Equilibrium Pricing of Contingent Claims in Tradable Permit Markets*

Masaaki Kijima[a], Akira Maeda[b], and Katsumasa Nishide[c†]

a *Graduate School of Social Science, Tokyo Metropolitan University*

b *Graduate School of Energy Science, Kyoto University*

c *Interdisciplinary Research Center, Yokohama National University*

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†Corresponding author. Interdisciplinary Research Center, Yokohama National University. 79-3 Tokiwadai, Hodogaya-ku, Yokohama, 240-8501 Japan.

TEL&FAX: +81-45-339-3698. E-mail: knishide@ynu.ac.jp

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Abstract. In this paper, we construct a permit market model to derive a pricing formula of contingent claims traded in the market in a general equilibrium framework. It is shown that prices of contingent claims exhibit significantly different properties from those in the ordinary financial markets. In particular, if the social cost function kinks at some level of abatement, the forward price as well as the spot price can be subject to the so-called price spike. However, this price-spike phenomenon can be weakened if the system of banking and borrowing is properly introduced.

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1 Introduction

The idea that market-based instruments (MBIs) are to be used for environmental policies has been popular in many developed countries as well as at the arena of international policy negotiations. In particular, the use of tradable permits was first proposed by Dale (1968) and has long histories of both academic studies and practices. Its well-known application to a nation-wide environmental problem includes the sulfur dioxide permit trading that was established by the Acid Rain Program of The Title IV of The 1990 U.S. Clean Air Act Amendments, and was made operational in 1995. Many economic analyses have been done, among which Ellerman *et al.* (2000) showed a comprehensive study on the assessment of the system.

The creation of an international market for greenhouse gas (GHG) permit was proposed in the course of negotiations for the Kyoto protocol and was concluded as the so-called Kyoto mechanisms in the protocol. Together with the international market for GHGs, European countries are making efforts to create domestic or regional GHG markets; the United Kingdom created its domestic market in 2002 as a part of UK's national climate policy; the European Union (EU) started the first phase of the EU Emissions Trading System (EU-ETS) in 2005.

Some reports provide concise surveys of the current situation in emission permit markets. Røine and Hasselknippe (2007) reported that the trading volume and the transaction amount of the EU-ETS was doubled in 2006 compared to the previous year, and is expected to be growing in 2007. Figure 1.1 in their report shows that non-emitters, who are not required to meet regulations, are also interested in carbon markets. In European Climate Exchange (ECX), the trades of futures and options were started in 2006, and their volumes are increasing rapidly. With these backgrounds, it becomes more and more important to understand how prices of permit market products are formed.

A first rigorous treatment of the theory of tradable permits was shown by Montgomery (1972). His seminal work was followed by many researches thereafter. Tietenberg (1985) summarized findings from studies that appeared until early 1980s. Stavins (2000) highlighted several important papers that have contributed to the academic literature.

It is worth pointing out that most of the past studies on tradable permits explored the mechanism, design, and policy implications of markets in which permits are physically traded and contracts are physically settled. Markets in the real world, however, do not restrict themselves to engaging in such spot trades and physical settlements. In fact, forward contracts, futures, options, etc. are available for future vintage permits in the real world. With these derivative products at hand, paper (non-physical) trades with cash settlements are possible, which enlarges the whole range of permit trades. Also, derivative products provide market participants with financial instruments that are used to hedge various risks. Derivative products or, more generally, contingent claims in permit markets are expected to be growing and become more important in the near future, as the system of tradable permits becomes mature.

Regardless of the usefulness of contingent claims in permit markets, there are quite few studies in the academic literature that explore the financial aspect of permit markets and derivative products. Of course, there are a lot of papers that study how the prices of contingent claims are determined in financial and commodity markets (see, for example, Duffie (2001)). However, these results cannot be used directly for the pricing of contingent claims traded in permit markets because of the special features.

In the microeconomic theory, the marginal cost of production determines the spot price of any goods in a market. Similarly, in permit markets, the current spot price is theoretically equal to the marginal cost of abatement. In contrast, when contingent claims are considered in permit markets, it is necessary to consider the so-called state price density, that is affected not only by the cost function to reduce the emission but also by the real (physical) probability measure and the utility functions of market participants.[1] Also, since permit markets are purely artificial and depend much on the design and implementation of regulations, prices in the markets have some specific characteristics that are not observed in the ordinary financial and commodity markets. It is therefore of great interest to investigate how the prices of contingent claims in permit markets are determined.

Analysis of financial aspects in permit markets is fairly new; see e.g., Benz and Trück (2008), Fehr and Hinz (2006), Seifert *et al.* (2008). Also, most of the existing papers examine only the properties of spot prices with no attention to the analysis of contingent claims. An exception is Maeda (2004), who examined a permit market in which all agents have mean-variance utilities, the cost function of emission abatement is quadratic, and future uncertainty is represented by Gaussian random variables. In this setting, he derived a pricing formula of forward contracts and illustrated the impact of banking and borrowing on the spot and forward prices. However, Maeda (2004) considered the forward

[1] In such previous papers as Rubin (1996) or Cronshaw and Kruse (1996), there is no uncertainty in the model and the effect of banking on the spot price is mainly studied. On the other hand, our model explicitly examines the properties of contingent claims by modeling market participants and the uncertainty in the economy.

price only, and other contingent claims cannot be priced in his setting.

Recently, Daskalakis *et al.* (2007) considered the pricing of options written on the spot price of emission permits; however, the spot price is exogenously given and the resulting formulas are of the Black–Scholes type in the ordinary financial markets. On the other hand, Chesney and Taschini (2008) assumed that the instantaneous emission rate Q_t in a CO2 market follows a geometric Brownian motion, as given, and derived the spot price of emission permits based on the comparison of Q_t and penalty P. In these papers, an underlying asset in permit markets is given exogenously and important characteristics that are special to permit markets are not used explicitly.

In this paper, we take the line of Maeda (2004) and extend it to construct a permit market model by which any contingent claims can be priced. To this end, we assume that all agents have exponential (CARA) utilities with distinct risk-aversion coefficients. The state price density of each future state is derived for any probability distribution and any cost function, whereby providing a fairly general formula that is used to price any contingent claims in the market. To our best knowledge, this paper is the first to study the pricing of contingent claims in permit markets in a general equilibrium framework.

A major contribution of this paper is as follows. First, our pricing formula depends on important features of permit markets, allowing us to analyze the qualitative properties of the pricing. More specifically, it depends on the probability distribution of the future abatement target, the aggressiveness of market participants, the social cost function to reduce pollutant emission, and the correlation between future emission amount and the aggregated exogenous income of agents. For example, if the future emission amount and the exogenous income are positively correlated, the price of forward contracts is lower than that with no correlation.

Next, through comparative statics studies, we find that the change of an exogenous parameter has two effects on the price of contingent claims. The first one is the direct effect caused by the change in the payoff, while the second one is the indirect effect that arises from the change in the state price density. In the ordinary financial markets, the state price density is exogenously given, and the change of exogenous parameters has no impact on the state price density. This is not the case in our permit market model, and in fact the two effects need to be considered simultaneously for the pricing of contingent claims traded in a permit market.

We also discuss some related topics that are specific to permit markets. One interesting finding is that if the social cost function kinks at some level of abatement, the forward price as well as the spot price may change drastically even when the probability distribution of the future abatement target is slightly updated. It is widely recognized in energy markets that the so-called price spike is often observed, i.e., the price dramatically changes due to the shortage of capacity to produce energy. Our observation indicates that similar price spikes can happen in permit markets, implying that the prices in permit markets may have some instability that cannot be overlooked. However, interesting enough, the price-spike phenomenon can be weakened if the system of banking and borrowing is properly introduced.

This paper is organized as follows. In the next section, we construct a permit market model in which regulated emitters and financial traders participate. Section 3 derives a pricing formula in a general equilibrium framework to evaluate any contingent claims traded in this market. In Section 4, we consider the case that all uncertainty is modeled by normal random variables, and analyze the properties of the pricing formula with numerical examples. Section 5 is devoted to the discussions of some related topics that are specific to permit markets, and Section 6 concludes this paper.

