Enterprise Risk Management in Insurance Groups: Measuring Risk Concentration and Default Risk

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Introduction (1)

- Trend towards consolidation in financial sector
- Financial conglomerate: financial group providing services and products in different sectors of financial markets
- Insurance group: financial group providing services and products in the insurance sector, not necessarily across sectors
- New types of risk
  - Risk concentrations: interdependencies and accumulation reduce diversification
- Crucial: proper risk assessment; enterprise risk management (ERM)
- Literature
  - Wang (1998, 2002): Overview of economic capital modeling, risk aggregation, use of copula theory in ERM
  - Kurizkes et al. (2003): ERM, capital adequacy in financial conglomerates under joint normality, measure diversification effect
  - McNeil et al. (2005): Modeling of depence using copulas

Introduction (2)

- Measure diversification on corporate level with economic capital of aggregated risk portfolio
  - Implicit assumption: different legal entities are merged into one
  - Only realistic in case of signed full-transfer-of-losses contract or if management decides in favor of cross-subsidization (e.g., for reputational reasons)
- But: intra-group transfers restricted by insurance law; limited liability of legal entities

⇨ Analysis from different perspectives:
  - Executive board of insurance group / shareholders: joint default, risk concentration
  - Policyholders / debtholders: default of individual entity
  - Solvency II / Swiss Solvency Test: diversification on group level?

Introduction (3)

- Aim of this paper:
  - Provide a detailed, more comprehensive picture of an insurance group's risk situation
  - Consider both: risk concentrations (full liability) and joint default probabilities (no liability) of legal entities
  - Analyze sensitivity of default probabilities and risk concentration

    ... under different distributional assumptions

    ... for different dependence structures (linear and nonlinear)

  ⇨ Provide additional information insight by simultaneous consideration

Economic capital

Economic capital on stand-alone basis

- Economic capital: amount necessary to buffer against unexpected losses from business activities to prevent default at a specific risk tolerance level α for a fixed time horizon (1 year)
- Necessary economic capital for legal entity given by

$$EC_i = VaR_{1-\alpha}\left(L_i\right) - E\left(L_i\right) \quad i = 1,\ldots,N.$$

L_i is the value of liabilities at t = 1 of company i = 1,...,N (legal entities in insurance group)

Aggregation (full transfer of losses between legal entities)

$$EC_{aggr} = VaR_{1-\alpha}\left(\sum_{i=1}^{N} L_i\right) - E\left(\sum_{i=1}^{N} L_i\right)$$

Risk concentration and default probabilities

Diversification versus concentration

- Risk concentration factor $$d = \frac{EC_{aggr}}{\sum_{i=1}^{N} EC_i}$$
- Detection of risk concentrations in insurance group
- But: hypothetical number since generally no full coverage of losses for entities in group

Determination of default probabilities

- Provides additional and valuable information about group's risk situation
- Assume no transfer of losses
- Joint default probabilities of exactly one (P1), two (P2), and three (P3) legal entities

Dependence structure (1)

Modeling the dependence structure between entities

- Nonlinear dependence with copulas: separate univariate margins and multivariate dependence structure
- Sklar's theorem:

$$P\left(X_1 < x_1, \ldots, X_N < x_N\right) = F_{X_1,\ldots,X_N}\left(x_1, \ldots, x_N\right) = C\left(F_{X_1}\left(x_1\right), \ldots, F_{X_N}\left(x_N\right)\right)$$

- Fix default probabilities of individual entities (adjust economic capital):

$$P\left(X_i < 0\right) = \alpha_i, i = 1, \ldots, N \qquad \left(P\left(A_i < L_i\right) = \alpha_i\right)$$

⇨ Joint default probabilities only depend on dependence structure (*C*) and on marginal default probabilities α_i:

$$P\left(X_1 < 0, \ldots, X_N < 0\right) = F_{X_1,\ldots,X_N}\left(0, \ldots, 0\right) = C\left(F_{X_1}\left(0\right), \ldots, F_{X_N}\left(0\right)\right) = C\left(\alpha_1, \ldots, \alpha_N\right)$$

Dependence structure (2)

Copulas

- Perfect dependence (comonotonicity)

$$M\left(u_1,...,u_N\right)=\min\left\{u_1,...,u_N\right\}$$

- Independence copula

$$\Pi\left(u_1,...,u_N\right)=\prod_{i=1}^{N}u_i$$

- Clayton copula (lower tail dependent)

$$C_{\theta,N}^{Cl}\left(u_1,...,u_N\right)=\left(\sum_{i=1}^{N}u_i^{-\theta}-N+1\right)^{-1/\theta} \qquad 0\leq\theta<\infty$$

