FAIR VALUATION OF THE SURRENDER OPTION EMBEDDED IN A GUARANTEED LIFE INSURANCE PARTICIPATING POLICY

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Abstract

In this paper we deal with the problem of valuing the surrender option embedded in a participating life insurance policy with a minimum interest rate guaranteed. This feature is an American-style put option that enables the policyholder to sell back the contract to the insurer at the surrender value. By means of a recursive binomial tree à la Cox, Ross and Rubinstein (1979) we compute, first of all, the total price of the contract, which includes also a "bonus" option. Then this price is split into the value of three components: the *basic contract*, the *bonus option* and the *surrender option*. The numerical implementation of the model allows us to catch some comparative statics properties and to tackle the problem of suitably fixing the contractual parameters in order to obtain the premium computed by insurance companies according to standard actuarial practice.

1 Introduction

Life insurance contracts and pension plans are often very complex contingent-claims that embed several financial options, both of European and of American style.

A typical example of European (put) option is implied by the maturity guarantees present in most types of equity-linked life insurance products. The importance of an accurate valuation of such guarantees is witnessed by a very

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large number of papers devoted to this issue that have followed the pioneering work by Brennan and Schwartz (1976, 1979a, 1979b) and Boyle and Schwartz (1977). For a categorization of the literature on equity-linked life insurance contracts with minimum guarantees see, e.g., Bacinello and Persson (1998).

Another example of European (call) option is implied by the bonus mechanism that characterizes policies with profits. This feature has been studied, for instance, by Briys and de Varenne (1997), Grosen and Jørgensen (2000, 2001), Miltersen and Persson (2000), and Bacinello (2000).

In particular, Bacinello (2000) analyses a life insurance endowment policy with a minimum interest rate guaranteed in which both the benefit and the periodical premiums are annually adjusted according to the performance of a special investment portfolio. Under the Black and Scholes (1973) framework Bacinello (2000) expresses, first of all, the fair price of such a policy in terms of one-year call options, and then derives a very simple closed-form relation that characterizes fair contracts. However, as a concluding remark, Bacinello (2000) points out that an important issue connected to participating policies which has not been dealt with in the paper is constituted by the presence of a surrender option. A surrender option is an American-style put option that entitles its owner (the *policyholder*) to sell back the contract to the issuer (the *insurer*) at the *surrender value*. The fair valuation of such an option, as well as an accurate assessment of the surrender values, are clearly crucial topics in the management of a life insurance company, both on the solvency and on the competitiveness side.

The aim of the present paper is just to fill this gap. More in detail, we consider the single-premium version of the contract analysed by Bacinello (2000) and define, first of all, a rule for computing the surrender values, which introduces an additional contractual parameter in the model. Then, by modelling the assets à la Cox, Ross and Rubinstein (1979), we obtain a recursive algorithm for computing the fair price of the whole contract. Of course, this algorithm explicitly uses death and survival probabilities, since the contract can be surrendered only if it has not been surrendered yet and the insured is still alive. As like as in Bacinello (2000), the fair price of the corresponding participating contract without the surrender option is expressible in closed-form, so that the value of the surrender option can be obtained residually. In this way the total price is split into the values of three components: the *basic contract* (i.e., without profits and surrender), the *bonus option*, and the *surrender option*. The numerical implementation of the model with a Pentium III 800 MHz shows that the results obtained can be quite accurate if the term of the contract is not very long. Moreover, the problem of choosing a set of contractual parameters that lead to a given level for the premium emphasized by Bacinello (2000) can also be numerically solved within the same model.

As far as we are aware, the problem of valuing the surrender option embedded in life insurance products has already been tackled, under different assumptions and by various methodologies, by Albizzati and Geman (1994), Grosen and Jørgensen (1997, 2000), and Jensen, Jørgensen and Grosen (2001).

Albizzati and Geman (1994) consider a financial contract with a guaranteed interest rate ("contrat à taux garanti") proportional to the initial yield on a zero-coupon bond with the same maturity. Taking into account both initial expenses and taxes, Albizzati and Geman (1994) compare, at any given future date, the (deterministic) final value of the contract with the final value of a new one, having the same maturity and acquired by reinvesting the (guaranteed) surrender value at the prevailing market conditions. The financial uncertainty is then given by the evolution of the price of a zero-coupon bond with a fixed maturity. In particular, under an Heath, Jarrow and Morton (1992) model with deterministic volatility, Albizzati and Geman (1994) derive a closed-form expression for the price of a European-style surrender option (i.e., of an option exercisable only at a fixed date) and then use pooling arguments for "averaging" this price with respect to all possible exercise dates.

Grosen and Jørgensen (1997) consider instead a unit-linked contract with a minimum interest rate guaranteed. This contract can be surrendered at any time before its maturity, and the minimum guarantee is effective also in case of early termination. Under the Black and Scholes (1973) framework, Grosen and Jørgensen (1997) express the total value of the minimum guarantee and the surrender option as the price of a standard American put option in an adjusted Black and Scholes (1973) economy in which the market rate is replaced by its spread over the minimum guaranteed interest rate.

Finally, a participating contract embedding a surrender option is also analysed, in the Black and Scholes (1973) framework, and priced by means of a "Monte Carlo + binomial lattice" approach in Grosen and Jørgensen (2000), by a finite difference approach in Jensen, Jørgensen and Grosen (2001).

The present paper is organized as follows. In Section 2 we describe the structure of the contract and define all the liabilities that the insurer has to face. Section 3 is devoted to the presentation of our valuation framework. In Section 4 we derive the fair value of the contract and of all its components describing, in particular, our recursive algorithm. Section 5 is devoted to the presentation of some numerical results that allow us i) to catch the comparative statics properties of the model, ii) to discuss about the possibility of suitably choosing the contractual parameters in order to obtain the premium computed by insurance companies according to standard actuarial practice. Finally, Section 6 concludes the paper hinting at some problems involved by the extension of the model to periodical premium contracts.