2 A Permit Market Model

We consider a single-period economy in which time 0 represents the present time and time 1 a future time. These two times mean the compliance deadlines for permit regulations. That is, emissions from regulated emitters who must obey the environmental legislation are measured and monitored at times 0 and 1. A regulatory authority imposes emission targets to which emitters are required to reduce their emission amount. Our model can be easily extended to a multi-period setting.

We assume that in the market there are two types of economic agents, regulated emitters and unregulated financial traders. While regulated emitters need emission permits for their own business activities, unregulated financial traders do not need to follow the regulation but can participate in the permit market. Each agent has no market power,[2] and behaves as a price taker. We denote the sets of emitters and financial traders by $\mathcal{M}_E$ and $\mathcal{M}_S$, respectively, and define $\mathcal{M} := \mathcal{M}_E \cup \mathcal{M}_S$. Each agent has an exogenous income at each time, and we denote by R_{kt} the income for agent $k \in \mathcal{M}$ at time t.

At time 0, there are two markets in the economy, the spot market and the contingent claims market. In the spot market, permits of time 0 are traded. On the other hand, the contingent claims market allows all the agents to trade contingent claims written on the

[2] In this paper, we only consider competitive markets for all types of trades. For discussions about the influence of market power, we refer to, e.g., Hahn (1994).

permits at time 1. It is assumed that the contingent claims market is complete in the sense that any contingent claim whose payoff realizes at time 1 is tradable. At time 1, all uncertainty reveals, all contingent claims traded at time 0 are settled, and only spot permits of time 1 are traded among emitters. We denote by S_t the spot price at time t. The risk-free interest rate is denoted by r, which is exogenously determined in the financial market, and measurable at time 0.

Let E_{it} be the amount of business-as-usual emission minus the initial permit endowment for emitter $i \in \mathcal{M}_E$ at time t. If E_{it} is positive, emitter i must reduce her emission by her own effort and/or purchase some amount of spot permit in the market with price S_t. If E_{it} is negative, emitter i can sell it in the spot market so as to earn extra money. The current abatement target denoted by E_{i0} is measurable at time 0, while E_{i1} is uncertain and is a random variable at time 0.[3]

We assume that the emission abatement effort is accompanied with some cost. The cost function is denoted by $c_{it}(X_{it})$, where X_{it} is the emission reduction amount by emitter i at time t. If emitter i decides X_{it} not to satisfy the requirement for her own, i.e. $X_{it} < E_{it}$, then she needs to buy the spot permit $E_{it} - X_{it}$ with price S_t to meet the requirement. This means that the total payment to comply the emission regulation is given by

$$c_{it}(X_{it}) + (E_{it} - X_{it})S_t.$$

Note that, when $X_{it} > E_{it}$, emitter i can sell the amount $X_{it} - E_{it}$ with price S_t in the spot market. Hence, the total payment can be negative, meaning that she earns the profit.

Suppose in addition that emitter $i \in \mathcal{M}_E$ enters the contingent claims market to purchase an ω-contingent claim $H_i(\omega)$ at time 0, which pays the amount $H_i(\omega)$ of cash when state ω is realized. The price of the contingent claim H_i is given by

$$\pi(H_i) = \mathbb{E}[\tilde{H}_i \tilde{\phi}],$$

where $\tilde{\phi}$ is a state price density and $\mathbb{E}$ denotes the expectation operator associated with the real probability measure $\mathbb{P}$.

Denoting by W_k the final wealth of agent $k \in \mathcal{M}$, it then follows that

$$\begin{aligned} W_i(\omega) =& (1+r)[R_{i0} - (E_{i0} - X_{i0})S_0 - c_{i0}(X_{i0}) - \pi(H_i)] \\ & + R_{i1}(\omega) - (E_{i1}(\omega) - X_{i1}(\omega))S_1(\omega) - c_{i1}(X_{i1}(\omega)) + H_i(\omega), \qquad i \in \mathcal{M}_E, \end{aligned} \tag{1}$$

[3]In this paper, uncertainty is described on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The realization of random variable Y for $\omega \in \Omega$ is denoted by $Y(\omega)$. Also, we use the notation $\tilde{Y}$ to emphasize that Y is a random variable. Hence, we denote the time-1 amount by $\tilde{E}_{i1}$ rather than E_{i1}.

for each state $\omega \in \Omega$, where the first line represents the cash flows at time 0 and the second line those at time 1 for state ω. Note from the second line in (1) that each emitter trades contingent claims to hedge not only the risk $\tilde{E}_{i1}$ of emission abatement, but also the risk $\tilde{R}_{i1}$ of exogenous income.[4]

On the other hand, financial trader $j \in \mathcal{M}_S$ has no incentive to enter the spot market and trades only in the contingent claims market. Thus, her final wealth is expressed as

$$W_j(\omega) = (1+r)[R_{j0} - \pi(H_j)] + [R_{j1}(\omega) + H_j(\omega)], \qquad j \in \mathcal{M}_S. \tag{2}$$

The term $R_{j1}(\omega) + H_j(\omega)$ clearly illustrates the motivation of financial trader j to hedge the risk in her exogenous income.

Suppose that both regulated emitters and financial traders are risk-averse and their preferences are represented by negative exponential utility functions with absolute risk-aversion coefficient γ_k for agent k. Hence, agent k maximizes the expected utility

$$\mathbb{E}\left[-\mathrm{e}^{-\gamma_k \tilde{W}_k}\right], \tag{3}$$

where $\tilde{W}_k$ is given by (1) for emitter i and by (2) for financial trader j. Note that the control variables for emitter i are not only $H_i(\omega)$ but also $\{X_{it}\}_{t=1,2}$, while financial trader j maximizes (3) with respect to $H_j(\omega)$ only.

In order to guarantee the existence and uniqueness of equilibrium, the following assumption is imposed.

Assumption 1 *For each i and t, the cost function $c_{it}(\cdot)$ is increasing, continuously differentiable, strictly convex with $c'_{it}(0) = 0$ and $c'_{it}(\infty) = \infty$.*

3 The Pricing of Contingent Claims

This section derives a formula to price any contingent claim traded in the market within an equilibrium framework.

To this end, we first consider an economy where banking and borrowing of permits are not allowed (the definitions of banking and borrowing will be given later). By using an arbitrage argument, we then examine how the introduction of banking and borrowing influences the pricing formula.

[4] We assume that the exogenous income R_{kt} reflects the profit optimization of agent k. That is, R_{kt} describes the profit after agent k has already taken optimal activities other than X_{it}.

3.1 A general equilibrium pricing formula

Suppose that banking and borrowing of permits are not allowed. Then, the time-0 and time-1 spot markets are completely separated, since different permits are traded and prices in each market are independently determined.

The market clearing conditions of the two spot trades are described as

$$\sum_{i \in \mathcal{M}_E} (E_{i0} - X_{i0}) = 0$$

and

$$\sum_{i \in \mathcal{M}_E} (E_{i1}(\omega) - X_{i1}(\omega)) = 0 \qquad \text{for almost all } \omega, \tag{4}$$

respectively. Also, since contingent claims are financial securities of zero supply, the market clearing condition of ω-contingent claims is written as

$$\sum_{k \in \mathcal{M}} H_k(\omega) = 0 \qquad \text{for almost all } \omega.$$

Recall that, for the pricing of contingent claims, it suffices to determine the state price density in the contingent claims market. To this end, from (1), the optimal reduction level at time t of emitter i satisfies

$$c'_{it}(X_{it}) - S_t = 0. \tag{5}$$

Since $c'_{it}(\cdot)$ is strictly increasing by Assumption 1, denoting its inverse function by $x_{it}(\cdot)$, we have the expression $X_{it} = x_{it}(S_t)$ for the optimal reduction level of emitter i. Plugging this expression into (4), we obtain

$$E_0 = \sum_{i \in \mathcal{M}_E} x_{i0}(S_0)$$

and

$$E_1(\omega) = \sum_{i \in \mathcal{M}_E} x_{i1}(S_t(\omega)),$$

respectively, where E_t denotes the time-t aggregated emission abatement target in the economy. Note that, under Assumption 1, the variables E_t and S_t are of one-to-one correspondence. It follows that there exists some function $s_t(\cdot)$ for which $S_t = s_t(E_t)$ in equilibrium for each time t. It is worth mentioning that the equilibrium spot price S_t

depends only on the aggregated abatement target E_t, and is independent of individual emission amounts.

