A Case Study of Operational Risk Measurement based on Loss Distribution Approach

NAKAGAWA Hidetoshi
MTB Investment Technology Institute Co., Ltd.
5-6, SHIBA 2-CHOME, MINATOK-KU
TOKYO 105-0014 JAPAN
Tel: +81 3 3457 1977
Fax: +81 3 3457 0910
nakagawa@mtec-institute.co.jp

Abstract

The management of operational risk has been one of the main issues for financial institution recently, although it is quite hard to properly measure the risk.

We concentrate on the Loss Distribution Approach(LDA), which separately estimates the distributions of likelihood and the severity from the reported loss cases and compounds both to obtain the distrtibution of the cumulative amount of losses during a certain period.

In this article, we discuss the implementation of LDA and illustrate the computation of the operational VaR from the actual loss data. We also present some consequences that give comparisons among various pair of severity distribution and parameter estimation and consider how to use external data to measure operational risk in the consistent way with internal loss data at last.

1 Introduction

The financial institution should consider various kinds of risks. For the market risk and credit risk, there have been many researches on both theoretical side and practical side. Although they might not be completed, some of them have been already implemented and applied to the measurement and control for market and credit risk. On the other hand, the definition of operational risk and liquidity risk has remained vague and the width and complexity of such risks prevent us to quantify the risk though many practitioners understand the importance and impact of such risks.

However the Basel Committee on Banking Supervision suggested that economic capital for operational risk should be allocated by the New Basel Capital Accord, scheduled to start in 2005, and it may be about 12% of the minimum regulatory capital. The final regulatory framework has not been determined yet, but the financial institution has been obliged to deal with the management of operational risk. According to the Consultative Documnet published by Basel Committee on Banking Supervision in January 2001 [4], a general definition of operational risk is thought as "*the risk of (direct or indirect)*[1] *loss resulting from inadequate or failed internal process, people and systems or from external events*".

The Basel Committee suggests some measurement approaches to capital charges for operational risk. The Basic Indicator Approach(BIA) obliges banks to hold capital for operational risk equivalent to a given percentage of a single indicator such as gross income. The Standardized Approach(SA) is almost the same as BIA, except that banks' activities have to be divided into eight business lines and the capital charge should be calculated each business line by multiplying the exposure indicator by a factor assigned to the underlying business line. Since these approaches, especially the former, can be applied to any bank regardless of its characteristics, the capital charge obtained from these approach may be one of the targets within the regulatory framework.

The Internal Measurement Approach(IMA) or the Loss Distribution Approach(LDA) allow banks to use their internal loss data to estimate the likelihood (the probability of loss event) and the severity(the loss given event). The likelihood and the severity should be given by a single value respectively in IMA while LDA demands their distributions. The Basel Committee call them the Advanced Measurement Approach(AMA) generically.

The purpose of this paper is to use the Loss Distribution Approach(LDA) to compute the operational VaR from the actual loss data. In the New Basel Capital Accord, they say "*Under the Loss Distribution Approach, the bank estimates, for each business line/risk type cell, the probability distribution functions of the single impact and the event frequency for the next (one) year using its internal data, and computes the probability distribution function of the cumulative operational loss.*"

The quantification model based on LDA is an analogy of the classical insurance risk model in risk theory. In short, after estimating the likelihood and the severity of loss events separately[2] , the aggregated amount of losses during a certain period can be represented in the compound form. Especially we use a compound Poisson model as the model of the aggregated amount of losses, which is called Cramér-Lundberg model in the collective risk theory. Frachot et al.([8]) give a very detailed description of the LDA implementation. We highly refer to their approach and illustrations for verification of our model.

Section 2 presents the simple model based on LDA. In section 3, we give the implementation of operational risk measurement based on our model using the actual loss data. On the way, we show some consequences that give comparisons among various pair of severity distribution and parameter estimation.

Section 4 discusses the remarks that we use external data to measure operational risk in the consistent way with internal loss data. The last section gives the conclusion, which comments the possibility of insurance for mitigating operational risk.

