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Measurement of Risk, Solvency Requirements and Allocation of Capital within Financial Conglomerates

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A New Millennium. A New Challenge for Actuaries

Purpose of this talk

- Examine overall approaches to capital needs;
- Suggest an overall approach;
- Suggest a new methodology;
- Examine allocation question;
- Examples.

Solvency: Top-down or Bottom-up?

- How should ***total solvency requirement*** be determined:
  - As ***sum of parts*** for each policy?
    - with some adjustment for correlation?
    - with some for adjustment for stress testing?
  - In ***aggregate*** over whole company?
    - With mechanism for allocation
      - to each line of business?
      - to each policy type?
      - to each individual policy?
    - This is a capital allocation problem

Bottom-up

- Traditional method in insurance:
  - margins contained in liability reserves through conservative assumptions
  - specific formulas for additional capital for specific risk exposures (e.g. RBC)
  - sensitivity and stress testing
  - Little consideration of non-product risk

Top-down

- Focus on solvency of entire enterprise
  - Could include insurance and other companies
  - Can include both product and non-product risks
  - Meets needs of insurance regulator; i.e. protection of policyholders
- But it requires
  - Looking at entire enterprise
  - Sophisticated models
  - Computer modelling and simulation

Top-down

- Traditional VaR models build up large model from components using ***multivariate Normal*** distribution
  - ***Correlation*** between parts can be reflected
  - But complicated ***interactions*** may not be adequately captured
- Integrated (internal) modelling is likely necessary
  - Exogenous factors (economic scenario generator; e.g. Wilkie model)
  - Company-specific factors (e.g. book of business)

Solvency measures

- Total balance sheet requirement is some amount? Usually actuaries think in terms of the ***probability of ruin*** or some other measure.
  - VaR uses quantile (e.g. 99%) meaning ruin probability is 1%
- If quantile is used, how to allocate capital to all business units?
  - Need a measurement tool that will allocate capital in a sensible way (and also give corresponding quantiles for each risk)

Criteria for Capital Allocation

- Consider a number of risks
- The total capital requirement for the combined risks should be smaller than sum for each free-standing risk.
  - Otherwise, there is an incentive to decompose company.
- The capital allocation to each risk should be smaller than the capital requirement for the same free-standing risk.
  - Otherwise, it may be advantageous to pull out specific risks from company.

Criteria for Capital Allocation (cont'd)

- Sum of capital allocation for each risk should be exactly the capital for the total risk.
- Allocation should be invariant under all decompositions of enterprise.
- Identical risks should have same allocation.
- Allocation for comonotonic risks should be additive.

Some formulas

- Consider sum of all loss random variables for the *n* business units

$$X = X_1 + X_2 + \ldots + X_n .$$

- Each X_j can be positive (loss) or negative (gain).
- Each X_j represents PV of losses for all (or some) future years.

Recommendation for total balance sheet requirement

- Use TailVaR (CTE) as risk measure
  - Find x_q satisfying

$$\Pr\{X > x_q\} = 1 - q$$

where X represents loss to the insurer.

  - Total balance sheet requirement is

$$\mathrm{E}\left[X \mid X > x_q\right]$$

TailVaR

$$\begin{aligned} \text{TailVaR} &= \mathrm{E}\left[X \middle| X > x_q\right] \\ &= x_q + \mathrm{E}\left[X - x_q \middle| X > x_q\right] \\ &= \text{VaR} + \text{expected "shortfall"} \end{aligned}$$

- Expected ***shortfall*** is the ***net stop-loss premium*** for excess losses given that a stop-loss claim occurs.
- The ***trigger point*** x_q can be thought of as the point at which the current assets are just sufficient (on average) to cover current liabilities.

Properties of TailVar

- TailVar is a ***coherent*** risk measure.
  - Subadditive. Capital for two risks is not larger than for each risk separately.
  - Risk with no uncertainty requires no extra capital.
  - Invariant under location and scale tranformations, e.g. changing currencies.
  - Additive for comonotonic risks.