2 The structure of the contract

Consider a life insurance endowment policy issued at time 0 and maturing T years after (at time T). As it is well known, under this contract the insurer is obliged to pay a specified amount of money (*benefit* or *sum insured*) to the beneficiary if the insured dies within the term of the contract or survives the maturity date. More precisely, we assume that, in case of death during the t-th year of contract, the benefit is paid at the end of the year, i.e., at time t ($t = 1, ..., T$); otherwise it is paid at maturity T.

We denote by C_1 the "initial" sum insured, payable in case of death during the first year of contract, and by C_t the benefit payable at time t ($t = 2, ..., T$). While C_1 is given, for $t>1$ C_t is contingent on the performance of a special portfolio of assets (*reference portfolio*, henceforth). The insurer directly manages this portfolio, and shares the profits with the policyholders. To see how this profit-sharing is realized, we first introduce the following notation: i (≥ 0) represents the annual compounded *technical* interest rate, g_t is the rate of return on the reference portfolio during the t-th year of contract, and η, between 0 and 1, identifies a participation coefficient. At the end of each policy year (except the last one), if the insured is still alive, the benefit is "adjusted" for the subsequent year at a rate δ_t so defined:

$$\delta_t = \max\left\{\frac{\eta g_t - i}{1+i}, 0\right\}, \quad t = 1, ..., T-1. \tag{1}$$

The following relation links then the benefits pertaining to consecutive years:

$$C_t = C_{t-1}(1 + \delta_{t-1}), \quad t = 2, ..., T. \tag{2}$$

Obviously, C_t can also be expressed directly, so that its path-dependence is immediately perceptible:

$$C_t = C_1 \prod_{k=1}^{t-1}(1 + \delta_k), \quad t = 2, ..., T. \tag{3}$$

We observe that it is standard, for insurance companies, to compute the (net) single premium as the expected value, with respect to a suitable mortality distribution, of the initial benefit C_1 discounted with the technical rate i. In this case a return at the rate i is credited to the policyholder since the beginning. Then, taking into account the adjustment mechanism and disregarding the surrender possibility, the total return granted to the policyholder during the t-th year of contract (except the year in which the benefit is paid) is given by

$$(1 + i)(1 + \delta_t) - 1 = \max\{i, \eta g_t\},$$

so that i can be interpreted as a minimum interest rate guaranteed. Moreover, if we consider the surrender option and the fact that the premium implicitly includes a compensation for it, we argue that the minimum interest rate guaranteed is even greater than i.

However, it must be pointed out that this interpretation of i is correct only if the premium is computed as described above. If instead the premium is different, for instance greater, we can still state that there is a minimum guarantee provision in the contract since δ_t cannot be negative, but no more that the total rate of return on the policy in a given year is bounded from below by i. In particular, in the following section, we will neglect any kind of interpretation for i and simply consider it a contractual parameter that intervenes in the definition of the liabilities. We will then compute the fair value of all the liabilities, and only in the numerical section we will discuss about the problem of suitably choosing the contractual parameters in order that the fair premium equals the one described above.

Coming now to the surrender conditions, we assume that surrender takes place (if the contract is still in force) at the beginning of the year, just after the announcement of the benefit for the coming year. Usually the surrender value depends on the level of the benefit at the surrender date and on this date as well, on maturity, sometimes also on the age attained by the insured, and finally on one or more contractual parameters. For instance, it could be the current benefit discounted from maturity to the surrender date with a suitable rate, or a percentage of the mathematical reserve of the policy. In the numerical section we will consider both these situations, in which there is only one contractual parameter (the discount rate or, respectively, the percentage to apply to the mathematical reserve). We denote this parameter by ρ, so that the surrender value at the beginning of the $(t+1)$-th policy year (i.e., at time t) can be represented as

$$R_t = f(C_{t+1},\, t,\, T,\, x+t,\, \rho), \quad t = 0, ..., T-1, \tag{4}$$

where the function f will be specified in Section 5.

We remark that, according to our assumptions, surrender can take place also at time 0, just after the payment of the single premium. However, if the contract is fairly priced (in particular, as we will see in the next section, if arbitrage opportunities are ruled out of the market), R_0 is obviously less than the premium, so that this is not an actual possibility for a rational and non-satiated policyholder. Moreover, the surrender rule f and the contractual parameter ρ can be fixed in such a way that the surrender values are penalizing (to different degrees) or not. In this connection, we point out that a too high level of penalization does not at all compromise the fairness of the contract since it very likely implies a zero-value for the surrender option, but it could seriously jeopardize the marketability of the policy. Therefore, as we have already stated in Section 1, the problem of choosing an adequate level for the contractual parameter ρ (given the surrender rule f) is also a crucial topic in the design of the product under scrutiny.

3 The valuation framework

The contract described in the previous section is a typical example of claim contingent both on mortality and on financial risk. While the mortality risk determines the moment in which the benefit is due, the financial risk affects both the amount of the benefit and the surrender decision. We assume, in fact, that financial and insurance markets are perfectly competitive, frictionless (in particular no taxes, no transaction costs such as, e.g., expenses and relative loadings of the insurance premiums, short-sale allowed), and free of arbitrage opportunities. Moreover, all the agents are supposed to be rational and non-satiated, and to share the same information. Therefore, in this framework, the surrender decision can only be the consequence of a rational choice, taken after comparison, at any time, between the total value of the policy (including the option of surrendering it in the future) and the surrender value.

