

Actuarial values for Long-term Care insurance products. A sensitivity analysis

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Agenda

- Motivation
- LTCI products
- The actuarial model
- Technical bases
- Sensitivity analysis
- Concluding remarks

Paper available at:

<http://www.cepar.edu.au/working-papers/working-papers-2015.aspx>

MOTIVATION

Long-term care insurance (LTCI) products deserve special attention

- LTCI provides benefits of remarkable interest in the current demographic and social scenario
- LTCI covers are “difficult” products

In particular:

- ▷ in many countries, shares of elderly population are rapidly growing because of increasing life expectancy and low fertility rates
- ▷ household size progressively reducing \Rightarrow lack of assistance and care services provided to old members of the family inside the family itself
- ▷ LTCI products are rather recent \Rightarrow senescent disability data are scanty \Rightarrow pricing difficulties arise
- ▷ high premiums (in particular, because of a significant safety loading) \Rightarrow obstacle to the diffusion of these products (especially stand-alone LTC covers only providing “protection”)

Uncertainty in technical bases, in particular biometric assumptions:

- probability of disablement, i.e. entering LTC state
- mortality of disabled people, i.e. lives in LTC state

Need for:

- ▷ accurate sensitivity analysis
- ▷ focus on product design \Rightarrow products whose premiums (and reserves) are not too heavily affected by the choice of the biometric assumptions

LTCI PRODUCTS

A classification

Long-Term Care insurance provides the insured with financial support, while he/she needs nursing and/or medical care because of chronic (or long-lasting) conditions or ailments (\Rightarrow disability implying dependence)

Severity of disability measured according to various scales (ADL, IADL) See for example: Pitacco [2014] and references therein

Types of LTC benefit:

- benefits with *predefined amount* (usually, lifelong annuities)
 - ▷ *fixed-amount* benefits
 - ▷ *degree-related* (or *graded*) benefits, i.e. amount graded according to the degree of dependence (e.g. according to ADL)
- reimbursement (usually partial) of nursery and medical expenses, i.e. *expense-related* benefits
- *care service* benefits (for example provided by the Continuing Care Retirement Communities, CCRCs)

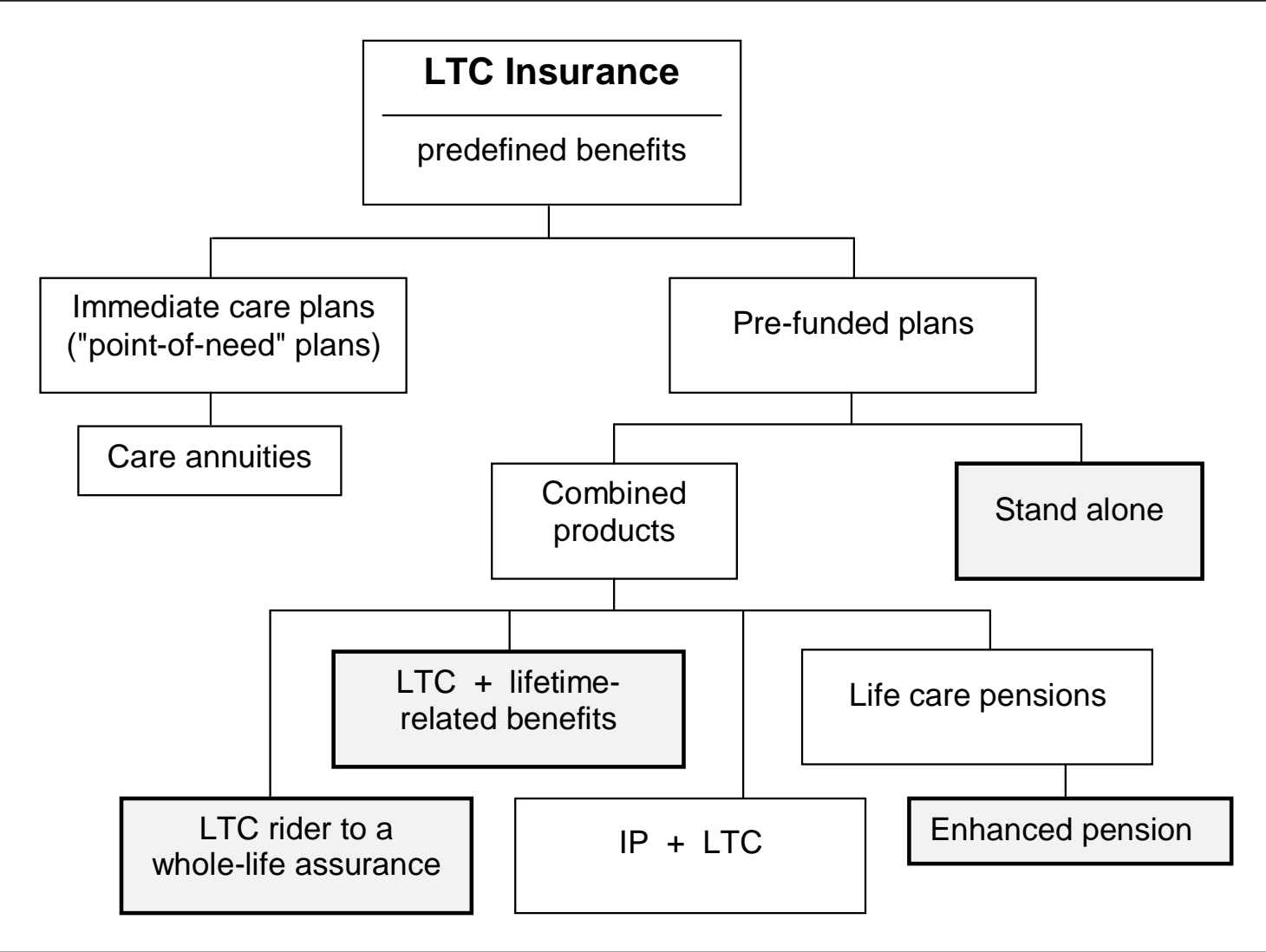
Classification of LTCl products which pay out benefits with predefined amount (see following Figure)

- *immediate care plans* relate to individuals already affected by disability
- *pre-funded plans*, i.e. relying on an accumulation phase
 - ▷ stand-alone
 - ▷ combined products

Several examples of insurance packages in which health-related benefits are combined with lifetime-related benefits

In the following, we focus on LTCl pre-funded plans

LTCI products (cont'd)



LTCI products with predefined benefits: a classification

We consider the following LTCl products:

- Stand-alone LTCl
- LTCl as an acceleration benefit in a whole-life assurance
- Package including LTC benefits and lifetime-related benefits
- Enhanced pension

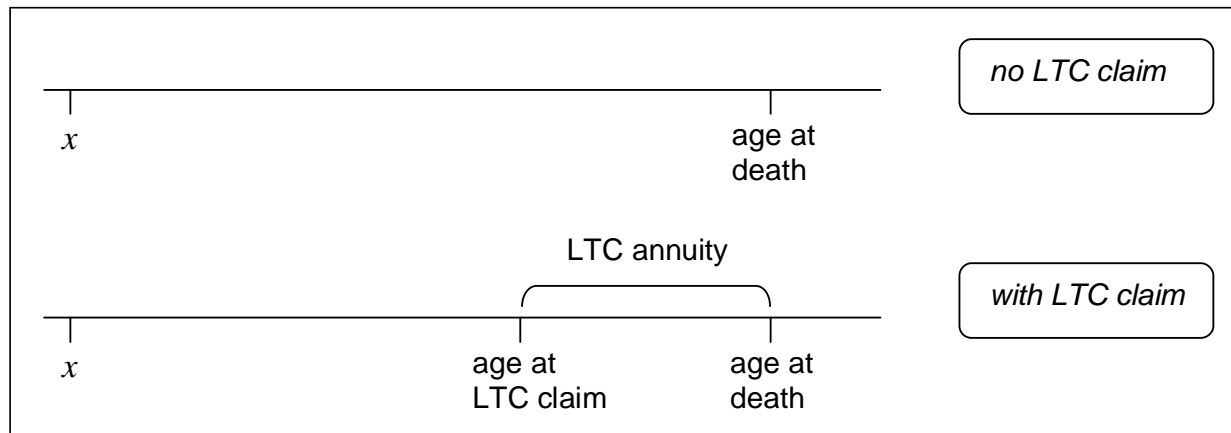
For more information, see Pitacco [2014] and references therein

In what follows: x = age at policy issue

Stand-alone LTCI

(Product P1)

