

# **LONGEVITY RISK IN LIFE ANNUITIES AND PENSIONS: HEDGING OR SHARING ?**

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# Agenda

1. Introduction
2. Hedging the longevity risk in life annuities and pensions
3. Sharing the longevity risk in life annuities and pensions
4. Concluding remarks

# 1 INTRODUCTION

Benefits provided by insurance and life annuity products (and pensions) imply a wide range of “guarantees”  $\Rightarrow$  risks borne by the insurance company (or the pension fund)

See, for example:

Pitacco [2012]

and references therein

Guarantees and inherent risks are clearly perceived in recent scenarios, in particular because of

- ▷ volatility in financial markets
- ▷ trends in mortality / longevity (and uncertainty in trends)

Then:

- appropriate modeling tools needed for pricing and reserving
  - logical and technical shift from expected present values, and their prominent role in actuarial mathematics, to more modern and complex stochastic approaches
- appropriate managing tools needed for hedging the risks

ERM (Enterprise Risk Management): a comprehensive approach

⇒ guidelines for:

- ▷ risk identification
- ▷ risk assessment
- ▷ choice of actions
- ▷ monitoring

Drawbacks in a rigorous approach:

- complexity is often an obstacle on the way towards sound pricing and reserving principles
- if sound pricing leads to very high premiums, the insurer's market share could become smaller
- possible poor effectiveness of hedging strategies

Alternative solution: appropriate product design aiming either

- at sharing risks between insurer and policyholders

or

- at transferring some risks to policyholders

An important example, as regards the market risk in participating (or with-profit) policies: shift from

- ▷ annual interest guarantee (  $\Rightarrow$  lock-in of past participation, i.e. cliquet-like option)

to

- ▷ point-to-point interest guarantee, in particular “to-maturity” guarantee

## 2 HEDGING THE LONGEVITY RISK IN LIFE ANNUITIES AND PENSIONS

Natural hedging:

- *across time*: packaging death benefits inside the life annuity product (e.g. “value-protected” annuities)
- *across LOBS*: hedging between products with negative sum at risk (life annuities) and products with positive sum at risk (term insurance, whole-life insurance, endowment insurance)

Low or zero cost, but effectiveness ?

Risk transfers:

- ▷ traditional reinsurance
- ▷ swap reinsurance
- ▷ transfer to capital markets (longevity bonds ?)

Possible high cost

### 3 SHARING THE LONGEVITY RISK IN LIFE ANNUITIES AND PENSIONS

*Conventional life annuity:*

- ▷ deterministic benefit, e.g. flat profile  $b$
- ▷ benefit payment also relies on “mortality credits”, i.e. release of reserves pertaining to died annuitants ( $\Rightarrow$  *mutuality*)
- ▷ longevity risk originated by possible number of deaths lower than expected, borne by the annuity provider

*Sharing the longevity risk*  $\Rightarrow$  linking the annual benefit to some longevity measure

Some specific solutions already adopted in insurance and pension practice

See, for example:

Pitacco et al. [2009]

## Sharing the longevity risk ... (cont'd)

### ***Sharing the (future) longevity risk during the accumulation phase***

Object: life annuity policies with some kind of GAR (e.g. GAR stated at policy issue: the traditional deferred life annuity)

Possible arrangements aiming at lowering the guarantee:

- GAR related to each single recurrent premium
- “conditional GAR”, i.e. GAR is kept provided that no important unanticipated improvement in mortality occurs during the accumulation period

Purpose: to charge low premium rates



### ***Sharing the risk in the decumulation phase***

Object: immediate life annuities

- (1) Assume high premium rates, allowing for important future mortality improvements  $\Rightarrow$  low benefits

In case of mortality improvements lower than expected, mortality profits arise

Profits can be distributed  $\Rightarrow$  raise in the annual benefit  
 $\Rightarrow$  participation in mortality profits

- (2) Assume high premium rates, allowing for important future mortality improvements  $\Rightarrow$  low benefits

Higher initial benefits, kept high provided that no important unanticipated mortality improvement occurs, otherwise  $\Rightarrow$  benefit reduction (with a guaranteed minimum benefit, possibly corresponding to theoretical high premium rates)

## Sharing the longevity risk ... (cont'd)

### ***Decumulation phase: a more systematic approach***

Previous arrangements: benefit  $b$  as a function of some measure of mortality trend