As it is standard in actuarial practice, we assume that mortality does not affect (and is not affected by) the financial risk, and that the mortality probabilities depend only on the age of the insured. We denote by x the entry age (at time 0), by ${}_tp_y$ the probability that the insured is still alive at age $y+t$ conditioned on the event that he/she is alive at age y, by ${}_{h/t}q_y$ the probability that the insured dies between ages $y+h$ and $y+h+t$ being alive at age y, and set $p_y={}_1p_y$ and $q_y={}_{0/1}q_y$. We assume that these probabilities are extracted from a *risk-neutral* mortality measure, i.e., that all insurance prices are computed as expected values with respect to this specific measure. If, in particular, the insurance company is able to extremely diversify its portfolio in such a way that mortality fluctuations are completely eliminated, then the above probabilities coincide with the "true" ones. Otherwise, if mortality fluctuations do occur, then the "true" probabilities are "adjusted" in such a way that the premium, expressed as an expected value, is implicitly charged by a *safety loading* which represents a compensation for accepting mortality risk. In this case the adjusted probabilities derive from a *change of measure*, as like as what happens in the Financial Economics environment; that is why we have called them "risk-neutral".

Coming now to the financial set-up, we assume that the rate of return on risk-free assets is deterministic and constant, and denote by r the annual compounded riskless rate. The financial risk which affects the policy under scrutiny is then generated by a stochastic evolution of the rates of return on the reference portfolio. In this connection, we assume that it is a well-diversified portfolio, split into *units*, and that any kind of yield is immediately reinvested and shared among all its units. Therefore the reinvested yields increase only the unit-price of the portfolio but not the total number of units, that changes when new investments or withdrawals are made.

These assumptions imply that the rates of return on the reference portfolio are completely determined by the evolution of its unit price. Denoting by G_τ this unit-price at time τ (≥ 0), we have then:

$$g_t = \frac{G_t}{G_{t-1}} - 1, \quad t = 1, ..., T-1. \tag{5}$$

For describing the stochastic evolution of G_τ, we choose the discrete model by Cox, Ross and Rubinstein (1979), universally acknowledged for its important properties. In particular it may be seen either as an "exact" model under which "exact" values for both European and American-style contingent-claims can be computed, or as an approximation of the Black and Scholes (1973) model to which it asymptotically converges.

More in detail, we divide each policy year into N subperiods of equal length, let $\Delta=1/N$, fix a volatility parameter $\sigma>\sqrt{\Delta}\ln(1+r)$, set $u=\exp(\sigma\sqrt{\Delta})$ and $d=1/u$. Then we assume that G_τ can be observed at the discrete times $\tau=t+h\Delta$, $t=0, 1, ...$; $h=0, 1, ..., N-1$ and that, conditionally on all relevant information available at time τ, $G_{\tau+\Delta}$ can take only two possible values: uG_τ ("up" value) and dG_τ ("down" value).

As it is well known, in this discrete setting absence of arbitrage is equivalent to the existence of a risk-neutral probability measure under which all financial prices, discounted by means of the risk-free rate, are martingales. Under this risk-neutral measure, the probability of the event $\{G_{\tau+\Delta} = uG_\tau\}$ conditioned on all information available at time τ (that is, in particular, on the knowledge of the value taken by G_τ), is given by

$$q = \frac{(1+r)^\Delta - d}{u-d}, \tag{6}$$

while

$$1-q = \frac{u-(1+r)^\Delta}{u-d}$$

represents the risk-neutral (conditioned) probability of $\{G_{\tau+\Delta} = dG_\tau\}$. We observe that, in order to prevent arbitrage opportunities, we have fixed σ in such a way that $d<(1+r)^\Delta<u$, which implies a strictly positive value for both q and $1-q$.

The above assumptions imply that g_t, $t=1, 2, ..., T-1$, are i.i.d. and take one of the following $N+1$ possible values:

$$\gamma_j = u^{N-j}d^j - 1, \quad j = 0, 1, ..., N \tag{7}$$

with (risk-neutral) probability

$$Q_j = \binom{N}{j} q^{N-j}(1-q)^j, \quad j = 0, 1, ..., N. \tag{8}$$

Moreover, also the adjustment rates of the benefit, δ_t, $t=1, 2, ..., T-1$, are i.i.d., and can take $n+1$ possible values, given by

$$\mu_j = \frac{\eta\gamma_j - i}{1+i}, \quad j = 0, 1, ..., n-1 \tag{9}$$

with probability Q_j, and 0 with probability $1-\sum_{j=0}^{n-1} Q_j$. Here

$$n = \left\lfloor \frac{N}{2} + 1 - \frac{ln(1+i/\eta)}{2ln(u)} \right\rfloor, \tag{10}$$

with $\lfloor y \rfloor$ the integer part of a real number y, represents the minimum number of "downs" such that a call option on the rate of return on the reference portfolio in a given year with exercise price i/η does not expire in the money.

4 The fair value of the contract and its components

Under the assumptions described in the previous section, in particular taking into account that all the probabilities introduced so far are risk-neutral and that the mortality uncertainty is independent of the financial one, the fair values of the European-style components of the contract can be computed in two separate stages: in the first stage the market value at time 0 of the benefit due at time t in case of death of the insured during the t-th year of contract is computed for all t=$1, 2, ..., T-1$, along with the market value of the benefit due at maturity T; in the second stage all these values are "averaged" with the probabilities of payment at each possible date. Observe that, for $t = 1, 2, ..., T-1$, these probabilities are given by ${}_{t-1/1}q_x$, while the probability that the benefit is due at maturity T is given by ${}_{T-1/1}q_x + {}_{T}p_x = {}_{T-1}p_x$.

4.1 The fair value of the basic contract: U^B

Recalling that we have called "basic contract" a standard endowment policy with benefit C_1 (without profits and without the surrender option), we have:

$$U^B = C_1 \left[\sum_{t=1}^{T-1} (1+r)^{-t}\, {}_{t-1/1}q_x + (1+r)^{-T}\, {}_{T-1}p_x \right]. \tag{11}$$

4.2 The fair value of the non-surrendable participating contract: U^P

To compute this value we need, first of all, to compute the market price at time 0 of the benefit C_t, due at time t=$1, 2, ..., T$. We denote this price by $\pi(C_t)$. While

$$\pi(C_1) = C_1(1+r)^{-1}, \tag{12}$$

for $t>1$

$$\pi(C_t) = E^Q[(1+r)^{-t}C_t],$$

where E^Q denotes expectation taken with respect to the (financial) risk-neutral measure introduced in the previous section.

