CyRCE:

A Credit Risk default model that measures concentration, single obligor limits and bank capital adequacy.

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Introduction

- One of the main concerns of regulators is having a proper assessment of the solvency of the Financial System.
- It is important to have an adequate picture of the level of risk of the Financial System as a whole, where risk is concentrated, and the individual banks' contribution to overall risk.
- This represents a technically formidable problem for several reasons:
    - Information
    - Credit risk methodologies:
    - No accepted paradigm
    - Numerical techniques with heavy computational requirements
    - How to draw the picture?

Existing Credit Risk Methodologies

- Big information requirements.
- Substantial computational effort to obtain loss distribution.
- No explicit relation between credit risk and
  - Capital adequacy
  - Concentration
  - Single obligor limits

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CyRCE: Properties

CyRCE is a default Credit Risk Methodology which avoids the use of computationally demanding numerical methods, by assuming that the loan portfolio loss distribution can be characterized by its mean and its variance:

- Closed form expression for Value at Risk (VaR)
- Explicit parametrization of all relevant credit risk elements.

Credit risk related Capital Adequacy can be established in terms of:

- Default rates.
- A measure of concentration and/or
- Single obligor limits.

The model is *"built up"* from a very simple case where all loans have *i.i.d* default probabilities, and extended to a general situation where default probabilities of loans can differ, be correlated and the portfolio can be segmented arbitrarily to detect risky concentration segments.

CyRCE: A simple Model

Let f_i be the i^{th} loan amount in the portfolio; $i = 1, 2, \ldots, N$

Define "*N*" binary *i.i.d* random variables:

$$X_i = \begin{cases} f_i & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases}$$

The mean and standard deviation of the total portfolio loss are:

$$\mu = pV$$

$$V = \sum_{i=1}^{N} f_i$$

$$\sigma = \sqrt{p(1-p)\sum_{i=1}^{N} f_i^2}$$

CyRCE: Value at Risk

Assume for the moment that the loss distribution can be approximated by the Normal distribution, so that, the *value at risk* with confidence level α is:

$$VaR_{\alpha} = pV + z_{\alpha}\sqrt{p(1-p)\sum_{i=1}^{N} f_i^{2}}$$

EXPECTED LOSS

μ

UNEXPECTED LOSS

$z_{\alpha}\sigma$

CyRCE: Capital Adequacy

CAPITALIZATION RATIO: $$\psi = \frac{\textit{Economic Capital}}{\textit{Value of Loan Portfolio}}$$

Capital adequacy requires:

$$\textbf{\textit{Economic Capital}} \geq \textbf{\textit{VaR}}_{\alpha} \Rightarrow \psi \geq p + z_{\alpha}\sqrt{p(1-p)H(\mathbf{F})}$$

where a measure of <u>concentration</u> emerges naturally:

$$H(\mathbf{F}) = \frac{\sum_{i=1}^{N} f_i^2}{\left(\sum_{i=1}^{N} f_i\right)^2}$$

Herfindahl-Hirschman concentration index

$\mathbf{F} = [\, f_1 \;\; f_2 \;\; \cdots \;\; f_N \,]^T$ loan portfolio vector

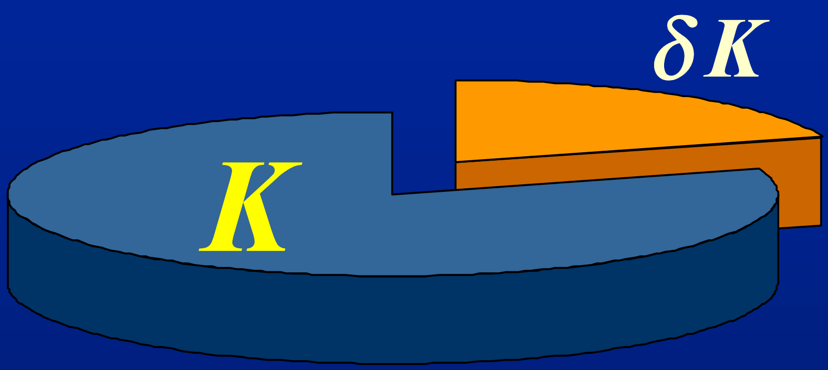
CyRCE: Concentration Limit

The VaR expression establishes a limit on concentration through H(F):

$$H(F)=\frac{\sum_{i=1}^{N} f_i^2}{\left(\sum_{i=1}^{N} f_i\right)^2}\le\frac{(\psi-p)^2}{z_\alpha^2\,p(1-p)}$$

From which single obligor limits can be obtained.

CyRCE: Single Obligor Limits and Concentration



$$\psi = \frac{K}{V}$$ is *the capitalization ratio*.

$$f_i \leq \delta K = \delta \frac{K}{V} \times V = \delta \psi V = \theta V$$

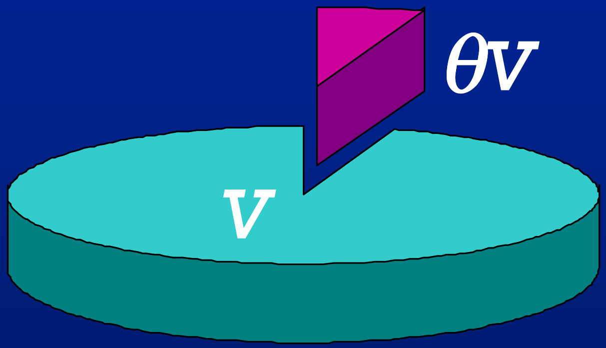
$i = 1, \ldots, N$

So, "single obligor limits" can be set on Capital or on total portfolio value <u>as long as</u>:

$$\theta = \delta \psi = \delta \textit{ (Capitalization Ratio)}$$

CyRCE: Single Obligor Limits and Concentration

HHI Property I:



$$f_i \le \theta V \quad i = 1, \ldots, N$$

$$\Rightarrow \quad H(\mathbf{F}) \le \theta$$

Then, it follows that:

$$f_i \le \theta V \quad i = 1, \ldots, N$$

$$\theta \le \frac{(\psi - p)^2}{z_\alpha^2 \, p(1 - p)}$$

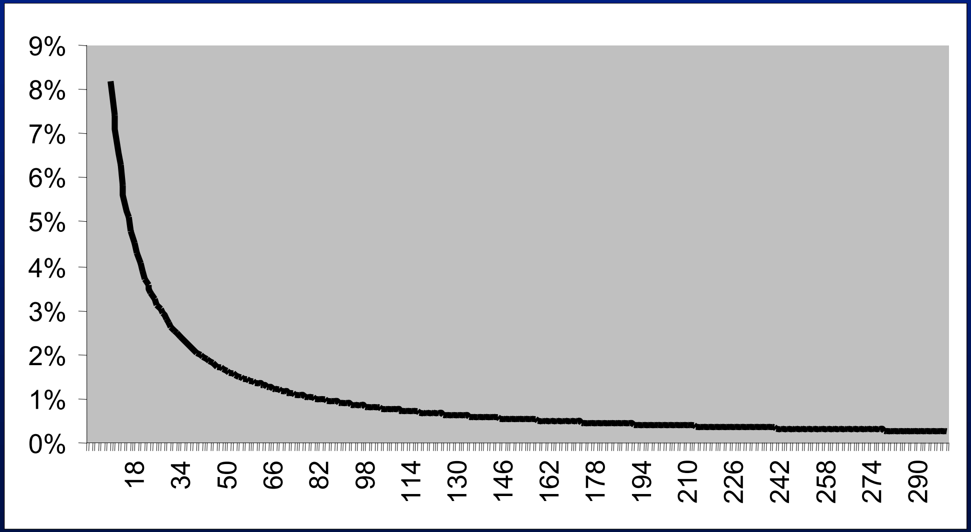
$$\Rightarrow \quad \psi$$

The condition is sufficient

CyRCE: The concentration index

HHI Property 2: $H(F) \leq \theta \Rightarrow \theta \leq$ Maximum loan $\leq \sqrt{\theta V}$

The tradeoff for lending the maximum loan to a single debtor, is at the expense of credit to all other debtors, which tends to zero as *N* increases.