Substituting the spot prices $S_t = s_t(E_t)$ into (1), we obtain

$$\begin{aligned}&(1+r)[R_{i0} - (E_{i0} - x_{i0}(s_0(E_0)))s_0(E_0) - c_{i0}(x_{i0}(s_0(E_0))) - \pi(H_i)] \\ &+ R_{i1}(\omega) - (E_{i1}(\omega) - x_{i1}(s_1(E_1(\omega))))s_1(E_1(\omega)) - c_{i1}(x_{i1}(s_1(E_1(\omega)))) + H_i(\omega).\end{aligned} \tag{6}$$

Let $c_t(\cdot)$ be the aggregated cost function to abate emission at time t, i.e.,

$$c_t(E_t) := \sum_{i \in \mathcal{M}_E} c_{it}(x_{it}(s_t(E_t))), \tag{7}$$

and define

$$R_t := \sum_{k \in \mathcal{M}} R_{kt}, \qquad \gamma := \left(\sum_{k \in \mathcal{M}} \frac{1}{\gamma_k} \right)^{-1}.$$

While R_t is the aggregated exogenous income of all market participants, γ represents the absolute risk-aversion index of the representative agent in the permit market.

We then have the following.

Proposition 1 *The state price density ϕ is given by*

$$\phi(\omega) = \frac{1}{1+r} \frac{\mathrm{e}^{\gamma\{c_1(E_1(\omega)) - R_1(\omega)\}}}{\mathbb{E}\left[\mathrm{e}^{\gamma\{c_1(\tilde{E}_1) - \tilde{R}_1\}}\right]}, \qquad \omega \in \Omega. \tag{8}$$

Hence, the contingent claim that pays $\tilde{C}$ at time 1 is evaluated as

$$\pi(C) = \frac{\mathbb{E}\left[\tilde{C}\mathrm{e}^{\gamma\{c_1(\tilde{E}_1) - \tilde{R}_1\}}\right]}{(1+r)\mathbb{E}\left[\mathrm{e}^{\gamma\{c_1(\tilde{E}_1) - \tilde{R}_1\}}\right]}. \tag{9}$$

Proof In equilibrium, the final wealth of emitter i at the compliance time $t = 1$ is given by (6) and that of financial trader j by (2). Then, applying the discussions given in Section 5 of Bühlmann (1980), we can verify that the state price density ϕ satisfies

$$\frac{1}{\gamma} \log \phi(\omega) = \{c_1(E_1(\omega)) - R_1(\omega)\} + K$$

for some constant K. Here, we have used the market clearing condition (4) for the equality. By the definition of state price density, we have $\mathbb{E}[\tilde{\phi}] = (1+r)^{-1}$. Therefore,

$$\mathrm{e}^{\gamma K}\mathbb{E}\left[\mathrm{e}^{\gamma\{c_1(\tilde{E}_1) - \tilde{R}_1\}}\right] = \frac{1}{1+r},$$

and the result follows at once. This completes the proof. □

Let $\tilde{Z}_1 := c_1(\tilde{E}_1) - \tilde{R}_1$. We can regard $\tilde{Z}_1$ as the aggregated risk associated with the permit market participants (both emitters and financial traders) at time 1. Note from Proposition 1 that the state price density is determined by $\tilde{Z}_1$ and γ. The risk-aversion parameter γ represents how the aggregated risk $\tilde{Z}_1$ affects the pricing formula. For example, if one of the market participants is risk-neutral, i.e. $\gamma_k = 0$ for some k, then γ becomes zero and the pricing formula of any contingent claim is equivalent to the so-called DCF method with no risk premium. On the other hand, when γ_k are all strictly positive, the total risk $\tilde{Z}_1$ in the market affects the price of each state.

The aggregated risk $\tilde{Z}_1$ consists of $c_1(\tilde{E}_1)$ and $\tilde{R}_1$, the former being the risk of social reduction cost and the latter being the risk of aggregated exogenous income. If the social cost to reduce emission is large, the market evaluates the prices of such states high, because emitters trade contingent claims to hedge the risk of abatement costs. This observation is consistent with the results in the standard finance theory.

Recall that contingent claims are traded not only to hedge the risk of emission uncertainty, but also to hedge the risk of income uncertainty. However, if $\tilde{R}_1$ is independent of $\tilde{E}_1$ and $\tilde{C}$ in the pricing formula (9), we obtain

$$\pi(C) = \frac{\mathbb{E}\left[\tilde{C}\mathrm{e}^{\gamma c_1(\tilde{E}_1)}\right]\mathbb{E}\left[\mathrm{e}^{-\gamma\tilde{R}_1}\right]}{(1+r)\mathbb{E}\left[\mathrm{e}^{\gamma c_1(\tilde{E}_1)}\right]\mathbb{E}\left[\mathrm{e}^{-\gamma\tilde{R}_1}\right]} = \frac{\mathbb{E}\left[\tilde{C}\mathrm{e}^{\gamma c_1(\tilde{E}_1)}\right]}{(1+r)\mathbb{E}\left[\mathrm{e}^{\gamma c_1(\tilde{E}_1)}\right]}.$$

Hence, in this case, the aggregated income risk $\tilde{R}_1$ has no impact on the prices in the permit market.[5]

With the pricing formula (9), we can price any contingent claim in the permit market. For later use, the next corollary provides the pricing formula of forward contracts.[6]

Corollary 1 *Denote by F the forward price of future permits at time 0. Then, we have*

$$F = \frac{\mathbb{E}\left[c_1'(\tilde{E}_1)\mathrm{e}^{\gamma\{c_1(\tilde{E}_1)-\tilde{R}_1\}}\right]}{\mathbb{E}\left[\mathrm{e}^{\gamma\{c_1(\tilde{E}_1)-\tilde{R}_1\}}\right]}. \tag{10}$$

Proof It is well known (see, e.g., Example 13.6 in Kijima (2002)) that F satisfies

$$\mathbb{E}\left[\tilde{\phi}(\tilde{S}_1 - F)\right] = 0.$$

[5]This pricing system is called the Esscher principle because of its formal connection with the Esscher transform (see, for example, Bühlmann (1980) or Kijima (2006)).

[6]Because the risk-free interest rate is constant in our setting, the price of a forward contract is the same as the price of the corresponding futures contract. See, e.g., Hull (2005).

Hence, it is sufficient to show that $S_t = c_t'(E_t)$. Differentiating the both sides of (7) with respect to E_t, we have

$$\begin{aligned} c_t'(E_t) =& \sum_{i \in \mathcal{M}_E} c_{it}'(x_{it}(s_t(E_t)))x_{it}'(s_t(E_t))s_t'(E_t) \\ =& s_t(E_t)\frac{\mathrm{d}s_t}{\mathrm{d}E_t}\frac{\mathrm{d}}{\mathrm{d}s_t}\sum_{i \in \mathcal{M}_E} x_{it}(s_t(E_t)), \end{aligned}$$

where we have used the relation $c_{it}'(x_{it}(s_t(E_t))) = s_t(E_t)$ for all $i \in \mathcal{M}_E$. The result follows at once from the fact that $E_t = \sum_{i \in \mathcal{M}_E} x_{it}(s_t(E_t))$ in equilibrium. □

In a similar manner, we can calculate the price of a call option with strike price K as

$$\frac{1}{1+r}\frac{\mathbb{E}\left[\{c_1'(\tilde{E}_1) - K\}_+ \mathrm{e}^{\gamma\{c_1(\tilde{E}_1) - \tilde{R}_1\}}\right]}{\mathbb{E}\left[\mathrm{e}^{\gamma\{c_1(\tilde{E}_1) - \tilde{R}_1\}}\right]}, \tag{11}$$

where $\{x\}_+ = \max\{x, 0\}$.