Recalling relation (3) of Section 2, and exploiting the stochastic independence of δ_k, $k=1,2,...,T-1$, we have, first of all

$$\pi(C_t) = C_1(1+r)^{-t}\prod_{k=1}^{t-1} E^Q[1+\delta_k].$$

Then, taking into account that δ_k, $k=1,2,...,T-1$, are also identically distributed, we have

$$\pi(C_t) = C_1(1+r)^{-t}\left(1 + \sum_{j=0}^{n-1}\mu_j Q_j\right)^{t-1}, \quad t = 2,3,...,T, \tag{13}$$

where Q_j, μ_j and n are defined in relations (8) to (10) of Section 3.

Observe that

$$\frac{1+i}{\eta(1+r)}E^Q[\delta_k] = \frac{1+i}{\eta(1+r)}\sum_{j=0}^{n-1}\mu_j Q_j$$

represents the market price, at the beginning of each year of contract, of a European call option on the rate of return on the reference portfolio with maturity the end of the year and exercise price i/η.

Finally, the fair value U^P is given by

$$U^P = \sum_{t=1}^{T-1}\pi(C_t)\,{}_{t-1/1}q_x + \pi(C_T)\,{}_{T-1}p_x. \tag{14}$$

4.3 The fair value of the bonus option: B

The value of this option is simply given by the difference between U^P and U^B:

$$\begin{aligned} B &= U^P - U^B \\ &= C_1\left\{\sum_{t=2}^{T-1}(1+r)^{-t}\left[\left(1+\sum_{j=0}^{n-1}\mu_j Q_j\right)^{t-1} - 1\right]{}_{t-1/1}q_x + \right. \\ &\qquad \left. + (1+r)^{-T}\left[\left(1+\sum_{j=0}^{n-1}\mu_j Q_j\right)^{T-1} - 1\right]{}_{T-1}p_x\right\}. \end{aligned} \tag{15}$$

4.4 The fair value of the whole contract: U^T

Under our assumptions, the stochastic evolution of the benefit $\{C_t, t{=}1, 2, ..., T\}$ can be represented by means of an $(n{+}1)$-nomial tree. In the root of this tree we represent the initial benefit C_1 (given); then each knot of the tree has $n{+}1$ branches that connect it to $n{+}1$ following knots. In the knots at time t we represent the possible values of C_{t+1}. The possible trajectories that the stochastic process of the benefit can follow from time 0 to time t ($t = 1, 2, ..., T{-}1$) are $(n{+}1)^t$, but not all these trajectories lead to different knots. The tree is, in fact, *recombining*, and the different knots (or *states of nature*) at time t are only $\binom{n+t}{n}$.

In the same tree we can also represent the surrender values defined by relation (4) of Section 2, the fair price of the whole contract, and a *continuation* price that we are going to define immediately. The last two prices can be computed by means of a backward recursive procedure operating from time $T{-}1$ to time 0. In particular, in each step and knot the fair price of the whole contract is given by the maximum between the surrender value and the continuation price.

To see this we denote, first of all, by $\{V_t,\ t = 0, 1, ..., T{-}1\}$ and $\{W_t,\ t = 0, 1, ..., T{-}1\}$ the stochastic processes with components the fair values of the whole contract, and the continuation values respectively, at the beginning of the $(t{+}1)$-th year of contract (time t), and let $U^T{=}\ V_0$. Then, observing that in each knot at time $T{-}1$ (if the insured is alive) the continuation value is given by

$$W_{T-1} = (1 + r)^{-1} C_T \tag{16}$$

since the benefit C_T is due with certainty at time T, we have

$$V_{T-1} = max\{W_{T-1},\ R_{T-1}\}. \tag{17}$$

Now assume to be, at time $t{<}T{-}1$, in a given knot K. For ease of notation we have not indexed so far either the benefit, or the surrender value, or the fair price of the whole contract, or the continuation price, in a given knot. Now, in order to catch the link between prices at time t and prices at time $t{+}1$, we denote by C^K_{t+1}, R^K_t, V^K_t, W^K_t all these values in the knot K, and by $V^{K(j)}_{t+1}$, $W^{K(j)}_{t+1}$ $j{=}0, 1, ..., n$, the fair value of the whole contract and the continuation value at time $t{+}1$ in each knot following K. More in detail, $V^{K(j)}_{t+1}$ ($W^{K(j)}_{t+1}$ respectively) , $j{=}0, 1, ..., n{-}1$, represent the value when $\delta_{t+1}{=}\mu_j$ (with risk-neutral probability Q_j), while $V^{K(n)}_{t+1}$ ($W^{K(n)}_{t+1}$) represents the value corresponding to $\delta_{t+1}{=}0$ (with probability $1{-}\sum_{j=0}^{n-1} Q_j$).

We observe that, in the knot K, to continue the contract means to receive, at time $t{+}1$, the benefit C^K_{t+1} if the insured dies within 1 year, or to be entitled to a contract whose total random value (including the option of surrendering it in the future) equals V_{t+1} if the insured survives. The continuation value at time t (in the knot K) is then given by the risk-neutral expectation of these payoffs, discounted for 1 year with the risk-free rate:

$$\begin{aligned} W_t^K &= (1+r)^{-1}\left\{q_{x+t}C_{t+1}^K + p_{x+t}\left[\sum_{j=0}^{n-1} V_{t+1}^{K(j)}Q_j \; + \right.\right. \\ &+ \left.\left. V_{t+1}^{K(n)}\left(1-\sum_{j=0}^{n-1}Q_j\right)\right]\right\}, \quad t=0,1,...T-2. \end{aligned} \tag{18}$$

To conclude, we have then

$$V_t^K = max\{W_t^K, R_t^K\}, \quad t=0,1,...T-2. \tag{19}$$

4.5 The fair value of the surrender option: S

The fair price at time 0 of the surrender option is given by the difference between U^T and U^P:

$$S = U^T - U^P. \tag{20}$$

5 Numerical results

In this section we present some numerical results for the fair value of the contract and of all its components. To obtain these results we have extracted the mortality probabilities from the Italian Statistics for Females Mortality in 1991, fixed C_1=1, T=5, N=250, and considered different values for the remaining parameters.