Perfect dependence for $\theta\rightarrow\infty$

Independence for $\theta\rightarrow 0$

Dependence structure (3)

- Gumbel copula (upper tail dependent)

$$C_{\theta,N}^{Gu}(u_1,\ldots,u_N) = \exp\left[\left(-\sum_{i=1}^{N}(-\log u_i)^{\theta}\right)^{1/\theta}\right] \qquad \theta \geq 1$$

Perfect dependence for $\theta \to \infty$

Independence for $\theta \to 1$

- Gauss copula (linear dependence)

$$C_{R}^{Ga}(u_1,\ldots,u_N) = \Phi_N\left(\Phi^{-1}(u_1),\ldots,\Phi^{-1}(u_N)\right)$$

R: correlation matrix with coefficients ρ_{ij} between the liabilities of entity *i* and entity *j*

Φ: standard univariate normal distribution function

Φ_N: joint distribution function of *X*

Dependence structure (4)

Linear dependence given normal distribution

- Stand-alone economic capital: $EC_i = \sigma\left(L_i\right) \cdot z_\alpha$

  z_α: α-quantile of the standard normal distribution

  σ stands for the standard deviation

- Aggregated economic capital:

$$EC_{aggr} = \sigma\left(L\right) \cdot z_\alpha = \sqrt{\begin{pmatrix} EC_1 \\ EC_2 \\ \vdots \\ EC_N \end{pmatrix}^T \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & 1 & \cdots & \rho_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1} & \rho_{N2} & \cdots & 1 \end{pmatrix} \begin{pmatrix} EC_1 \\ EC_2 \\ \vdots \\ EC_N \end{pmatrix}}.$$

- Diversification effect on EC_{aggr} depends on N, EC_i, R

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Input parameters – basis for simulation study

TABLE 1

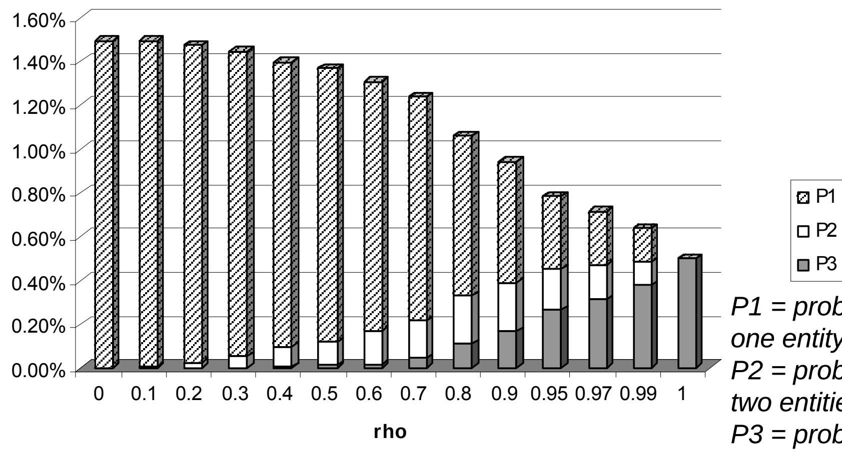
Economic capital for individual entities in an insurance group for different distributional assumptions given a default probability $\alpha = 0.50\%$ and $E(L_i) = 100$, $i = 1, 2, 3$.

Legal entity	*Distribution type*	*Case (A)*		*Case (B)*	
		$\sigma(L_i)$	EC_i	$\sigma(L_i)$	EC_i
	"normal"				
Bank	Normal	15.00	38.64	35.00	90.15
Life insurer	Normal	15.00	38.64	5.00	12.88
Non-life insurer	Normal	15.00	38.64	5.00	12.88
Sum			115.91		115.91
	"non-normal"				
Bank	Normal	15.00	38.64	35.00	90.15
Life insurer	Lognormal	15.00	45.22	5.00	13.59
Non-life insurer	Gamma	15.00	42.84	5.00	13.35
Sum			126.70		117.09