Capital allocation under TailVar

- Total loss for the enterprise is sum of losses for each risk

$$X = X_1 + X_2 + \ldots + X_n$$

- Total balance sheet requirement for risk *j* is

$$E\left[X_j \middle| X > x_q\right]$$

- Allocation to each line of business is the expected contribution to the "shortfall" when the trigger point is exceeded.

Properties

- Capital requirement is this allocation minus reserves, however calculated.
- Allocation incorporates all sources of variation and correlations.
- Allocation is invariant under all methods of subdivision of the company.
- Allocation is easily calculated as a part of simulation exercise.
- TailVar is a coherent risk measure.

Numerical Example

- Consider two identical risks, each Normally distributed with mean 0 and variance 1. For each risk separately:

Prob $1-q$	Total Bal Reqt	VaR x_q	Expected Shortfall
10.00%	1.75	1.28	0.47
1.00%	2.67	2.33	0.34
0.10%	3.37	3.09	0.28
0.01%	3.95	3.72	0.23

Example (cont'd)

Prob $1\text{-}q$	Correlation Corefficient	VaR x_q	Total Balance Sheet Reqt	Allocation to each risk
1%	100%	4.65	5.33	2.67
1%	75%	4.35	4.99	2.49
1%	50%	4.03	4.62	2.31
1%	25%	3.68	4.21	2.11
1%	0%	3.29	3.77	1.88
1%	-25%	2.85	3.26	1.63
1%	-50%	2.33	2.67	1.33
1%	-75%	1.64	1.88	0.94
1%	-100%	0.00	0.00	0.00

Analytic Results for Normal Distribution

$$K = E\left[X \middle| X > x_q\right] = \mu + a\sigma^2$$

where

$$a = \frac{\phi(x_q)}{1 - \Phi(x_q)}.$$

Analytic Results for Multivariate Normal Model

- It is sufficient to consider only the case with

$n = 2$ by combining all the risks, except the first one, into the random variable X_2 .

- Then

$$X = X_1 + X_2$$

and

$$K_1 = E\left[X_1 \middle| X > x_q\right] = \mu_1 + a\sigma_1^2 (1 + \rho_{1,2} \frac{\sigma_2}{\sigma_1})$$

Analytic Results for Multivariate Normal Model

- If $\rho_{1,2} < 0$

then $K_1 = E\left[X_1 \middle| X > x_q\right] < \mu_1 + a\sigma_1^2$

- If $\rho_{1,2} < -\frac{\sigma_1}{\sigma_2}$

then $K_1 = E\left[X_1 \middle| X > x_q\right] < \mu_1$

Numerical Illustration

$Mean_1$	$StdDev_1$	$Mean_2$	$StdDev_2$	Corr	Prob	TailVaR	Pr(TailVaR)	$Alloc_1$	$Pr(Alloc_1)$	$Alloc_2$	$Pr(Alloc_2)$
0	1	0	1	0	0.99	3.77	0.996	50%	0.97	50%	0.97
0	1	0	1	0.5	0.99	4.62	0.996	50%	0.99	50%	0.99
0	1	0	1	1	0.99	5.33	0.996	50%	0.996	50%	0.996
0	1	0	1	-0.5	0.99	2.67	0.996	50%	0.909	50%	0.909
0	1	0	1	-1	0.99	0	0.5	50%	0.5	50%	0.5
0	1	0	2	0.5	0.99	7.05	0.996	29%	0.978	71%	0.994
0	1	0	4	0.5	0.99	12.21	0.996	14%	0.959	86%	0.995
0	2	0	4	0.5	0.99	14.1	0.996	29%	0.978	71%	0.994
0	1	0	2	-0.5	0.99	4.62	0.996	0%	0.5	100%	0.99
0	1	0	4	-0.5	0.99	9.61	0.996	-8%	0.959	108%	0.995
0	2	0	4	-0.5	0.99	9.23	0.996	0%	0.978	100%	0.99