This graph shows how fast decreases the amount of other loans under the hypothesis of a maximum loan when *N* increases.

CyRCE: Simple Model continued

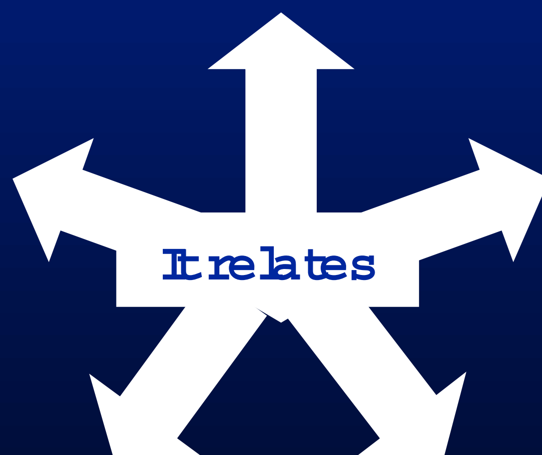
The expression

$$\psi \geq p + z_{\alpha} \sqrt{p(1-p)H(F)}$$

is attractive because:

Loan portfolio value at risk through: "z_{α}"

Capital Adequacy: "ψ"



Single Obligor Limits

Single loan default probabilities: "p"

Loan portfolio concentration: "$H(F)$"

CyRCE: Example

The average default probability for the 25 loans is 10.89%.

$$H(F) = \frac{4{,}728^2 + 7{,}728^2 + \ldots + 6{,}480^2}{130{,}164^2} = 0.0661$$

Obligor	Amount	Obligor	Amount
A1	4,728	A14	5,042
A2	7,728	A15	15,411
A3	5,528	A16	1,933
A4	5,848	A17	2,317
A5	3,138	A18	2,411
A6	3,204	A19	2,598
A7	4,831	A20	358
A8	4,912	A21	1,090
A9	5,435	A22	2,652
A10	5,320	A23	4,929
A11	5,765	A24	6,467
A12	20,239	A25	6,480
A13	1,800	**Total**	**130,164**

Assuming Normality and a 5%, confidence level, Capital adequacy requires:

$$\psi \geq 0.1089 + 1.96\sqrt{(0.1089)(1-0.1089)(0.0661)} = 26.6\%$$

Or:

$$K \geq (26.6\%)(130{,}164) = \$34{,}603$$

CyRCE: Example (continued)

Suppose K = 35,000, then:

Capitalization ratio: $\psi = \dfrac{K}{V} = \dfrac{35{,}000}{130{,}164} = 26.9\% \geq 26.6\%$ required

¿Is the portfolio too concentrated?

$$H(\mathbf{F}) = 0.0661 \leq \frac{(\psi - p)^2}{z_\alpha^2 p(1-p)} = \frac{(0.2689 - 0.1089)^2}{(1.96)^2(0.1089)(0.8911)} = 0.0687$$

¿What's the largest possible loan?

$$f^* = \left(\sqrt{0.0687}\right)(130{,}164) = \$34{,}108$$

¿Would there be single obligor limit compliance?

$$f_i \leq 0.0687 \times 130{,}164 = \$8{,}942$$

CyRCE: A General Model

- All loans have different default probabilities: $p_1, \ldots, p_N$.

  The mean and standard deviation of the portfolio loss per loan are:

$$\mu_i = p_i f_i \qquad \sigma_i = \sqrt{p_i(1-p_i)}\, f_i \qquad i = 1,\ldots,N$$

2. All portfolio losses can be correlated to each other though a covariance matrix.

$$\sigma_{i,j} = \text{Default Covariance between loan } i \text{ and loan } j = \sigma_i \sigma_j \rho_{i,j}$$

$\rho_{i,j}$: default correlation between loan *i* and loan *j*

CyRCE: Value at Risk

Approximating by the Normal distribution, the *value at risk* with confidence level α is:

$$VaR_\alpha = \underbrace{\sum_i p_i f_i}_{\text{EXPECTED LOSS}} + z_\alpha \sqrt{\underbrace{\sum_i \sigma_i^2}_{\text{VARIANCE}} + \underbrace{\sum_{i \neq j} \sigma_i \sigma_j \rho_{ij}}_{\text{COVARIANCE}}}$$

Using matrix notation:

$$VaR_\alpha = \underbrace{\pi^{\mathrm{T}} \mathbf{F}}_{\text{EXPECTED LOSS}} + z_\alpha \underbrace{\sqrt{\mathbf{F}^{\mathrm{T}} \mathbf{M} \mathbf{F}}}_{\text{UNEXPECTED LOSS}}$$

$\pi^{\mathrm{T}} = [p_1 \ldots p_N]^{\mathrm{T}}$

M: covariance matrix

CyRCE: Capital Adequacy

Capital Adequacy relation is now:

$$\psi \geq \frac{\pi^{T}\mathbf{F}}{V} + z_{\alpha}\sqrt{\frac{\mathbf{F}^{T}\mathbf{M}\mathbf{F}}{\mathbf{F}^{T}\mathbf{F}}}H(\mathbf{F})$$

Weighted average default probability of the loan portfolio $\bar{p}$

$$R(\mathbf{F},\mathbf{M}) = \frac{\mathbf{F}^{T}\mathbf{M}\mathbf{F}}{\mathbf{F}^{T}\mathbf{F}}$$

IHH

Herfindahl-Hirschman index

Rayleigh's quotient

Summarizes the variance-covariance effect for portfolio losses

CyRCE: Single obligor limits and Concentration

Under the general model, the concentration bound is:

$$\psi \geq \bar{p} + z_{\alpha}\sqrt{R(\mathbf{F},\mathbf{M})\,H(\mathbf{F})}$$

$$H(\mathbf{F}) \leq \frac{(\psi - \bar{p})^{2}}{z_{\alpha}^{2}\,R(\mathbf{F},\mathbf{M})}$$

$$\frac{\left[\text{Capitalization Ratio} - \text{Weighted average default probability of the loan portfolio}\right]^{2}}{\text{Confidence Level}^{2} \times \text{Variance-Covariance effect}}$$

CyRCE: Example

Rating	A	B	C	D	E	F	G
Mean (%)	1.65	3.00	5.00	7.50	10.00	15.00	30.00
Stand. Dev.(%)	12.74	17.06	21.79	26.34	30.00	35.71	45.83

Obligor	Amount	Obligor	Amount
A1	4,728	A14	5,042
A2	7,728	A15	15,411
A3	5,528	A16	1,933
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A11	5,765	A24	6,467
A12	20,239	A25	6,480
A13	1,800	**Total**	**130,164**

$$H(\mathrm{F}) = 6.61\%$$

$$\bar{p} = 10.89\% \qquad R(\mathrm{F}, \mathrm{M}) = 0.401$$

Assuming Normality and a 5% confidence level, Capital adequacy requires:

$$\psi \geq 0.1089 + 1.96\sqrt{(0.401)(0.0661)} = 42.78\%$$

Or:

$$K \geq (42.78\%)(130{,}164) = \$55{,}685$$

CyRCE: Example (continued)

Suppose K = 60,000, then

Capitalization ratio: $\psi = \frac{K}{V} = \frac{60{,}000}{130{,}164} = 46.10\% \geq 42.78\%$ required

¿Is the portfolio too concentrated?