We observe that our choice for N implies a daily change in the unit price of the reference portfolio since there are about 250 trading days in a year. Moreover, this choice guarantees a very good approximation to the Black and Scholes (1973) model. In fact, if we assumed that the unit price of the reference portfolio follows a geometric Brownian motion with volatility parameter σ, then the market value, at the beginning of each year of contract, of a European call option written on the rate of return on the reference portfolio with maturity the end of the year and exercise price i/η would be given by

$$\phi(a) - \frac{1+i/\eta}{1+r}\,\phi(b),$$

where

$$a = \frac{\ln(1+r) - \ln(1+i/\eta)}{\sigma} + \frac{\sigma}{2}, \qquad b = a - \sigma,$$

and ϕ denotes the cumulative distribution function of a standard normal variate. In a very large amount of numerical experiments carried out with different sets of parameters we have found that the difference between this Black and Scholes (1973) price and the one obtained in our model (with N=250), and the difference between the fair values of the bonus option in the two models, are both less than 1 basis point (bp). However, this high number of steps in each year requires a large amount of CPU time; that is why we have not fixed a high value for T. In particular, the CPU time required by a Pentium III 800 MHz for computing the fair value of the contract and of all its components when T=5 and N=250 is about 3 minutes.

As already mentioned in Section 2, we have specified two alternative rules for computing the surrender values. According to the former, the surrender value at the beginning of each year of contract is given by the current benefit discounted from maturity to the surrender date with an annual compounded rate ρ_1:

$$R_t = C_{t+1}(1+\rho_1)^{-(T-t)}, \quad t = 0, 1, ..., T-1. \tag{21}$$

According to the latter rule, the surrender value is a rate ρ_2 of the mathematical reserve of the policy. Traditionally this reserve is computed as an expected value, with respect to the mortality measure, of the current benefit discounted by means of the technical rate i. Then, in this case,

$$R_t = \rho_2 C_{t+1}\left[\sum_{h=1}^{T-t}(1+i)^{-h}\ {}_{h-1/1}q_{x+t} + (1+i)^{-(T-t)}\ {}_{T-t}p_{x+t}\right], \quad t = 0, 1, ..., T-1. \tag{22}$$

In order to get some numerical feeling about our fingers and to catch some comparative statics properties of the model, we have fixed a basic set of values for the parameters x, r, i, η, σ, ρ_1, ρ_2, and then we have moved each parameter one at a time. For comparison, we have also computed the expected value, with respect to the mortality measure, of the initial benefit C_1 discounted with the technical rate i, that we denote by U:

$$U = C_1\left[\sum_{t=1}^{T-1}(1+i)^{-t}\ {}_{t-1/1}q_x + (1+i)^{-T}\ {}_{T-1}p_x\right]. \tag{23}$$

As already discussed in Section 2, if U is the single premium paid by the policyholder, then i can be interpreted as a minimum interest rate guaranteed.

The basic set of parameters, fixed in such a way that the fair price of the whole contract U^T is very close to U, is as follows:

$$x = 50,\ r = 0.05,\ i = 0.02,\ \eta = 0.5,\ \sigma = 0.15,\ \rho_1 = 0.035,\ \rho_2 = 0.985.$$

With these parameters we have obtained the following results:

$$U^B = 0.7845,\ B = 0.1084,\ U^P = 0.8930,\ U = 0.9062.$$

Moreover, if the surrender values are computed according to relation (21), then the fair price of the surrender option $S_{(1)} = 0.0128$ and that of the whole contract $U^T_{(1)} = 0.9058$. If instead the surrender values are expressed by relation (22), then $S_{(2)} = 0.0123$ and $U^T_{(2)} = 0.9053$.

Also without the aid of numerical results it is quite obvious that the fair value of the basic contract U^B is increasing with respect to the age of the insured x, decreasing with the market rate r, and constant with respect to the remaining parameters i, η, σ, ρ_1, ρ_2. As for the bonus option B, it is increasing with the participation coefficient η and the volatility parameter σ, decreasing with the age of the insured x and the technical rate i, constant with the surrender parameters ρ_1 and ρ_2; it is instead a priori undetermined the behaviour of B with respect to the market rate r. The fair value of the non-surrendable participating contract U^P is increasing with respect to η and σ, decreasing with i, constant with respect to ρ_1 and ρ_2, undetermined with x and r. The single premium U is increasing with x, decreasing with i, constant with respect to r, η, σ, ρ_1, ρ_2. Finally, the fair value of the surrender option $S_{(j)}$ and that of the whole contract $U^T_{(j)}$ are a priori undetermined with respect to all the parameters except ρ_j $(j = 1, 2)$. More precisely, if the surrender values are expressed by relation (21), then $S_{(1)}$ and $U^T_{(1)}$ are both decreasing with ρ_1; if instead relation (22) holds, then $S_{(2)}$ and $U^T_{(2)}$ are increasing with ρ_2. From this behaviour we can argue that, when the fair value of the non-surrendable participating contract U^P is not greater than U, it is possible to find (numerically) a value of the surrender parameter ρ_j such that $U^T_{(j)} = U$. As we will see from the following tables, also the remaining parameters can be chosen in such a way that $U^T_{(j)} = U$.

More in detail, in Table 1 we present the results obtained when x varies between 40 and 60 and in Table 2 those obtained when r varies between 2% and 10% with step 0.5%. In Table 3 i varies between 0 and 5% with step 0.5%; in Table 4 η varies between 5% and 100% with step 5%; in Table 5 σ varies between 5% and 50% with step 5%. Finally, in Table 6 and in Table 7 we move the surrender parameters ρ_1 and ρ_2, from 0 to 5% and from 97% to 100% respectively, with step 0.5%.