The author would like to thank Hideyuki Torii, Osamu Egawa, Shingo Suzuki and Shu Zeng who develop an operational risk management system,

OperationalRisk Browser at Numerical Technologies Inc. and also appreciates Eiji Ihori and Keisuke Nakagiri who work for Mitsubishi Trust and Banking Corp. They have studied the methodology to quantify the operational risk with the author. A part of this paper is scheduled to be contained in the book “THE PRACTICE OF OPERATIONAL RISK MANAGEMENT” written in Japanese.

2 The model

The model suggested here is not especially new but has been actively studied for a long time in the category of collective risk theory of the insurance field. One must think that the characteristics of the loss event related to the operational risk are similar to that of the accident event in non-life insurance. The point is that the loss due to the accident may be quite large though the likelihood of the accident is not so high. Therefore, it seems extremely natural to make advantage of the model researched in the insurance field to develop a quantification model of the operational risk.

Immediately you see that the below model is essentially a compound Poisson model, which is often called Cramér-Lundberg model in the risk theory except for the terminology of the operational risk.

The period during which the operational risk will be measured is fixed, for example one year in the future. Though the New Basel Capital Accord told to quantify the operational risk for each cell, which is specified as a given pair of a business line and a event type, we assume that there is only one cell, that is, both business lines and event types are not considered. We avoid the general description of the loss distribution model and present a compound Poisson model directly. See [8] for the more general description.

Denote by N the random variable which represents the number of loss event occurred during a given period. N is assumed to follow a simple Poisson distribution with the mean parameter λ.

For $k = 1, \cdots, N$, let X_k be the random variable that stands for the severity of k-th loss event. It is assumed that $X_1, X_2, \cdots$, are independent and every X_k is independent of N. Roughly speaking, there is no tendency for the occurrence of loss events and the size of severity and the number of loss events are uncorrelated. Besides, $X_1, X_2, \cdots$, have the same distribution function $F(x) = P(X_k \leq x)$. Since the value of X_k is supposed nonnegative, F should be the function whose domain is the real number more than or strictly larger than zero. For example, log-normal, Weibull, Gamma, Pareto and so on.

Then the aggregated severity during the period, Z is defined as

$$Z = X_1 + X_2 + \cdots + X_N = \sum_{k=1}^{N} X_k. \tag{1}$$

We denote by $G(z)$ the distribution of Z. In our setting, G is expressed in terms of Poisson distribution and the convolution of F in the following way.

$$G(z) = \sum_{n=1}^{\infty} e^{-\lambda} \frac{\lambda^n}{n!} F^{n*}(z), \tag{2}$$

where F^{n*} is the n-fold convolution of F.

Then the operational VaR with a confidence level q is defined by

$$\mathrm{OPVaR}(q) = \inf\{z | G(z) \geq q\} \tag{3}$$

If it is possible to know $G(z)$ explicitly, we can easily compute the operational VaR. However, it is hard to achieve the explicit expression of the distribution of Z in general, so we will use Monte Carlo simulation to get the operational VaR later.

3 Verification of model by case

This section verifies the model suggested in the previous section by using actual reporting data on human errors or system breakdown. At first, we overview the characteristics of actual data, then show the result of parameter estimation for a distribution of severity per event, and finally operational VaR is calculated by using simple Monte Carlo simulation.

3.1 Analysis of reporting data

In this analysis, we used a part of the malfunction reporting data which the Mitsubishi Trust Bank had collected for the Quantitative Impact Study 1 (QIS1) report.

The reporting data is divided into six business lines like Agency services, Asset management, Commercial banking, and so on. The event type, that is, causes of loss event is not considered this time.

We just take notice of the date when a loss event was reported (not occurred nor exposed), the severity and the business line which the loss event belonged to. Actual data included the near-miss case, that is, the case where the error resulted in zero loss. We omitted zero-loss cases in our analysis though such cases should be used in some way.

Moreover, we transformed the actual amount of loss into the zero dimension quantity by dividing it by an exposure indicator given every business line and then changing its scale.

One reason is, of course, that it is not possible to make public raw figures of loss due to the character of data; we think that it is an essential data processing method in order to understand the severity of the loss event and to make a comparison of the impact among different lines or banks, since the impact of severity really differs according to the number of workers, trading volume, the total asset and so on, that is, the "size" each business line and bank. Hereafter in this paper, "the severity" means the quantity after some transformations.

Furthermore, we notice that we do not especially consider the case that several events was combined and was reported as one loss event although it seems that it is necessary to do some processing about it.

