ENVIRONMENT & FINANCE

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Why we should make the environment a part of the financial environment.

Abstract

This study represents a proposal to create a kind of financial markets out of environmental improvements. We shall explain how to do it, and why the financial approach is practically the only one able to stop and invert the environmental degradation. Particularly we shall concentrate our attention in deforestation and show how to produce effectively reforestation. As a model of actual and future (after applying our approach) situation we consider $X(t)$, 0-th dimensional squared Bessel process with drift. We justify our choice as a limit of branching phenomena. To put the project into practice funds are needed, but contrary to an infinity of failed projects the use of resources would be apolitical, transparent and efficient. In the game of reforestation versus deforestation we solve the optimization problem of the first move needed to start the project. On the other hand because we are aiming for the creation of a developed market, we also solve the classical Merton problem with underlying asset $X(t)$ instead of Geometric Brownian motion.

1. Introduction

Deforestation, loss of biodiversity and CO_2 emissions are only some of many environmental problems that must be treated urgently. We will not present statistics, since they can be found all over. Our goal is to invert things, and commence with reforestation taking this word a little bit freely.

Reforestations is still by far one of the easiest to use words in the environmental language. Words like restorations, and particularly "integrity" (emerging from ethics and landing in environmental science without sufficient justifications) make things very vague.

There is also a list of problems in the practicality of sustainable development. Here are some of the reasons.

1. The utopian vision about the possibility of improvements using the intrinsic value of nature without involving any money.
2. Decisions considering the environment are taken where possible results practically cannot be evaluated, and thus are inadequate, and on the other hand (as stated by global integrity project) to conserve integrity we must be guided by a knowledge of our actions consequences, for example see Westra et al. (2000) which is no more than a vicious circle.
3. Agents responsible for new projects often go around the law, which frequently is not precise enough.
4. The call for "back to nature" is pure nonsense, particularly if referred to undeveloped (read poor) countries.
5. Using governamental programs in undeveloped countries is once again wasting money. Drawing examples from India, where governmental programs are supposed to work cf. Oats (1999), and trying to apply them in say, Africa or Latin America, shows poignant lack of knowledge about differences between religions, culture and regions.
6. An international Analog Forestry Project can offer only local and reduced solutions. We cannot expect that huge numbers of humans could make their lives "picking berries" (it is of course an oversimplification made on purpose by the author of this study).
7. Environmentalist groups frequently posses only a dim interest.

We have reviewed some topics from the environment just to show that our program probably is the only one that could work. But before explaining why it could be done, we still need one more comment.

Nowadays, environmentalists blame Bush's presidency because of its anti environmentalism. This simply shows that their work has been very fragile.

Our project is based on ”positive action awards”, and there is a need for funds, either public or comming from environmental organizations.

Many resources are frequently wasted because of:

1. Extremely expensive consulting.
2. Failed environmental projects frequently supporting corrupt agents involved.

In the last years we have observed the development of modern finance in fields related to our idea. Firstly, the creation of derivatives products, where the underlying asset cannot be directly priced such as:

1. Real Options.
2. Options on climate and others.

Secondly, there have been many recent developments on market frictions such as; transaction costs, asymetries, or insiders (agents with additional knowledge).

All these allow for the formulation and solution of new challenging problems.

We will now look into the current environmental programs that are related to finances. In our opinion, the most important ones are:

1. Financial instituions have made a commitment that they will not support new projects if the environment is at stake (hard to analyze).
2. Insurance companies reduce premiums to cleaner industries.
3. Permits to pollute in micro or macro scale.

The first two topics are agreements between parties and can be distorted. As stated by Merton and Bodie we pretend for the environment to become a part of the financial innovation spiral. Among other benefits it would provide transparent ways to transfer economic resources.

2. The model before the financial intervention

To model the number of trees in a given region, we propose a 0-th dimensional squared Bessel process $X(t)$ with negative drift defined by:

$$dX(t) = 2\sqrt{\sigma X(t)}dW(t) + 2\beta X(t)dt$$

$X(0) = x_0 > 0, \beta < 0.$

In the sequel we will set $\sigma = 1$, for simplicity sake.