LTCI benefit: a lifelong annuity with predefined annual amount, possibly graded according to the severity of LTC status



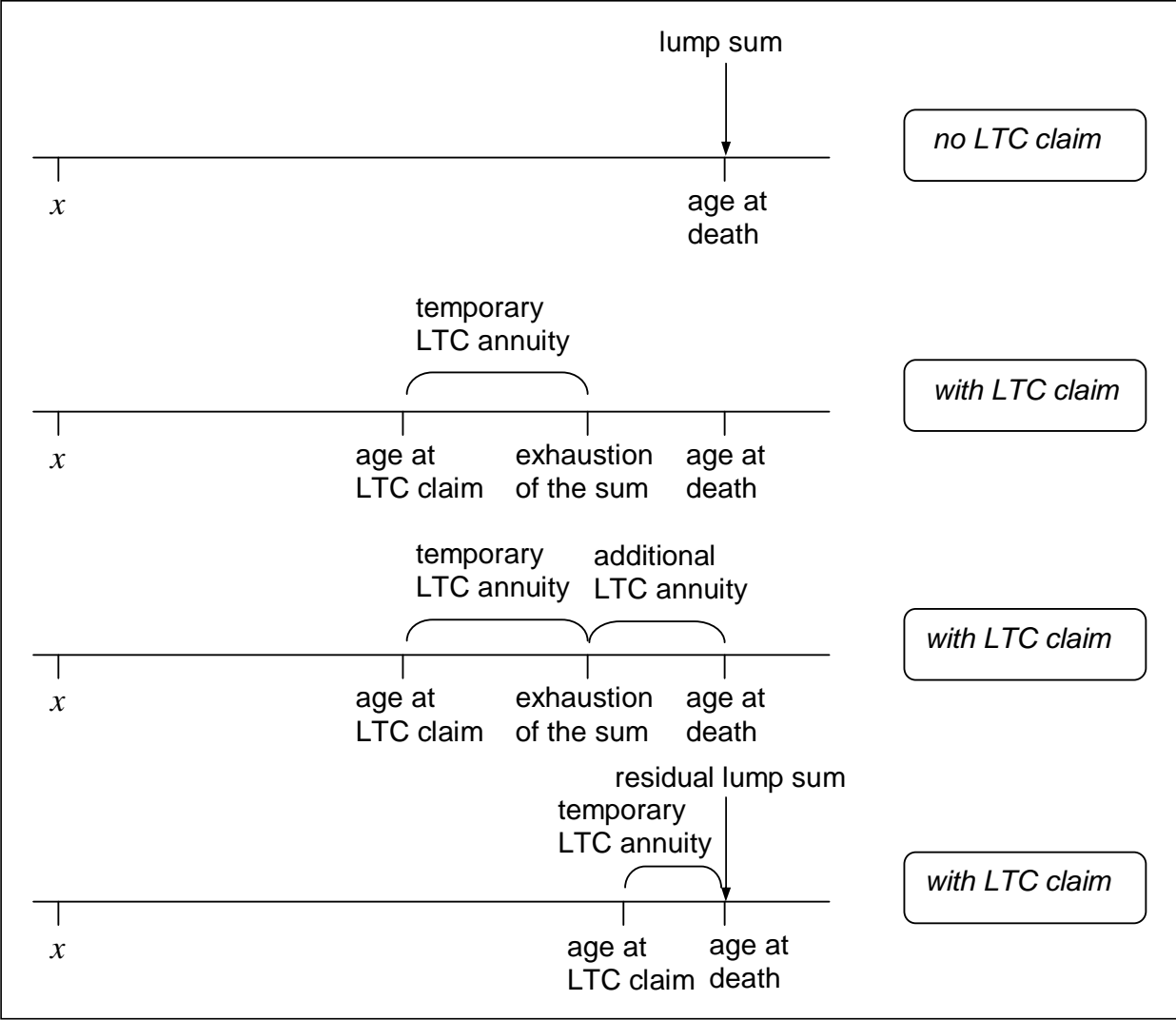
LTCI as an acceleration benefit in a whole-life assurance

(Product P2(s))

Annual LTC benefit = $\frac{\text{sum assured}}{s}$ paid for s years at most

Possibly complemented by a (deferred) lifelong LTC annuity in the case of sum exhaustion

LTCI products (cont'd)



Package including LTC benefits and lifetime-related benefits

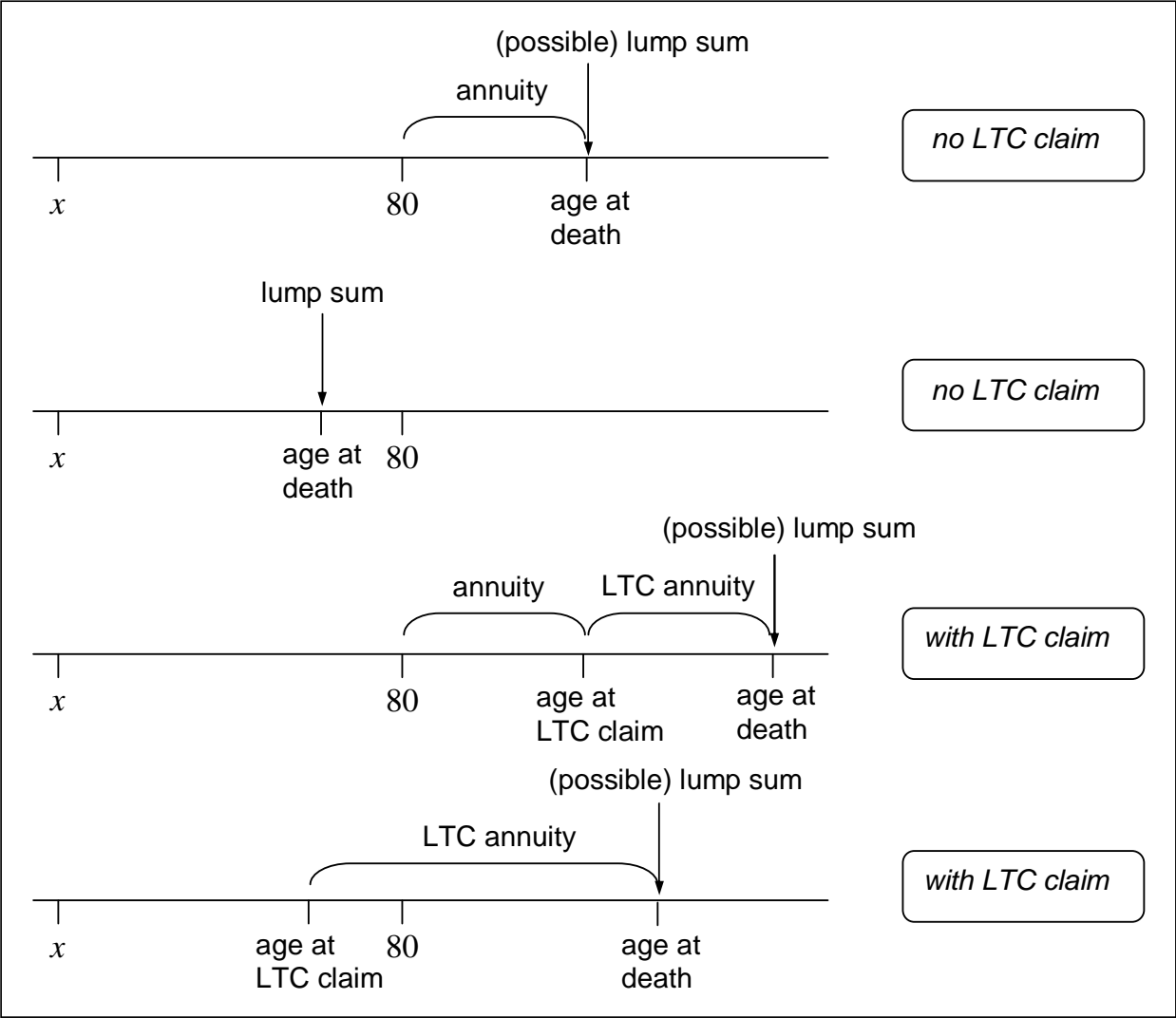
(Products $P3a(x + n)$ and $P3b(x + n)$)

Benefits:

1. a lifelong LTC annuity (from the LTC claim on)
2. a deferred life annuity from age $x + n$ (e.g. $x + n = 80$), while the insured is not in LTC disability state
3. a lump sum benefit on death, alternatively given by
 - 3a. a fixed amount, stated in the policy
 - 3b. the difference (if positive) between a fixed amount and the total amount paid as benefit 1 and/or benefit 2

Benefits 1 and 2 are mutually exclusive

LTCI products (cont'd)



Enhanced pension (Life care pension)

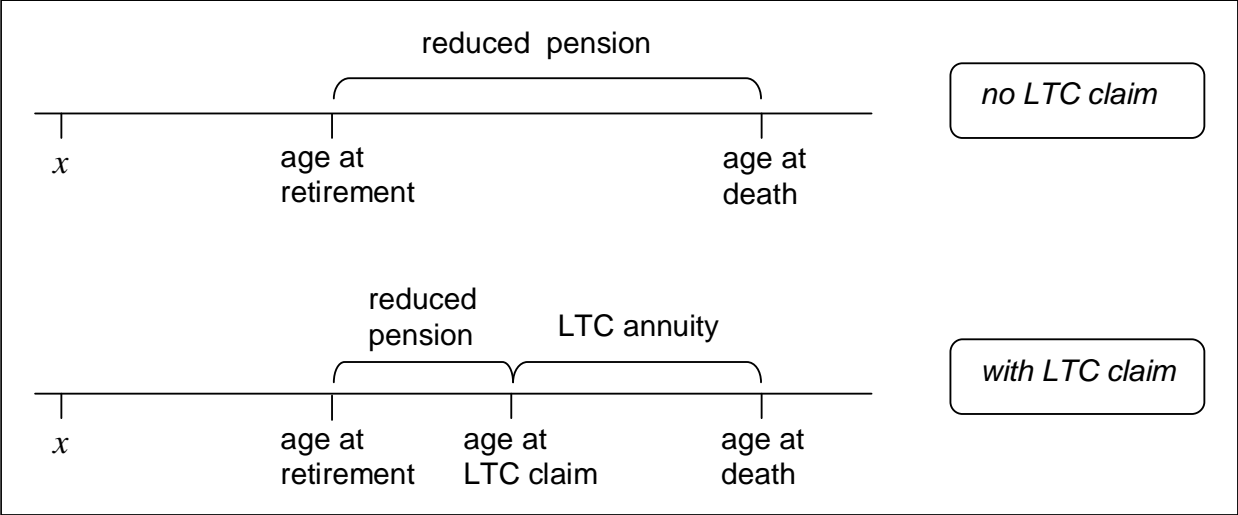
(Product P4(b' , b''))

LTC annuity benefit defined as an uplift with respect to the basic pension b

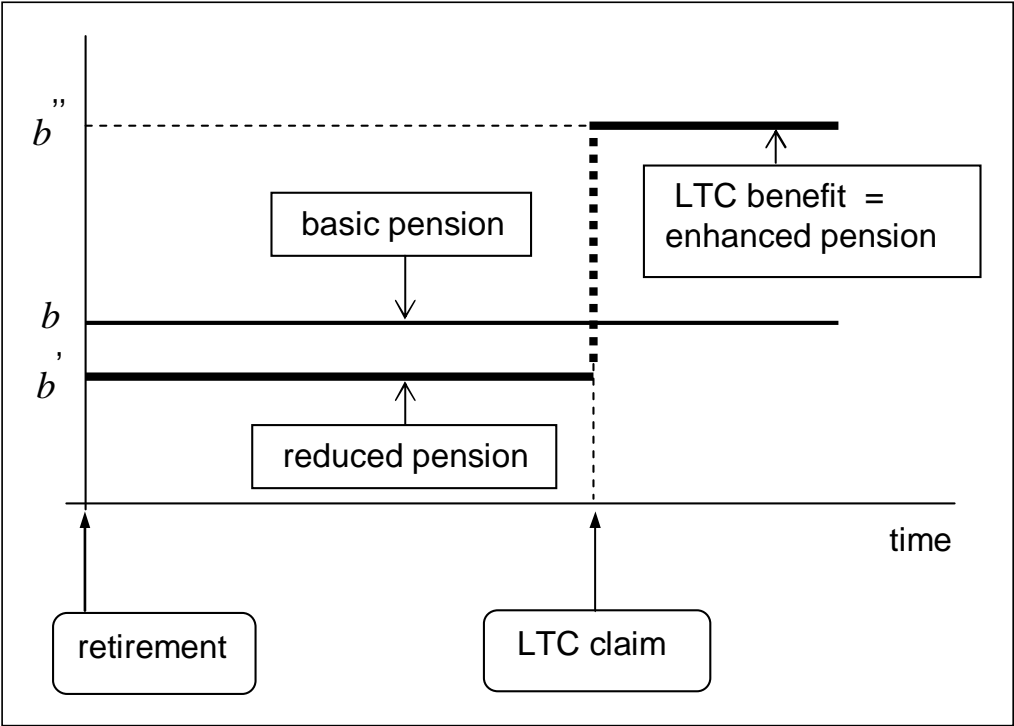
Uplift financed by a reduction (with respect to the basic pension b) of the benefit paid while the policyholder is healthy

- ▷ reduced benefit b' paid as long as the retiree is healthy
- ▷ uplifted lifelong benefit b'' paid in the case of LTC claim
(of course, $b' < b < b''$)

LTCI products (cont'd)



LTCI products (cont'd)



THE ACTUARIAL MODEL

Multistate models for LTCI

States:

a = active = healthy

i = invalid = in LTC state

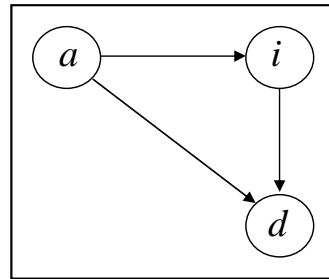
d = died

i' = in low-severity LTC state

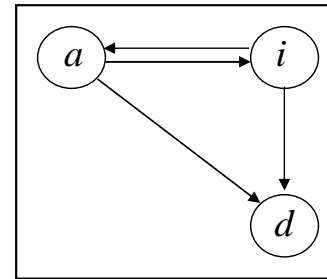
i'' = in high-severity LTC state

For more information on time-discrete models see Pitacco [2014],
and on time discrete and time-continuous models see Haberman and Pitacco [1999]

The actuarial model (cont'd)

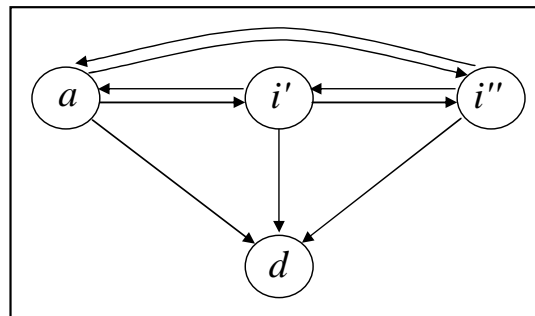


(a)

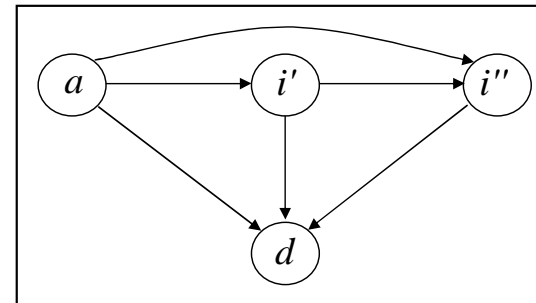


(b)

Three-state models



(a)



(b)

Four-state models

In what follows we adopt the three-state model (a), in a time-discrete context

Biometric functions (needed)

Refer to three-state model (a)

For an active age x :

$q_x^{aa} =$ prob. of dying before age $x + 1$ from state a

$w_x =$ prob. of becoming invalid (disablement) before $x + 1$

For an invalid age x :

$q_x^i =$ prob. of dying before age $x + 1$

TECHNICAL BASES

Assumptions

q_x^{aa} : life table (first Heligman-Pollard law)

w_x : a specific parametric law

$q_x^i = q_x^{aa} + \text{extra-mortality}$ (i.e. additive extra-mortality model)

Life table

First Heligman-Pollard law:

$$\frac{q_x^{aa}}{1 - q_x^{aa}} = a^{(x+b)^c} + d e^{-e (\ln x - \ln f)^2} + g h^x$$

Technical bases (cont'd)

| a | b | c | d | e | f | g | h |
|---------|---------|---------|---------|-------|-------|--------------|---------|
| 0.00054 | 0.01700 | 0.10100 | 0.00014 | 10.72 | 18.67 | 2.00532 E-06 | 1.13025 |

