (MC) simulation [3, 4]. This enables for instance the calculation of the fair premium and the answers to the above mentioned risk management questions, especially the calculation of risk measures as VaR or expected shortfall. The combination of a classical engineering method for risk analysis on the one hand (FTA) and actuarial risk management techniques on the other, leads to the generation of total claim distribution for technical systems and insurance portfolios. The method is very flexible: It can be applied to any technical system for that FTA is possible. Even other distribution of relevant quantities – concerning the technical systems and portfolios of it – can be calculated (not only claim costs), as for instance the working time of outsourced tasks (for instance to consulting engineers). In the following we introduce the above described method and demonstrate its application to an example of insurance practice: The dentist’s chair.

2. The Quantification of Risk for Technical Systems

The Fault Tree Analysis (FTA) is a standard engineering method for risk minimisation and quality control [1]. The aims of the FTA are the following:

- Revealing all possible problems or faults of the technical system.
- Showing critical interaction mechanisms of the (sub)system.
- Detecting external conditions or influences that cause a fault of the system.
- Explaining the causes and/or sources of the system fault.

The FTA starts with a unexpected event, the so called "top event", that has to be prevented. In our example the top event is the damage or fault of the dentist's chair. Once the top event is defined, the FTA looks for all (probabilistic) paths that will lead to it. In this way all problematic situations can be found. The result of the FTA is given as a graphic representation, a FTA tree with logic connections as nodes. The FTA tree of our case study, the dentist chair is shown systematically in figure 1. There are several levels of nodes. They are generated in finding all possible paths to the top event. To every node the probability for the different paths is assigned. For instance in our example (see fig. 1) the first level is the distinction between internal (15% probability) and external (85% probability) sources of faults. After having developed the total FTA tree, one knows all possible claim events and its probability. For more details about the FTA tree see [1, 5]. The tree consists of many branches (22 branches in our example, see tab. 1), the disjoint but not independent claim events. They make up all possible claim segments of the technical system.

3. Quantitative Risk Management, the Simulation and its Results

The scope of this work is the combination of a quantitative FTA, a standard engineering method of technical risk analysis, on the one hand with methods of actuarial risk evaluation on the other hand to a method of quantitative risk management. The aim is to determine the total claim distribution of technical systems (devices or processes). This is necessary to calculate the fair premium, to take sensible reinsurance actions, to calculate risk measures (for instance the VaR or the expected shortfall) and therefore to allocate the correct risk capital. We demonstrate the method on a concrete example from insurance practice: a medical device, the dentist's chair. The description of the FTA analysis of the dentist's chair can be found above and in more detail in [5]. In the following we introduce the method of quantitative risk management. One attribute of this method is its high flexibility. It can be used to answer even other questions – for the example at hand this could be: The loss of income of a dentist, the claim size of a claim portfolio or the total working load of an engineering office responsible for risk management of the chairs. All these quantities can be calculated with our new method, as the "total claim size" can be defined depending on the context (for instant as loss, working time,…).

3.1 The Integration of Methods and the Total Claim Size Distribution

We integrate methods from different scientific fields (engineering and actuarial science) to a holistic, quantitative method of risk analysis for technical subjects. The basis of this integration is the following model:

N is the total claim number

$$N = \sum_{i=1}^{I} n_i \text{ , } (1)$$

where n_i is the claim number of branch *i*. The FTA tree has altogether I branches. The total claim number *N* is either a constant (for fixed number of claims) or a random variable, for instance in the case of a claim portfolio.
The total claim size *S* can be calculated as:

$$S = \sum_{i=1}^{N} \left(\sum_{j=1}^{n_i} s_j^{(i)} \right) \text{ (2)}$$

where $s_j^{(i)}$ is a single claim size of branch *i* and $j = 1,...,n_j$ counts the claims of branch i.