FIGURE 1

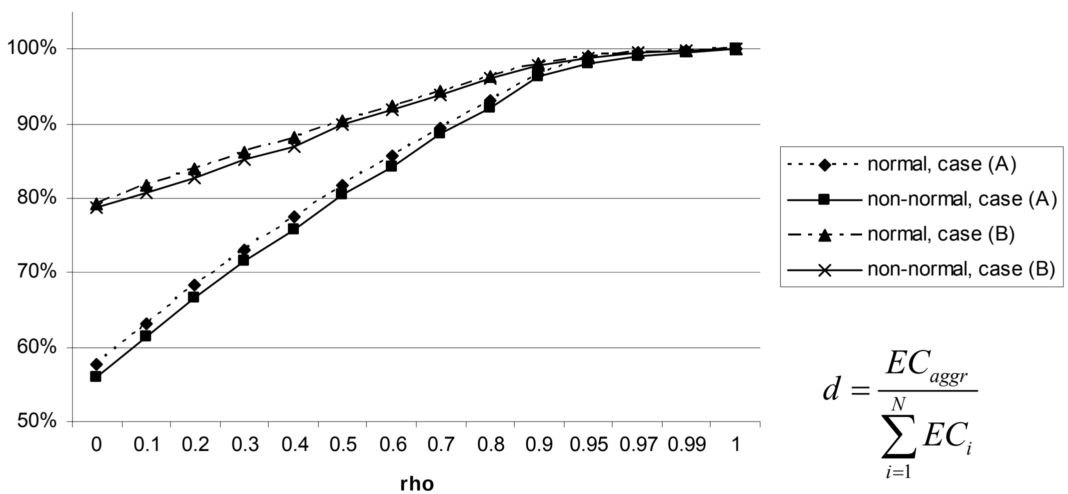
Default probabilities and risk concentration factor for linear dependence on the basis of Table 1

a) Joint default probabilities for linear dependence



P1 = probability that exactly one entity defaults
P2 = probability that exactly two entities default
P3 = probability that all three entities default

b) Risk concentration factor for linear dependence



$$d = \frac{EC_{aggr}}{\sum_{i=1}^{N} EC_i}$$

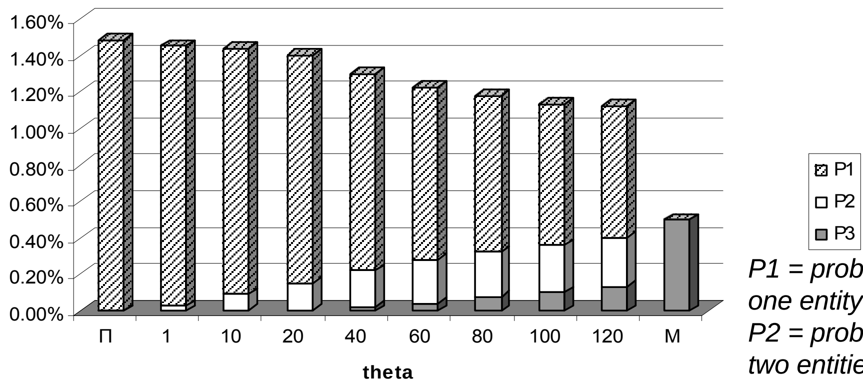
Gauss copula

- Joint default probabilities only depend on dependence structure and individual default probabilities; not on distributional assumptions
- With increasing dependence, risk concentration factor increases, P3 increases, and P1 decreases
- Given liabilities have same standard deviations, distributional assumptions (normal vs. non-normal) have only marginal influence on risk concentration
- Large risk contribution of bank in case (B) leads to higher risk concentration in insurance group as a whole, compared to case (A)
- For perfect correlation (rho = 1): concentration factor is at maximum of 100% for all models; P3 = 0.50%, P1 = P2 = 0.

FIGURE 2

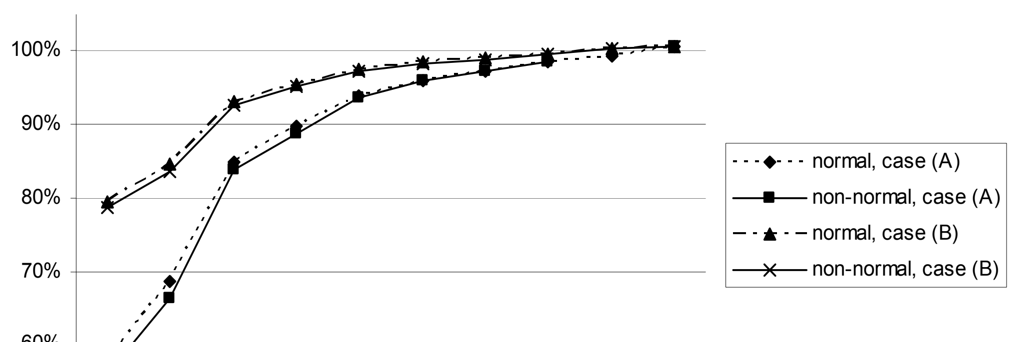
Default probabilities and risk concentration factor for Clayton copula on the basis of Table 1.