The first Heligman-Pollard law: parameters

| $\overset{\circ}{e}_0$ | $\overset{\circ}{e}_{40}$ | $\overset{\circ}{e}_{65}$ | Lexis | q_0^{aa} | q_{40}^{aa} | q_{80}^{aa} |
|------------------------|---------------------------|---------------------------|-------|------------|---------------|---------------|
| 85.128 | 46.133 | 22.350 | 90 | 0.00682 | 0.00029 | 0.03475 |

The first Heligman-Pollard law: some markers

Disablement (LTC claim)

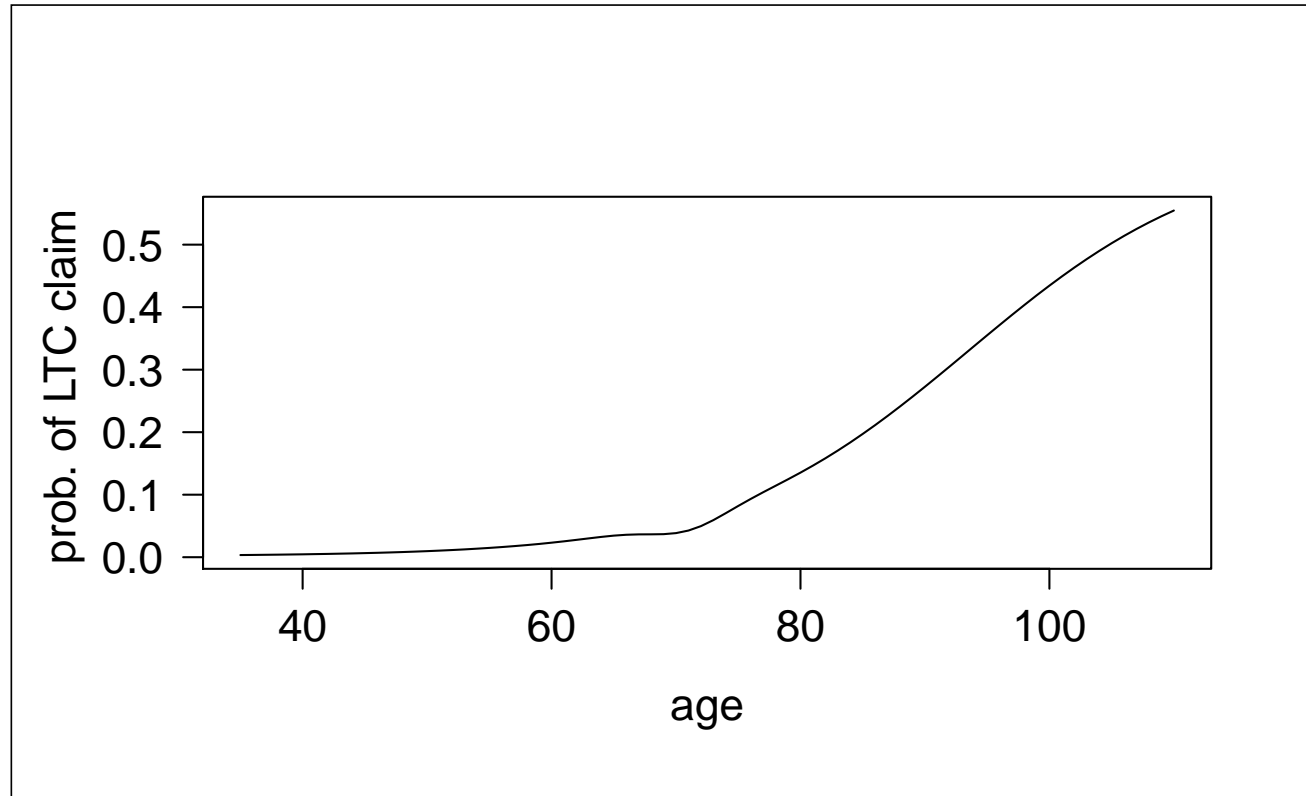
Assumption by Rickayzen and Walsh [2002]

$$w_x = \begin{cases} A + \frac{D - A}{1 + B^{C-x}} & \text{for females} \\ \left(A + \frac{D - A}{1 + B^{C-x}} \right) \left(1 - \frac{1}{3} \exp \left(- \left(\frac{x - E}{4} \right)^2 \right) \right) & \text{for males} \end{cases}$$

| Parameter | Females | Males |
|-----------|----------|---------|
| <i>A</i> | 0.0017 | 0.0017 |
| <i>B</i> | 1.0934 | 1.1063 |
| <i>C</i> | 103.6000 | 93.5111 |
| <i>D</i> | 0.9567 | 0.6591 |
| <i>E</i> | n.a. | 70.3002 |

Parameters Rickayzen-Walsh

Technical bases (cont'd)



Probability of disablement (Males)

Extra-mortality

Assumption by Rickayzen and Walsh [2002]

$$q_x^{i(k)} = q_x^{[\text{standard}]} + \Delta(x, \alpha, k)$$

with:

$$\Delta(x, \alpha, k) = \frac{\alpha}{1 + 1.1^{50-x}} \frac{\max\{k - 5, 0\}}{5}$$

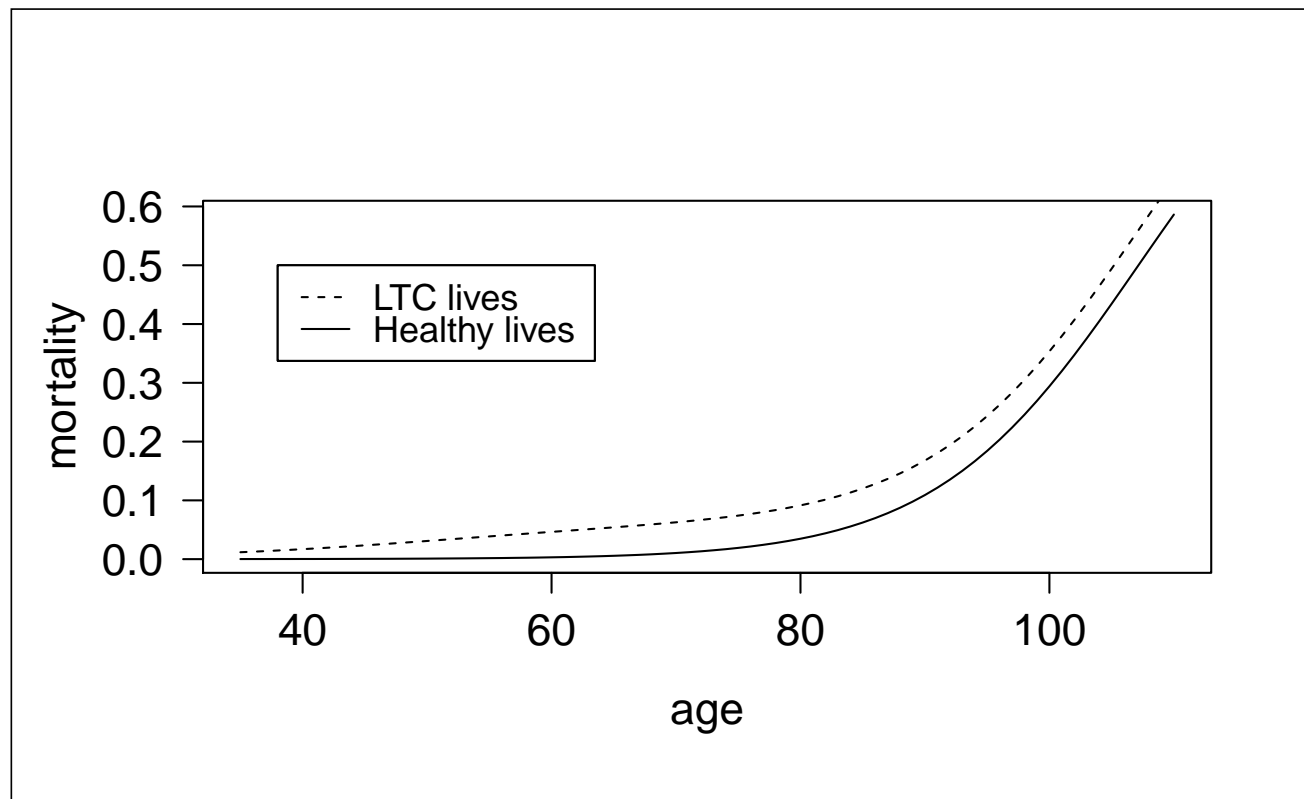
where:

- parameter k expresses LTC severity category
 - ▷ $0 \leq k \leq 5 \Rightarrow$ less severe \Rightarrow no impact on mortality
 - ▷ $6 \leq k \leq 10 \Rightarrow$ more severe \Rightarrow extra-mortality
- parameter α (assumption by Rickayzen [2007])

$$\alpha = 0.10 \quad \text{if } q_x^{[\text{standard}]} = q_x^{aa} \text{ (mortality of insured healthy people)}$$

Our (base) choice: $\alpha = 0.10$, $k = 8$; hence:

$$q_x^i = q_x^{aa} + \Delta(x, 0.10, 8) = q_x^{aa} + \frac{0.06}{1 + 1.1^{50-x}}$$



Mortality assumptions (Males)

SENSITIVITY ANALYSIS

Sensitivity analysis concerning:

- ▷ probability of disablement (i.e. entering into LTC state)
- ▷ extra-mortality of lives in LTC state

Notation:

$\Pi_x^{[PX]}(\delta, \lambda)$ = actuarial value (single premium) of product PX, according to the following assumptions:

- $\delta \Rightarrow$ disablement

$$\bar{w}_x(\delta) = \delta w_x$$

where w_x is given by the previous Eq. (assumption by Rickayzen and Walsh [2002])

- $\lambda \Rightarrow$ extra-mortality

$$\bar{\Delta}(x; \lambda) = \lambda \Delta(x, \alpha, k) = \Delta(x, \lambda 0.10, 8)$$

and hence:

$$q_x^i(\lambda) = q_x^{aa} + \bar{\Delta}(x; \lambda)$$

Sensitivity analysis (cont'd)

For products P1, P2, P3, normalize and define the ratio:

$$\rho_x^{[PX]}(\delta, \lambda) = \frac{\Pi_x^{[PX]}(\delta, \lambda)}{\Pi_x^{[PX]}(1, 1)}$$

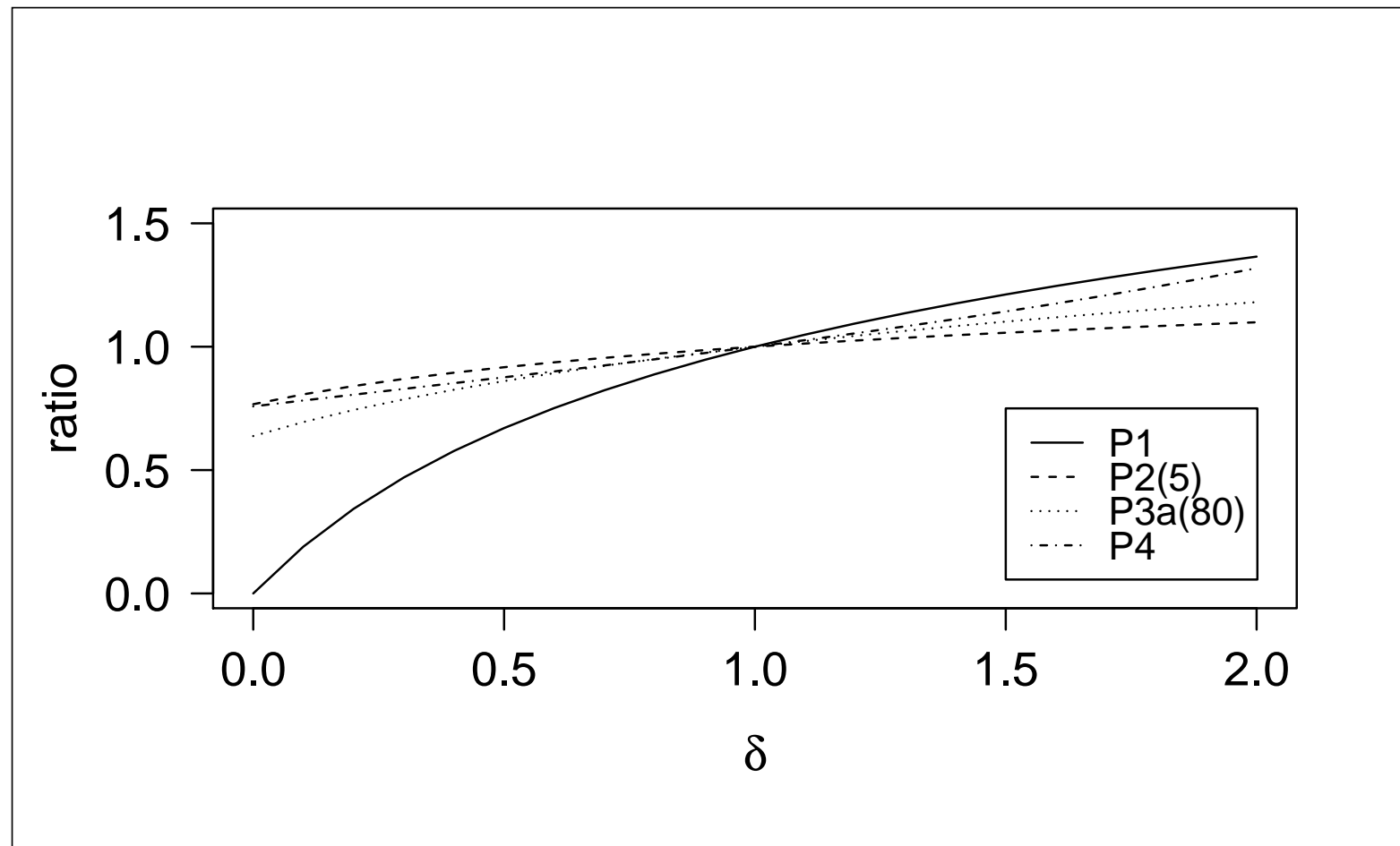
For product P4, with given b and b'' , normalize and define the ratio:

$$\rho_x^{[P4]}(\delta, \lambda) = \frac{b'(1, 1)}{b'(\delta, \lambda)}$$

For all the products, we first perform *marginal* analysis, i.e. tabulating the functions:

$$\Pi_x^{[PX]}(\delta, 1), \rho_x^{[PX]}(\delta, 1); \quad \Pi_x^{[PX]}(1, \lambda), \rho_x^{[PX]}(1, \lambda)$$

Sensitivity analysis: disablement assumption (parameter δ)



Sensitivity analysis (cont'd)

| δ | $\Pi_{50}^{[P1]}(\delta, 1)$ | $\rho_{50}^{[P1]}(\delta, 1)$ |
|----------|------------------------------|-------------------------------|
| 0.0 | 0.00000 | 0.0000000 |
| 0.1 | 97.44457 | 0.1897494 |
| 0.2 | 176.07799 | 0.3428686 |
| 0.3 | 241.25240 | 0.4697798 |
| 0.4 | 296.47515 | 0.5773125 |
| 0.5 | 344.12555 | 0.6700999 |
| 0.6 | 385.86840 | 0.7513839 |
| 0.7 | 422.90118 | 0.8234961 |
| 0.8 | 456.10675 | 0.8881558 |
| 0.9 | 486.15044 | 0.9466585 |
| 1.0 | 513.54361 | 1.0000000 |
| 1.1 | 538.68628 | 1.0489592 |
| 1.2 | 561.89632 | 1.0941550 |
| 1.3 | 583.42997 | 1.1360865 |
| 1.4 | 603.49644 | 1.1751610 |
| 1.5 | 622.26854 | 1.2117151 |
| 1.6 | 639.89052 | 1.2460296 |
| 1.7 | 656.48397 | 1.2783412 |
| 1.8 | 672.15229 | 1.3088514 |
| 1.9 | 686.98406 | 1.3377327 |
| 2.0 | 701.05581 | 1.3651339 |