We implement this model by the help of MC simulation techniques. In this way we receive concrete results. In Section 3.2 we will discuss the claim size distribution and risk measure of a medical device, a dentist's chair. The preconditions to perform this method to a technical system are the following: First of all it has to be clear what quantity has to be looked at, some examples are the claim amount an insurance company has to pay, the dentist's economic loss, the time engineering consultants need to handle the claim. The FTA tree of the system under consideration according to Section 2, including the probabilities on the nodes of the tree, has to be known. Finally the single claim size distribution of each FTA branch has to be known. This is an actuarial standard task. It can be received by data analysis (if enough data accessible) or by the judgment of experts or by a combination of both (Bayes theory) [2]. Another parameter of the model is the claim volume as constant claim number or as the parameter(s) of a claim number distribution. After the preconditions of the model have been clarified, the implementation to a MC simulation is described in the following. We consider first a single MC run. Obeying the nature of MC techniques the simulation consists of a huge amount of runs for instance 1000 or 10000.
The fault tree is implemented via a series of multinomial stochastic processes. The initial parameter of the program is the number of claims. It can be chosen either as a constant or as a random variable, simulated according to a given claim number process – a Poisson process in our example. Therefore the method is very flexible: It can describe portfolios of different sizes. The given number of claims is distributed in a series of multinomial stochastic processes along the branches of the FTA tree. Every level of the tree is in accordance with one MC multinomial process. After that one knows how many claims per branch (see tab.1) of the FTA tree have to be sampled. This is the number of claims n_i (see eq. 1) for branch no. i. For every claim of branch i a single claim size has to be generated according to the **single claim distribution** of that branch. The estimation of single claim distributions is a standard actuarial task. It gives the severity distribution of the claim costs. We need the single claim distribution for every branch of the FTA tree, this means that the kind and type of the claim is very well defined and described. This restricts on the one hand the availability of the claim data, on the other hand it is very helpful for an expert judgment. We give an example: The empirical distribution function for the claim type "mains water damage / destruction of the chair's surface "can be given in good approximation as a piecewise constant function:
20% of the claims cost an amount of less than 500 EUR,
30% of the claims cost between 500EUR and 1200EUR
50% of the claims cost between 1200EUR and 2000EUR.

The single claim size distribution for each branch (for our example see tab. 1) is implemented in the MC program. The program samples n_i claim sizes for every branch i.
The **total claim** is finally calculated as the sum of the single claims (see equation (2)). This is, of course, quite an easy task in a MC simulation: The claims of all segments (branches) are added to the total claim.
To sum up: For every MC run a total claim number is simulated (either a constant or a random number). It is distributed to the several branches according to the FTA tree. For every branch the appropriate number and type of claim sizes are generated and finally all these claims are added to the total claim. In this way every MC run generates one realisation of the total claim. The MC run is repeated very often to get many realisations (for example 1000 or 10000) of the total claim. Thereof statistical quantities as the mean, the standard deviation and the empirical distribution function can be calculated and all risk measures (for instance the VaR or the expected shortfall) are accessible. The number of runs determines the accuracy of the simulated quantities. In the following the above introduced method is applied to the example “dentist’s chair”. Additionally the simulation results will be presented.

3.2 The results of Monte Carlo Simulation – some examples

The fault tree shown in figure 1 is the basis of the following simulations. From the structure and the probabilities of the fault tree follow the claim numbers at each branch of the FTA tree. For the simulation example at hand we use single claim distributions as given in table 1. There are shown the types of the single claim distributions together with their parameters. The details of table 1 are: The number of the branch (column 1; consistent to Fig.1 – top-down order), the probability for the events (column 2), the numerical value for the single claim expectations (column 3) and standard deviations (column 4) as well as the distribution type (column 5). For every branch we have chosen an appropriate single claim distribution – here some examples: For small claim sizes, as for instance in the case of interrupted water or power supply (branch No. 9-11), we have chosen an exponential single claim distribution with the exponential parameter equal to the expectation of a single claim. For medium sized claims log-normal distributions are used. Their two free parameters are estimated via their first two statistical moments. Medium sized can be found for instance in case of a fire where the chair can still be used after restoration (branch No. 19) or in the case of a software bug (branch No. 2). For cases where the chair can not be restored, we use an empirical, piecewise constant claim distribution density. Those branches are marked with “Spezial” in column 5 of table 1. An example for this case is the total loss of the chair due to fire. The corresponding claim size density is:
2% of the claims cost less than 2500 EUR,
2% of the claims cost between 2500EUR and 10000EUR,
3% of the claims cost between 10000EUR and 56600EUR,
93% of the claims cost between 56600EUR and 70000EUR.
This means that almost every claim (93%) is a total loss of the chair additionally surplus costs (for instance cost of disposal). Only 7% of the claims result in no or little amount of payment (for instance due to the contract, regress).
Consider the application of this method in insurance practice: It is a standard task for actuaries to estimate the single claim distribution. This has to be done for every branch of the FTA tree (see table 1), for instance via methods of data analysis. Another way of calibrating the single claim distribution to the real world is the judgment of experts (engineers). For this the use of the piecewise constant claim distribution density (see the above example) is especially suited. For practical use and the communication with experts / engineers it is much easier to focus on a well defined claim segment (as it is the case for every FTA branch) than

