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MEASUREMENT OF RISK, SOLVENCY REQUIREMENTS AND ALLOCATION OF CAPITAL WITHIN FINANCIAL CONGLOMERATES

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Abstract

This paper addresses the allocation of solvency capital in multi-line financial businesses. Although this paper is uniformly applicable to financial enterprises of all types, the terminology in the paper is mainly that of insurance. The TailVaR risk measure is extended in a natural way to allocating capital to each of the business units. This method of allocation allocates capital in a way that is invariant over the method of decomposing the enterprise into business units. Analytic results are derived in the case of multivariate Normal risks. The key result of this paper is that the TailVaR-based proportional allocation of total required capital is identical to that based on mean-variance considerations analogous to the CAPM in the case of the multivariate Normal distribution. The allocation methodology results are then applied to a real bancassurance portfolio of 10 lines of business to illustrate the various concepts discussed in the paper.

This paper is to presented to the AFIR/ICA in March 2002 in Cancun, Mexico.

1. Introduction

The subject of the determination of risk capital has been of active interest to researchers, of interest to regulators of financial institutions, and of direct interest to commercial vendors of financial products and services.

At the international level, the actuarial and accounting professions and insurance regulators through the International Accounting Standards Board, the International Actuarial Association, and the International Association of Insurance Supervisors are all active in developing a framework for accounting and capital requirements for insurance companies. Similarly, the Basel Committee, and Bank of International Settlements have been developing capital standards for use by banks.

The concept of *Value-at-Risk* (VaR) has become the standard risk measure used to evaluate exposure to risk. In general terms, the VaR is the amount of capital required to ensure, with a high degree of certainty, that the enterprise doesn't become technically insolvent. The degree of certainty chosen is arbitrary. In practice, it can be a high number such as 99.95% for the entire enterprise, or it can be much lower, such as 95%, for a single unit within the enterprise. This lower percentage may reflect the inter-unit diversification that exists.

The promotion of concepts such as VaR has prompted the study of risk measures by numerous authors (e.g. Wang, 1996, 1997). Specific desirable properties of risk measures were proposed as axioms in connection with risk pricing by Wang, Young and Panjer (1997) and more generally in risk measurement by Artzner *et al* (1999).

In this paper, we consider a random variable X_j representing the negative of the possible profits, i.e. the possible losses, arising from a business unit identified with subscript j. Then the total or aggregate losses for n units combined is simply the sum of the losses for all units

$$X = X_1 + X_2 + \ldots + X_n.$$

The probability distribution of the aggregate losses depends not only on the distributions of the losses for the individual units but also on the inter-relationships between them. Correlation is one such measure of inter-relationship. Correlation is, however, a simple linear relationship that may not capture many aspects of the relationship between the variables. However, it does perform perfectly for describing inter-relationships in the case where the losses from the individual blocks form a multivariate Normal distribution. Although the Normal assumption is used extensively in connection with the modelling of changes in the logarithm of prices in the stock market, it may not be entirely appropriate for modeling many processes including insurance loss processes. As in the field of finance where most theory is based on Brownian motion resulting in Normal distributions, the Normal distribution model serves as a benchmark and provides insight into key relationships.

There are two broad approaches to the application of risk measurement to complex organizations such as insurance companies and banks. One approach is to develop a mathematical model for each of the risk exposures separately and assign a capital requirement to each exposure based on the study of that risk exposure. This is often called the *risk-based capital* (RBC) approach in insurance. The total capital requirement is the adjusted sum of the capital requirements for each risk exposure. Some offset may be possible due to the recognition that there may be a diversification or hedging effect of risks that are not perfectly correlated. The second approach uses a integrated model of the entire organization (the *internal model* approach. In this approach, a mathematical model is developed to describe the entire organization) integrating all business units in the company. The model incorporates all interactions between business units in the company. In either approach, an allocation of the total capital back to the units is necessary for a variety of business management or solvency management reasons. This paper focuses on that allocation.

2. Risk Measures

A *risk measure* is a mapping from the random variables representing the risks to the real line. A risk measure gives a single number that quantifies the risk exposure in a way that is meaningful for the problem at hand. The standard deviation of a distribution is such a measure of risk. One of the other most commonly used risk measures in the fields of finance and statistics is the *quantil*e or *Value-at-Risk.* This risk measure is the size of loss for which there is a small (e.g. 1%) probability of exceedence. For some time, it has been recognized that this measure suffers from serious deficiencies if losses are not Normally distributed.