In more general terms  $\Rightarrow$  *Adjustment process*  $\Rightarrow$  benefit  $b_t$  due at time  $t$ :

$$b_t = b_0 \alpha_t^{[m]}$$

with  $\alpha_t^{[m]}$  = coefficient of adjustment over  $(0, t)$ , according to mortality trend measure  $[m]$

At annuity inception: random behavior of mortality  $\Rightarrow$  random annual benefit  $B_t$ , at time  $t$

Various interesting contributions regarding life annuity design, and in particular practicable models for the adjustment process

See:

Denuit et al. [2011], Goldsticker [2007], Kartashov et al. [1996], Lütjens et al. [2001], Piggott et al. [2005], Richter and Weber [2011], Rocha et al. [2011], Sherris and Qiao [2011], van de Ven and Weale [2008], Wadsworth et al. [2001]

## Sharing the longevity risk ... (cont'd)

Basic problems in defining the adjustment process:

1. choice of the age pattern of mortality referred to
2. choice of the link between annual benefits and mortality

Reasonable aim: sharing the *aggregate* longevity risk (i.e. the systematic component of the longevity risk), leaving the volatility (i.e. the random fluctuation component) with the annuity provider

1. Examples of *mortality referred to*

- (a) Actual number of surviving annuitants

$$n_{x+1}, n_{x+2}, \dots$$

- (b) Actual number of survivors in the “reference” cohort

$$l_{x+1}, l_{x+2}, \dots$$

- (c) Expected number of surviving annuitants, according to (initial) information  $\mathcal{F}$  (for example:  $\mathcal{F}$  = life table)

$$\mathbb{E}[N_{x+1} | \mathcal{F}], \mathbb{E}[N_{x+2} | \mathcal{F}], \dots$$

## Sharing the longevity risk ... (cont'd)

- (d) Expected number of survivors in the reference cohort, according to (initial) information  $\mathcal{F}$

$$\mathbb{E}[L_{x+1} | \mathcal{F}], \mathbb{E}[L_{x+2} | \mathcal{F}], \dots$$

- (e) Expected number of surviving annuitants, according to (current) updated information  $\mathcal{F}'$

$$\mathbb{E}[N_{x+t} | \mathcal{F}'], \mathbb{E}[N_{x+t+1} | \mathcal{F}'], \dots$$

for example:  $\mathcal{F}' = \{\mathcal{F}; n_{x+1}, \dots, n_{x+t-1}\}$ ; See: Olivieri and Pitacco [2009a]

- (f) Expected number of survivors in the reference cohort, according to (current) updated information  $\mathcal{F}^*$

$$\mathbb{E}[L_{x+t} | \mathcal{F}^*], \mathbb{E}[L_{x+t+1} | \mathcal{F}^*], \dots$$

for example:  $\mathcal{F}^* =$  new projected life table

## Sharing the longevity risk ... (cont'd)

*Reference cohort*: a cohort in a population, which should have

- ▷ age-pattern of mortality
- ▷ mortality trend

close to those in the portfolio or pension fund

Reference cohort should be referred to (instead of annuitants in the portfolio or pension plan) for objectivity and transparency reasons

However, *basis risk* arises when linking adjustments to a reference cohort, because of possible mortality trend different from the one experienced in the portfolio or pension fund

### 2. Definition of the *adjustment coefficients*

Various approaches can be adopted

In particular the definition can be

- ▷ *retrospective*: directly involving observed mortality, in terms of  
 $n_{x+1}, n_{x+2}, \dots$   
or  
 $l_{x+1}, l_{x+1}, \dots$
- ▷ *prospective*: relying on updated mortality forecasts, e.g.  
 $\mathbb{E}[L_{x+t} | \mathcal{F}^*], \mathbb{E}[L_{x+t+1} | \mathcal{F}^*], \dots$

Quantities involved:

- $\ddot{a}_{x+t}^{[\mathcal{F}]}$  = actuarial value of an annuity, according to information  $\mathcal{F}$
- $V_t^{[\mathcal{F}]}$  = individual reserve at time  $t$
- $V_t^{[P, \mathcal{F}]}$  = portfolio reserve at time  $t$ , according to information  $\mathcal{F}$
- $A_t$  = assets available at time  $t$