3.2 Arbitrage with banking and borrowing permits

We next introduce banking and borrowing in our model, and examine their effects on the pricing of contingent claims. Note that the existing literature such as Cronshaw and Kruse (1996), Rubin (1996) and Schennach (2000) studies welfare effects of the banking and borrowing. To our best knowledge, this paper is the first to analyze the impact on the prices of contingent claims when banking and borrowing are introduced to the permit markets in a general equilibrium framework.[7]

Banking in permit markets means that unused permits in one period can be saved and used in later periods, whereas borrowing in the current period requires the reduction of the same amount of permits in the future periods. When the banking and borrowing are allowed, the current and future permits are regarded as the same good (perfect substitute). Therefore, the current and future spot markets are connected and can be seen as an integrated market. Consequently, the prices of time-1 contingent claims have some intertemporal relationship with the spot price at time 0.

Denote the banking at time 0 by B_{i0}. When B_{i0} is positive, emitter i additionally abates and banks her permit to time 1, while negative B_{i0} means that emitter i borrows her emission from time 1. Hence, the final wealth of emitter i changes from (1) to

$$\begin{aligned} W_i(\omega) =& (1+r)[R_{i0} - (E_{i0} - X_{i0} + B_{i0})S_0 - c_{i0}(X_{i0}) - \pi(H_i)] \\ &+ R_{i1}(\omega) - (E_{i1}(\omega) - X_{i1}(\omega) - B_{i0})S_1(\omega) - c_{i1}(X_{i1}(\omega)) + H_i(\omega). \end{aligned} \tag{12}$$

[7] See Maeda (2004) for related works on the banking.

Let the aggregated amount of net banking by B_0, i.e.,

$$B_0 := \sum_{i \in \mathcal{M}_E} B_{i0}.$$

Then, the market clearing conditions of emission permits change from (4) to

$$\sum_{i \in \mathcal{M}_E} (E_{i0} + B_0 - X_{i0}) = 0$$

and

$$\sum_{i \in \mathcal{M}_E} (E_1(\omega) - B_0 - X_{i1}(\omega)) = 0,$$

respectively. Note that these two conditions are equivalently expressed as

$$\sum_{t=0}^{1} \sum_{i \in \mathcal{M}_E} (E_{it} - X_{it}) = 0.$$

Hence, under the system of banking, the abatement target must be equal to the social self-reduction over the whole periods.

As in Section 3, we obtain the spot prices in the permit market with banking and borrowing as

$$S_0 = s_0(E_0 + B_0) = x_0^{-1}(E_0 + B_0)$$

and

$$S_1(\omega) = s_1(E_1(\omega) - B_0) = x_1^{-1}(E_1(\omega) - B_0).$$

The aggregated risk of the market participants is given by

$$Z_1(\omega) = c_1(E_1(\omega) - B_0) + R_1(\omega).$$

Thus, the state price density of this market is obtained as

$$\phi_{\mathrm{WB}}(\omega) = \frac{1}{1+r} \frac{\mathrm{e}^{\gamma\{c_1(E_1(\omega)-B_0)+R_1(\omega)\}}}{\mathbb{E}\left[\mathrm{e}^{\gamma\{c_1(\tilde{E}_1-B_0)+\tilde{R}_1\}}\right]}.$$

Of course, when $B_0 = 0$, the state price density $\phi_{\mathrm{WB}}(\omega)$ agrees with the one given in Proposition 1. However, when banking and borrowing are allowed in the market, the time-1 price system is connected with the time-0 price through the aggregated net banking amount B_0. Note that the aggregated amount B_0 is not yet determined. Below, we shall obtain B_0 in equilibrium using the no-arbitrage argument.

To this end, we denote by F_{WB} the forward price of future permits at time 0 when the system of banking and borrowing is introduced. Following the arguments in Section 4 of Maeda (2004), we have $F_{\mathrm{WB}} = (1+r)S_0$ from the no-arbitrage condition. To see this, consider the contingent claim $H_i(\omega) = -S_1(\omega)B_{i0}$. Then, from (12), the final wealth becomes

$$\begin{aligned} W_i(\omega) =&(1+r)\left[R_{i0} - (E_{i0} - X_{i0})S_0 - c_{i0}(X_{i0}) + B_{i0}\left\{\frac{F_{\mathrm{WB}}}{1+r} - S_0\right\}\right] \\ &+ R_{i1}(\omega) - (E_{i1}(\omega) - X_{i1}(\omega))S_1(\omega) - c_{i1}(X_{i1}(\omega)), \end{aligned}$$

where we have used the identity $F_{\mathrm{WB}} = (1+r)\mathbb{E}[\tilde{S}_1\tilde{\phi}_{\mathrm{WB}}]$. As a result, if $F_{\mathrm{WB}} > (1+r)S_0$, emitter i can make an infinitely large profit with no risk from banking by setting $B_{i0} \to \infty$. If $F_{\mathrm{WB}} < (1+r)S_0$, on the other hand, borrowing permits allows an arbitrage opportunity for each emitter.

The no-arbitrage condition is expressed as $S_0 = \mathbb{E}[\tilde{S}_1\tilde{\phi}_{\mathrm{WB}}]$. It follows from the fact $s_t(E) = c_t'(E)$ that

$$(1+r)c_0'(E_0 + B_0) = \frac{\mathbb{E}\left[c_1'(\tilde{E}_1 - B_0)\mathrm{e}^{\gamma\{c_1(\tilde{E}_1 - B_0) + \tilde{R}_1\}}\right]}{\mathbb{E}\left[\mathrm{e}^{\gamma\{c_1(\tilde{E}_1 - B_0) + \tilde{R}_1\}}\right]}. \tag{13}$$

The aggregated amount B_0 of net banking in equilibrium is determined by (13). Note that the solution in B_0 always exists and is unique under Assumption 1.

4 The Case of Normally Distributed Risks

As mentioned in the introductory section, permit markets are rapidly growing. The number of market participants is quite large, so that the social emission abatement E_t is given by the sum of a large number of relatively small E_{it}'s. The same argument can apply to the aggregated income R_t. Therefore, due to the central limit theorem, it is expected that $(\tilde{E}_t, \tilde{R}_t)$ can be approximated by a bivariate normal distribution. In this section, we show that the pricing formula becomes significantly simpler when the risks of emission abatement and exogenous income are normally distributed.

Throughout this section, we assume that banking and borrowing are not allowed. The case of banking and borrowing can be treated in a similar manner.

4.1 The pricing formula

Suppose that $(\tilde{E}_1, \tilde{R}_1)$ follows a bivariate normal distribution with

$$\begin{pmatrix} \tilde{E}_1 & \tilde{R}_1 \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mu_E & \mu_R \end{pmatrix}, \begin{pmatrix} \sigma_E^2 & \rho\sigma_E\sigma_R \\ \rho\sigma_E\sigma_R & \sigma_R^2 \end{pmatrix}\right]. \tag{14}$$

Under the normal setting, the pricing formula (8) can be simplified significantly and is given by the Esscher transform as follows.

Proposition 2 *Consider a contingent claim that pays $g(S_1)$ at time 1, and let $h(\cdot) = g(c_1'(\cdot))$. Then, the price of the claim is given by*

$$\pi(g) = \frac{\mathbb{E}\left[h(\tilde{Z}^*)\mathrm{e}^{\gamma c_1(\tilde{Z}^*)}\right]}{(1+r)\mathbb{E}\left[\mathrm{e}^{\gamma c_1(\tilde{Z}^*)}\right]}, \tag{15}$$

where $\tilde{Z}^*$ is a normal random variable with mean $\mu_Z := \mu_E - \gamma\rho\sigma_E\sigma_R$ and variance σ_E^2.

Proof Let (X, Y) be any bivariate normal variables, and let $f(\cdot)$ be any function for which the following expectations exist. It is well known (see, e.g., Exercise 3.16 in Kijima (2002)) that

$$\mathbb{E}\left[f(X)\mathrm{e}^{-Y}\right] = \mathbb{E}\left[\mathrm{e}^{-Y}\right]\mathbb{E}[f(X - \mathbb{C}[X, Y])]. \tag{16}$$

The proposition follows from simply substituting $X = E_1$ and $Y = \gamma R_1$ into this equation. □

Proposition 2 illustrates how the exogenous income R_1 affects the pricing formula in equilibrium for the normal case. Consider, for example, the case that the correlation ρ between $\tilde{E}_1$ and $\tilde{R}_1$ is positive. Then, the increase in correlation has the same influence on the prices of contingent claims as the decrease in μ_E, the expected abatement level.