Analytic Results for Multivariate Normal Model

- For *n* risks:

$$K_j = E\left[X_j \middle| X > x_q\right] = \mu_j + a\sigma_j^2 \left(1 + \rho_{j,-j} \frac{\sigma_{-j}}{\sigma_j}\right)$$

or:

$$K_j - \mu_j = \rho_{j,X} \frac{\sigma_j}{\sigma_X} (K - \mu)$$

Analytic Results for Multivariate Normal Model

- Finally,

$$K_j - \mu_j = \beta_j (K - \mu)$$

- This looks like CAPM with "internal" beta

$$\beta_j = \rho_{j,X} \frac{\sigma_j}{\sigma_X}$$

Example - A Real Case

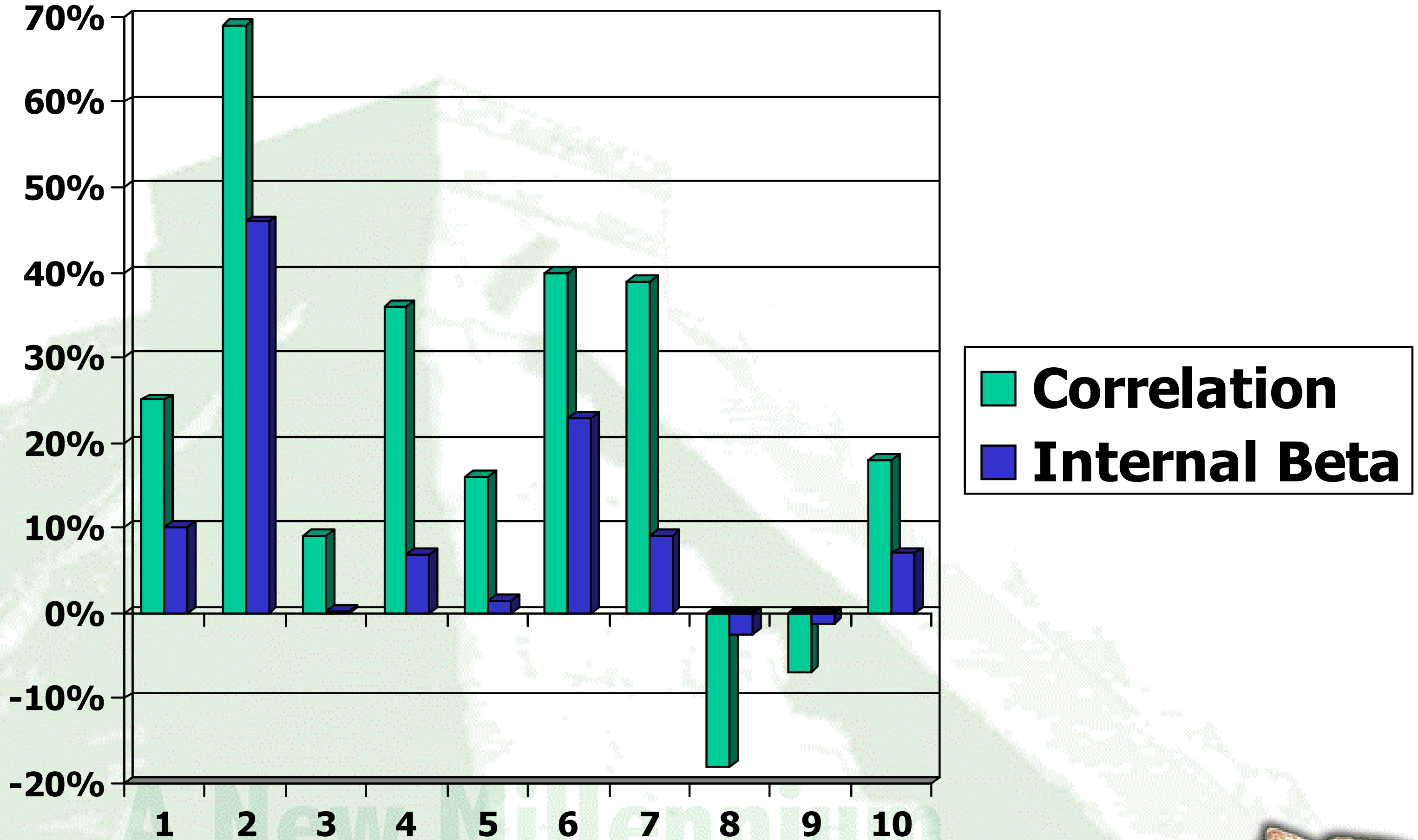
					Correlation matrix					
Product	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	-0.00	0.12	-0.02	0.18	-0.26	-0.12	0.11	0.08	-0.03
2	-0.00	1	0.05	0.27	0.02	0.08	0.16	-0.21	-0.17	-0.15