To begin with, we show a summary of reported data.

Table 1 shows the numbers of loss events each business lines during some period. The business line is indicated by the number, but we notice that the alphabetical order of the business line does not correspond to the number.

	BL1	BL2	BL3	BL4	BL5	BL6	Total
The whole period	400	160	86	5	104	122	877

Table 1: The numbers of loss events each business lines

The number of loss events is necessary when estimating the mean parameter of the Poisson distribution, which is assumed as a model of the likelihood distribution. Indeed, it may be desirable to utilize the ratio of the number of loss events occurring to the total number of transactions since the actual number itself seems senseless; the more the business grows in the future, the more errors are likely to happen. Besides, the mean parameter is given by averaging the loss frequency among several periods.

However, this time we use the actual number of events as parameter for Poisson distribution by assuming that the size of business will be stable.

Next, we examine the history of the average loss per day and the aggregated loss per day and whether there is a remarkable trend on occurrence of loss events. Figure 1 displays the average loss per day during the period and Figure 2 shows aggregated loss per day during the same period.

Since it is supposed that all the loss events independently happen in both likelihood and severity, it is essential to examine the existence of outstanding bias on the data. It can be observed that this sample data does not have a remarkable bias.

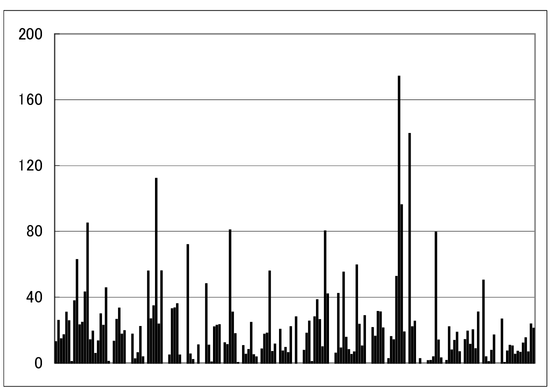

Figure 1: The average of historical severity per day

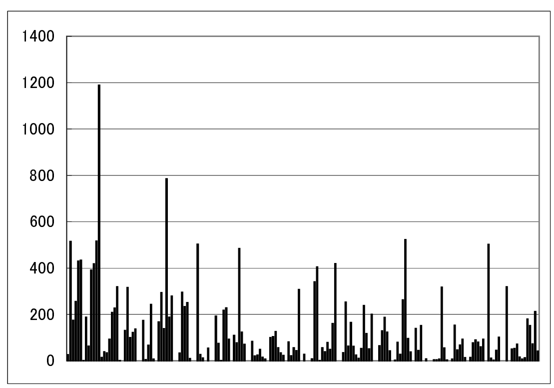

Figure 2: The aggregated severity per day

3.2 Parameter estimation of severity distribution

Now, we use the reported data of the business line 1(BL1) and the line 6(BL6) to estimate parameters of some candidates for severity distribution, that is, the distribution of the amount of the loss for one event. As for BL1, there are a lot of numbers of accident data as seen in Table 1 while in fact BL6 contains many samples during a narrow range, so BL6 is chosen so as to examine the influence.

Log-normal(LN), Weibull, Gamma and General Pareto(GPD) distribution are chosen as candidates for severity distribution here. The specification of each distribution in this paper is as follows.

1. Log-normal distribution : $LN(\mu, \sigma), \ \ \mu > 0, \sigma > 0$

   dens. func. : $f(x; \mu, \sigma) = \dfrac{1}{\sqrt{2\pi}\sigma x} \exp\Big(-\dfrac{(\log x - \mu)^2}{2\sigma^2}\Big) \qquad x > 0$

2. Weibull distribution : $Weibull(\alpha, \beta), \ \ \alpha > 0, \beta > 0$

   dist. func. : $F(x; \alpha, \beta) = 1 - \exp\Big\{-\Big(\dfrac{x}{\beta}\Big)^{\alpha}\Big\}$

3. Gamma distribution : $Gamma(\alpha, \beta), \ \ \alpha > 0, \beta > 0$

   dens. func. : $f(x; \alpha, \beta) = \dfrac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} \exp\Big(-\dfrac{x}{\beta}\Big) \qquad x \geq 0,$

   where $\Gamma(x)$ is a Gamma function.