a) Joint default probabilities for Clayton copula



P1 = probability that exactly one entity defaults
P2 = probability that exactly two entities default
P3 = probability that all three entities default

b) Risk concentration factor for Clayton copula



$$d = \frac{EC_{aggr}}{\sum_{i=1}^{N} EC_i}$$

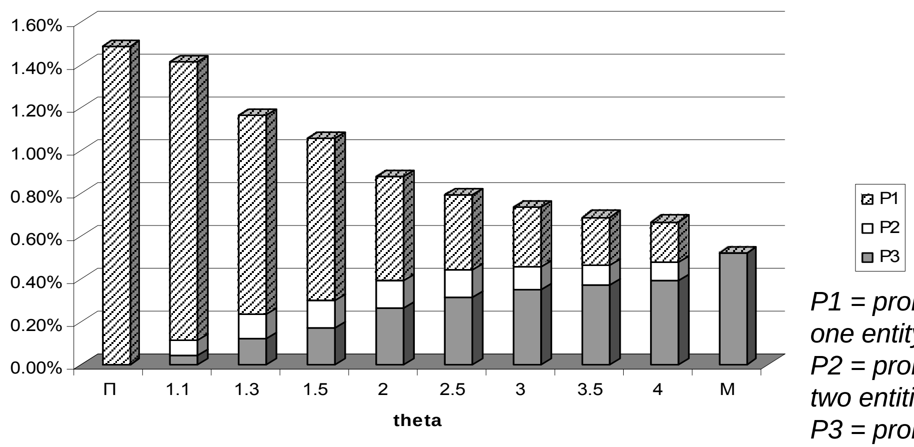
Clayton copula

- Similar to Gauss copula
- Probability of default for any company (P1) decreases much more slowly with increasing dependence parameter theta
- For perfect comonotonicity (M), P3 = 0.50%, concentration factor at 100%

FIGURE 3

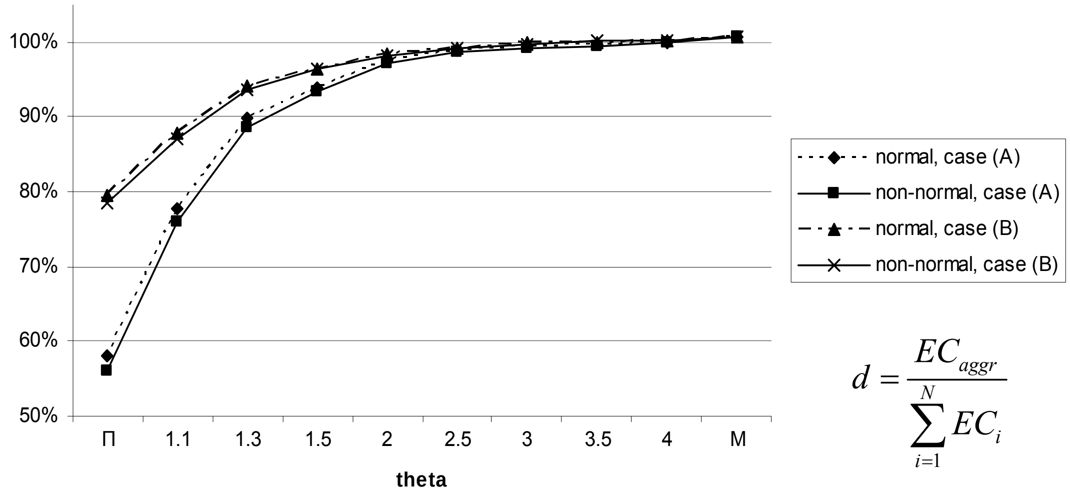
Default probabilities and risk concentration factor for Gumbel copula on the basis of Table 1.

a) Joint default probabilities for Gumbel copula



P1 = probability that exactly one entity defaults
P2 = probability that exactly two entities default
P3 = probability that all three entities default