## Sharing the longevity risk ... (cont'd)

(a) Example 1 of the retrospective approach. Define:

$$\alpha_t^{[1]} = \frac{\mathbb{E}[L_{x+t} | \mathcal{F}]}{\mathbb{E}[L_x | \mathcal{F}]} \frac{n_x}{n_{x+t}}$$

Result:  $V_{t+}^{[P, \mathcal{F}]}$  expected value at time 0 of the portfolio reserve

(b) Example 2 of the retrospective approach. Define:

$$\alpha_t^{[2]} = \frac{A_t}{V_t^{[P, \mathcal{F}]}}$$

Result:  $V_{t+}^{[P, \mathcal{F}]} = A_t =$  available assets

Note that:

- ▷ both volatility and aggregate longevity risk borne by the annuitants
- ▷ market risk also borne by the annuitants
- ▷ arrangement characterizing (pure) Group Self-Annutization (GSA)

## Sharing the longevity risk ... (cont'd)

(c) Example of the prospective approach. Define:

$$\alpha_t^{[3]} = \frac{\ddot{a}_{x+t}^{[\mathcal{F}]}}{\ddot{a}_{x+t}^{[\mathcal{F}^*]}}$$

Result:  $b_t \ddot{a}_{x+t}^{[\mathcal{F}^*]} = b_0 \ddot{a}_{x+t}^{[\mathcal{F}]}$

and hence:  $V_{t+}^{[P, \mathcal{F}^*]} = V_t^{[P, \mathcal{F}]}$

Possible arrangements combining in  $\alpha_t^{[m]}$  :

- ▷ longevity indexing
- ▷ financial profit participation



### ***Some numerical results***

- One cohort, all individuals initial age  $x = 65$
- Mortality/longevity adjustments every  $k = 5$  years
- Maximum age for mortality/longevity adjustment (apart from the GSA, i.e.  $\alpha_t^{[2]}$ ): 95 (i.e., time 30)
- $\frac{A_{\omega-x}}{A_0}$ : remaining assets at cohort's exhaustion, as a percentage of the initial assets (initial assets are funded just through premiums)
- Traditional premium calculation (equivalence principle):

$$A_0 = n_x \times \mathbb{E}[a_{K_x} | \mathcal{F}]$$

## Sharing the longevity risk ... (cont'd)

Experienced mortality: 90% of the best-estimate (as at time 0, i.e.  $\mathcal{F}$ )

No extra-return on investments

New projected life table at time 10, yielding a higher life expectancy

$t$	no adj	$\alpha_t^{[1]}$	$\alpha_t^{[2]}$	$\alpha_t^{[3]}$
0	1.000	1.000	1.000	1.000
5	1.000	0.996	0.996	1.000
10	1.000	0.993	0.872	0.880
15	1.000	1.007	1.031	1.000
20	1.000	1.007	1.054	1.000
25	1.000	1.000	1.105	1.000
30	1.000	0.997	1.243	1.000
35	1.000	1.000	1.684	1.000
40	1.000	1.000	3.372	1.000
$\frac{b_{95-x}}{b_0}$	100.00%	98.03%	129.70%	87.98%
$\frac{A_{\omega-x}}{A_0}$	-8.554%	-7.580%	0.180%	9.467%

## 4 CONCLUDING REMARKS

Mortality / longevity risk  $\Rightarrow$  a rigorous stochastic approach should be adopted (not relying on expected values only)

However

- implementation of complex stochastic models may constitute an obstacle on the way towards sound pricing
- facing the risks by charging very high premiums can reduce the insurer's market share

Alternative solution: appropriate product designs which aim at sharing risks between annuity provider and annuitants, or between insurer and policyholders

Weakening guarantees and simplifying the products do not exempt insurers and annuity providers from a sound (but hopefully simpler) assessment of the risk profile of portfolios and pension funds

## Concluding remarks (cont'd)

Life annuities: severe solvency requirements (see Solvency 2)  
because of the aggregate longevity risk

See, for example:

Olivieri [2011], Olivieri and Pitacco [2009a], Olivieri and Pitacco [2009b]

Sharing the longevity risk  $\Rightarrow$  less “absorbing” annuity and pension  
products (in particular as regards solvency regulation)

Main problems

- to find appropriate “reference” longevity
- to link effectively benefits to reference longevity

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