4. General Pareto distribution(GPD) : $GPD(\xi, \beta), \ \ \xi \in \mathbf{R}\backslash\{0\}, \beta > 0$

   dist. func. : $F(x; \xi, \beta) = 1 - \Big(1 + \dfrac{\xi}{\beta}x\Big)^{-\frac{1}{\xi}}$

   $(F(x; \beta) = 1 - \exp\Big(-\dfrac{x}{\beta}\Big) \text{ if } \xi = 0)$

Moreover, parameters of each distribution are estimated based on several methods; the least-square method(LSM), the maximum likelihood method(MLM), and the method of moments(MOM). As for GPD, we apply the method of probability-weighted moments(MOPWM) instead of MOM.

For MLM and MOM, the estimate values are numerically calculated through a common procedure by using some numerical technique. On the other hand, the cumulative empirical probabilities on each loss should be suitably specified for LSE and MOPWM. Originally the cumulative empirical probability for a certain loss stands for the ratio of the numbers of the loss less than the target loss to the total number of events. However we remark that cumulative empirical probabilities are specified as follows. When the entire number of data is N(400 for BL1 and 122 for BL6), $\dfrac{k}{N+1}$ is the cumulative empirical probability for k-th smallest amount of the loss. It follows that the value corresponding to the largest N'th data becomes a little smaller than one.

The estimate of LSE is given by solving the problem minimizing the sum of the square difference between the cumulative empirical probability and the corresponding probability obtained from the estimated distribution.

MOPWM is the method devised in order to make the method of moments applicable to the distribution whose moments do not exist. The idea of MOPWM is the following.

Denote by $X_1, \cdots X_N$ the amounts of loss in the ascending order and assume that they follow a distribution function $F(x;\theta)$, where θ denotes the set of parameters. Let $p_1, \cdots, p_N$ be the cumulative empirical probabilities corresponding to $X_1, \cdots X_N$ in the ascending order too.

Instead of considering the moments $E[X^r]$, $r = 1, 2, \cdots$, we take account of either $E[XF(X;\theta)^r]$ or $E[X\{1 - F(X;\theta)\}^r]$, $r = 1, 2, \cdots$. The probability-weighted moment estimates is obtained by solving the equations that the probability-weighted moments above coincides with the corresponding cumulative empirical statistics such as $\frac{1}{N}\sum_{k=1}^{N} X_k p_k^r$. Refer to Embrechts et al. [7] for the more explanation for MOPWM.

For each pair of severity distribution function and estimation method, the P-P plot is showed below (see Figure 3 - 8). The vertical axis means the probability implied by estimated distribution function while the horizonal axis stands for the cumulative empirical probability distribution on each loss.

Naturally, the model can seem better if the P-P plot is as close to the diagonal as possible. The minimum mean square method gives a better fit than the others in such a meaning.

However, we should observe the upper-right area more carefully where the cumulative empirical probability is close to one because VaR of 95% or 99% is paid attention to at the quantification of the operational risk.

In point of fitting at the tail of distribution, the method of method (the method of probability-weighted moments for GPD) seems to give desirable results as a whole.

Table 2 represents the value of 95% and the value of 95% of the original data, and ones based on the estimated parameters for each pair of severity distribution and estimation method. It follows that the result largely varies for the same distribution according to the estimation method.

Next, we investigate the tail of the severity distribution by observing the relation between the theoretical maximum value of the estimated distribution and the largest points of original data.

First of all, we think about the distribution of the theoretical maximum value of several independent variables following the estimated distribution. Figure 9 - 14 show that the relation of high-ranking values of the original data and a survival function of the theoretical maximum value. When $F_{max}(x)$ is assumed to be a distribution function of maximum value of N variables, $M_N = \max(X_1, \cdots, X_N)$, the survival function stands for $\bar{F}_{max}(x) = 1 - F_{max}(x) = P(M_N > x)$. That is, it means the probability by which the maximum value M_N exceeds x. (Think about the distribution of