This is of course also 0-th dimensional Cox, Ingersoll & Ross model for interest rates. $X(\cdot)$ will eventually become zero before t and if so, it will stay there forever. If not, the remining probability has the density:

$$f_{(x,y)}(t) = \frac{\beta}{2\sinh\beta t} * (\frac{x}{y})^{\frac{1}{2}} * \exp(2\beta t) * exp[\beta\frac{xexp(\beta t) + yexp(-\beta t)}{2\sinh\beta t}] * I_1(\frac{\beta\sqrt{xy}}{\sinh\beta t})$$

$$I_1(z) = \frac{z}{2}\sum\frac{(\frac{z}{2})^{2n}}{n!(n+1)!}$$

In this study we will not do any statistics and will justify the model by a "heavy traffic" approximation instead. This can be done in many ways. For example as an initial model we choose:

i. The corresponding picewice deterministic Markov process (PDMP).
ii. The Bachelier model, where the drift and volatility is changing properly.

Consider the PDMP process $U(t)$ with the infinitesimal generator:

$$\mathcal{G}f(x) = \lambda x\{-(1+\theta)p_1 f'(x) - \int_{-\infty}^{\infty}[f(x-y) - f(x)]P(dy)\}$$

The process $U(t)$ can be described heuristically as follows: $cU(t)$ is the rate of deforestation,$c < 0$ and the process moves in the deterministic way. Further, according to the exponential law λx, x being the actual number of trees, occurs reforestation (or deforestation) X_i, with probability law P, X_i are i.d.i.r.v. $E(X_i) = p_1$.

(Usually, and for many reasons, it is easier to reforestate in less devastated areas). If is convenient to parametrize $c = -\lambda(1+\theta)p_1$.

Suppose that the low P has finite second moment p_2, and change $\lambda \to \lambda n$,$x_i \to \frac{x_i}{\sqrt{n}}$, $\theta \to \frac{\theta}{\sqrt{n}}$ (heavy traffic approximation). Then we have:

Proposition 1.

$$X_n(t) \Longrightarrow_{n\to\infty} X(t)$$

with $\sigma = \frac{\lambda p_2}{4}$ and $\beta = \frac{-\lambda p_1\theta}{2}$.

Proof

We will give a sketch of the proof. For details we refere to the book by Ethier & Kurtz (1985). It is enough to prove that:

$$\mathcal{G}_n(f(x)) \xrightarrow[n \to \infty]{} \frac{1}{2} 4\sigma x f''(x) + 2\beta x f'(x) \tag{1}$$

for any function $f \in C_k^\infty$ (with bounded support).

We may assume that $\mid y \mid < c\sqrt{n}$ for some c. Using Taylor expansion, results (1) in a streightforward way except maybe the fact that:

$$\frac{1}{\sqrt{n}} \int_0^{c\sqrt{n}} \mid y \mid^3 P(dy) \xrightarrow[n \to \infty]{} 0$$

But this fact results from a kind of Kronecker's lemma:

$\frac{1}{k} \int_0^{ck} \mid y^3 \mid P(dy) \to 0$, because $\int_{-\infty}^{\infty} y^2 P(dy) < \infty$._

If we change signs in the infinitesimal generator $\mathcal{G}$, and drop "x", then the resulting process becomes classical Lundberg surplus model in risk theory. In this case, the well known result is that the heavy traffic approximation leads to:

$$Y(t) = y_0 + \sqrt{\lambda p_2} W(t) - \theta \lambda p_1 t.$$

That is classical Bachelier model.Making properly λ state dependent one can easily obtain the same result as before.

3. Financial Intervention

Consider the "option" that pays:

$$\int_0^1 (X(s) \wedge k) ds = k - \int_0^1 (k - X_s)_+ ds$$

For simplicity we assume zero interest rate. Here k is the maximum capacity in a given area. We will call it call option or "good" option (the payment is larger if more trees). The owner of the option paying the premium c, will possibly make some effort to obtain larger payment. That is: planting trees and protect them, if his investment will produce larger payments considering all expenses.