$$H(\mathrm{F}) = 0.0661 \leq \frac{(\psi - \bar{p})^2}{z_\alpha^2 R(\mathrm{F}, \mathrm{M})} = \frac{(0.4610 - 0.1089)^2}{(1.96)^2 (0.401)} = 0.0805$$

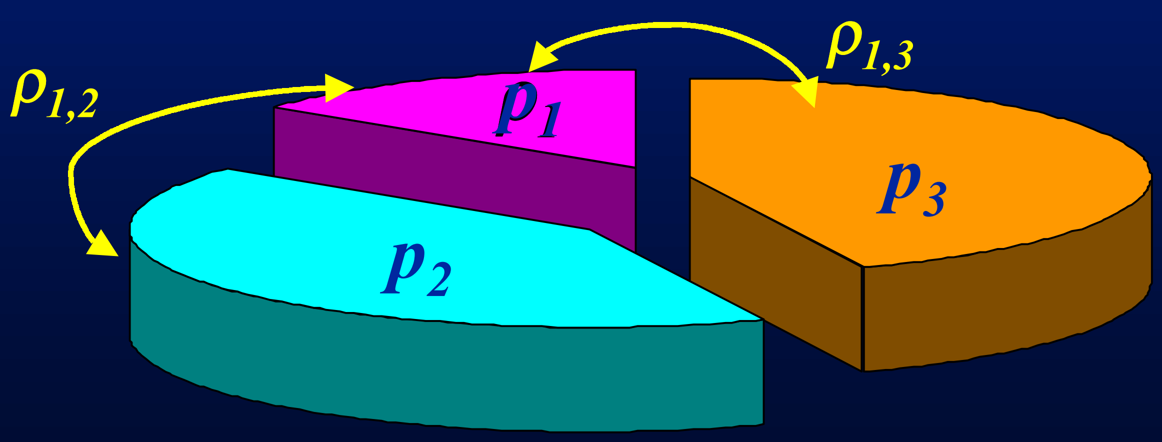
¿What's the single obligor limit?

$$f^* = (0.0805)(130{,}164) = \$10{,}482$$

CyRCE: Portfolio Segmentation

The loan portfolio can be partitioned <u>arbitrarily</u> in segments such that:

- Each group has its own default probabilities.
- The covariance matrix M includes two kinds of covariation:
  - Idiosincratic: among defaults within the same segment.
  - Extra-group: among defaults of different segments.



CyRCE: Value at Risk

The *value at risk* with confidence level α for each segment *j* is:

$$VaR_{\alpha}^{j} = \pi_j^{\mathrm{T}} \mathbf{F}_j + z_{\alpha} \sqrt{\mathbf{F}^{\mathrm{T}} \mathbf{R}_j \mathbf{F}}$$

EXPECTED LOSS

UNEXPECTED LOSS

π_j is the vector of default probabilities of segment *j*

F_j is the vector of the amounts of the loans in segment *j*

R_j is the matrix of the idiosincratic covariances in segment *j* and the default covariances between the loans of segment j with the loans of other segments.

CyRCE: Value at Risk (continued)

The matrix R_j has the following structure:

$$R_j = \begin{bmatrix} 0 & \cdots & C_{1,j} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ C_{j,1} & \cdots & M_j & \cdots & C_{j,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & C_{N,j} & \cdots & 0 \end{bmatrix}$$

M_j = Matrix of the idiosincratic covariances for the loans in segment j.

$C_{j,i}$ = Matrix of default covariances between the loans of segment j with the loans of segment i.

CyRCE: Capital Adequacy

After a bit of algebra, the Capital Adequacy relation per segment is:

$$\psi_j \geq \frac{\pi_j^{\mathrm{T}} \mathbf{F}_j}{V_j} + z_\alpha \sqrt{R(\mathbf{F}_j, \mathbf{M}_j) H(\mathbf{F})_j + 2 \sum_{j \neq i} \frac{\mathbf{F}_j^{\mathrm{T}} \mathbf{C}_{j,i} \mathbf{F}_i}{V_j^2}}$$

Weighted average default probability of the loans in segment j $\bar{p}_j$

Rayleighs quotient for segment j

HHI_j

Adjustment for Correlation

CyRCE: Single obligor limits and Concentration

The concentration bound per segment is:

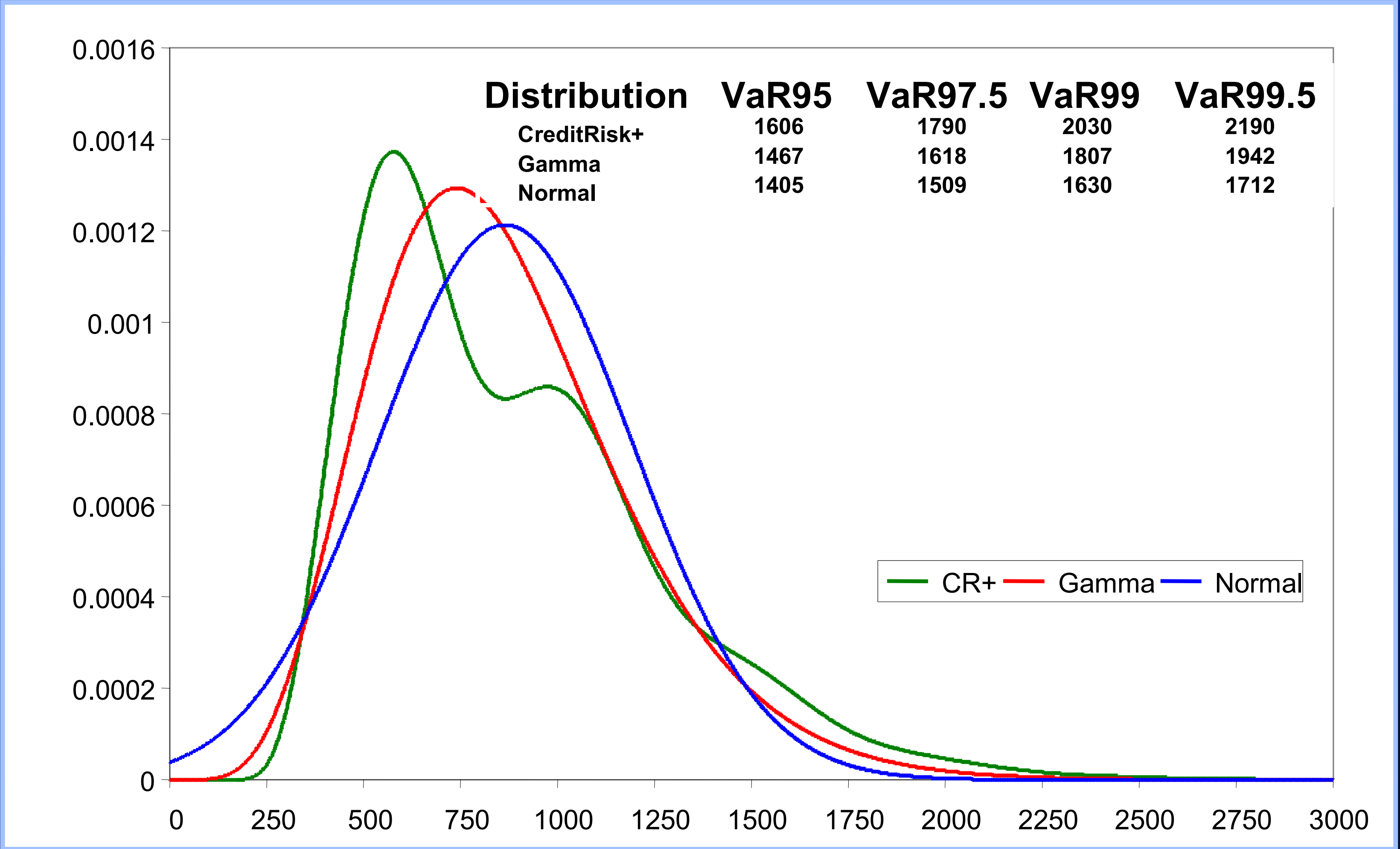
$$H(\mathrm{F}_j) \le \frac{(\psi_j - \bar{p}_j)^2}{z_\alpha^2 \, R(\mathrm{F}_j, \mathrm{M}_j)} - 2\sum_{j \neq i} \frac{\mathrm{F}_j^{\mathrm{T}} \mathrm{C}_{j,i} \mathrm{F}_i}{V_j^2 \, R(\mathrm{F}_j, \mathrm{M}_j)}$$

Bound

Adjustment for Correlation

Numerical comparison to CreditRisk+

For illustration purposes, an arbitrary sample of 1,320 loans was picked from SENICREB and VaR was calculated following both CreditRisk+ methodology and CyRCE methodology, using a Normal and a Gamma distribution.