drawing conclusions about the technical system's claim distribution as a whole. This fact is an important advantage of the here introduced new method.

In the following we discuss some results of the MC simulation: First we determine the total claim distribution for one claim, this means N=1 (constant, no random variable). We aggregate the 22 single claim distributions as given in table 1 to the total claim distribution of the technical system "dentist's chair". The simulation result is depicted in figure 2: The total claim distribution (black line), the theoretical expectation of the total claim (i.e. the sum product of columns 2 and 3 in table 1; equals 30344 EUR; blue vertical line) and the Normal distribution with fitted first and second moment (red line). Figure 2 shows a considerable deviation between the total claim distribution and the Normal distribution. The 50% - quantile of the total claim distribution is higher than its expectation. Almost 60% of the total claim sizes are higher than the expectation of the total claim size. The total claim distribution is clearly skewed to the right (to higher values). The profile of the total claim distribution is reasonable, because the total claim consists of many different kinds of single claim sizes (different types of single claim distributions and different orders of magnitude; see table 1). The FTA-branches responsible for major claims (branches No. 15-18 and 21, tab.1) cause the skewness of the total claim distribution.
Now we will investigate claim portfolios. This means that N is no longer a constant, but a random variable. We compare small claim portfolio with E[N]=15 claims to a large on with E[N]=150 claims. E[N] is the expectation of the claim number. We compare them with the case of a constant claim number. We assume N to be Poisson-distributed with the Poisson-parameter equal to E[N]. The results of the MC simulation are shown in figure 3. We see the empirical total claim distribution. The black (green) line represents the case of the constant (Poisson-distributed) claim numbers. The upper (lower) part of figure 3 shows the total claim distribution of the large (small) portfolio. In all four cases the depicted total claim distributions of the portfolio are much more similar to Normal distributions than in the case of the case N=1 – claim distribution (fig. 2). This effect is due to the central limit theorem: The higher the claim number N (portfolio size), the better is the Normal distribution approximation. The conditions for the validity of the central limit theorem are approximately fulfilled in the application at hand: The portfolio consists of many total claims and the total claim consists of many single claims. All single claim distributions have a finite second moment. The portfolio claims result from independent events (note: the single claim events resulting in one total claim event are dependent due to the FTA-tree structure). It is quite obvious that the width of the distribution is considerably higher in the case where N is a random variable compared to the case of constant N (see the differences between green and black lines in fig. 3).
Finally figure 4 considers the tail of the portfolio claim size distribution for Poisson-distributed claim numbers with E[N]=15 (lower part) and E[N]=150 (upper part). The tail of the claim distribution is decisive for the evaluation of the major claim risk, because it contains information about claims much higher than the expectation. From figure 4 follows the portfolio – VaR (see blue, sketched line): For instance we find for the small portfolio a VaR to the 95% (99%) – level of approx. 680 TEUR (785 TEUR). For the large portfolio we find a VaR to the 95% (99%) – level of approx. 5.25 Mio. EUR (5.55 Mio. EUR). Of course, one receives even better approximations of the VaR values through more detailed analysis of the simulation data – this is beyond the scope of this article.
To summarize: We have demonstrated with an concrete example from insurance practice how questions of quantitative risk management can be answered by the help of our new method for technical systems. Due to the combination of FTA, actuarial analysis of single claims and simulation methods, we succeed in analyzing the risk of technical systems and even portfolios of them.