3	0.12	0.05	1	0.01	-0.11	0.10	0.03	-0.12	-0.09	-0.12
4	-0.02	0.27	0.01	1	0.22	0.05	0.09	-0.11	0.13	-0.23
5	0.18	0.02	-0.11	0.22	1	-0.11	0.01	-0.03	0.14	-0.01
6	-0.26	0.08	0.10	0.05	-0.11	1	0.07	-0.09	-0.46	-0.16
7	-0.12	0.16	0.03	0.09	0.01	0.07	1	-0.25	0.08	0.14
8	0.11	-0.21	-0.12	-0.11	-0.03	-0.09	-0.25	1	-0.16	-0.16
9	0.08	-0.17	-0.09	0.13	0.14	-0.46	0.08	-0.16	1	0.21
10	-0.03	-0.15	-0.12	-0.23	-0.01	-0.16	0.14	-0.16	0.21	1
Corr. with Sum	0.25	0.69	0.09	0.36	0.16	0.40	0.39	-0.18	-0.07	0.18
SD (Loss Ratio)	7.47%	3.73%	16.12%	2.51%	82.14%	8.05%	3.36%	11.85%	12.29%	5.17%
Premium in$MM	$36.00	$120.40	$1.30	$52.42	$0.70	$48.09	$47.40	$8.08	$8.64	$50.15
SD in $MM	$2.69	$4.49	$0.21	$1.32	$0.57	$3.87	$1.59	$0.96	$1.06	$2.59