Distribution	VaR95	VaR97.5	VaR99	VaR99.5
CreditRisk+	1606	1790	2030	2190
Gamma	1467	1618	1807	1942
Normal	1405	1509	1630	1712

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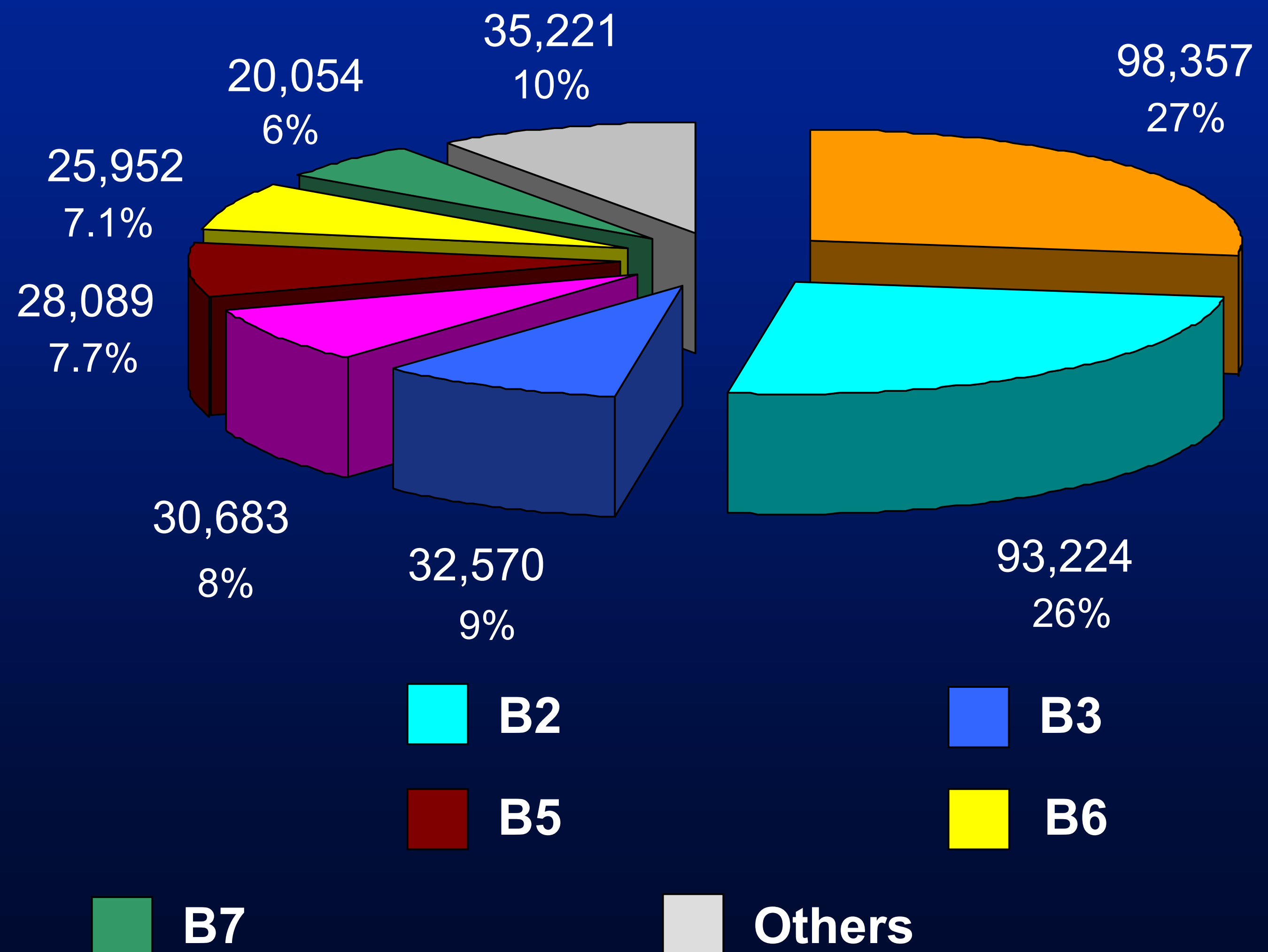
Distribution of the loan portfolio by major banks: SYSTEM

SYSTEM = 364,150 MP

November 2001



Loan Portfolio Segmentation

SYSTEM | **BANKS** | **ECONOMIC SECTOR**

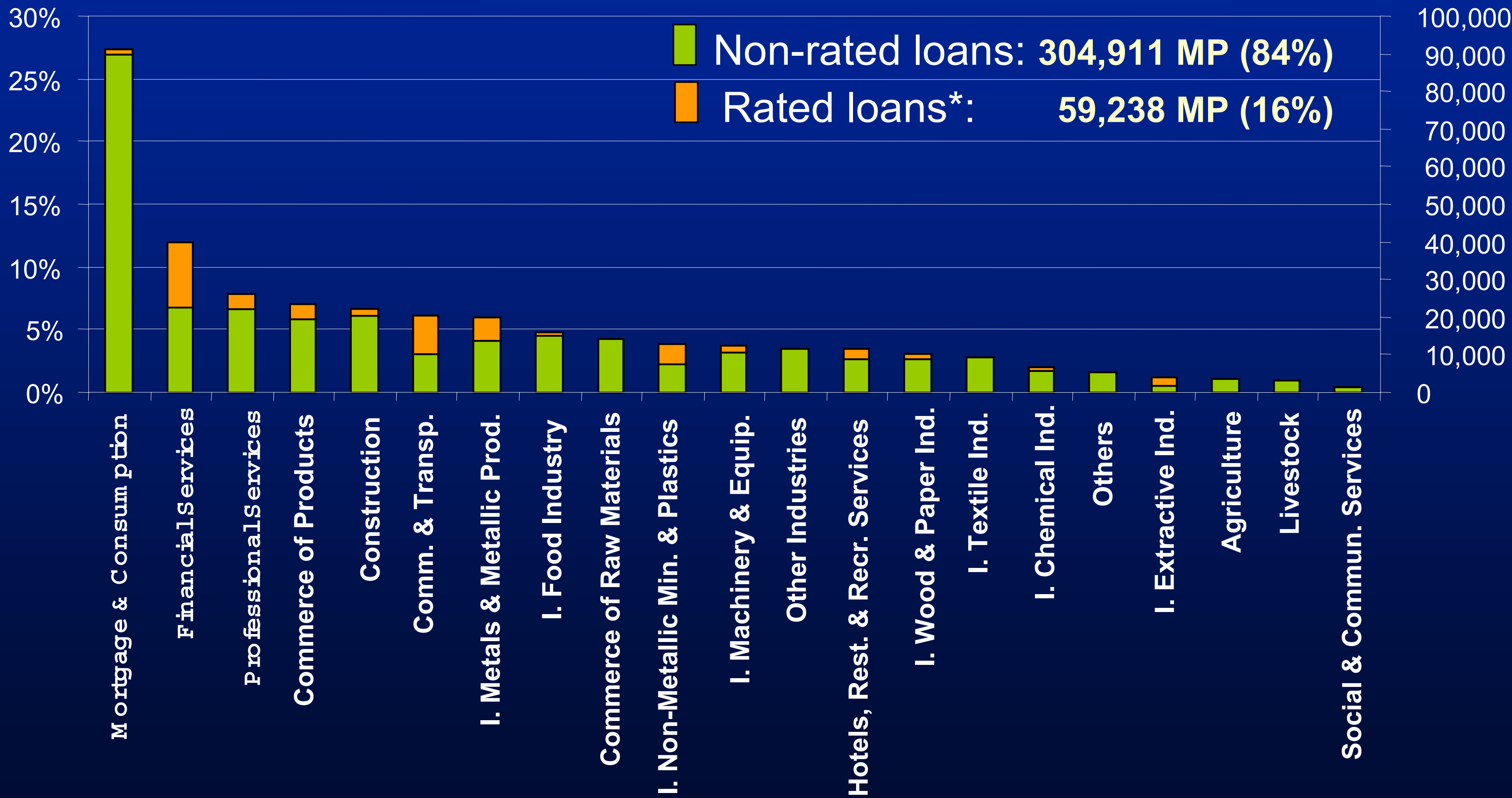
- **AGR/LIVESTOCK**
- **INDUSTRY**
- **CONSTRUCTION**
- **COMMERCE**
- **COMMUNICATION & TRANSPORT**
- **SERVICES**
- **OTHERS**
- **RATED LOANS**