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No. Bra.	Probability Branch	EW Claim	StdDev Claim	Distribution type
1	0,2250%	10.000	10.000	exp
2	0,5250%	20.000	4.000	LogNorm
3	0,1781%	30.000	4.500	LogNorm
4	3,3844%	30.000	4.500	LogNorm
5	1,7813%	15.000	3.000	LogNorm
6	1,7813%	15.000	3.000	LogNorm
7	0,7125%	20.000	4.000	LogNorm
8	6,4125%	20.000	4.000	LogNorm
9	0,7438%	4.500	4.500	exp
10	0,7438%	4.500	4.500	exp
11	1,3813%	4.500	4.500	exp
12	1,3813%	4.500	4.500	exp
13	10,6250%	10.000	1.000	exp
14	10,6250%	30.000	4.500	LogNorm
15	0,5950%	45.027	8.830	Spezial
16	2,3800%	60.018	13.333	Spezial
17	1,7850%	45.027	8.830	Spezial
18	3,5700%	60.018	13.333	Spezial
19	12,4950%	35.000	3.500	LogNorm
20	3,8675%	25.000	3.750	LogNorm
21	5,8013%	45.027	8.830	Spezial
22	29,0063%	35.000	3.500	LogNorm

Tab. 1: Single Claim Distributions - Overview

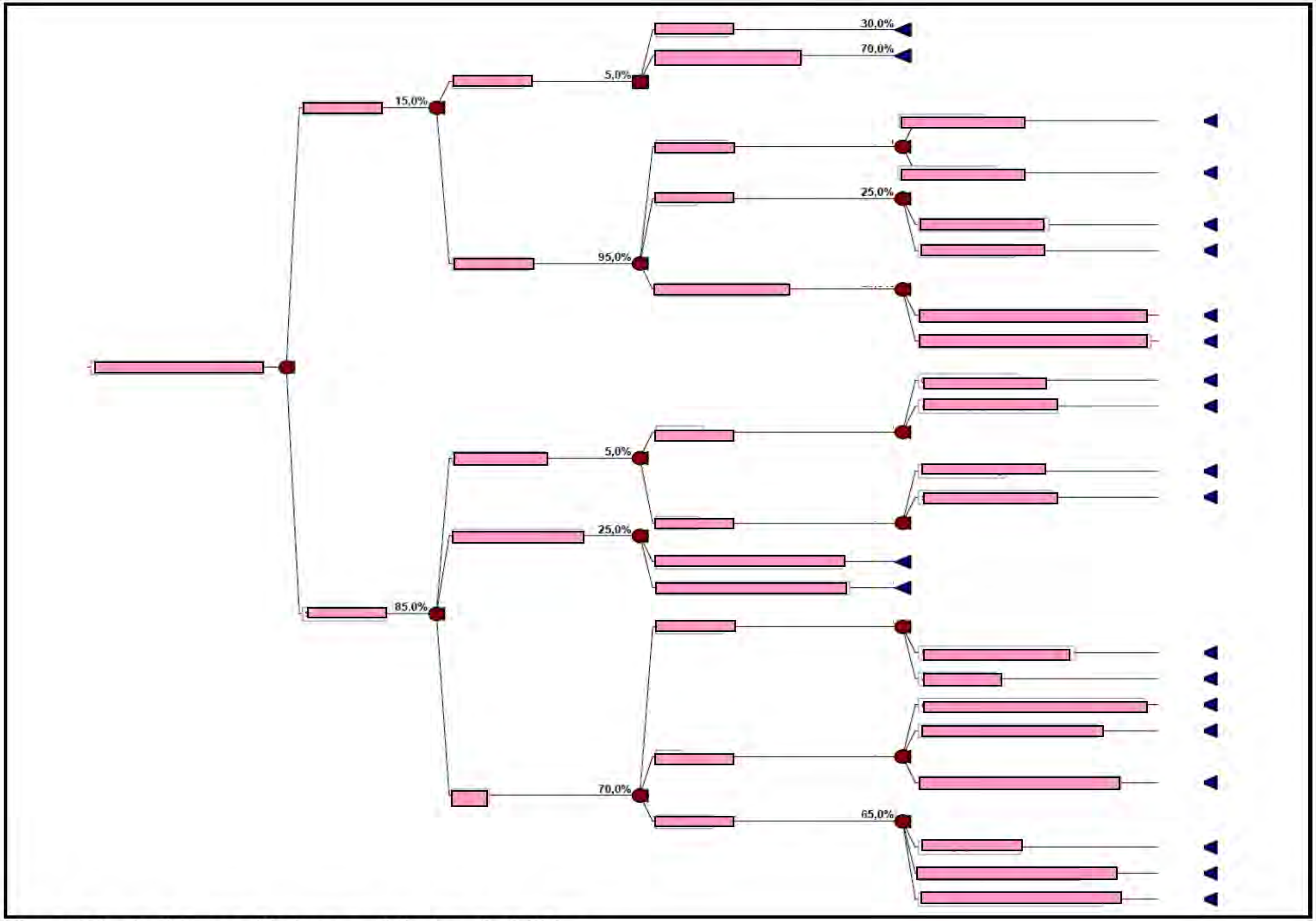

Fig. 1: The fault tree with (some of the) probabilities

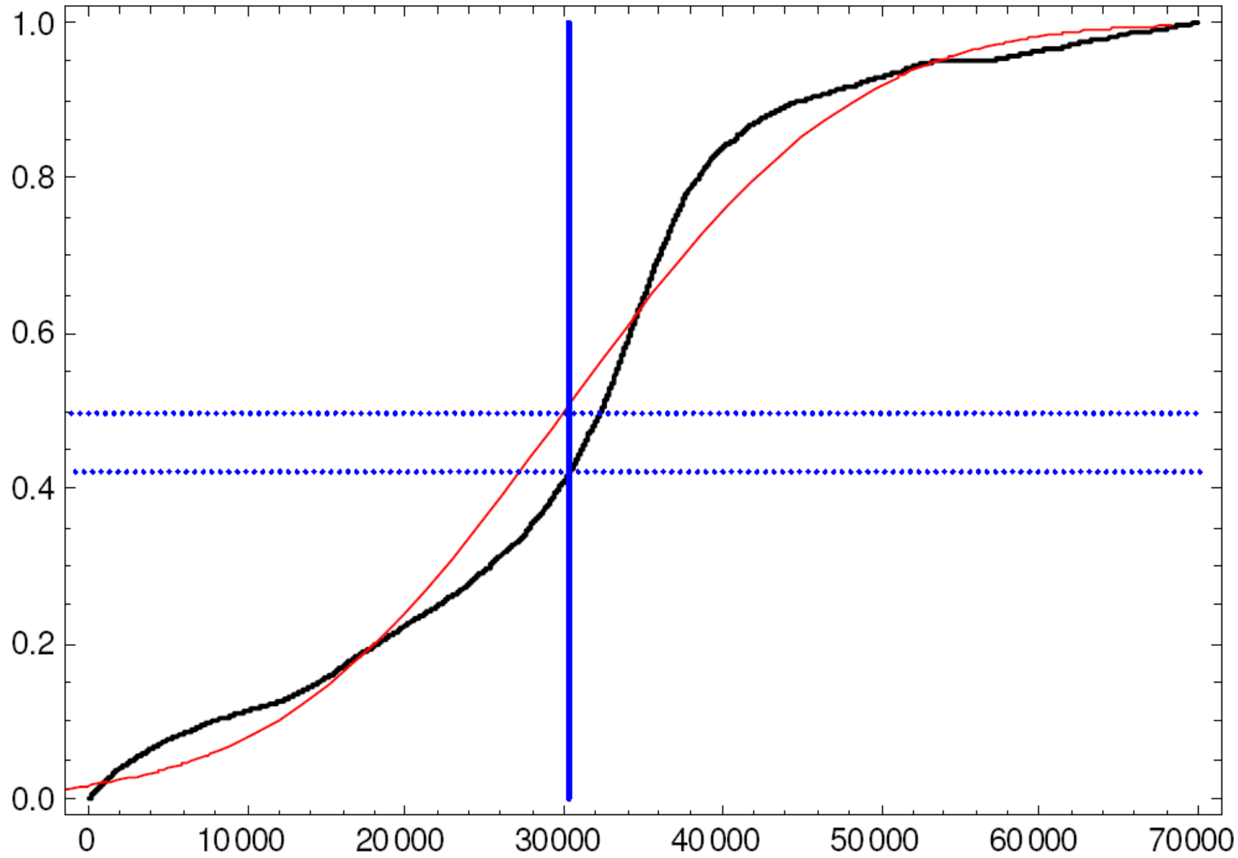

Fig. 2: The empirical total claim distribution (black) compared to a Normal Distribution (red) with fitted first and second moments and the expectation (blue line).