Product P1 (Stand-alone); $x = 50, b = 100$

Sensitivity analysis (cont'd)

| δ | $\Pi_{50}^{[P2(1)]}(\delta, 1)$ | $\rho_{50}^{[P2(1)]}(\delta, 1)$ | $\Pi_{50}^{[P2(5)]}(\delta, 1)$ | $\rho_{50}^{[P2(5)]}(\delta, 1)$ |
|----------|---------------------------------|----------------------------------|---------------------------------|----------------------------------|
| 0.0 | 492.1453 | 0.7446436 | 492.1453 | 0.7668209 |
| 0.1 | 522.4302 | 0.7904664 | 517.9195 | 0.8069802 |
| 0.2 | 547.3508 | 0.8281727 | 539.5114 | 0.8406230 |
| 0.3 | 568.3981 | 0.8600184 | 558.0108 | 0.8694472 |
| 0.4 | 586.5416 | 0.8874705 | 574.1426 | 0.8945825 |
| 0.5 | 602.4415 | 0.9115280 | 588.4118 | 0.9168156 |
| 0.6 | 616.5641 | 0.9328964 | 601.1825 | 0.9367139 |
| 0.7 | 629.2483 | 0.9520882 | 612.7241 | 0.9546971 |
| 0.8 | 640.7467 | 0.9694859 | 623.2411 | 0.9710837 |
| 0.9 | 651.2520 | 0.9853810 | 632.8914 | 0.9861200 |
| 1.0 | 660.9139 | 1.0000000 | 641.7995 | 1.0000000 |
| 1.1 | 669.8509 | 1.0135223 | 650.0652 | 1.0128789 |
| 1.2 | 678.1584 | 1.0260919 | 657.7693 | 1.0248828 |
| 1.3 | 685.9139 | 1.0378264 | 664.9783 | 1.0361152 |
| 1.4 | 693.1814 | 1.0488226 | 671.7475 | 1.0466625 |
| 1.5 | 700.0145 | 1.0591615 | 678.1234 | 1.0565969 |
| 1.6 | 706.4581 | 1.0689111 | 684.1455 | 1.0659801 |
| 1.7 | 712.5507 | 1.0781294 | 689.8475 | 1.0748645 |
| 1.8 | 718.3251 | 1.0868664 | 695.2586 | 1.0832956 |
| 1.9 | 723.8097 | 1.0951649 | 700.4040 | 1.0913127 |
| 2.0 | 729.0293 | 1.1030626 | 705.3059 | 1.0989504 |

Product P2 (Acceleration benefit); $x = 50, C = 1\,000$

Sensitivity analysis (cont'd)

| δ | $\Pi_{50}^{[P3a(80)]}(\delta, 1)$ | $\rho_{50}^{[P3a(80)]}(\delta, 1)$ | $\Pi_{50}^{[P3b(80)]}(\delta, 1)$ | $\rho_{50}^{[P3b(80)]}(\delta, 1)$ |
|----------|-----------------------------------|------------------------------------|-----------------------------------|------------------------------------|
| 0.0 | 700.5211 | 0.6379255 | 524.3054 | 0.6681005 |
| 0.1 | 762.7792 | 0.6946205 | 564.2116 | 0.7189513 |
| 0.2 | 816.5343 | 0.7435723 | 598.8261 | 0.7630591 |
| 0.3 | 863.9507 | 0.7867518 | 629.5434 | 0.8022009 |
| 0.4 | 906.4564 | 0.8254594 | 657.2615 | 0.8375209 |
| 0.5 | 945.0332 | 0.8605891 | 682.5844 | 0.8697888 |
| 0.6 | 980.3808 | 0.8927781 | 705.9351 | 0.8995436 |
| 0.7 | 1 013.0142 | 0.9224956 | 727.6214 | 0.9271776 |
| 0.8 | 1 043.3239 | 0.9500969 | 747.8754 | 0.9529864 |
| 0.9 | 1 071.6132 | 0.9758584 | 766.8772 | 0.9771996 |
| 1.0 | 1 098.1236 | 1.0000000 | 784.7703 | 1.0000000 |
| 1.1 | 1 123.0514 | 1.0227003 | 801.6718 | 1.0215369 |
| 1.2 | 1 146.5586 | 1.0441071 | 817.6790 | 1.0419342 |
| 1.3 | 1 168.7817 | 1.0643443 | 832.8740 | 1.0612966 |
| 1.4 | 1 189.8365 | 1.0835178 | 847.3271 | 1.0797136 |
| 1.5 | 1 209.8231 | 1.1017185 | 861.0993 | 1.0972629 |
| 1.6 | 1 228.8288 | 1.1190259 | 874.2436 | 1.1140122 |
| 1.7 | 1 246.9299 | 1.1355096 | 886.8072 | 1.1300214 |
| 1.8 | 1 264.1943 | 1.1512313 | 898.8317 | 1.1453438 |
| 1.9 | 1 280.6825 | 1.1662462 | 910.3545 | 1.1600268 |
| 2.0 | 1 296.4487 | 1.1806036 | 921.4091 | 1.1741132 |

Products P3a and P3b (Insurance packages); $x = 50, C = 1\,000, b' = 50, b'' = 150$

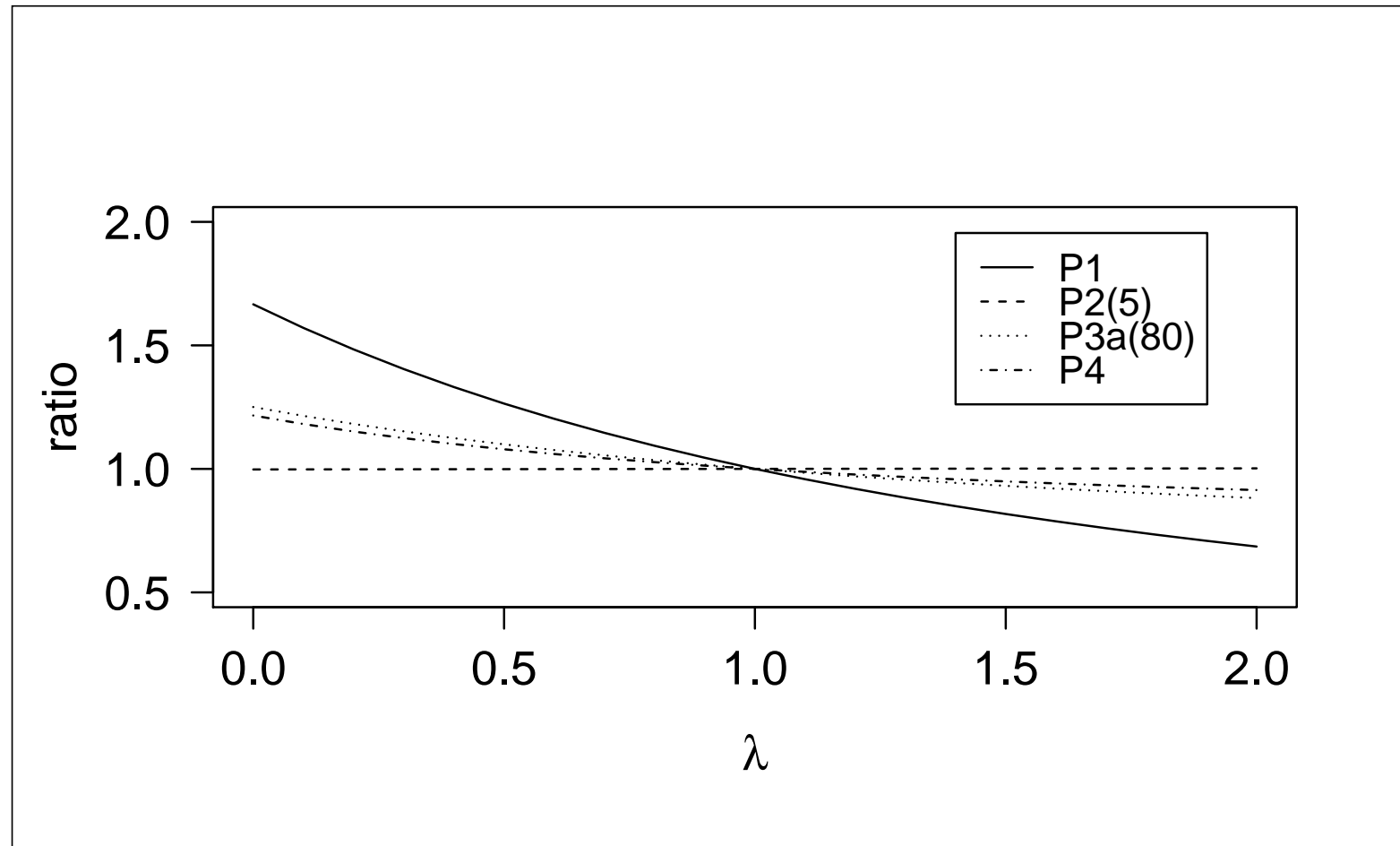
Sensitivity analysis (cont'd)