- Agriculture
- Livestock, Forestry, Fishing & Hunting
- Extractive
- Food
- Textile
- Wood/Paper
- Products
- Chemical
- Raw Materials
- Non-Metallic Minerals & Plastics
- Mortgage/Consumption/Credit Cards
- Metals & Metallic Prod.
- Machinery & Equipment
- Other Industries
- Financial
- Professional/Technical/Personal
- Recreational/Hotels/Restaurants
- Example: Foreign Loans
- Social & Communal
- Group by five differentiated rates

Distribution of the loan portfolio by Economic Activity: SYSTEM

November 2001

Total Loan Portfolio: 364,150 MP



*Rated by Standard & Poor's, Moody's y Fitch

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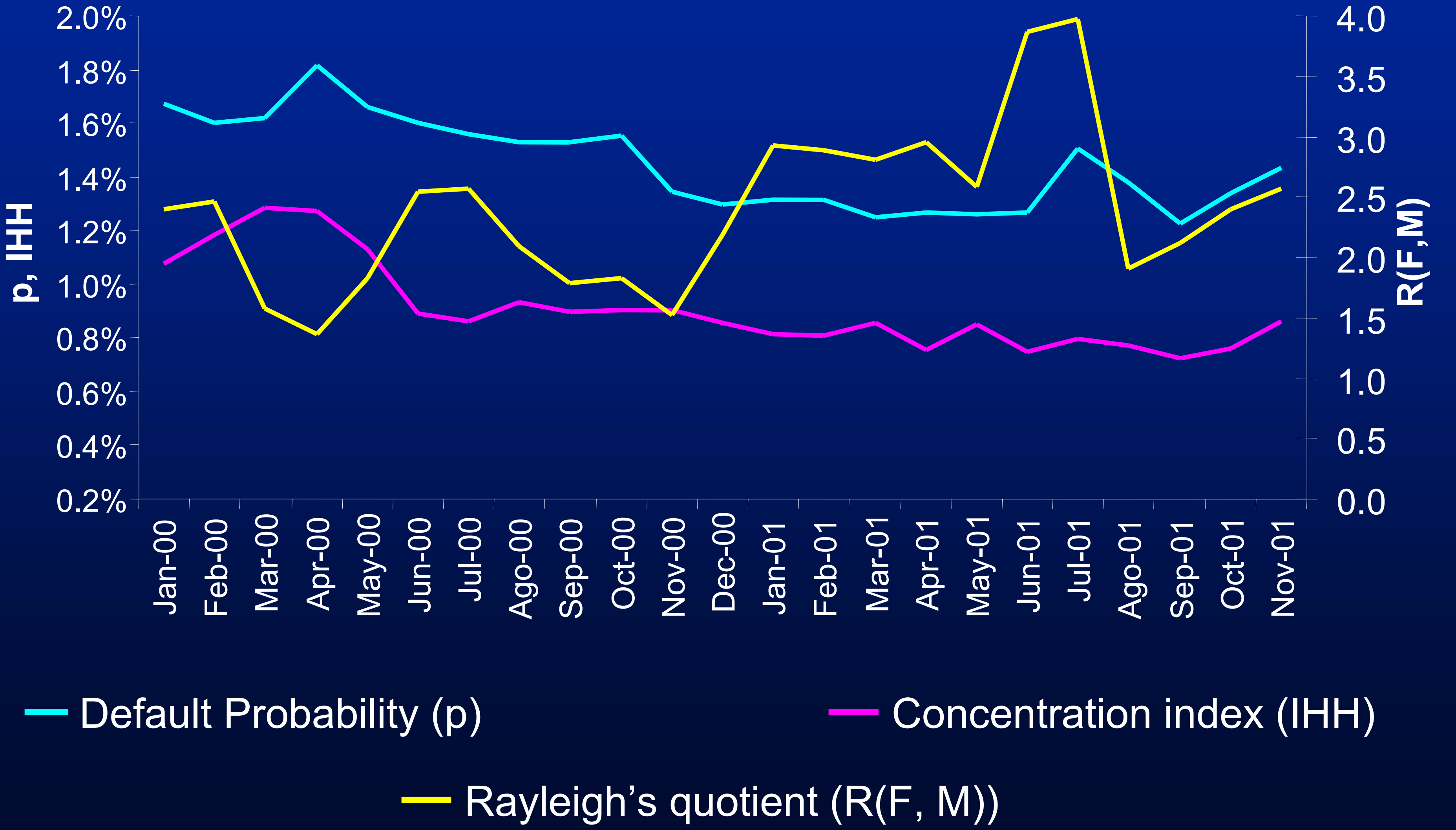
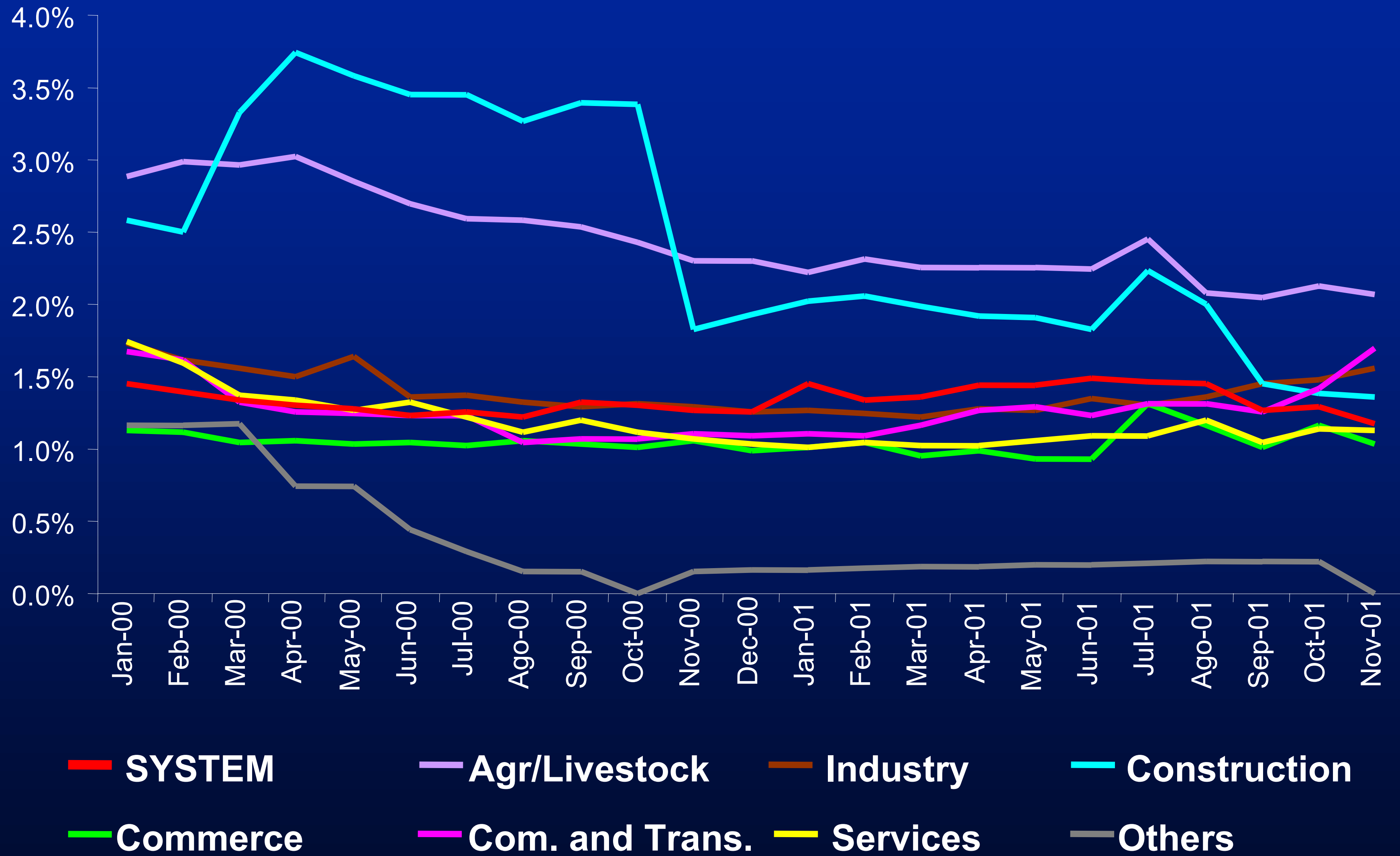
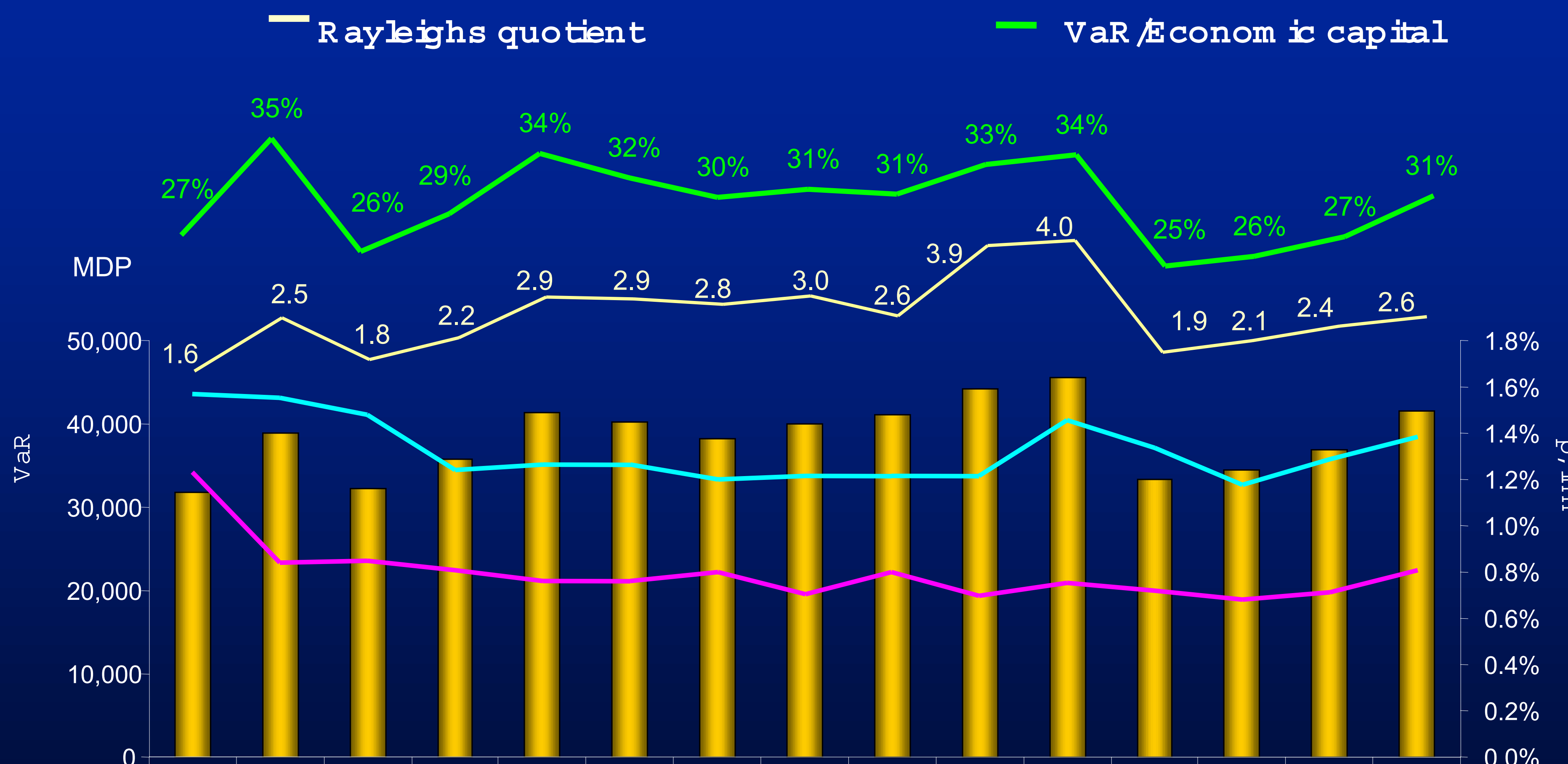
Default Probability, Concentration Index and Rayleigh's Quotient: SYSTEM