		BL1		BL6	
		95%	99%	95%	99%
Actual data		80.3	154.7	29.0	45.2
LN	LSM	237.8	1194.9	29.3	70.2
	MLM	143.7	621.0	33.7	90.9
	MOM	67.5	150.2	22.6	46.2
Weibull	LSM	106.1	250.2	17.6	27.8
	MLM	40.1	148.8	14.0	34.1
	MOM	88.4	142.8	27.1	43.3
Gamma	LSM	76.5	139.3	17.3	27.0
	MLM	10.2	21.7	7.7	12.2
	MOM	94.2	188.8	30.9	58.0
GPD	LSM	772.5	24184.3	19.7	35.3
	MLM	157.7	1414.2	24.5	53.8
	MOPWM	66.1	188.8	23.6	47.5

Table 2: 95 percentile and 99 percentile for each pair of severity distribution and estimation method

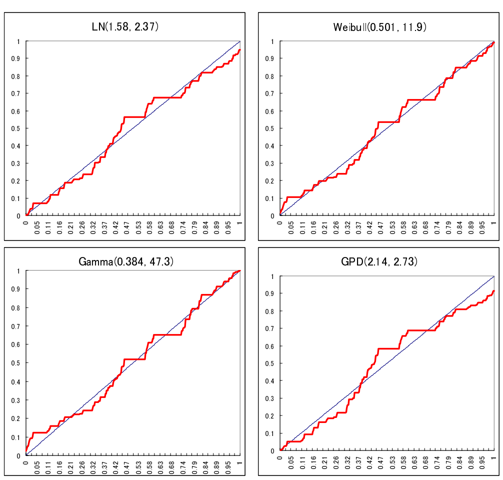

Figure 3: PP-plot for BL1:LSM

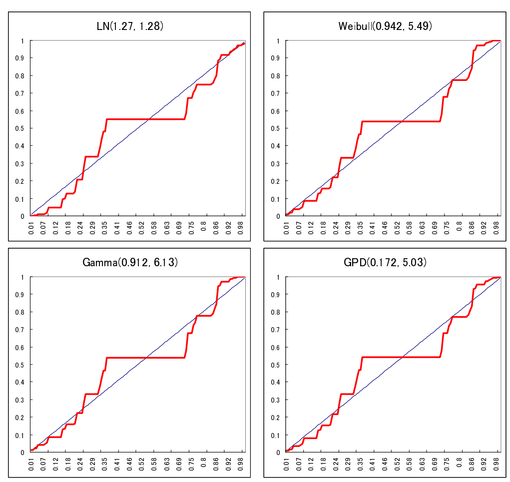

Figure 4: PP-plot for BL6:LSM

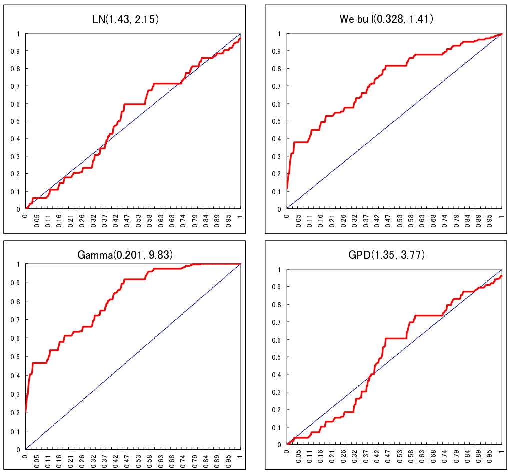

Figure 5: PP-plot for BL1:MLM

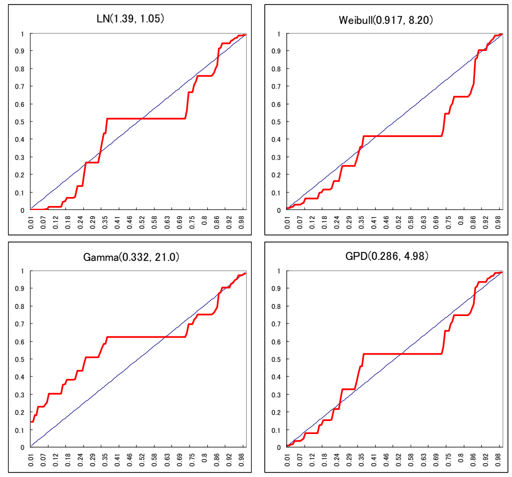

Figure 6: PP-plot for BL6:MLM

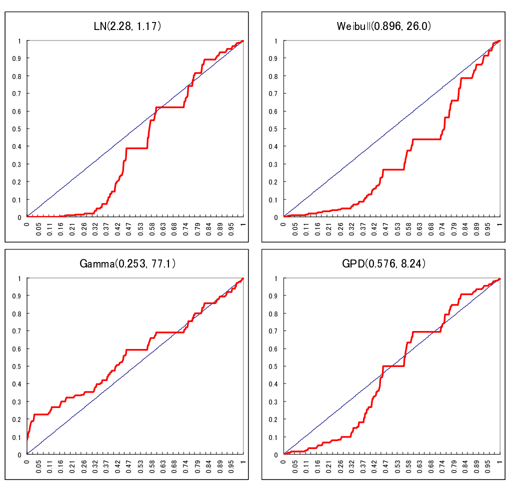

Figure 7: PP-plot for BL1:MOM

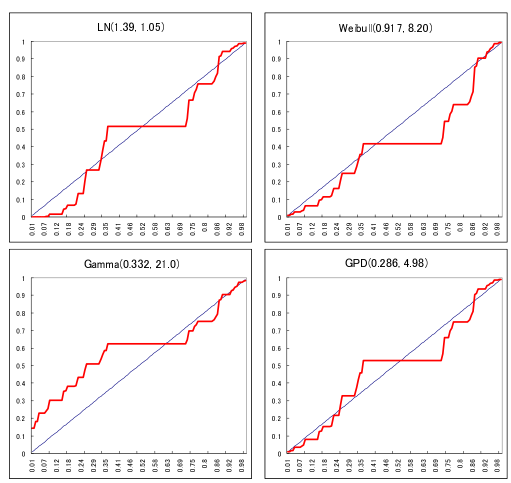

Figure 8: PP-plot for BL6:MOM

the maximum value in 400 independent variables for BL1 while the maximum value of 122 independent variables for BL6.)

As the right end of the curve of survival function of the maximum is closer to 1(for instance, see the LN case of Figure 12), we can judge the tail of the estimated distribution is fatter than actual data, for it means that the theoretical maximum exceeds the actual largest severity by quite a high probability. In short, the risk is too overvalued.