Consider N options sold, and all holders acting in an optimal way which now will be explained. The cost of planting (it is worth to do it at the begining) is easy to set, and is C_0 per each tree, and protection would change the parameter $\beta \to B$, $B > \beta$ in our state variable. B can be positive.

This protection has the continous cost $\delta = \delta(\beta, B, X(t))$. This part is difficult to set without empirical studies but here we assume that $\delta = X(t)\tilde{\delta}(\beta, B)$. Now an active option holder will spend $\frac{c}{N}(x - x_0)$ for initial planting, and $\frac{1}{N}\int_0^1 \tilde{\delta}(\beta, B)X(t)dt$ for the protection.

Assuming linear utility function (for simplicity), the optimal action is determined by maximization of:

$$E(\int_0^1 (X_s \wedge k) ds - C - \frac{C_0}{N}(x - x_0) - \frac{1}{N}\int_0^1 \tilde{\delta}(\beta, B) X(s) ds)$$

with respect to B and x, C being the price of the option.

The dynamic of the process is:

$$dX(t) = 2\sqrt{X(t)}dW(t) + 2BX(t)dt$$
$$X(0) = x$$

We have chosen linear utility because in this case we can solve the problem with the relatively easy use of numerical methods.

Of course talking about awarding for planting trees we refer to undeveloped countries. It is well known that rural communities in undeveloped countries have a hierarchical structure dominated by powerful individuals.

Some authors, for example Oats (1999), indentify this as a failure of a community based approach in the past. Precisely this hierarchical structure can help our purpose, and these powerfull individuals could act as natural agents.

To start the project we could give this options free from any charges (of course in reduced range). We will try to apply this approach in Mexico and this could give us better idea about models, parameters, and costs.

Let $\sum$ be a found destinated to reforestation in given region.

Now we can sell $N = \frac{\sum}{k-C}$ options. Appearently N does not depend on the model. But C must make the buisness worthy, so it depends on the choosen model.

This is the first move in the game between reforestation and deforestation.

Second move in the game would be selling put options "bad options" with the payment $\int_0^1 (k - X(s))_+ ds$.

The price of this option should be high so to price it we could apply the original model with parameters (x_0, β) or even charge more. This will be the object of another study. We finish this section with the comment that environmentalists should buy bad option and do nothing. This would increase the fund. The description of the game we leave to another study.

4. Comments about financial markets on environment

We are aiming toward the creation of markets out of environment so we have to answer the following question:

Are there any opposite interests that help to create the market?

Referring to this question and taking once again reforestation as an example, we can clearly identify opposing views. On the one hand, reforestation is desired by

1. General public
2. Tourism, although excessive environmental tourism can be harmful as shown by the Galapagos example.
3. Wooden industry with long term vision.
4. Benefits of "analog forestry"

On the other hand it is clear that not everyone embraces it, such as

1. Cattle ranches and milk industries. (Sometimes these industries give cattle to small farmers for free).
2. Myopic wooden industries.

Some kind of opposite interest can be possibly found in any environmental topic. Assume now that we have the market. In this case it makes sense to talk about the value of $X(t)$ itself, even of buying $X(t)$. Of course if work δ is implied it would act as change from $r \to r + \delta$ in risk neutral valuation.

The natural question is, can we solve the classical Merton problem of optimal investment if, instead of geometric brownian motion, the underlying asset is $X(t)$?

The answer is yes and we will show how to do it.

Who could be the emisor of environmental options? In the worldwide scale it should be done by United Nations dependencies.

In the case of "AAA confidence" toward a given government, this government could be in charge of the management of resources and issue options.

5. Merton Problem

We will give here a sketch of the soluton. Assume that we can invest a part of our wealth X in the risky asset with the dinamic:

$$dX(t) = 2\sqrt{X(t)}dW(t) + 2\beta X(t)dt$$

and the risk free one with the rate r.

We want to find the dynamical optimal portafolio that maximizes the utility of the final wealth.

$$E(u(y^{\pi}))$$

where $u(y) = \frac{y^{1-\alpha}}{1-\alpha}, \alpha > 0, \alpha \neq 1$.