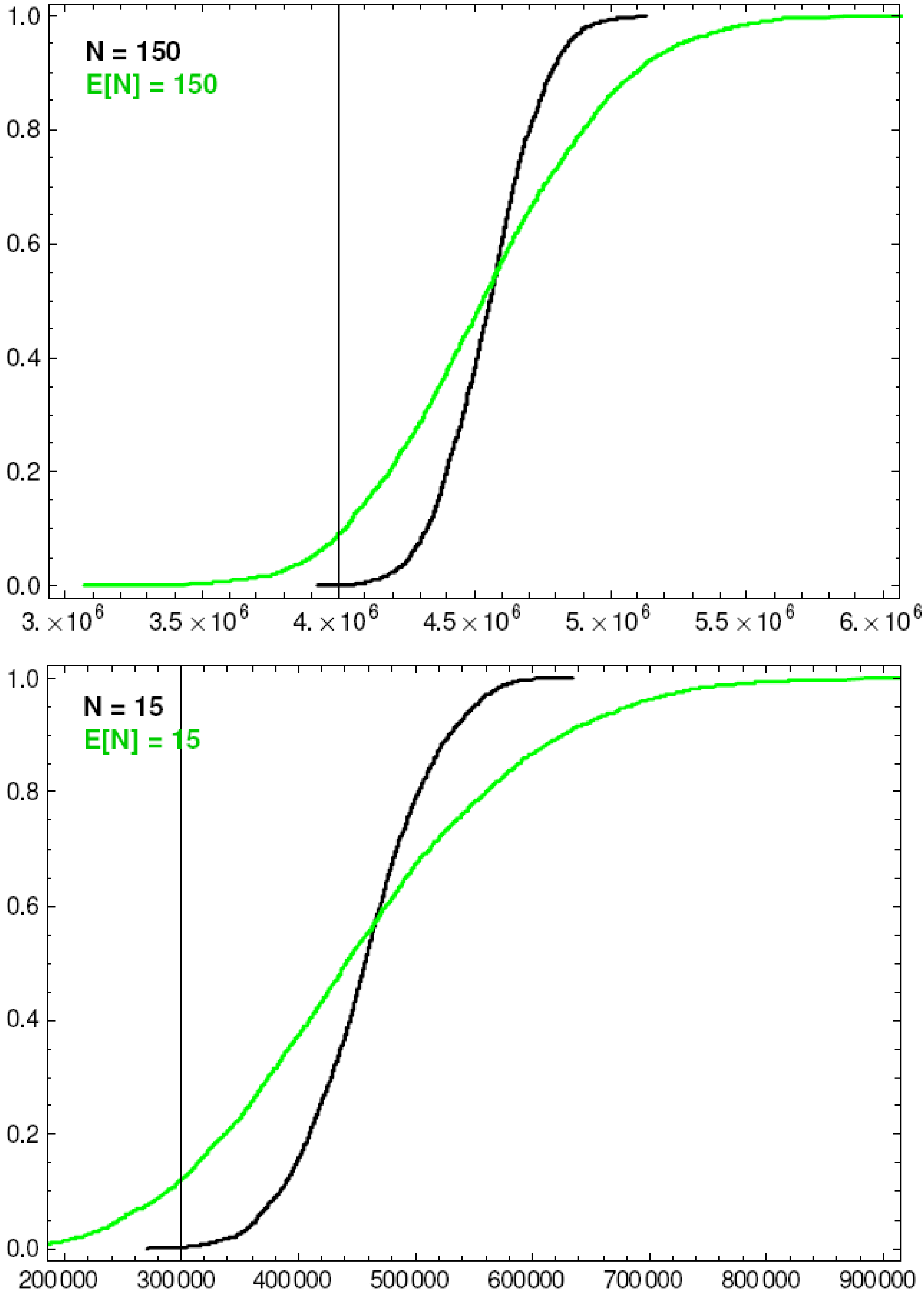

Fig. 3: Die The empirical total claim distribution for two claim portfolios of different size: Comparison of the cases with constant (black) and Poisson- distributed (green) claim number.