On the othe hand, from the case of Gamma in Figure 12 it follows that it is hardly probable that the theoretical maximum exceeds the sixth largest loss, that is, the risk is too undervalued.

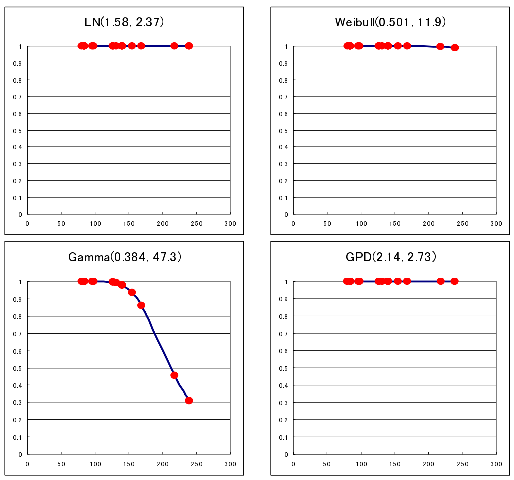

Figure 9: Survival probability of theoretical maximum for actual high-ranked data for BL1:LSE

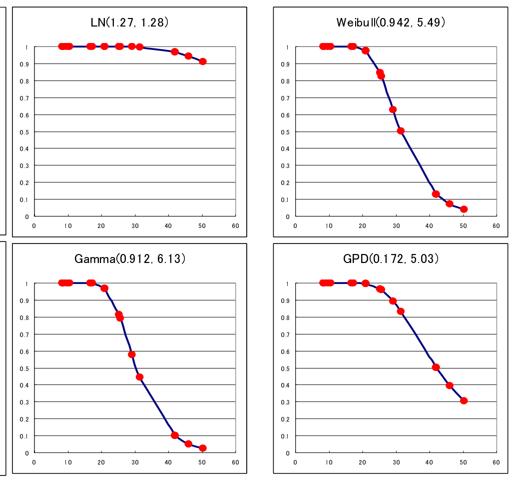

Figure 10: Survival probability of theoretical maximum for actual high-ranked data for BL6:LSE

3.3 The relation between the threshold level in the POT approach and calculated VaR

Next, the consequence of the parameter estimation when we apply the POT approach in the extreme value theory by using only the samples that are more than a certain threshold among BL1 data [3] . The POT approach is the method to approximate the excess distribution function given the threshold by GPD(ξ, β). For a given distribution function $F(x)$, we define the distribution function of the excess over the threshold u by

$$F_u(y) = P(X - u \leq y | X > u) = \frac{F(y+u) - F(u)}{1 - F(u)}, \qquad (4)$$

where $u \leq y \leq x_F = \sup\{c | F(c) < 1\}$.