In our case the state price density is the process H defined as the solution of:

$$dH_t = -H_t(rdt + \theta_t dW_t) = -H_t(rdt + \theta\sqrt{X_t}dW_t), H_0 = 1$$

$$\theta_t = \frac{2\beta - r}{2}\sqrt{X_t} = \theta\sqrt{X_t}$$

It is well know that the terminal optimal wealth is

$$Y_T^* = (u')^{-1}(\nu H_T)$$

where ν is a Langrange multiplier which satisfies $E(H_T Y_T^*) = x$, and the current optimal wealth is

$$Y_t^* H_t = E(Y_T^* \mid \mathcal{F}_t)$$

and the optimal portafolio is obtained from a predictable representation theorem.

For our utility function ν can be find by an inversion of Pitman & Yor formula, cf. Revuz & Yor (1998)p.444,(we drop the drift first using Girsanov theorem) and:

$$Y_t^* H_t = \nu^{\gamma} E(H_T^{\gamma+1} \mid \mathcal{F}_t)$$

where $\gamma = \frac{-1}{\alpha}$.

The pair (X, H) is markovian, therefore

$$H_T = H_t e^{-r(T-t)} exp(-\int_t^T \theta\sqrt{X_s}dW_s - \frac{1}{2}\int_t^T \theta^2 X_s ds)$$

$$= H_t e^{-r(T-t)} exp(-\int_t^T \frac{\theta}{2}dX_s - \beta\theta\int_t^T X_s ds - \frac{1}{2}\int_t^T \theta^2 X_s ds)$$

$$= H_t e^{-r(T-t)} exp(-\frac{\theta}{2}(X_T - X_t - t) - (\beta\theta + \frac{1}{2}\theta^2)\int_t^T X_s ds).$$

Therefore

$$E(H_T^{\gamma+1} \mid \mathcal{F}_t) = H_t^{\gamma+1} e^{-(\gamma+1)r(T-t)} \varphi(t, X_t),$$

where $\varphi(t, X_t) = E(exp(1+\gamma)[-\frac{\theta}{2}(X_T - X_t - t) - \beta\theta\int_t^T X_s ds - \frac{1}{2}\int_t^T \theta^2 X_s ds] \mid \mathcal{F}_t)$

and can be calculated easily by Pitman& Yor formula.

Now the optimal wealth

$$X_t^* = \nu^\gamma H_t^\gamma \varphi(t, X_t)$$

$$H_t = e^{-rt} exp[-\frac{\theta}{2}(X_t - X_0) - (\beta\theta + \frac{1}{2}\theta^2)\int_0^t X_s ds]$$

so the problem can be solved explicitely.

Further Valuation Topics.

In this section we show how to solve valuation problems related to Section 3. We do not have a market, so valuation means in the "real world".

Unfortunately we can only give solutions in terms of the Lapace transform. We are interested in calculations of

i. $E_x(\int_0^1 (X_s \wedge k) ds I\{inf_{0\leq u\leq 1} X(s) \geq l)\}$

ii. $E_x(e^{-\lambda\int_0^1 X(s)ds} I\{inf_{0\leq u\leq 1} X(s) \geq l)\}$

We start with a motivation for i. and ii. In the i. the term inside the expectations is clearly a better kind (in our case) of awards. The ii. can be interpreted as the price of a bond if there is no "default". On the other hand perhaps a better way of awording could be paying for positive action, so our barrier option in i. should activate if $X(s)$ is large (the best way of awarding could be a kind of double barrier option).

Analytically it leads to the same kind of calcuations. For calculations of i. and ii. we assume known the density f_{x,τ_l}, where $X(0) = x$ and τ_l hitting time of the barrier l.

The problem of finding this density numerically could be of general interest, and we will try to solve it in another study. Because the dimension of the process is zero, the Laplace transform of hitting time of zero is

$$\varphi_x(\alpha) = \alpha\int_0^\infty e^{-\alpha t} exp\frac{-x\beta}{1 - e^{-2\beta t}} dt$$

Here $exp\frac{-x\beta}{1-e^{-2\beta t}}$ is the probability that $X(t) = 0$, cf. Revuz & Yor (1998), and because zero is absorbing barrier this is also $P(\tau \leq t)$.