This result can be explained as follows. As noted earlier, contingent claims are traded in the market not only to hedge the risk of abatement uncertainty, but also to hedge the risk of her exogenous income. This means that, when the correlation ρ is positive, each emitter or financial trader buys (sells, respectively) a contingent claim of event ω at which her exogenous income will be low (high). Hence, in total, risks in the permit market are priced as if the expected level of the abatement target μ_E fell to μ_Z. A similar argument can be applied to the effect of σ_R to the pricing formula.

The pricing formula given in Proposition 2 provides the comparative statics results for the price of any contingent claims with respect to the exogenous parameters. The next corollary can be proved by simply differentiating (15). For the state price density $\tilde{\phi}$, the probability measure

$$\mathbb{P}^{\phi}(A) = (1+r)\mathbb{E}[\tilde{\phi}1_A], \qquad A \in \mathcal{F}$$

is called the *risk-neutral measure*, where 1_A denotes the indicator function, meaning that $1_A = 1$ if $\omega \in A$ and $1_A = 0$ otherwise. We denote by $\mathbb{E}_{\phi}$ and $\mathbb{C}_{\phi}$ the expectation and covariance operators under the risk-neutral measure, respectively.

Corollary 2 *Let*

$$\Delta_1 := \mathbb{E}_{\phi}[h'(\tilde{Z})] + \gamma\mathbb{C}_{\phi}[h(\tilde{Z}), c_1'(\tilde{Z})] \tag{17}$$

and

$$\Delta_2 := \mathbb{E}_{\phi}\left[h'(\tilde{Z})\frac{\tilde{Z}-\mu_Z}{\sigma_E}\right] + \gamma\mathbb{C}_{\phi}\left[h(\tilde{Z}), c_1'(\tilde{Z})\frac{\tilde{Z}-\mu_Z}{\sigma_E}\right]. \tag{18}$$

Then, we have

$$\frac{\partial\pi(g)}{\partial\mu_E} = \frac{1}{1+r}\Delta_1, \quad \frac{\partial\pi(g)}{\partial\rho} = -\frac{1}{1+r}\gamma\sigma_E\sigma_R\Delta_1, \quad \frac{\partial\pi(g)}{\partial\sigma_R} = -\frac{1}{1+r}\gamma\rho\sigma_E\Delta_1$$

and

$$\frac{\partial\pi(g)}{\partial\sigma_E} = -\frac{1}{1+r}\left(\gamma\rho\sigma_R\Delta_1 + \frac{1}{1+r}\Delta_2\right).$$

Suppose that the expected abatement level μ_E increases marginally. From the expression of Δ_1, we find that the increase in μ_E has two effects. The first term of Δ_1 represents how the expected payoff changes under the risk-neutral measure, whereas the second term describes the effect of the change in the state price density, since the change in μ_E induces the change of the risk-neutral measure. This effect can be written as the correlation between the payoff and the spot price at time 1 (recall that the spot price is the first-order derivative of the aggregated cost function). For example, when the price of a call option is considered, both terms in Δ_1 become positive. Therefore, the increase in μ_E not only raises the price of a call option directly, but also raises the price by the change of the state price density. Similar observations apply for the parameters ρ and σ_R.

On the other hand, the impact of σ_E can be divided into four terms. Equation (18) describes the effect through the variable $\tilde{E}_1$, while (17) shows the effect from the correlation term $\gamma\rho\sigma_E\sigma_R$. The first terms of Δ_1 and Δ_2 represent the marginal change in the expected payoff under the risk-neutral measure, while the second terms are caused by the change in the state price density.

In the standard theory of financial contingent claims such as the Black–Scholes formula, only the direct effect is considered. However, when prices of contingent claims in a permit market are analyzed, the indirect effect should also be considered, because the state price density depends much on the exogenous parameters that characterize the permit market.

4.2 The case of quadratic cost function

In this subsection, we derive and analyze the forward price when the cost function is quadratic.

Suppose that the cost function of each emitter at time t is given by

$$c_{it}(x) = \begin{cases} \frac{\hat{c}_{it}}{2}x^2, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

where $\hat{c}_{it}$ is a positive constant. The parameter $\hat{c}_{it}$ characterizes the marginal cost function. Namely, a lower value of $\hat{c}_{it}$ means that emitter i can abate her emission with a lower cost.

Under this setting, the optimal emission of emitter i at time 1 is given by $X_{it}(\omega) = \frac{S_t(\omega)}{\hat{c}_{it}}$ as far as S_t is positive. From market clearing condition (4), we then obtain

$$s_t(E_t) = \hat{c}_t \tilde{E}_t \quad \text{and} \quad c_t(E_t) = \frac{1}{2}\hat{c}_t \tilde{E}_t^2 \quad \text{for } \tilde{E}_t \geq 0,$$

where

$$\hat{c}_t := \left(\sum_{i \in \mathcal{M}_E} \frac{1}{\hat{c}_{it}} \right)^{-1}.$$

When E_t is negative, on the other hand, the spot price should be zero in equilibrium. To see this, suppose that the total abatement target is negative, while some emitter must reduce a positive amount of emission. Namely, $E_t \leq 0$ while $E_{it} > 0$ for some $i \in \mathcal{M}_E$. An emitter with negative abatement target is willing to sell her permits, as long as the spot price is strictly positive, because selling her permits always increases her utility. As a result, since the total amount of sell orders surpasses that of buy orders, the spot price should be zero in the competitive market.

From Proposition 1, the state price density in this market is given by

$$\phi(\omega) = \frac{1}{1+r} \frac{\mathrm{e}^{\gamma\left\{\frac{1}{2}\hat{c}_1 E_1^2(\omega) 1_{\{E_1(\omega) \geq 0\}} - R_1(\omega)\right\}}}{\mathbb{E}\left[\mathrm{e}^{\gamma\left\{\frac{1}{2}\hat{c}_1 \tilde{E}_1^2 1_{\{\tilde{E}_1 \geq 0\}} - \tilde{R}_1\right\}}\right]}. \tag{19}$$

Note that the indicator function $1_{\{E_1(\omega)\geq 0\}}$ explicitly appears in the formula. Any contingent claim in this market can be priced by the state price density (19).

The next proposition provides the forward price in this market when the risks are normally distributed.

Proposition 3 *Let $\mu_Z := \mu_E - \gamma\rho\sigma_E\sigma_R$ and $\alpha := \gamma\hat{c}_1\sigma_E^2$, and suppose that the cost function to reduce emission is quadratic. Then, the forward price F is given by*

$$F = \frac{\hat{c}_1\left(\frac{\mu_Z}{1-\alpha}N(d_0) + \frac{\sigma_E}{\sqrt{1-\alpha}}n(d_0)\right)}{N(d_0) + \sqrt{1-\alpha}\exp\left\{-\frac{\alpha\mu_Z^2}{2\sqrt{1-\alpha}}\right\}N\left(-\frac{\mu_Z}{\sigma_E}\right)}, \tag{20}$$

where $n(\cdot)$ and $N(\cdot)$ denote the density and distribution functions of the standard normal, respectively, and $d_0 := \frac{\mu_Z}{\sigma_E\sqrt{1-\alpha}}$.

Proof The formula (20) is fairly complicated and not easy to analyze. The reason for the complicated form is due to the indicator function $1_{\{E_1(\omega)\geq 0\}}$ in (19). However, when μ_E is large enough, which is to be expected in practice, the effect from the indicator function will disappear. In fact, for large μ_E, we have $N(d_0) \approx 1$ and the other terms are approximately zero. It follows from (20) that

$$F = \frac{\hat{c}_1\mu_Z}{1-\alpha} = \frac{\hat{c}_1\left(\mu_E - \gamma\rho\sigma_E\sigma_R\right)}{1-\gamma\hat{c}_1\sigma_E^2} \tag{21}$$

for large μ_E. In what follows, we use the simplified formula (21) for the forward price when the risks are normally distributed and the cost functions are quadratic.[8]

Before proceeding, we check the accuracy of the formula (21) by some numerical example. Figure 1 compares the forward prices calculated from (20) and (21). It is observed that the simplified formula (21) provides a very good approximation to the exact formula for $\mu_E \geq 0.3$.