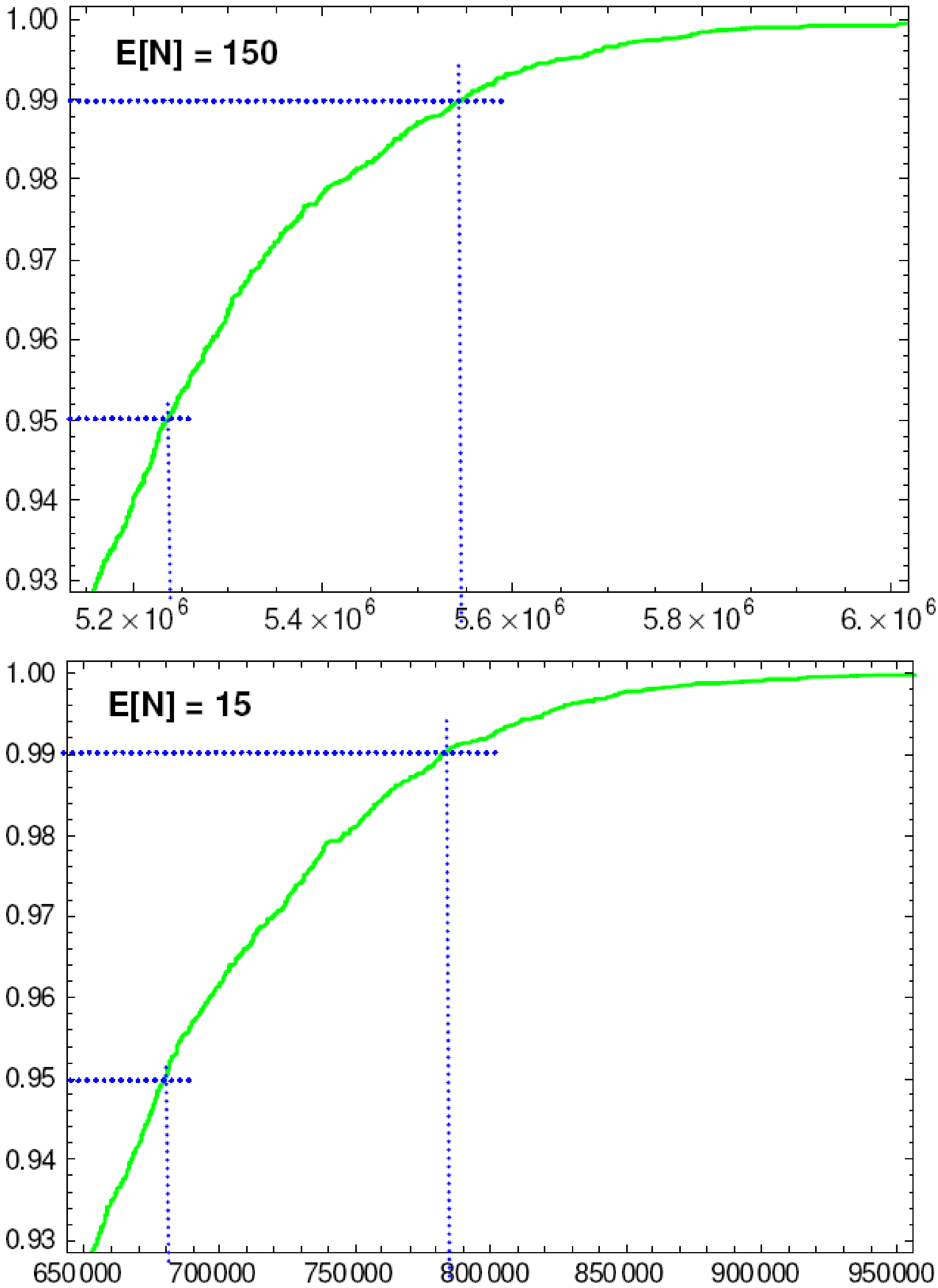

Fig. 4 : VaR calculation of the total claim on the basis of the claim severity distribution for Poisson – distributed claim numbers with expectation 150 (above) and 15 (below).