Now we denote by $G_{\xi,\beta(u)}(x)$ the generalized Pareto distribution with two

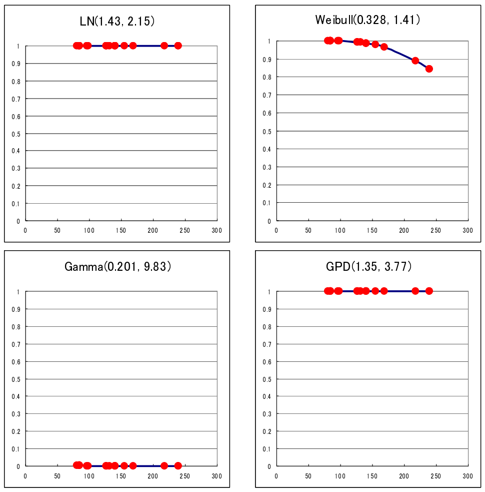

Figure 11: Survival probability of theoretical maximum for actual high-ranked data for BL1:MLM

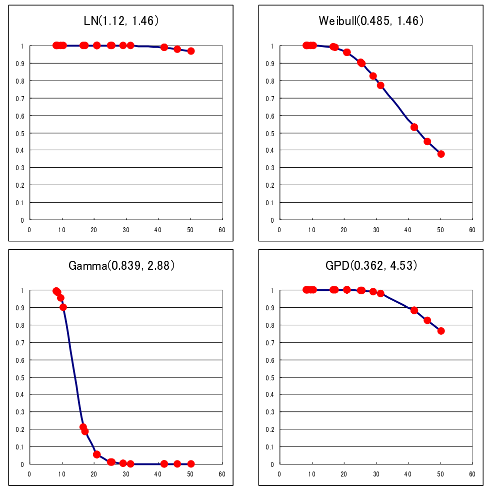

Figure 12: Survival probability of theoretical maximum for actual high-ranked data for BL6:MLM

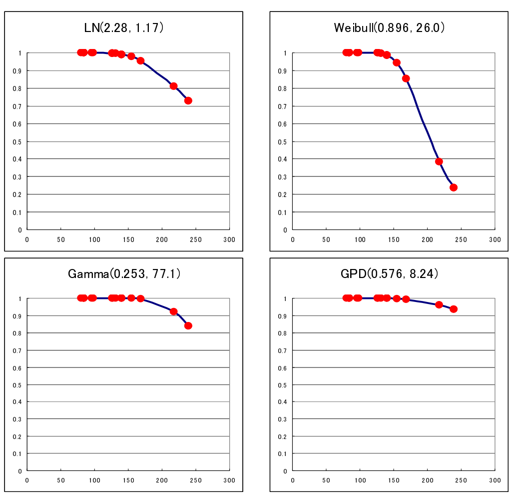

Figure 13: Survival probability of theoretical maximum for actual high-ranked data for BL1:MOM(MOPWM)

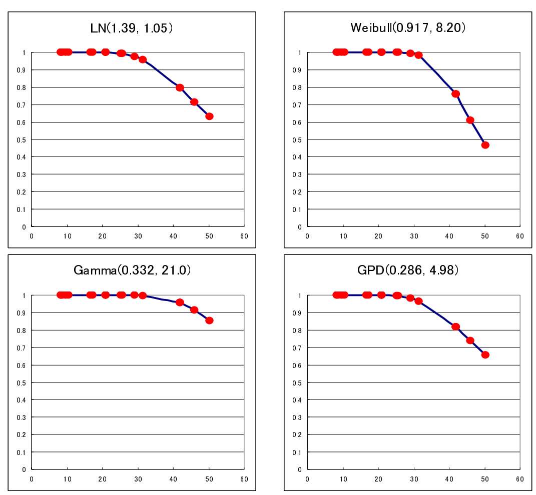

Figure 14: Survival probability of theoretical maximum for actual high-ranked data for BL6:MOM(MOPWM)