Following Artzner *et al*. (1999), a *coherent risk measure* is defined as one that has the following properties for any two bounded loss random variables X and Y. Throughout this paper, the risk measure is denoted by the function $\rho(.)$. For this paper, it is convenient to think of $\rho(X)$ as the amount of solvency capital required for the risk X.

1. ***Subadditivity:***

$$\rho(X + Y) \le \rho(X) + \rho(Y)$$

This means that the capital requirement for two risks combined will not be greater than for the risks treated separately. This is necessary, since otherwise companies would have an advantage to disaggregate into smaller companies.

2. ***Monotonicity:***

If $X \le Y$ for all possible outcomes, then $\rho(X) \le \rho(Y)$.

This means that if one risk always has greater losses that another risk, the capital requirement should be greater.

3. ***Positive Homogeneity:***

For any positive constant λ, $\rho(\lambda X) = \lambda\rho(Y)$.

This means that the capital requirement is independent of the currency in which the risk is measured.

4. ***Translation invariance***

For any positive constant α, $\rho(X + \alpha) = \rho(X) + \alpha$.

This means that there is no additional capital requirement for an additional risk for which there is no uncertainty. In particular, by making X identically zero, the total capital required for a certain outcome is exactly the value of that outcome.

Risk measures satisfying these criteria are deemed to be coherent. There are many such risk measures. The classical VaR does not satisfy these criteria.

The *q*-quantile or VaR

The *q*-quantile, x_q, is the smallest value satisfying

$$\Pr\{X > x_q\} = 1 - q .$$

As a risk measure, x_q is the Value-at-Risk and is used extensively in financial risk management of trading risk over a fixed (usually relatively short) time period. It is not a coherent risk measure (see Artzner *et al*, 1997)

The conditional tail expectation or TailVaR

The conditional tail expectation is given by

$$E\left[X \middle| X > x_q\right]$$

This is called conditional tail expectation by Wirch (1997) and TailVaR by Artzner (1999). It can be seen that this will be larger that the VaR measure for the same vale of *q* described above since it is the VaR x_q plus the expected excess loss; i.e.,

$$E\left[X \middle| X > x_q\right] = x_q + E\left[X - x_q \middle| X > x_q\right].$$

TailVaR is a coherent measure in the sense of Artzner *et al* (1997). The papers by Artzner (1999) and Wirch and Hardy (1999) on coherent risk measures are potential sources of ideas for the application of this kind of risk measure.

Overbeck (2000) also discusses VaR and TailVaR as risk measures. He argues that VaR is an "all or nothing" risk measure, in that if the extreme event causing ruin occurs, there is no capital to cushion losses. He also argues that TailVaR provides a definition of "bad times” which are those where losses exceed some threshold, not using up all available capital. TailVaR the provides the expected excess loss over that threshold, when the threshold has been exceeded. One can define the threshold x_q as we have done above in the definition

$$\rho(X) = E\left[X \middle| X > x_q\right].$$

Alternatively, one can define the threshold by first establishing the quantity $\rho(X)$ by any method, and then solve to determine the threshold x_q which defines the “bad times” of Overbeck (2000).

3. Allocation of Capital

Consider now that the random variable X and the allocation of capital to the individual risks

$$X_1, X_2, \ldots, X_n$$

when the capital requirement $\rho(X)$ has been determined for the total risk X. Denault (2001) address this question by defining a set of desirable properties for an allocation methodology. He defines a *coherent allocation method* as one that possesses these properties.

Let $K = \rho(X)$ represent the risk measure for the total risk X. Let K_i denote the allocation of K to the i-th risk. The properties are:

1. ***Full allocation***

$$K_1 + K_2 + \ldots + K_n = K$$

This means that all of the capital is allocated to the risks.

2. ***No undercut***

$$K_a + K_b + \ldots + K_z \leq \rho(X_a + X_b + \ldots + X_z)$$

for any subset $\{a, b, \ldots, z\}$ of $\{1, 2, \ldots, n\}$.

This means that any decomposition of the total risk will not increase the capital from that if the risks stood alone.

3. ***Symmetry***

Within any decomposition, substitution of one risk X_i with and otherwise identical risk X_j will result in no change in the allocations.

4. ***Riskless allocation***

The capital allocation (in excess of the mean) to a risk that has no uncertainty is zero.

These properties seem to be reasonable and intuitive requirements for an allocation method. They are, however, not sufficient to characterize a single allocation method.