TABLE 1

C_1=1, N=250, T=5, r=0.05, i=0.02, η=0.5, σ=0.15, ρ_1=0.035, ρ_2=0.985

x	U^B	B	U^P	$S_{(1)}$	$U^T_{(1)}$	$S_{(2)}$	$U^T_{(2)}$	U
40	0.7839	0.1088	0.8927	0.0129	0.9056	0.0124	0.9052	0.9059
41	0.7840	0.1088	0.8927	0.0129	0.9056	0.0124	0.9052	0.9059
42	0.7840	0.1088	0.8928	0.0129	0.9056	0.0124	0.9052	0.9059
43	0.7841	0.1087	0.8928	0.0129	0.9056	0.0124	0.9052	0.9060
44	0.7841	0.1087	0.8928	0.0129	0.9057	0.0124	0.9052	0.9060
45	0.7842	0.1086	0.8928	0.0128	0.9057	0.0124	0.9052	0.9060
46	0.7843	0.1086	0.8929	0.0128	0.9057	0.0124	0.9052	0.9061
47	0.7843	0.1086	0.8929	0.0128	0.9057	0.0124	0.9052	0.9061
48	0.7844	0.1085	0.8929	0.0128	0.9057	0.0124	0.9053	0.9061
49	0.7845	0.1085	0.8929	0.0128	0.9057	0.0124	0.9053	0.9062
50	0.7845	0.1084	0.8930	0.0128	0.9058	0.0123	0.9053	0.9062
51	0.7846	0.1084	0.8930	0.0128	0.9058	0.0123	0.9053	0.9062
52	0.7848	0.1083	0.8930	0.0128	0.9058	0.0123	0.9053	0.9063
53	0.7849	0.1082	0.8931	0.0128	0.9058	0.0123	0.9054	0.9063
54	0.7850	0.1081	0.8931	0.0127	0.9059	0.0123	0.9054	0.9064
55	0.7851	0.1080	0.8932	0.0127	0.9059	0.0123	0.9054	0.9065
56	0.7853	0.1079	0.8932	0.0127	0.9059	0.0122	0.9055	0.9065
57	0.7855	0.1078	0.8933	0.0127	0.9060	0.0122	0.9055	0.9066
58	0.7857	0.1077	0.8934	0.0127	0.9060	0.0122	0.9056	0.9067
59	0.7859	0.1076	0.8934	0.0126	0.9061	0.0122	0.9056	0.9068
60	0.7861	0.1074	0.8935	0.0126	0.9061	0.0121	0.9057	0.9069

From the results reported in Table 1 we can notice that the age of the insured seems to have a very small influence on the premiums, at least in the range of values here considered. The basic premium U^B is about 78% of the initial benefit C_1, the bonus option is rather expensive (about 11% of this benefit), whereas the surrender option is very cheap (between 1.2% and 1.3% of C_1). Moreover, in all the examples here reported the fair value of the whole contract is (slightly) less than U, so that some contractual parameter (for instance ρ_j) should be modified in order that $U^T_{(j)} = U$. Finally, the increasing trend of the basic premium U^B "beats" the decreasing trend of the bonus option B, so that $U^P = U^B + B$ increases with x. Also the surrender options $S_{(j)}, j = 1, 2$, decrease in value with x, but not so strongly to capsize the behaviour of $U^T_{(j)} = U^P + S_{(j)}$, increasing with x.

TABLE 2

C_1=1, N=250, T=5, x=50, i=0.02, η=0.5, σ=0.15, ρ_1=0.035, ρ_2=0.985

$\boxed{U=0.9062}$

r	U^B	B	U^P	$S_{(1)}$	$U^T_{(1)}$	$S_{(2)}$	$U^T_{(2)}$
0.020	0.9062	0.0955	1.0017	0.0000	1.0017	0.0000	1.0017
0.025	0.8844	0.0977	0.9821	0.0000	0.9821	0.0000	0.9821
0.030	0.8633	0.0999	0.9631	0.0000	0.9631	0.0000	0.9631
0.035	0.8427	0.1020	0.9448	0.0000	0.9448	0.0000	0.9448
0.040	0.8228	0.1042	0.9270	0.0044	0.9314	0.0040	0.9309
0.045	0.8034	0.1063	0.9097	0.0087	0.9184	0.0082	0.9179
0.050	0.7845	0.1084	0.8930	0.0128	0.9058	0.0123	0.9053
0.055	0.7662	0.1105	0.8767	0.0168	0.8935	0.0163	0.8930
0.060	0.7484	0.1126	0.8610	0.0206	0.8816	0.0316	0.8926
0.065	0.7311	0.1146	0.8457	0.0242	0.8700	0.0469	0.8926
0.070	0.7143	0.1166	0.8309	0.0278	0.8587	0.0617	0.8926
0.075	0.6979	0.1185	0.8165	0.0312	0.8477	0.0761	0.8926
0.080	0.6820	0.1205	0.8025	0.0394	0.8420	0.0901	0.8926
0.085	0.6666	0.1224	0.7890	0.0530	0.8420	0.1036	0.8926
0.090	0.6515	0.1243	0.7758	0.0662	0.8420	0.1168	0.8926
0.095	0.6369	0.1261	0.7630	0.0790	0.8420	0.1296	0.8926
0.100	0.6226	0.1279	0.7505	0.0915	0.8420	0.1421	0.8926

From Table 2 we notice that all the results reported are very sensitive with respect to the market rate r, and this is not surprising at all. The value of the basic contract ranges from 90.62% of C_1 (when $r = i = 2\%$) to 62.26% (when $r = 10\%$), and that of the bonus option from 9.55% of C_1 to 12.79%, thus exhibiting an increasing trend. However, once again this trend is beaten by the trend of U^B, so that $U^P = U^B + B$ decreases with r (from 100.17% of C_1 to 75.05%). Moreover, observe that, when $r = i = 2\%$, the non-surrendable participating contract is quoted "over par". The surrender options $S_{(j)}, j = 1, 2$, are both increasing in value with r, but this behaviour does not capsize the decreasing trend of $U^T_{(j)} = U^P + S_{(j)}$. In particular, both $S_{(1)}$ and $S_{(2)}$ are valueless if $r \leq 3.5\%$, $S_{(1)}$ reaches the level of 9.15% of C_1 and $S_{(2)}$ that of 14.21% when $r = 10\%$. Finally, there exists a level of r, between 4.5% and 5%, such that $U^T_{(j)} = U$ for $j = 1, 2$.