[Figure 1 is inserted here.]

Our important findings from the forward price (21) are the following. Note that the expected value of S_1 under the physical measure $\mathbb{P}$ is given by $\hat{c}_1\mu_E$. The term $\alpha = \gamma\hat{c}_1\sigma_E^2$ in the denominator characterizes the risk premium of the forward price. Hence, the more

[8] The formula (Equation (7)) obtained in Maeda (2004) is slightly different from (21), since Maeda (2004) assumed that regulated emitters do not take into account their exogenous incomes in their decision frame, that is, $R_{it} = 0, i \in \mathcal{M}_E$ in our model. This is a matter of definition of agent types. Adjusting this difference turns out that these formulas are basically the same.

risk-averse traders in the market, or the more the variance of the future emission amount, the higher the forward price. On the other hand, the term $\gamma\rho\sigma_E\sigma_R$ reflects the effect of the correlation between the emission and the aggregated exogenous income. If the correlation is positive, it decreases the forward price, because a short position of the forward contract is used in order to hedge the risk of exogenous income.

Finally, by using a similar procedure to the derivation of (20), we can calculate the price of options written on the spot price at time 1. The call option price with strike $K > 0$ is given by

$$\begin{aligned}&\frac{\mathbb{E}\left[\left\{\hat{c}_1\tilde{E}_1 - K\right\}_+ \mathrm{e}^{\gamma\left\{\frac{1}{2}\hat{c}_1\tilde{E}_1^2 1_{\{\tilde{E}_1\geq 0\}} - \tilde{R}_1\right\}}\right]}{(1+r)\mathbb{E}\left[\mathrm{e}^{\gamma\left\{\frac{1}{2}\hat{c}_1\tilde{E}_1^2 1_{\{\tilde{E}_1\geq 0\}} - \tilde{R}_1\right\}}\right]}\\ =&\frac{1}{1+r}\frac{\left(\frac{\hat{c}_1\mu_Z}{(1-\alpha)} - K\right)N(d_K) + \frac{\hat{c}_1\sigma_X}{\sqrt{1-\alpha}}n(d_K)}{N(d_0) + \sqrt{1-\alpha}\exp\left\{-\frac{\alpha\mu_Z^2}{2\sqrt{1-\alpha}}\right\}N\left(-\frac{\mu_Z}{\sigma_E}\right)},\end{aligned} \tag{22}$$

where

$$d_K = \frac{\mu_Z - \frac{(1-\alpha)K}{\hat{c}_1}}{\sigma_E\sqrt{1-\alpha}}.$$

Note the resemblance of formulas between (22) and (20). From (10) and (11), the forward price can be considered as the (non-discounted) call option price with zero strike price.

5 Implications on Forward Contracts

Since permit markets are purely artificial, the design and implementation of such markets are the most important issues. In particular, for regulated emitters, contingent claims that are monotonically increasing in S_1 are useful tools to hedge the risk of emission abatement costs. In this section, we discuss about some topics that are specific in permit markets by examining the price of forward contracts.

5.1 Price spikes

We first provide a numerical example to show that the so-called *price spike* often observed in energy markets may happen even in permit markets.

Assume that the cost function to reduce pollutant emission by emitter i is given by

$$c_{it}(x) = \begin{cases} 0, & x \le 0, \\ \frac{\hat{c}_{iL}}{2}x^2, & 0 < x \le \bar{X}_B, \\ \frac{\hat{c}_{iH}}{2}x^2 - (\hat{c}_{iH} - \hat{c}_{iL})\bar{X}_B x + \frac{\hat{c}_{iH}-\hat{c}_{iL}}{2}\bar{X}_B^2, & x > \bar{X}_B, \end{cases} \tag{23}$$

where $\hat{c}_{iL}$, $\hat{c}_{iH}$ and $\bar{X}_B$ are positive constants with $\hat{c}_{iL} < \hat{c}_{iH}$. For simplicity, we assume that the *kink point* $\bar{X}_B$ is common among all emitters.

Note that the cost function (23) is quadratic in the whole region, while the marginal cost kinks at $\bar{X}_B$. The ratio $\hat{c}_{iH}/\hat{c}_{iL}$ represents the magnitude of the kink at that point. If $\hat{c}_{iL} = \hat{c}_{iH}$, the cost function is smooth in the whole region. Of course, the cost function satisfies Assumption 1.

Let $\hat{c}_L$ and $\hat{c}_H$ be $\left(\sum_{i\in\mathcal{M}_E} 1/\hat{c}_{iL}\right)^{-1}$ and $\left(\sum_{i\in\mathcal{M}_E} 1/\hat{c}_{iH}\right)^{-1}$, respectively, and define $\kappa := \hat{c}_H/\hat{c}_L$. Then, the social cost function is obtained as

$$c_1(E_1(\omega)) = \hat{c}_L \times \begin{cases} 0, & E_1(\omega) \le 0, \\ \dfrac{1}{2}E_1(\omega)^2, & 0 < E_1(\omega) \le \bar{X}_B, \\ \dfrac{\kappa}{2}E_1(\omega)^2 - (\kappa-1)\bar{X}_B E_1(\omega) + \dfrac{\kappa-1}{2}\bar{X}_B^2 & E_1(\omega) > \bar{X}_B, \end{cases}$$

and the spot price at time 1 in equilibrium is given by

$$S_1(\omega) = \hat{c}_L \times \begin{cases} 0, & E_1(\omega) \le 0, \\ E_1(\omega), & 0 < E_1(\omega) \le \bar{X}_B, \\ \kappa E_1(\omega) - (\kappa-1)\bar{X}_B, & E_1(\omega) > \bar{X}_B. \end{cases}$$

The forward price for this case is expressed in terms of the standard normal density and its cumulative distribution function,that can be calculated by standard numerical methods with ease. Figure 2 illustrates the relationship between the forward price and the expected emission abatement level. Here, we set the parameters as $\gamma = 1.0$, $\sigma_E = 0.3$, $\rho = 0$, $\hat{c}_L = 1.0$, and $\bar{X}_B = 3.0$.

[Figure 2 is inserted here.]

It is explicitly observed that, when $\kappa = \hat{c}_H/\hat{c}_L > 1$, the forward price kinks around $\mu_E = 2.2$ and the kink level of the forward price is much lower than the kink level $\bar{X}_B = 3.0$ of the spot price. Also, as κ increases, the magnitude of kinks increases drastically.

The price kink also happens with respect to σ_E, the parameter that represents the uncertainty of the future emission amount. Figure 3 shows how the forward price changes in σ_E. Here, we set $\mu_E = 2$ and the other parameters to be the same as in Figure 2.

[Figure 3 is inserted here.]

It is well known that, in energy markets, the so-called price spike often appears, meaning that the price dramatically rises due to the shortage of capacity to produce energy. In our model, when the cost function is subject to a kink, such a price spike can happen also in permit markets. For example, suppose that market participants slightly update the probability distribution of the future emission and forecast either the mean level μ_E or the uncertainty σ_E to become higher than before. Then, the forward price can rise dramatically. That is, a small change in the forecast about the future emission may lead to a big price change in forward contracts. Note from the above observation that the price spike for the forward contract is more likely to occur than the spot price.

Next, we consider the case that the system of banking and borrowing is introduced to the above model. Figure 4 depicts the relationship between the mean emission level μ_E and the forward prices with and without banking. Here, we set $\kappa = 3.0$ and the level of the current social abatement target E_0 to be 2.11, at which $B_0 = 0$ in equilibrium for $\mu_E = 2.0$. As a comparison, we also depict the future spot price in the case of no banking.

[Figure 4 is inserted here.]