| δ | $b'(\delta, 1)$ | $\rho_x^{[P4]}(\delta, 1)$ |
|----------|-----------------|----------------------------|
| 0.0 | 100.00000 | 0.7582433 |
| 0.1 | 96.96404 | 0.7819840 |
| 0.2 | 94.13166 | 0.8055136 |
| 0.3 | 91.47026 | 0.8289506 |
| 0.4 | 88.95221 | 0.8524165 |
| 0.5 | 86.55461 | 0.8760288 |
| 0.6 | 84.25873 | 0.8998988 |
| 0.7 | 82.04926 | 0.9241317 |
| 0.8 | 79.91365 | 0.9488283 |
| 0.9 | 77.84153 | 0.9740858 |
| 1.0 | 75.82433 | 1.0000000 |
| 1.1 | 73.85486 | 1.0266668 |
| 1.2 | 71.92708 | 1.0541833 |
| 1.3 | 70.03587 | 1.0826500 |
| 1.4 | 68.17685 | 1.1121713 |
| 1.5 | 66.34626 | 1.1428576 |
| 1.6 | 64.54086 | 1.1748267 |
| 1.7 | 62.75783 | 1.2082052 |
| 1.8 | 60.99468 | 1.2431301 |
| 1.9 | 59.24927 | 1.2797513 |
| 2.0 | 57.51967 | 1.3182330 |

Product P4 (Enhanced pension); $x = 65$, $b = 100$, $b'' = 150$

Sensitivity analysis (cont'd)

Sensitivity analysis: extra-mortality assumption (parameter λ)



Sensitivity analysis (cont'd)

| λ | $\Pi_{50}^{[P1]}(1, \lambda)$ | $\rho_{50}^{[P1]}(1, \lambda)$ |
|-----------|-------------------------------|--------------------------------|
| 0.0 | 855.7094 | 1.6662838 |
| 0.1 | 806.6737 | 1.5707987 |
| 0.2 | 761.9567 | 1.4837234 |
| 0.3 | 721.0856 | 1.4041370 |
| 0.4 | 683.6467 | 1.3312339 |
| 0.5 | 649.2769 | 1.2643073 |
| 0.6 | 617.6576 | 1.2027364 |
| 0.7 | 588.5080 | 1.1459748 |
| 0.8 | 561.5807 | 1.0935405 |
| 0.9 | 536.6571 | 1.0450079 |
| 1.0 | 513.5436 | 1.0000000 |
| 1.1 | 492.0686 | 0.9581828 |
| 1.2 | 472.0797 | 0.9192592 |
| 1.3 | 453.4411 | 0.8829652 |
| 1.4 | 436.0319 | 0.8490650 |
| 1.5 | 419.7439 | 0.8173482 |
| 1.6 | 404.4804 | 0.7876263 |
| 1.7 | 390.1547 | 0.7597305 |
| 1.8 | 376.6889 | 0.7335090 |
| 1.9 | 364.0128 | 0.7088255 |
| 2.0 | 352.0634 | 0.6855570 |

Product P1 (Stand-alone); $x = 50, b = 100$

Sensitivity analysis (cont'd)

| λ | $\Pi_{50}^{[P2(1)]}(1, \lambda)$ | $\rho_{50}^{[P2(1)]}(1, \lambda)$ | $\Pi_{50}^{[P2(5)]}(1, \lambda)$ | $\rho_{50}^{[P2(5)]}(1, \lambda)$ |
|-----------|----------------------------------|-----------------------------------|----------------------------------|-----------------------------------|
| 0.0 | 660.9139 | 1 | 640.3371 | 0.9977214 |
| 0.1 | 660.9139 | 1 | 640.4879 | 0.9979563 |
| 0.2 | 660.9139 | 1 | 640.6376 | 0.9981896 |
| 0.3 | 660.9139 | 1 | 640.7863 | 0.9984213 |
| 0.4 | 660.9139 | 1 | 640.9341 | 0.9986515 |
| 0.5 | 660.9139 | 1 | 641.0808 | 0.9988801 |
| 0.6 | 660.9139 | 1 | 641.2265 | 0.9991071 |
| 0.7 | 660.9139 | 1 | 641.3712 | 0.9993326 |
| 0.8 | 660.9139 | 1 | 641.5150 | 0.9995566 |
| 0.9 | 660.9139 | 1 | 641.6577 | 0.9997791 |
| 1.0 | 660.9139 | 1 | 641.7995 | 1.0000000 |
| 1.1 | 660.9139 | 1 | 641.9404 | 1.0002194 |
| 1.2 | 660.9139 | 1 | 642.0802 | 1.0004374 |
| 1.3 | 660.9139 | 1 | 642.2191 | 1.0006538 |
| 1.4 | 660.9139 | 1 | 642.3571 | 1.0008688 |
| 1.5 | 660.9139 | 1 | 642.4941 | 1.0010822 |
| 1.6 | 660.9139 | 1 | 642.6302 | 1.0012943 |
| 1.7 | 660.9139 | 1 | 642.7653 | 1.0015048 |
| 1.8 | 660.9139 | 1 | 642.8995 | 1.0017140 |
| 1.9 | 660.9139 | 1 | 643.0328 | 1.0019216 |
| 2.0 | 660.9139 | 1 | 643.1652 | 1.0021279 |

Product P2 (Acceleration benefit); $x = 50, C = 1\,000$

Sensitivity analysis (cont'd)

| λ | $\Pi_{50}^{[P3a(80)]}(1, \lambda)$ | $\rho_{50}^{[P3a(80)]}(1, \lambda)$ | $\Pi_{50}^{[P3b(80)]}(1, \lambda)$ | $\rho_{50}^{[P3b(80)]}(1, \lambda)$ |
|-----------|------------------------------------|-------------------------------------|------------------------------------|-------------------------------------|
| 0.0 | 1 373.1426 | 1.2504444 | 1 030.1514 | 1.3126789 |
| 0.1 | 1 333.7360 | 1.2145591 | 992.0364 | 1.2641106 |
| 0.2 | 1 297.7979 | 1.1818323 | 957.9426 | 1.2206663 |
| 0.3 | 1 264.9490 | 1.1519186 | 927.4057 | 1.1817544 |
| 0.4 | 1 234.8573 | 1.1245157 | 900.0200 | 1.1468579 |
| 0.5 | 1 207.2314 | 1.0993584 | 875.4306 | 1.1155246 |
| 0.6 | 1 181.8156 | 1.0762136 | 853.3264 | 1.0873583 |
| 0.7 | 1 158.3843 | 1.0548760 | 833.4345 | 1.0620108 |
| 0.8 | 1 136.7389 | 1.0351648 | 815.5147 | 1.0391763 |
| 0.9 | 1 116.7039 | 1.0169200 | 799.3555 | 1.0185853 |
| 1.0 | 1 098.1236 | 1.0000000 | 784.7703 | 1.0000000 |
| 1.1 | 1 080.8603 | 0.9842793 | 771.5943 | 0.9832104 |
| 1.2 | 1 064.7915 | 0.9696463 | 759.6816 | 0.9680305 |
| 1.3 | 1 049.8081 | 0.9560017 | 748.9029 | 0.9542957 |
| 1.4 | 1 035.8128 | 0.9432570 | 739.1434 | 0.9418596 |
| 1.5 | 1 022.7189 | 0.9313331 | 730.3010 | 0.9305921 |
| 1.6 | 1 010.4485 | 0.9201591 | 722.2849 | 0.9203775 |
| 1.7 | 998.9319 | 0.9096716 | 715.0140 | 0.9111125 |
| 1.8 | 988.1065 | 0.8998136 | 708.4160 | 0.9027050 |
| 1.9 | 977.9161 | 0.8905337 | 702.4263 | 0.8950725 |
| 2.0 | 968.3098 | 0.8817858 | 696.9867 | 0.8881411 |

Products P3a and P3b (Insurance packages); $x = 50, C = 1\,000, b' = 50, b'' = 150$