TABLE 3

C_1=1, N=250, T=5, x=50, r=0.05, η=0.5, σ=0.15, ρ_1=0.035, ρ_2=0.985

$\boxed{U^B=0.7845}$

i	B	U^P	$S_{(1)}$	$U^T_{(1)}$	$S_{(2)}$	$U^T_{(2)}$	U
0.000	0.1489	0.9335	0.0134	0.9468	0.0515	0.9850	1.0000
0.005	0.1380	0.9226	0.0132	0.9358	0.0383	0.9609	0.9755
0.010	0.1274	0.9120	0.0131	0.9250	0.0255	0.9374	0.9517
0.015	0.1178	0.9023	0.0129	0.9153	0.0169	0.9192	0.9286
0.020	0.1084	0.8930	0.0128	0.9058	0.0123	0.9053	0.9062
0.025	0.0999	0.8845	0.0127	0.8972	0.0079	0.8924	0.8844
0.030	0.0917	0.8763	0.0126	0.8889	0.0036	0.8799	0.8633
0.035	0.0844	0.8689	0.0125	0.8814	0.0000	0.8689	0.8427
0.040	0.0772	0.8618	0.0124	0.8741	0.0000	0.8618	0.8228
0.045	0.0708	0.8554	0.0123	0.8676	0.0000	0.8554	0.8034
0.050	0.0646	0.8492	0.0122	0.8613	0.0000	0.8492	0.7845

From Table 3 we can observe that the technical rate i has a strong influence on the value of the bonus option B (as expected), which ranges from 14.89% of C_1 (when $i = 0$) to 6.46% (when $i = r = 5\%$). The same happens for U and the fair price of the surrender option $S_{(2)}$. Recall, in fact, that i negatively affects the surrender values when computed according to relation (22). The value of the surrender option $S_{(1)}$, instead, does not seem to be very sensitive with respect to i. Anyway, all the prices reported in Table 3 are decreasing with i and, in particular, $S_{(2)} = 0$ when $i \geq 3.5\%$. Finally, a value of i between 2% and 2.5% is such that $U^T_{(j)} = U$ for $j = 1, 2$.

TABLE 4

C_1=1, N=250, T=5, x=50, r=0.05, i=0.02, σ=0.15, ρ_1=0.035, ρ_2=0.985

$\boxed{U^B\text{=0.7845, }U\text{=0.9062}}$

η	B	U^P	$S_{(1)}$	$U^T_{(1)}$	$S_{(2)}$	$U^T_{(2)}$
0.05	0.0003	0.7848	0.0571	0.8420	0.1078	0.8926
0.10	0.0053	0.7898	0.0522	0.8420	0.0894	0.8792
0.15	0.0147	0.7993	0.0427	0.8420	0.0668	0.8660
0.20	0.0261	0.8107	0.0313	0.8420	0.0423	0.8530
0.25	0.0387	0.8232	0.0188	0.8420	0.0170	0.8402
0.30	0.0520	0.8365	0.0120	0.8485	0.0116	0.8481
0.35	0.0655	0.8501	0.0122	0.8622	0.0117	0.8618
0.40	0.0796	0.8642	0.0124	0.8766	0.0119	0.8761
0.45	0.0939	0.8785	0.0126	0.8911	0.0121	0.8906
0.50	0.1084	0.8930	0.0128	0.9058	0.0123	0.9053
0.55	0.1233	0.9078	0.0130	0.9208	0.0125	0.9204
0.60	0.1385	0.9230	0.0132	0.9362	0.0128	0.9358
0.65	0.1538	0.9384	0.0135	0.9518	0.0130	0.9513
0.70	0.1694	0.9539	0.0137	0.9676	0.0132	0.9671
0.75	0.1851	0.9697	0.0139	0.9836	0.0134	0.9831
0.80	0.2011	0.9856	0.0141	0.9998	0.0136	0.9993
0.85	0.2172	1.0018	0.0144	1.0161	0.0139	1.0156
0.90	0.2336	1.0181	0.0146	1.0327	0.0141	1.0322
0.95	0.2501	1.0347	0.0148	1.0495	0.0143	1.0490
1.00	0.2669	1.0514	0.0151	1.0665	0.0145	1.0659

As far as the participation coefficient η is concerned, we notice, from Table 4, a very strong influence on the value of the bonus option, that ranges from 0.03% of C_1 (when $\eta = 5\%$) to 26.69% (when $\eta = 100\%$). Observe, moreover, that the non-surrendable participating contract is quoted over par when $\eta \geq 85\%$. Also the values of the surrender options and, especially, $S_{(2)}$, are quite sensitive with respect to η. In particular $S_{(1)}$, equal to 5.71% of C_1 when $\eta = 5\%$, decreases until 1.2% of C_1 when $\eta = 30\%$, then increases very slightly and reaches the value of 1.51% of C_1 when $\eta = 100\%$. Anyway, the non-monotonicity of $S_{(1)}$ does not capsize the increasing trend of $U^T_{(1)} = U^P + S_{(1)}$. As for $S_{(2)}$, it decreases from 10.78% of C_1 (when $\eta = 5\%$) to 1.16% (when $\eta = 30\%$), and then slightly increases up to 1.45% of C_1 for $\eta = 100\%$. This behaviour influences also the trend of $U^T_{(2)} = U^P + S_{(2)}$, that does not result monotonic too. Finally, a value of η between 50% and 55% makes $U^T_{(j)} = U$ for $j = 1, 2$.