Now if $l < x$ then

$$\varphi_{\tau_l}(\alpha) = \frac{\varphi_x(\alpha)}{\varphi_l(\alpha).}$$

Another solution for the Laplace transform is presented by Going & Yor (1999).

Calculations of ii.

The knowledge of ii. could lead to the knowledge of the joint law of $\int_0^t X(s)ds$ and τ_l. Here we will use the parametrization:

$$dX(t) = 2\sqrt{X(t)}dW(t) - 2\beta X(t)dt, X(0) = x.$$

Using Girsanov theorem we have

$$E_x(e^{-\lambda\int_0^1 X(s)ds}I\{infX_s \geq l\}) = e^{Ax}E_x^Q[e^{-AX(1)}I\{infX(s) \geq l\}]$$

where under Q

$$dX(t) = 2\sqrt{X(t)}dW(t) + 2\sqrt{\beta^2 + 2\lambda}X(t)dt$$

and $A = \frac{\beta+\sqrt{\beta^2+2\lambda}}{2}$.

So if we know the density $f_{x,\tau_l} then,$

$$E_x^Q(e^{-AX(1)}I\{inf_{0\leq u\leq 1}X(s) = l)\} = \int_0^1 E^Q(e^{-AX(1)} \mid X_\tau = l)f_{x,\tau_l}(u)du$$

Of course the term $E_x^Q(e^{-AX(1)})$ is explicite by Pitman & Yor formula jointly with Girsanov theorem.

Calculations of i.

$$E_x\int_0^1(X(s)\wedge k)dsI\{\tau_l \geq 1\} = \int_0^1 E_x(X_s\wedge k)ds - \int_0^1 E_x[(X_s\wedge k)I\{\tau_l < s\}]ds - \int_0^1 E_x(X_s\wedge k)I\{\tau_l \in (s,1)\}ds$$

$$= \int_0^1\int_0^s E_l[(X_s \wedge k) \mid \tau_l = u]f_{x,\tau_l}(u)duds + \int_0^1 E_x(X_s \wedge kE(I\{\tau_l \in (s,1)\} \mid \mathcal{F}_s)ds$$

Here F is the filtration of $X(s)$ (it is also brownian filtration). Therefore, knowing the law of τ_l we can get the explicite solution.

Another kind of paying premium can be obtained by modelling positive action as "anti-default". It means that there is no payment if there is no anti-default which is modelled as default (occurs if $sup_{0\leq u\leq 1}l^*$). Of course it is a structural approach. It means that default is mesurable in the filtration of the asset. There is also a natural way to introduce the intensity of the anti-default. This approach will lead to explicite solutions.

Define $\tau = inf\{t : \int_0^t \lambda X(s) = \theta\}$ where θ is exponential random variable with parameter 1 independent of the process.

Now we have:

Proposition 2.

$$\forall_{s\leq t}P(\tau > s \mid \mathcal{F}_t) = exp(-\int_0^s \lambda X(u)du)$$

Proof:

We have $\{\tau > s\} = \{\theta > \int_0^s \lambda X(u)du\}$. From the independence and the $\mathcal{F}_t$ mesurability of $\int_0^t \lambda X(u)du$.

$$P(\tau > s \mid \mathcal{F}_t) = P(\int_0^s \lambda X(u) < \theta \mid \mathcal{F}_t) = e^{-\int_0^s \lambda X(u)du}$$

Now if $\mathcal{G}_t = \mathcal{F}_t \vee H_t$, where H_t is information about default (for details cf. Elliot et al. 2000)

$E(I_{T<\tau} \mid \mathcal{G}_t) = I_{t<\tau}\frac{E(I(\tau>T)|\mathcal{F}_t)}{E(I(\tau>t)|\mathcal{F}_t)} = I_{t<\tau}E(e^{-\int_t^T \lambda X(u)du} \mid \mathcal{F}_t)$

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