Weighted average default probability by Economic Activity

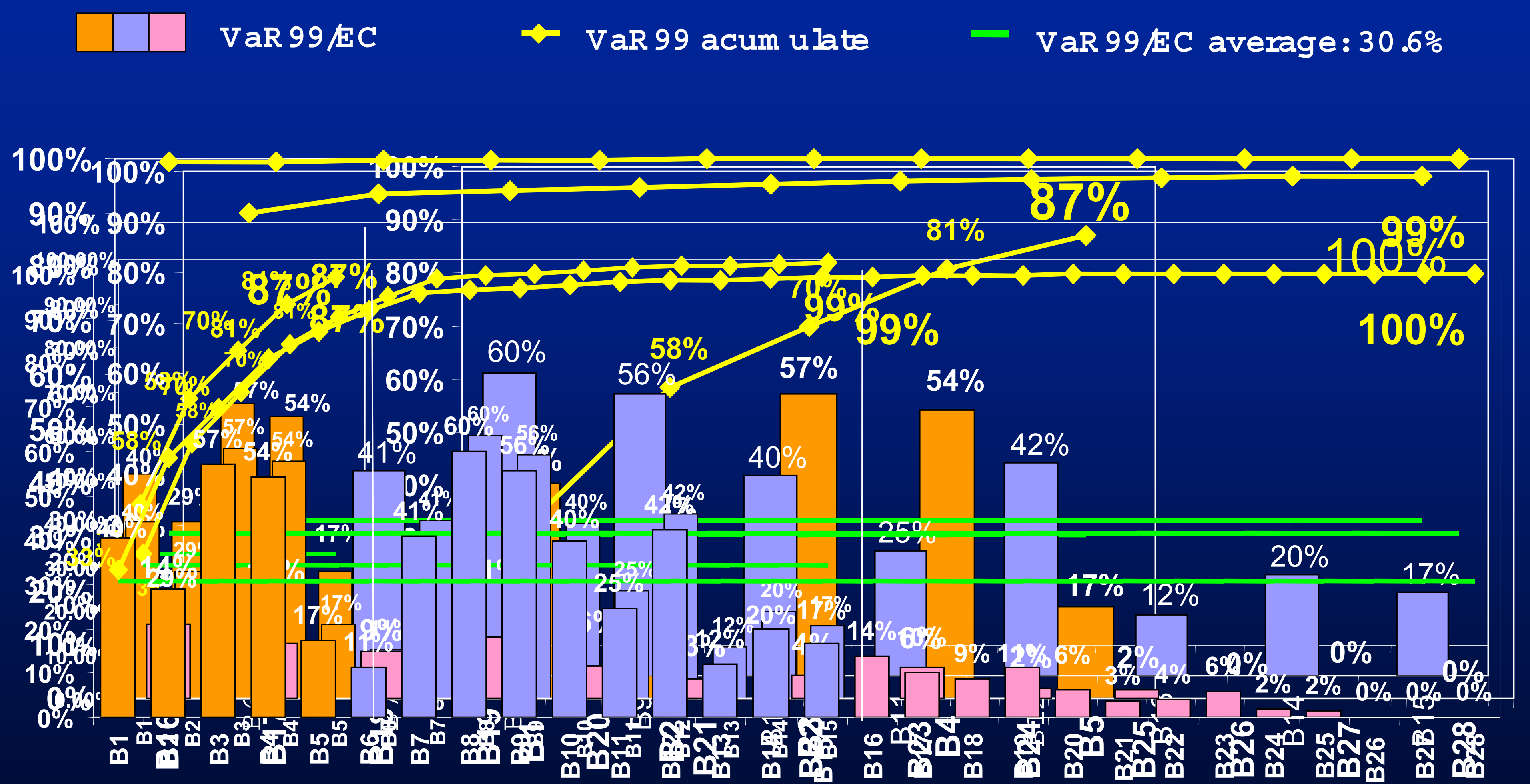


Risk, Default Probability, Concentration Index and Rayliegh's quotient: SYSTEM

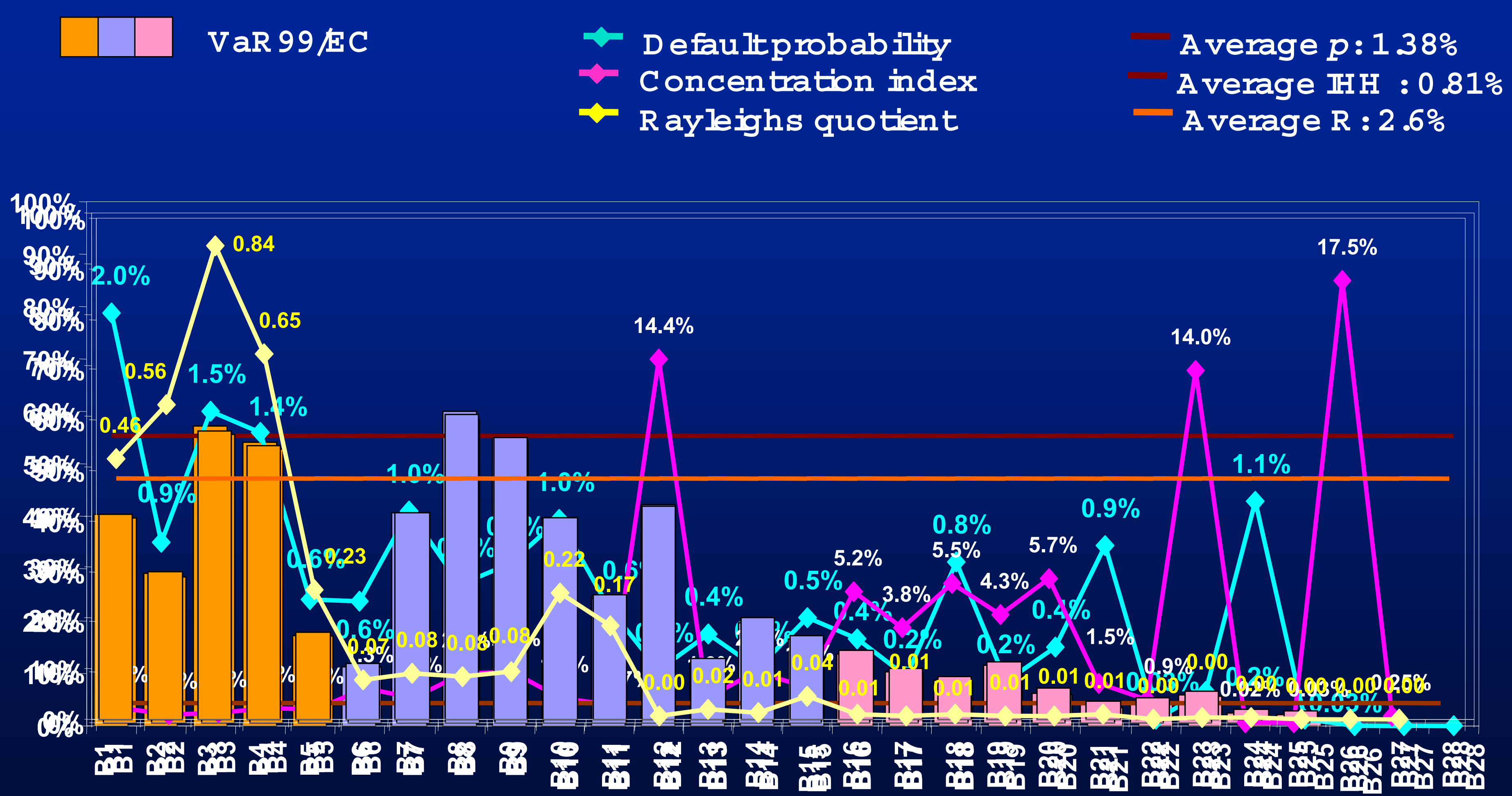


	Mar-00	Jun-00	Sep-00	Dec-00	Jan-01	Feb-01	Mar-01	Apr-01	May-01	Jun-01	Jul-01	Ago-01	Sep-01	Oct-01	Nov-01
VaR	31,744	38,938	32,283	35,828	41,344	40,230	38,183	40,032	41,029	44,305	45,519	33,279	34,477	36,965	41,584
p	1.6%	1.6%	1.5%	1.2%	1.3%	1.3%	1.2%	1.2%	1.2%	1.2%	1.5%	1.3%	1.2%	1.3%	1.4%
IHH	1.2%	0.8%	0.8%	0.8%	0.8%	0.8%	0.8%	0.7%	0.8%	0.7%	0.7%	0.7%	0.7%	0.7%	0.8%