It is observed that the forward price with banking is less sensitive to the expected abatement target, meaning that the banking and borrowing eliminates most of the spike effect on the forward price. This is so, because the change in the forward price is partly absorbed by the change in the spot price via the aggregated banking or borrowing. Note from Figure 4 that, although the kink in the cost function has an impact on the forward price with banking, the kink only appears around $\mu_E = 3.7$, which is not only much higher than that without banking, but also higher than X_0, the level at which the future spot price kinks.

These results have an important implication about the design of tradable permit markets. That is, in energy markets, the price spike is inevitable, because energy production has an apparent capacity limitation and current energies are usually nonexchangeable with future energies. In permit markets, on the other hand, current permits can be stored and substituted for future permits through banking and borrowing. When effectively implemented by regulated emitters, the system of banking and borrowing mitigates the instability of both spot and forward prices, while not changing the total emission abatement amount over the periods.

5.2 Contango vs. normal backwardation

In commodity markets, the relationship between spot and forward prices has been extensively studied. When the forward price is greater than the expected future spot price, it is called a contango market, while we say that the market shows a normal backwardation when the forward price is smaller than the expected future spot price.[9]

In usual commodity markets, it is often said that market contango and backwardation are attributed to properties of the traded commodity itself and economic environments surrounding the commodity. For example, contango in gold markets is often referred to the cost of carry, while backwardation often appears in oil markets because of the convenience yield. Therefore, it is practically and economically important to find what kind of factors are influential in creating contango or backwardation for the understanding of commodity markets. Our forward price formula (21) provides an interesting implication in this context.

Suppose first that the social emission amount $\tilde{E}_1$ and the aggregated exogenous income $\tilde{R}_1$ are mutually independent. Then, from the discussions in Esary *et al.* (1967), the forward price F (without banking) is always higher than $\mathbb{E}[S_1]$, irrespective of the distribution of $\tilde{E}_1$. This means that the permit market will show contango when dependence between $\tilde{E}_1$ and $\tilde{R}_1$ is not so strong. The difference between F and $\mathbb{E}[S_1]$ is determined by the risk premium $\alpha := \gamma \hat{c}_1 \sigma_E^2$.

Next, let us consider a more general case that $\tilde{E}_1$ and $\tilde{R}_1$ are positively correlated. From (21), it is readily verified that the market is contango (backwardation, respectively) if

$$\frac{\rho \sigma_R}{\sigma_E} < (>) \hat{c}_1 \mu_E \equiv \mathbb{E}[S_1].$$

Let $\tilde{R}_{E1} := \sum_{i \in \mathcal{M}_E} R_i$ and $\tilde{R}_{S1} := \sum_{j \in \mathcal{M}_S} R_j$, so that $\tilde{R}_1 = \tilde{R}_{E1} + \tilde{R}_{S1}$. Then, we easily see that

$$\frac{\rho \sigma_R}{\sigma_E} = \frac{\mathbb{C}[\tilde{E}_1, \tilde{R}_1]}{\sigma_E^2} = \frac{\mathbb{C}[\tilde{E}_1, \tilde{R}_{E1}] + \mathbb{C}[\tilde{E}_1, \tilde{R}_{S1}]}{\sigma_E^2}. \tag{24}$$

Note from the bilinearity of the covariance operator that $\mathbb{C}[\tilde{E}_1, \tilde{R}_1]$ increases as the number of market participants increases, provided $\mathbb{C}[\tilde{E}_1, \tilde{R}_k] > 0$ for $k \in \mathcal{M}$. Of course, this argument applies not only to regulated emitters but also unregulated financial traders.

[9] For definitions of these terms, see standard finance textbooks such as Hull (2005). These terms have been used in several ways to describe characteristics of various futures markets. For a comprehensive survey of the historic use of these terms, we refer to Duffie (1989).

Hence, in particular, the permit market will become in normal backwardation if there are many financial traders in the market. The property that the characteristic of market participants determines the price structure of the market is quite unique and distinct from other markets.

5.3 Risk sensitivity

In this subsection, we investigate the impact of σ_E, the risk parameter of the social emission in the future, on the forward price F. Note that, in many commodity markets, the forward price is monotonic with respect to the volatility of the spot asset price (see, for example, Schwartz (1997)). However, as we shall show below, the monotonicity result does not hold in our permit market.

Throughout this subsection, it is assumed that the correlation ρ is non-negative, which is natural in reality. Then, as σ_E increases, both the numerator and the denominator in (21) are monotonically decreasing. Note that, as $\sigma_E \nearrow \sqrt{\frac{1}{\gamma \hat{c}_1}}$, the denominator diverges while the numerator stays finite, whence the decreasing effect of the denominator dominates that of the numerator for σ_E large enough. In other words, as for the ordinary commodity markets, the forward price F is monotonically increasing in σ_E large enough. However, the behavior of F is not clear when σ_E is not large.

In fact, the forward price can be non-monotonic when $\tilde{E}_1$ and $\tilde{R}_1$ are positively correlated. Figure 5 depicts a graph of the forward price with respect to σ_E when $\rho = 0.5$.

[Figure 5 is inserted here.]

It is observed that the forward price is initially decreasing and then increasing in σ_E.

The result is consistent with comparative statics results given in Corollary 2. The corollary says that the marginal change in each parameter has two effects on the change in the price of any contingent claim. The first effect is caused by the change in expected payoff. Here, the increase in σ_E induces the decrease in μ_Z, meaning that the forward price becomes lower. More precisely, as σ_E increases, the covariance $\mathbb{C}[E_1, R_1]$ increases, so that market participants want to sell the asset to hedge the risk of income uncertainty. Therefore, when the correlation is strongly positive and σ_E is small, this first effect becomes significant.

The second effect appears in the change of risk premium $\alpha = \gamma \hat{c}_1 \sigma_E^2$ and is caused by the change in the state price density. As σ_E increases, uncertainty in the spot price of permits increases. Hence, since all market participants are risk-averse, they become more willing to hold the forward contract and the forward price becomes higher. Figure 5 indicates that the second effect dominates the first one when σ_E is large enough.

5.4 Sensitivity analysis of other parameters

Differentiating the simplified formula (21) of the forward price with respect to each parameter, we obtain the following results.

Corollary 3

1. *The forward price is always increasing in $\hat{c}_1$.*
2. *The forward price is always decreasing in ρ.*
3. *The forward price is decreasing in σ_R when ρ is positive.*
4. *The impact of γ is indeterminate.*

These results also echo Corollary 2. The marginal change in each parameter yields the marginal change in the expected payoff and that in the state price density. The effect of correlation corresponds to the marginal change in the expected payoff, since the correlation has a similar effect to μ_E. On the other hand, the effect of the risk-premium is regarded as the marginal change in the state price density, because the state price density determines the risk-premium.

The above analyses can be applied to the market design and other related problems. Suppose, for example, that the government wants to prohibit financial traders from participating in a permit market. Suppose that financial traders are less risk-averse than general business firms and their incomes are less correlated with the emission amount. Then, from Corollary 3, their impact on the forward price is not so simple, and the government must take both the risk-premium and the correlation effects into account.

6 Concluding Remarks

In this paper, we construct a permit market model in which all agents have CARA utility functions with distinct risk-aversion parameters. The state price density is derived for any probability distribution and any cost function to price any contingent claims in the permit market. To our best knowledge, this paper is the first to study the pricing of contingent claims in permit markets in a general equilibrium framework.

The pricing formula depends on the probability distribution of the future abatement target, the aggressiveness of market participants, the social cost function to reduce pollutant emission, and the correlation between future emission amount and the aggregated exogenous income of agents. Using numerical examples, we illustrate how these parameters change the price of forward contracts.

Through comparative statics studies, we find that the change of parameters has two effects on the price of contingent claims. The first one is the direct effect caused by the change in the payoff, while the second one is the indirect effect that arises from the change in the state price density. In the ordinary financial markets, the state price density is exogenously given, and the change of exogenous parameters has no impact on the state price density.

We also discuss some related topics that are specific to permit markets. One interesting finding is that if the social cost function kinks at some level of abatement, the forward price as well as the spot price may be subject to the so-called price spike, whence the prices in permit markets may have some instability that cannot be overlooked. However, this price-spike phenomenon can be weakened if the system of banking and borrowing is properly introduced.