TABLE 5

C_1=1, N=250, T=5, x=50, r=0.05, i=0.02, η=0.5, ρ_1=0.035, ρ_2=0.985

$\boxed{U^B\text{=0.7845, } U\text{=0.9062}}$

σ	B	U^P	$S_{(1)}$	$U^T_{(1)}$	$S_{(2)}$	$U^T_{(2)}$
0.05	0.0408	0.8253	0.0166	0.8420	0.0673	0.8926
0.10	0.0740	0.8586	0.0123	0.8709	0.0206	0.8792
0.15	0.1084	0.8930	0.0128	0.9058	0.0123	0.9053
0.20	0.1440	0.9286	0.0133	0.9419	0.0128	0.9414
0.25	0.1804	0.9649	0.0138	0.9788	0.0133	0.9783
0.30	0.2175	1.0020	0.0144	1.0164	0.0139	1.0159
0.35	0.2559	1.0405	0.0149	1.0554	0.0144	1.0549
0.40	0.2953	1.0799	0.0155	1.0954	0.0149	1.0948
0.45	0.3356	1.1201	0.0161	1.1362	0.0155	1.1356
0.50	0.3767	1.1612	0.0167	1.1779	0.0161	1.1773

Most of the comments concerning the behaviour of the premiums with respect to the participation coefficient η are still valid when referred to the volatility coefficient σ. From Table 5, in fact, we can observe that B is very sensitive with respect to σ, and ranges from 4.08% of C_1 (when $\sigma = 5\%$) to 37.67% (when $\sigma = 50\%$). Also $S_{(2)}$ is quite sensitive, and not monotonic, with respect to σ, whereas $S_{(1)}$, not monotonic as well, does not seem to be very sensitive. The premium $U^T_{(1)} = U^P + S_{(1)}$ is increasing, while $U^T_{(2)} = U^P + S_{(2)}$ is not monotonic. The non-surrendable participating contract is quoted over par when $\sigma \geq 30\%$, and there exists a value of σ, between 15% and 20%, such that $U^T_{(j)} = U$ for $j = 1, 2$.

TABLE 6

C_1=1, N=250, T=5, x=50, r=0.05, i=0.02, η=0.5, σ=0.15

$\boxed{U^B\text{=0.7845, } B\text{=0.1084, } U^P\text{=0.8930, } U\text{=0.9062}}$

ρ_1	$S_{(1)}$	$U^T_{(1)}$
0.000	0.1070	1.0000
0.005	0.0824	0.9754
0.010	0.0585	0.9515
0.015	0.0353	0.9283
0.020	0.0260	0.9189
0.025	0.0215	0.9145
0.030	0.0172	0.9101
0.035	0.0128	0.9058
0.040	0.0085	0.9015
0.045	0.0042	0.8972
$\geq$0.050	0.0000	0.8930

From Table 6 we notice that, when the surrender values are computed according to relation (21), the influence of the discount rate ρ_1 is very strong, as expected. In particular, if $\rho_1 = 0$, i.e., if there are no penalties and surrender is treated as like as death, then the value of the surrender option $S_{(1)}$ is 10.7% of C_1 and the whole contract is quoted exactly at par. When instead $\rho_1 \geq 5\%$, then $S_{(1)} = 0$. Finally, there exists a value of ρ_1, between 3% and 3.5%, such that $U^T_{(1)} = U$.

TABLE 7

C_1=1, N=250, T=5, x=50, r=0.05, i=0.02, η=0.5, σ=0.15

U^B=0.7845, B=0.1084, U^P=0.8930, U=0.9062

ρ_2	$S_{(2)}$	$U^T_{(2)}$
≤0.970	0.0000	0.8930
0.975	0.0032	0.8962
0.980	0.0078	0.9008
0.985	0.0123	0.9053
0.990	0.0169	0.9098
0.995	0.0214	0.9144
1.000	0.0260	0.9189

When the surrender values are computed according to relation (22) the surrender option $S_{(2)}$, although being quite sensitive with respect to ρ_2, reaches the maximum value of only 2.6% of C_1 when $\rho_2 = 100\%$, and is valueless for $\rho_2 \leq 97\%$. Moreover, a value of ρ_2 between 98.5% and 99% makes $U^T_{(2)} = U$ (see Table 7).

6 Concluding remarks

In this paper we have analysed a single premium life insurance endowment policy in which the benefit is annually adjusted according to the performance of a special investment portfolio. In addition to this participating mechanism, that is coupled with the provision of a minimum return guaranteed, the contract is also equipped with a surrender option, i.e., with an American-style option to sell it back before expiration at a price computed according to a predetermined formula (*surrender value*). Then this policy can be divided in three components: the *basic contract*, the *bonus option* and the *surrender option*. Assuming that the unit price of the reference portfolio follows the discrete model by Cox, Ross and Rubinstein (1979), we obtain a closed-form expression for the fair value of the first two components, and present a recursive algorithm for computing the fair value of the third one. The numerical implementation of the model allows us to address also the problem of suitably choosing the contractual parameters in order that the fair price of the whole contract equals the premium computed by insurance companies according to standard actuarial practice.

The policy here analysed is very often paid by annual premiums. However, the extension of the valuation model here proposed in order to compute the annual premium is not at all trivial. The fair price of the whole contract depends, in fact, on the value of the surrender option, which in turn depends on the annual premium. Moreover, even though this total price were given, in order to compute the annual premium it should be split into an annuity with instalments paid only if the insured is still alive and the contract has not been surrendered yet. Then the annual premium determines also the value of this annuity, through the surrender decision. A real vicious circle arises in this way and, what is more, the numerical solution of the problem, at least with a satisfactory level of accuracy, is thwarted by the high computational complexity of the model. This problem constitutes then an important topic to be addressed in the near future.

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