Results for each line and combined portfolio

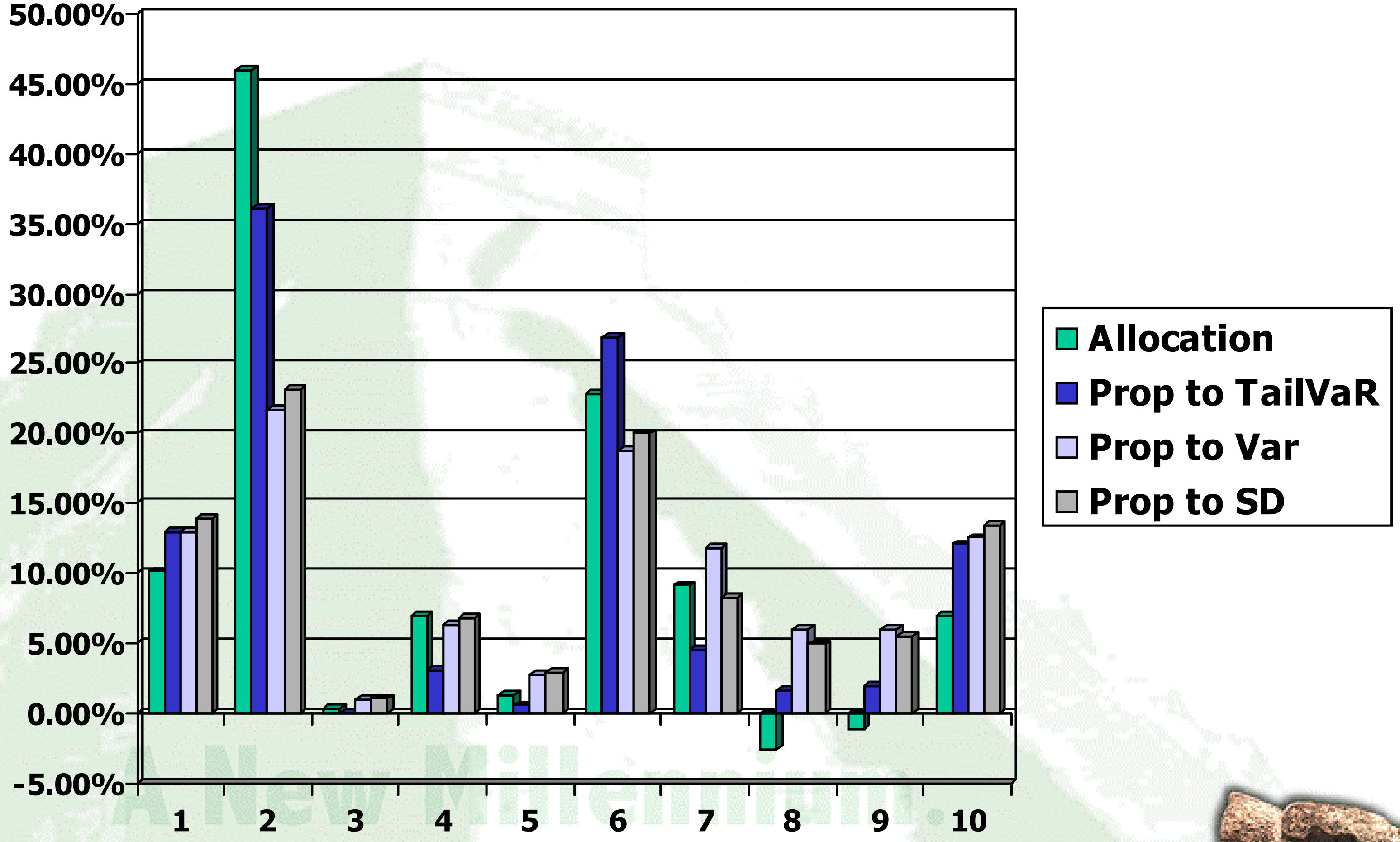
Line	Mean	99.865%	Capital
1	25.69	33.75	8.06
2	37.84	51.30	13.46
3	0.85	1.48	0.63
4	12.70	16.65	3.95
5	0.15	1.87	1.72
6	24.05	35.67	11.62
7	14.41	21.73	7.32
8	4.49	8.24	3.75
9	4.39	8.11	3.72
10	9.56	17.35	7.79
Total	134.13	196.15	62.02

All lines	Mean	99.865%	Capital
combined	134.13	161.39	27.24

Correlations and Internal Beta



Allocation Comparisons



A New Millennium

A New Challenge for Actuaries

“Regulatory” price of risk

- Consider the standard deviation as the unit of risk.
- The regulatory price of risk is the amount of capital required for each risk unit:

$$\frac{K_j - \mu_j}{\sigma_j}$$

- This is analogous in the one-period CAPM to

$$\frac{r_j - r_f}{\sigma_j}$$

“Regulatory” price of risk

Line	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Pre	3.00	3.00	3.01	3.00	3.00	3.00	4.59	3.92	3.50	3.00
Post	1.03	2.79	0.37	1.44	0.64	1.61	1.56	-0.73	-0.30	0.74

Conclusions

- Capital should be allocated exactly as would be done by the CAPM, except that the total capital is based on TailVaR while CAPM is based on variance.
- Methodology provides a coherent framework for BOTH determination of total capital as well as allocation to business units.
- Note: Other methods exist, but are often based on optimization of some objective function. None use our approach.

Implementation issues

- This top-down approach requires major computing resources in practice:
  - Simulation models with some analytics.
  - Consistent approach with trading risk management practices used currently.
- Long term-direction, but with coherent theoretical framework.
  - Applicable to any combination of institutions in a conglomerate.
  - Useful for both regulation and internal risk management.

Further ongoing work

- Sensitivity to non-normality
  - Especially if some risks have much heavier tails than others
- Allocation of capital to each future year in the horizon
  - Can be done