Our model can also be used to study the effect of environmental taxes or subsidies on the prices of contingent claims. Suppose that a government imposes regulated emitters on an environmental tax $\tau_t(X)$ for the self-abatement effort X, where $\tau_t(X)$ is regarded as a subsidy when it is negative. Then, the cost function for X after tax is described as $c_{it}(X) + \tau_t(X)$, and the first-order condition corresponding to (5) is given by $c'_{it}(X_{it}) + \tau'_t(X_{it}) = S_t$. From the above discussions, when the government imposes the environmental tax in the form of lump sum payment, it does not affect the self-effort abatement X_{it}, meaning that the equilibrium prices of contingent claims are the same as those before the introduction of tax. Hence, if the government wants to design the tax so as to affect the prices properly, one possible way is to offer emitters some subsidy whose amount is dependent on the self-effort abatement. This sort of investigation, among others, is our future works.

As permit markets are globally growing and developing, it becomes more and more important to understand how prices of permit market products are formed. This study provides new insights as a first step for the problem.

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Figures

Figure 1: The figure presents the comparison between (20) and (21) with various μ_E. We set $\gamma = 1$, $\sigma_E = 0.3$, $\rho = 0$, and $\hat{c}_1 = 1.0$. It is observed that the simplified formula (21) can be regarded as a good approximation for the formal formula (20) under plausible parameter settings.

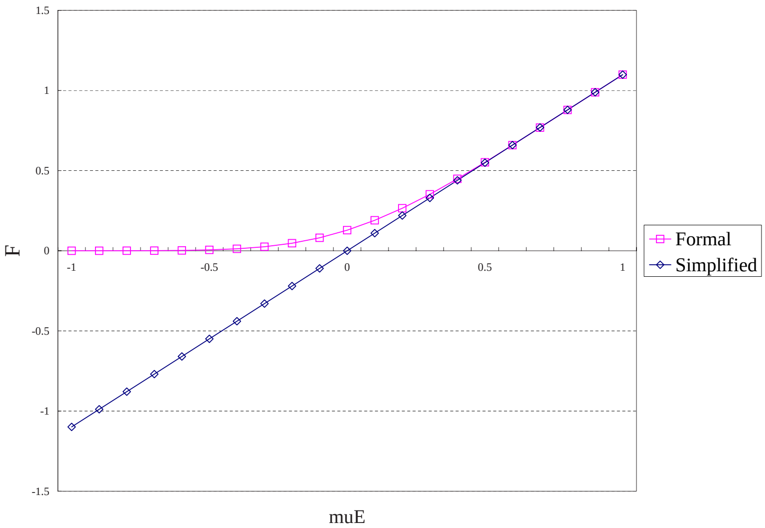

Figure 2: The relation between F and μ_E with different values of κ. The graph shows that when $\kappa > 1$, the forward price kinks at around $\mu_E = 2.2$. This figure effectively illustrates how the probability distribution of $\tilde{E}_1$ as well as the form of the cost function $c_1(\cdot)$ affects the pricing of contingent claims.

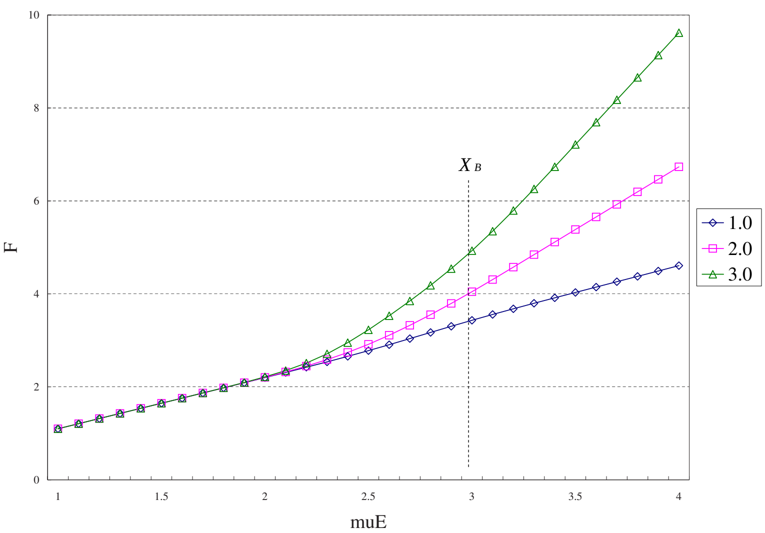

Figure 3: The relation between F and σ_E with different values of κ. We here set $\mu_E = 2.0$. As in Figure 2, the price difference is notable when $\sigma_E \geq 0.3$.

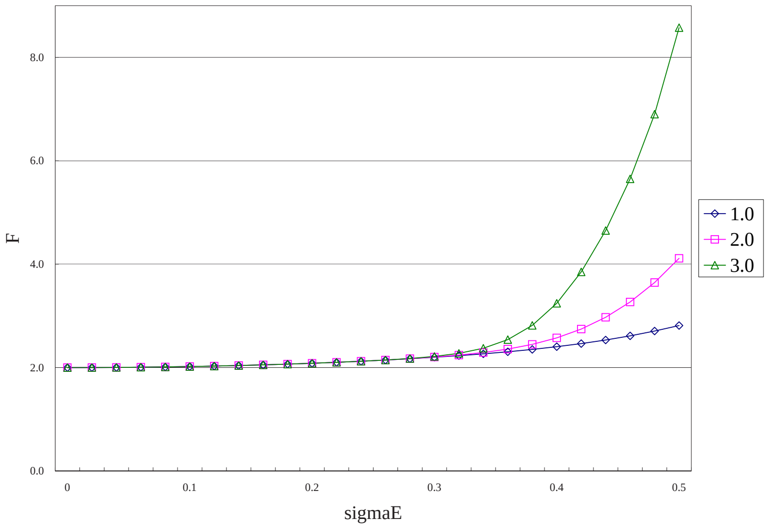

Figure 4: The relation between the expected emission level and the forward price when permit banking is introduced. We set $\hat{c}_H = 1.0$ and $r = 0.05$, and $E_0 = 2.11$, at which $B_0 = 0$ in equilibrium for $\mu_E = 2.0$. Other parameters are the same as in Figure 2. We observe from the figure that the system of banking and borrowing lowers the sensitivity of μ_E to the forward price. Also note that the effect of the kink appears at around $\mu_E = 3.5$, much higher than in the case of Figure 2 and $\bar{X}_B$.

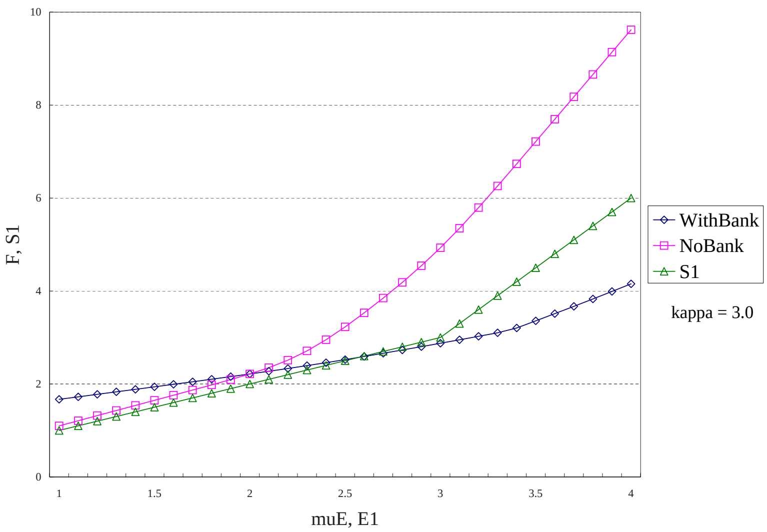

Figure 5: The relation between F and σ_E. We set $\rho = 0.5$ and $\sigma_R = 0.5$, and the values of the other parameters are the same as in Figure 1. The figure shows that the forward price is initially decreasing and then increasing at around $\sigma_E = 0.06$.