4. TailVaR Allocation

Overbeck (2000) discusses a natural extension of TailVaR, among other methods, as a basis of allocating capital to the blocks or lines of credit risk. Denault (2001) briefly mentions TailVaR as well, but focus on game-theoretic methods of allocating capital.

TailVaR allocation is based on the idea that the capital each risk should be based the contribution of the specific risk to the total capital, that is

$$K_j = E\left[X_j \middle| X > x_q\right].$$

This formula is not only simple; it is also intuitive. The capital required for each risk is precisely the expected contribution to the shortfall when a shortfall occurs. Overbeck (2000) calls this the "contribution to shortfall" method. This allocation method satisfies the four desired properties in the last section. Although Overbeck (2000) and Denault (2001) consider other plausible methods, this one seems to have the greatest natural appeal when the risk measure for the total of all risks is based on TailVaR.

5. Application to Multivariate Normal Risks

The Normal distribution is used extensively in financial applications. It arises naturally in connection with Brownian motion forms the basis of many economic models. In this section, we use the Normal distribution to model the distribution of the present value of losses for a risk. The risk could be an entire company, such as an insurance company or other financial institution, or it could be a much smaller unit such as a block of insurance policies.

Consider the aggregate risk

$$X = X_1 + X_2 + \ldots + X_n$$

where the X_js form a multivariate Normal distribution. Note that X itself also follows a Normal distribution. Denoting its mean and variance by μ and σ, it is straightforward to show that the TailVaR can be written as

$$K = E\left[X \middle| X > x_q\right] = \mu + a\sigma^2$$

where

$$a = \frac{f(x_q)}{1 - F(x_q)}$$

and $f(.)$ and $F(.)$ are the probability density function and the cumulative distribution function of the Normal distribution with mean μ and standard deviation σ.

To consider the individual allocations, it is sufficient to consider only the case with $n = 2$ by isolating one random variable (say X_1) and combining all the risks, except X_1, into the random variable X_2. This will simplify the notation considerably.

So consider the aggregate risk

$$X = X_1 + X_2$$

In this case, with a bit of calculation, one finds the allocation to risk 1

$$K_1 = E\left[X_1 \middle| X > x_q\right] = \mu_1 + a\sigma_1^2 (1 + \rho_{1,2} \frac{\sigma_2}{\sigma_1}) \cdot$$

where $\rho_{1,2}$ represents to correlation coefficient between X_1 and X_2.

Table 1 illustrates how the allocation works for different combinations of parameters. The column headings in Table 1 are as follows:

***Mean*₁**	Mean of X_1
***StdDev*₁**	Standard deviation of X_1
***Mean*₂**	Mean of X_2
***StdDev*₂**	Standard deviation of X_2
Corr	Correlation coefficient between X_1 and X_2
Prob	Cumulative probability value
TailVaR	Value of TailVaR
Pr*(TailVaR)*	Cumulative probability value at TailVaR
***Alloc*₁**	Allocation to risk 1
Pr*(Alloc*₁*)*	Cumulative probability at this allocation for risk 1
***Alloc*₂**	Allocation to risk 1
Pr*(Alloc*₂*)*	Cumulative probability at this allocation for risk 2

For the bivariate Normal model considered here, the size of the TailVaR for the total risk is, of course, dependent on the correlation coefficient.

If the two risks are uncorrelated, the capital allocation for the each risk is of the same form as the TailVaR for each if the risks taken separately on a stand-alone basis except that the factor a is based on the distribution of the sum of the two risks.

TABLE 1
Allocation of capital to two risks – 11 cases

$Mean_1$	$StdDev_1$	$Mean_2$	$StdDev_2$	*Corr*	*Prob*	*TailVaR*	Pr(*TailVaR*)	$Alloc_1$	Pr($Alloc_1$)	$Alloc_2$	Pr($Alloc_2$)
0	1	0	1	0	0.99	3.77	0.996	50%	0.97	50%	0.97
0	1	0	1	0.5	0.99	4.62	0.996	50%	0.99	50%	0.99
0	1	0	1	1	0.99	5.33	0.996	50%	0.996	50%	0.996
0	1	0	1	-0.5	0.99	2.67	0.996	50%	0.909	50%	0.909
0	1	0	1	-1	0.99	0	0.5	50%	0.5	50%	0.5
0	1	0	2	0.5	0.99	7.05	0.996	29%	0.978	71%	0.994
0	1	0	4	0.5	0.99	12.21	0.996	14%	0.959	86%	0.995
0	2	0	4	0.5	0.99	14.1	0.996	29%	0.978	71%	0.994
0	1	0	2	-0.5	0.99	4.62	0.996	0%	0.5	100%	0.99
0	1	0	4	-0.5	0.99	9.61	0.996	-8%	0.959	108%	0.995
0	2	0	4	-0.5	0.99	9.23	0.996	0%	0.978	100%	0.99

If the two risks are identical, the proportion allocated to each risk is always 50% of the total allocation independent of the correlation (see first five cases in Table 1). In general, this should also hold for any multivariate distribution with identical marginals.