Sensitivity analysis (cont'd)

| λ | $b'(1, \lambda)$ | $\rho_x^{[P4]}(1, \lambda)$ |
|-----------|------------------|-----------------------------|
| 0.0 | 62.34898 | 1.2161277 |
| 0.1 | 64.17119 | 1.1815946 |
| 0.2 | 65.86125 | 1.1512738 |
| 0.3 | 67.43103 | 1.1244723 |
| 0.4 | 68.89119 | 1.1006390 |
| 0.5 | 70.25128 | 1.0793302 |
| 0.6 | 71.51992 | 1.0601847 |
| 0.7 | 72.70488 | 1.0429056 |
| 0.8 | 73.81315 | 1.0272469 |
| 0.9 | 74.85106 | 1.0130027 |
| 1.0 | 75.82433 | 1.0000000 |
| 1.1 | 76.73813 | 0.9880920 |
| 1.2 | 77.59716 | 0.9771534 |
| 1.3 | 78.40567 | 0.9670771 |
| 1.4 | 79.16755 | 0.9577704 |
| 1.5 | 79.88630 | 0.9491531 |
| 1.6 | 80.56513 | 0.9411556 |
| 1.7 | 81.20698 | 0.9337169 |
| 1.8 | 81.81451 | 0.9267834 |
| 1.9 | 82.39015 | 0.9203081 |
| 2.0 | 82.93615 | 0.9142494 |

Product P4 (Enhanced pension); $x = 65$, $b = 100$, $b'' = 150$

Joint sensitivity analysis (parameters δ, λ)

For the generic product PX, and a given age x , find (δ, λ) such that:

$$\rho_x^{[PX]}(\delta, \lambda) = \rho_x^{[PX]}(1, 1) = 1 \quad (*)$$

Eq. (*) implies

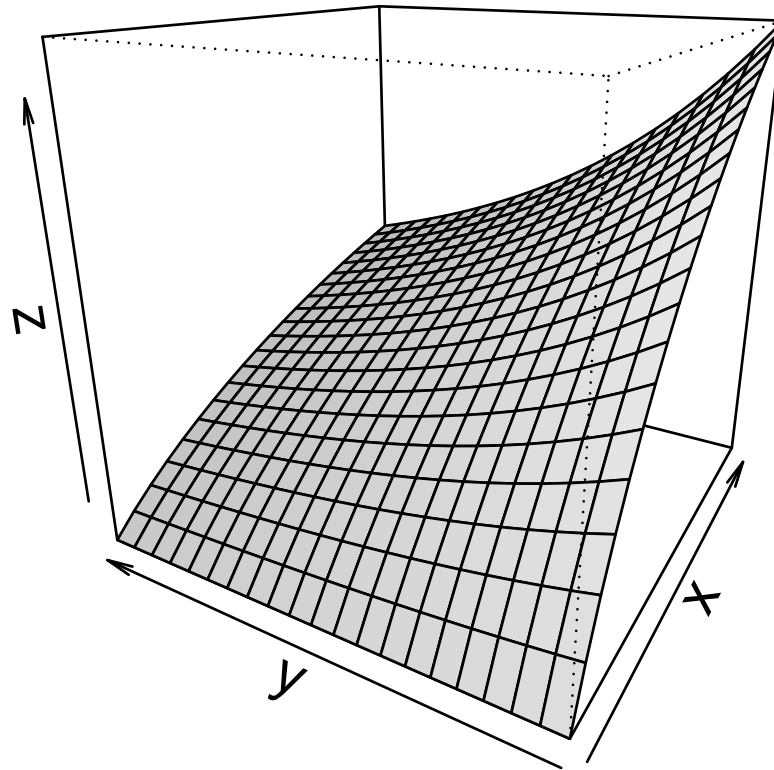
- for products P1, P2, P3:

$$\Pi_x^{[PX]}(\delta, \lambda) = \Pi_x^{[PX]}(1, 1)$$

- for product P4:

$$b'(\delta, \lambda) = b'(1, 1)$$

Sensitivity analysis (cont'd)



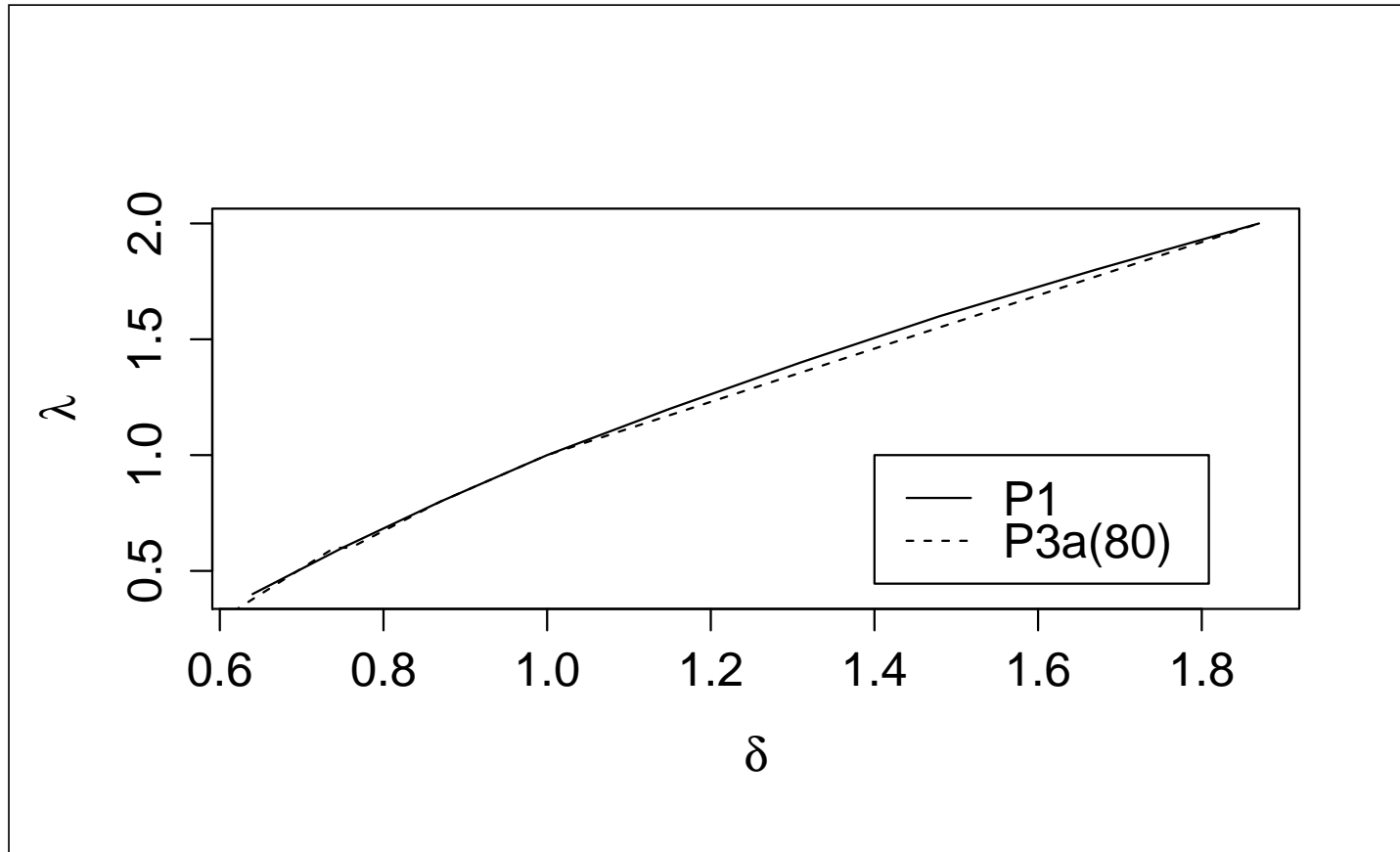
Product P3a(80)

$X = \delta \Rightarrow$ *disablement*

$Y = \lambda \Rightarrow$ *extra-mortality*

$Z = \Pi \Rightarrow$ *premium*

Sensitivity analysis (cont'd)



Offset effect: isopremium lines

CONCLUDING REMARKS

Combined LTCI products: mainly aiming at reducing the relative weight of the risk component by introducing a “saving” component, or by adding the LTC benefits to an insurance product with an important saving component

Combined insurance products in the area of health insurance:

- Insurer’s perspective
 - ▷ a combined product can result profitable even if one of its components is not profitable
 - ▷ a combined product can be less risky than one of its components (less exposed to impact of uncertainty risk related to the choice of technical bases)
- Client’s perspective ⇒ purchasing a combined product can be less expensive than separately purchasing all the single components (in particular: reduction of acquisition costs charged to the policyholder)

Concluding remarks (*cont'd*)

In particular

- LTC covers as riders to life insurance; see:
 - ▷ acceleration benefit in whole life assurance
 - ▷ LTC annuity in enhanced pension
- LTC covers in insurance packages; see:
 - ▷ packages including old-age deferred life annuity and death benefit

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*Many thanks
for your kind attention !*