b) Risk concentration factor for Gumbel copula



$$d = \frac{EC_{aggr}}{\sum_{i=1}^{N} EC_i}$$

Gumbel copula

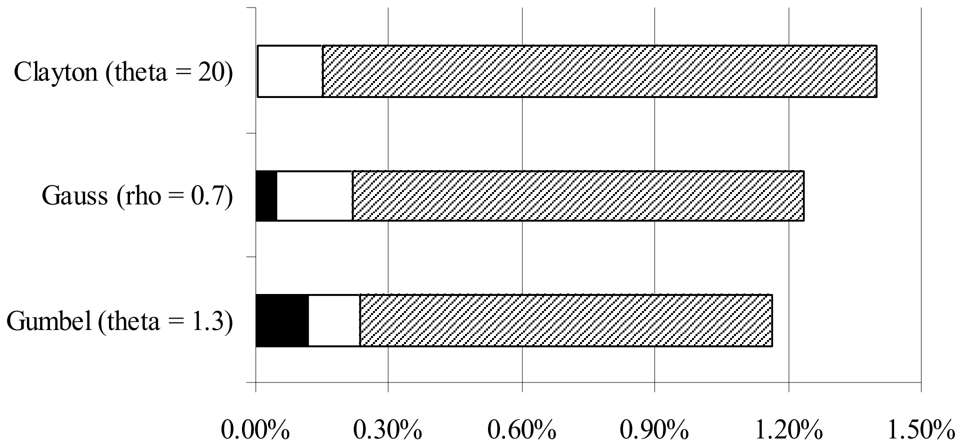
- Probability of default for any company (P1) decreases much faster than in case of Clayton copula with increasing dependence parameter theta:
  ⇨ Due to upper tail dependence of Gumbel copula

- Outcomes for Gauss, Clayton, Gumbel copula are difficult to compare

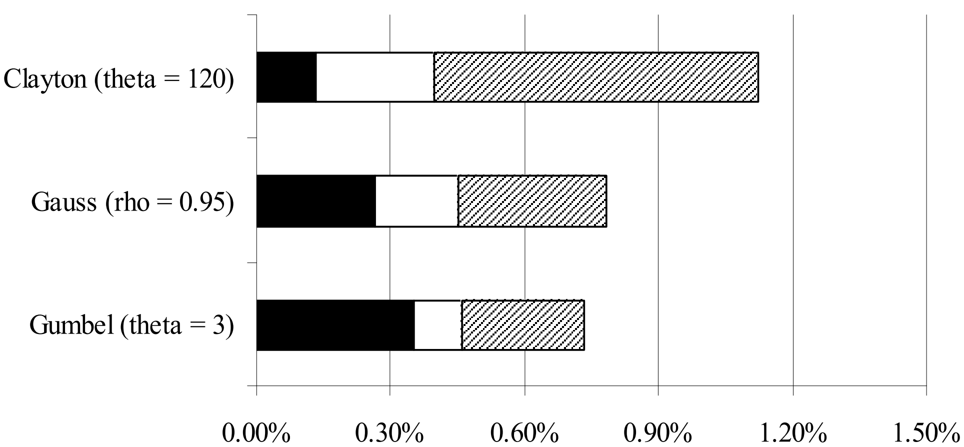
- Even though results look similar at first glance, they can differ tremendously

FIGURE 4
Comparison of joint default probabilities for one (P_1), two (P_2), and three (P_3) companies for different dependence structures; case (A), normal distributions.

a) Risk concentration factor d = 90%.



b) Risk concentration factor d = 99.40%.



- Same risk concentration factor for all dependence structures, but default probabilities differ substantially
- Higher concentration factor (b) has lower sum (P1+P2+P3) but much higher P3
- Clayton copula has highest sum (P1+P2+P3) and lowest P3

In contrast:
- Gumbel copula has lowest sum (P1+P2+P3) and highest P3

⇨ Substantial model risk involved in calculating risk concentration and default risk

Summary (1)

- Assessed and related risk concentration and default risk of (three) legal entities in an insurance group
  - ⇨ Sensitivity analysis provides insight in the group's risk situation: highly relevant for ERM on corporate group level
- Diversification concepts assume that entities are fully liable
  - ⇨ Useful in determining risk concentration in insurance group
- Additionally calculate joint default probabilities, given no transfer of losses between legal entities in a group
  - ⇨ Only depend on individual default probability and coupling dependence structure
- Compare Gauss, Clayton, Gumbel copulas for normal and non-normal marginal distributions

Summary (2)

- For all dependency models, increasing dependence led to:
  - ⇨ Risk concentration factor and joint default probability of all three entities (P3) increase
  - ⇨ Probability of single default decreases
  - ⇨ Sum of joint default probabilities (P1+P2+P3) decreases
- Large risk contribution (in terms of volatility) of one entity led to much higher risk concentration for insurance group
- Distributional assumptions (normal / non-normal) had minor effect due to same expected value and same standard deviation
- Even if different dependence structures imply same risk concentration factor, joint default probabilities for different sets of subsidiaries can vary tremendously:
  - ⇨ Lower tail-dependent Clayton copula: lowest default probability P3
  - ⇨ Upper tail-dependent Gumbel copula: highest default probability P3