A negative correlation will decrease the required capital to each risk compared to the case of independent risks. Compare case 1 and 4 or cases 6 and 9 or cases 8 and 11 in Table 1.

It is interesting to note that the total capital allocated to risk 1 can be less than the mean. The second last line (case 10) in Table 1 illustrates this point. This can only occur in the situation where the correlation coefficient is negative and satisfies

$$\rho_{1,2} \leq -\frac{\sigma_1}{\sigma_2}.$$

This means that in the usual situation where the standard deviation of the risk 1 is small relative to the standard deviation of the sum of the other risks (represented collectively by risk 2), if the correlation coefficient is sufficiently negative, any hedging provided by the risk will result in credit being given to that risk for hedging.

When the above inequality is replaced by equality, as in the last line (case 11) in Table 1, the allocation to risk 1 is exactly zero.

6. Allocation and the CAPM in the Multivariate Normal Case

We now revert back to the original notation involving all n risks. The subscript j refers to the j-th risk while the negative subscript $-j$ refers to all but the j-th risk so that

$$X_{-j} = X_1 + X_2 + \ldots + X_{j-1} + X_{j+1} + \ldots + X_n .$$

Then, by replacing subscript 1 by j and subscript 2 by $-j$, the allocation formula can be rewritten as

$$K_j = E\left[X_j \middle| X > x_q\right] = \mu_j + a\sigma_j^2 (1 + \rho_{j,-j} \frac{\sigma_{-j}}{\sigma_j}).$$

Now, denoting the correlation coefficient and the covariance between the j-th risk represented by X_j and the sum of all risks represented by X , by the symbols $\rho_{j,X}$ and $\sigma_{j,X}$ respectively, and recognizing that

$$\sigma_{j,X} = \sum_{i=1}^{n} \sigma_{i,j} = \sigma_j^2 + \sigma_{j,-j}$$

and

$$\sigma_X^2 = \sigma_j^2 + \sigma_{-j}^2 + 2\rho_{j,-j}\sigma_j\sigma_{-j} ,$$

it is easily shown that

$$K_j = E\left[X_j \middle| X > x_q\right] = \mu_j + a\sigma_{j,X} .$$

With a simple substitution, the capital allocation (in excess of the mean) to the j-th risk can be written in terms of the total capital requirement (in excess of the mean) for the sum of all risks as

$$K_j - \mu_j = \rho_{j,X} \frac{\sigma_j}{\sigma_X} (K - \mu) = \frac{\sigma_{j,X}}{\sigma_X^2} (K - \mu).$$

By letting

$$\beta_j = \rho_{j,X} \frac{\sigma_j}{\sigma_X} = \frac{\sigma_{j,X}}{\sigma_X^2}$$

denote the “internal beta,” the allocation formula reduces to

$$K_j - \mu_j = \beta_j (K - \mu).$$

The internal beta plays the same role as the "beta" in the capital asset pricing model (CAPM). In our case, the beta represents the proportion of the total capital allocated to the specific risk, noting that the "proportion" could be positive or negative, but that the sum of the proportions is exactly 1. This method of allocation is sometimes referred to as "covariance-based" allocation, without specific reference to CAPM; although, the connection with the CAPM is apparent.

One can also consider the "solvency price of risk" measuring the amount of capital required per unit of risk taken, analogous to the market price of risk in the CAPM framework. In the case of the Normal distribution, the relevant measure of risk is the standard deviation. Thus the solvency price of risk for risk j is given by

$$\frac{K_j - \mu_j}{\sigma_j} = \rho_{j,X} \frac{(K - \mu)}{\sigma_X}$$

demonstrating that the appropriate price of risk could be negative in the case where the specific risk acts as a natural hedge against movements in the sum X ;i.e. where the correlation coefficient is negative.

It is interesting that, at least in the case of the multivariate distribution, the allocation formula derived by starting with the TailVaR risk measure is identical to that of the CAPM, which is usually derived based on mean-variance considerations with no reference to VaR or TailVaR. At this point, it remains unknown how these results can be generalized to other distributions.