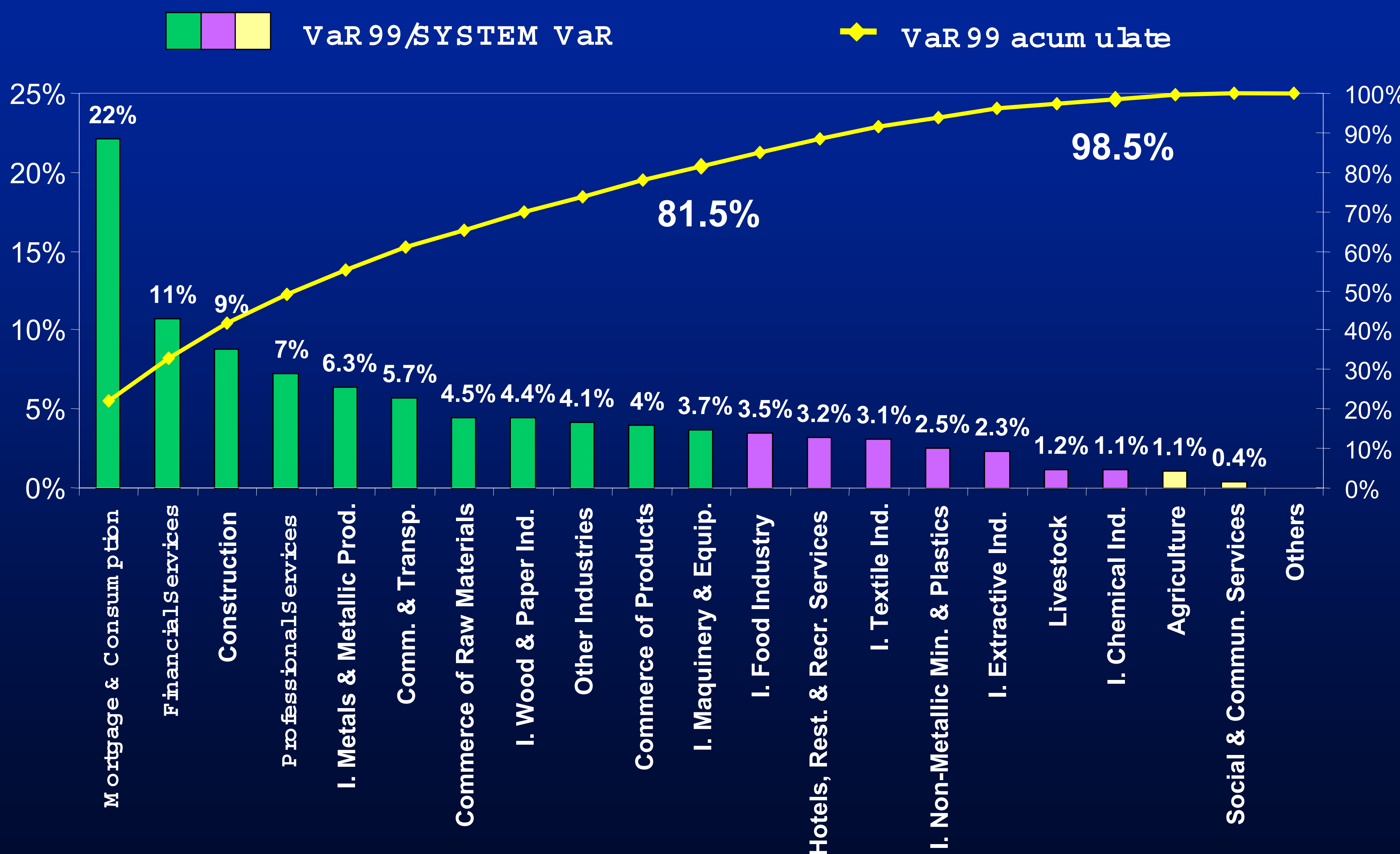
Contribution to System Risk by Institution: November 2001



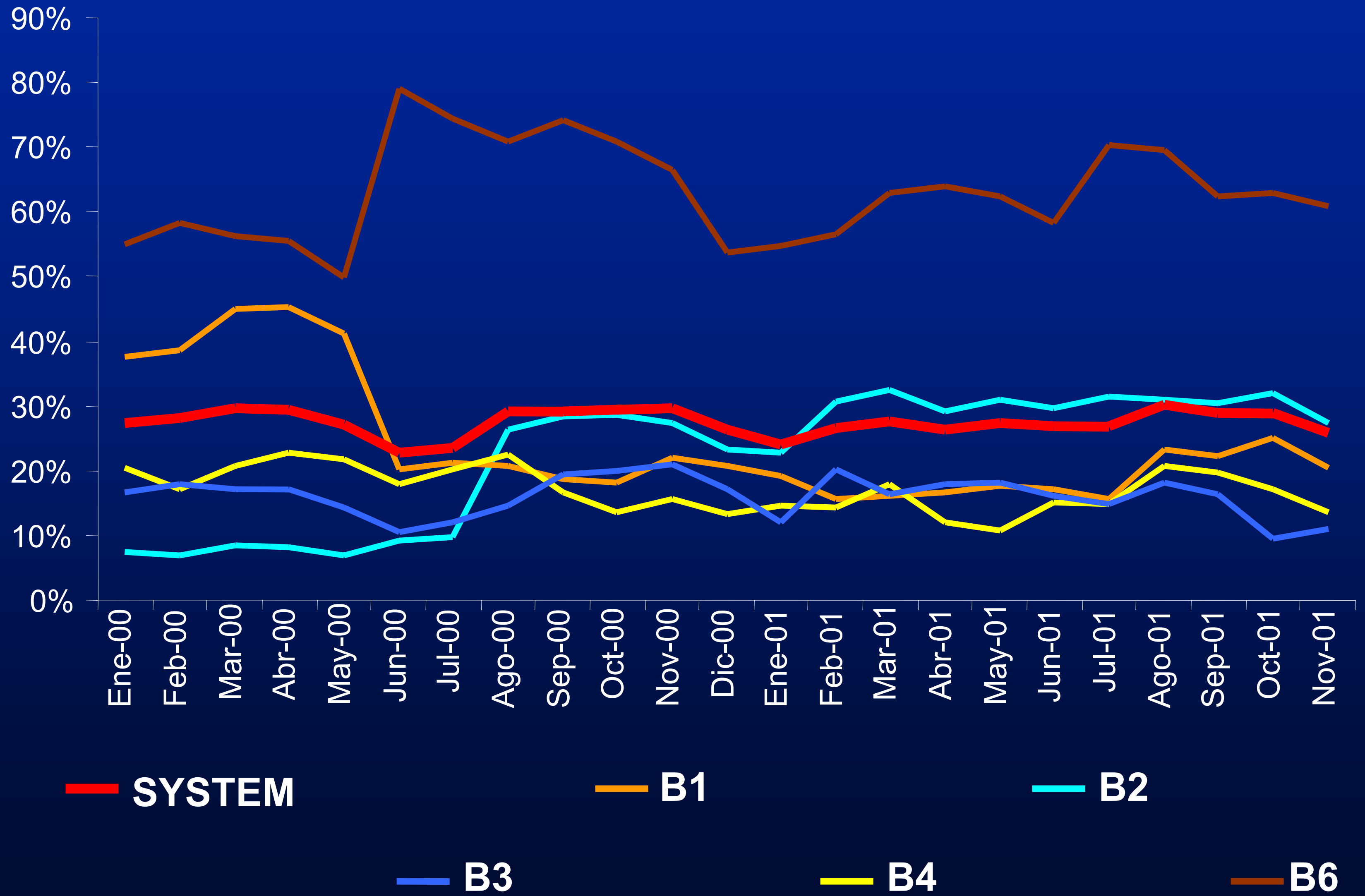
Default Probability, Concentration Index and Rayleigh's Quotient: November 2001



Contribution to System Risk by Economic Activity: November 2001



Capital adequacy: $(EC - VaR)/\text{Loan portfolio} \geq 0$



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