7. Application to a Real Bancassurance Company

In order to provide some more insight into actuarial application of the above methodology, we have applied it to real company data[1]. The are 10 business units (lines of business), representing a range of insurance and other related financial products. The ten relevant random variables are present values of the amounts necessary to assure solvency over a fixed time horizon with a confidence level of 99.865%, corresponding to three standard deviations above the mean for a Normal model. The joint distribution of these ten random variables is developed using a complex simulation model incorporating a variety of sources of variation. The correlation matrix of $\rho_{i,j}$, and standard deviations of loss ratios are given in Table 2.

It is interesting to note from the Table 2 that, while correlations are generally small, there are numerous negative correlations. In particular, risks 6 and 9 have a correlation of -0.46, the largest (in absolute value) correlation between any two lines of business. In Table 2, the line labelled "Corr. With Sum" contains the correlation coefficients $\rho_{j,X}$ between the individual lines and the company as a whole (the sum of the ten random variables). Note that lines 8 and 9 are negatively correlated with the company as a

[1] With thanks to Stuart Wason, MMC Enterprises Inc.

TABLE 2
Correlations and standard deviations - 10 lines of business

	Correlation matrix									
Line of Business	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
1	1	-0.00	0.12	-0.02	0.18	-0.26	-0.12	0.11	0.08	-0.03
2	-0.00	1	0.05	0.27	0.02	0.08	0.16	-0.21	-0.17	-0.15
3	0.12	0.05	1	0.01	-0.11	0.10	0.03	-0.12	-0.09	-0.12
4	-0.02	0.27	0.01	1	0.22	0.05	0.09	-0.11	0.13	-0.23
5	0.18	0.02	-0.11	0.22	1	-0.11	0.01	-0.03	0.14	-0.01
6	-0.26	0.08	0.10	0.05	-0.11	1	0.07	-0.09	-0.46	-0.16
7	-0.12	0.16	0.03	0.09	0.01	0.07	1	-0.25	0.08	0.14
8	0.11	-0.21	-0.12	-0.11	-0.03	-0.09	-0.25	1	-0.16	-0.16
9	0.08	-0.17	-0.09	0.13	0.14	-0.46	0.08	-0.16	1	0.21
10	-0.03	-0.15	-0.12	-0.23	-0.01	-0.16	0.14	-0.16	0.21	1
Corr. with Sum	0.25	0.69	0.09	0.36	0.16	0.40	0.39	-0.18	-0.07	0.18
SD (Loss Ratio)	7.47%	3.73%	16.12%	2.51%	82.14%	8.05%	3.36%	11.85%	12.29%	5.17%
Premium in$MM	$36.00	$120.40	$1.30	$52.42	$0.70	$48.09	$47.40	$8.08	$8.64	$50.15
SD in $MM	$2.69	$4.49	$0.21	$1.32	$0.57	$3.87	$1.59	$0.96	$1.06	$2.59

whole, while line 2 is strongly positively correlated with the company as a whole due to its dominant size as measured by premium income.

The mean and 99.865% quantile of the loss distributions for each of the ten lines of business are given in Table 3. The difference between these is the capital (in excess of the mean) required using the 99.865% quantile when treating each line as a separate company on a stand-alone basis. The sum of these capital amounts is $62.02 million. When the ten lines are combined into one company the resulting corresponding capital requirement is $27.24 million, reflecting a significan benefit gained by pooling risks.

TABLE 3
Required capital for stand-alone and combined lines

Line	Mean	99.865%	Capital
1	25.69	33.75	8.06
2	37.84	51.30	13.46
3	0.85	1.48	0.63
4	12.70	16.65	3.95
5	0.15	1.87	1.72
6	24.05	35.67	11.62
7	14.41	21.73	7.32
8	4.49	8.24	3.75
9	4.39	8.11	3.72
10	9.56	17.35	7.79
Sum	**134.13**	**196.15**	**62.02**
Combined	**134.13**	**161.39**	**27.24**

The TailVaR allocation method developed in this paper is used to allocate the $27.24 million of capital. The values of the proportion allocated to the ten lines (the values of the beta) are given in Table 4. Note that both lines 8 and 9 have a negative allocation and that the allocation to risk 2 is by far the largest. Table 4 also gives the correlations of each line of business with the total company. To obtain the values of beta, the correlation coefficients are scaled down by the ratio of the standard deviation of the line to the standard deviation of the total company.

TABLE 4
Allocation percentages

Line	1	2	3	4	5	6	7	8	9	10
Beta	10.13%	45.95%	0.29%	6.97%	1.36%	22.85%	9.16%	-2.56%	-1.16%	7.02%
Corr.	25.37%	68.96%	9.17%	35.67%	15.94%	39.73%	38.67%	-18.04%	-7.34%	18.23%

It is interesting to compare this method of allocation with other more naïve methods. Table 5 gives results when the total required capital for the combined company is allocated in proportion to the TailVaR, the variance and the standard deviation for each line considered on a stand-alone basis. None of these methods can give a negative allocation. Table 5 shows that line 2 has a much greater allocation than for any of these naïve methods. This is due to the high positive correlation between this line and the total company. None of the other methods involves correlation or other interaction.

TABLE 5
Comparison of allocation methods

Line	1	2	3	4	5	6	7	8	9	10
Beta	10.13%	45.95%	0.29%	6.97%	1.36%	22.85%	9.16%	-2.56%	-1.16%	7.02%
Prop. to TailVaR	12.96%	36.10%	0.08%	3.10%	0.59%	26.88%	4.56%	1.64%	2.02%	12.07%
Prop. to Var	13.00%	21.70%	1.02%	6.37%	2.78%	18.73%	11.80%	6.05%	6.00%	12.56%
Prop. to SD	13.89%	23.18%	1.08%	6.80%	2.97%	20.00%	8.24%	4.95%	5.49%	13.40%

It is also interesting to examine the "solvency price of risk for each line" both on a stand-alone" basis as well as on a combined basis. These are given in Table 6. Note that the solvency price of risk on a stand-alone basis is about 3 for seven of the ten lines of business. This suggests that the marginal distribution for these risks is close to the Normal distribution, since the price of risk was defined to be exactly 3 based on the 99.865% quantile being exactly 3 standard deviations beyond the mean for the Normal distribution. Lines of business 7, 8 and 9 are somewhat skewed since the since the solvency price of risk exceeds 3 significantly. This empirical observation suggests that further study of allocation methods under non-Normal assumptions is likely warranted.

TABLE 6
Solvency price of risk

Line	1	2	3	4	5	6	7	8	9	10
Stand Alone	3.00	3.00	3.01	3.00	3.00	3.00	4.59	3.92	3.50	3.00
Combined	1.03	2.79	0.37	1.44	0.64	1.61	1.56	-0.73	-0.30	0.74

Table 6 also illustrates the reduction in the solvency price per unit of risk that results from diversification and correlations. Note particularly that two of the three skewed distributions actually have a negative net cost (due to the negative correlation) when combined with the entire company. This suggests that the skewness of those two lines may actually work in favour of the company, from a solvency perspective rather than against it. This is somewhat counter-intuitive.

8. Observations and Conclusions

TailVaR is only one of many possible coherent risk measures. However, it is particularly well suited to solvency applications. Historically, solvency analysis has been frequently focused on the quantile, whether using “probability of ruin” or VaR; that is, focused only on the extreme tail, intentionally ignoring the shape of the distribution. The quantile methods have been shown to be inadequate in many ways. The simple extension to TailVaR retains the focus on the tail and adds one element of the shape of the tail in excess of the quantile, namely, the mean excess or shortfall.

The TailVaR-based allocation using the average contribution to the shortfall seems to be a nature method in the sense that it fairly allocates solvency costs to the sources of those costs. In the case where a line of business is negatively correlated with the company as a whole, the line will receive a “credit” in its allocation as a “reward” for providing a partial hedge. This method satisfies desirable properties for a coherent allocation. In particular, the method will allocate to any business unit an amount of allocated capital that does not depend on the way in which the organization is decomposed into smaller and smaller units.

The key result of this paper is that in the case of the multivariate Normal distribution, the percentage allocation to each line of business using this method of allocation is the same as would be obtained by using a covariance-variance-based approach, as in the CAPM. The multivariate Normal distribution serves as a benchmark in many finance and insurance applications.

The numerical example of the company with 10 lines of business illustrates some of the key concepts. One observation arising from the example is that some distributions may depart significantly from the Normal distribution, but that this departure may work either for or against the company as a whole depending on the sign of the relevant correlation with the company as a whole.

As a final note, the reader is cautioned that this measure of risk allocation should not be used to measure return-on-equity or to reward managers of the individual business units. The reward for hedging and diversification across business units (the 10 lines in the example) belong with the manager of these unit mangers. The managers of each business unit should be rewarded on the basis of the pooling of the sub-units within the business unit.

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