parameters ξ and β, that is,

$$G_{\xi,\beta(u)}(x) = \begin{cases} 1 - \left(1 + \frac{\xi x}{\beta}\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - \exp\left(-\frac{x}{\beta}\right) & \text{if } \xi = 0 \end{cases} \tag{5}$$

where $\beta > 0$, and $x \geq 0$ if $\xi \geq 0$ and $0 \leq x \leq -\dfrac{1}{\xi}$ if $\xi < 0$.

If F satisfies some technical conditions, Pickands, Belkema and de Haan proved that there exists a positive function $\beta(u)$ such that

$$\lim_{u \uparrow x_F} \sup_{0<y<x_F-u} |F_u(y) - G_{\xi,\beta(u)}(y)| = 0$$

Therefore if u is large enough, we can find a $\beta \equiv \beta(u)$ and approximate the tail of the distribution F by

$$F(x) = (1 - F(u))G_{\xi,\beta}(x-u) + F(u), \qquad x > u.$$

Suppose that N is the total number of samples and $c(u)$ is the number of samples exceeding the threshold u. Then we might see $F(u) = \frac{c(u)}{N}$, so the estimate of q-VaR(for example $q = 0.95$ or 0.99) can be approximated by the following expression:

$$\mathrm{VaR}_q(F) = u + \frac{\widehat{\beta}}{\widehat{\xi}}\left(\left(\frac{N}{c(u)}(1-q)\right)^{-\widehat{\xi}} - 1\right), \tag{6}$$

$\hat{\xi}$ and $\hat{\beta}$ are the estimates of ξ and β respectively.

Figure 15 - 17 display how the estimated value of shape parameter ξ, 95% and 99%VaR change when the threshold is changed under the constrained condition that the estimated value of ξ does not become negative. The number on the top of the figures corresponds to the number of excess samples over the threshold level on the bottom. As for each estimation, it is remarked that the estimated value of ξ splashes greatly near the threshold of 29, 42, and 57 probably by the influence that comparatively many samples concentrated near the figures.

In spite of the characteristic of the original data, it is observed that the estimated value based on MOPWM and the values of VaR following it are relatively stable for a threshold. Moreover, except for some ranges in the least square method and the maximum likelihood method, it is noticed that both 95% and 99% VaR are computed close to the actual level of severity since 95% and 99% of the original data of BL1 are 80.3 and 154.7 respectively.

While the effectiveness of the POT approach should be studied for more different samples, it may become a very effective method for VaR calculation in the operational risk management.

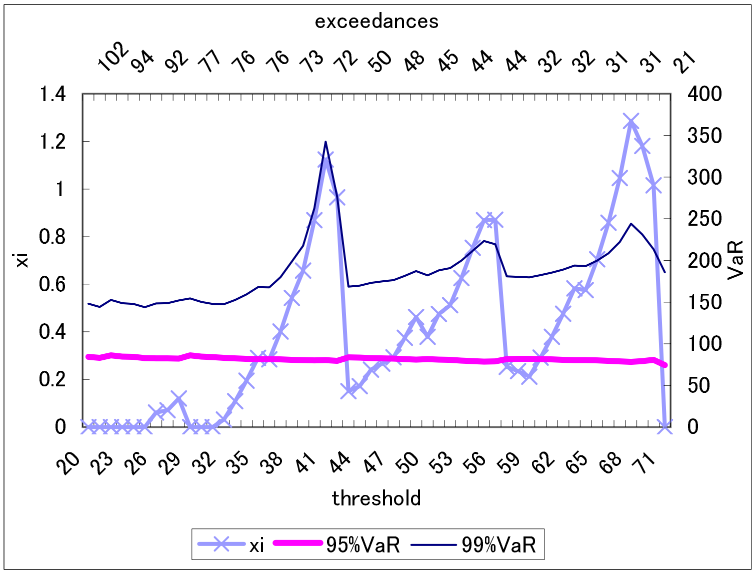

Figure 15: The change of the estimated value of shape parameter ξ, 95% and 99%VaR when the threshold is changed（BL1:LSM）

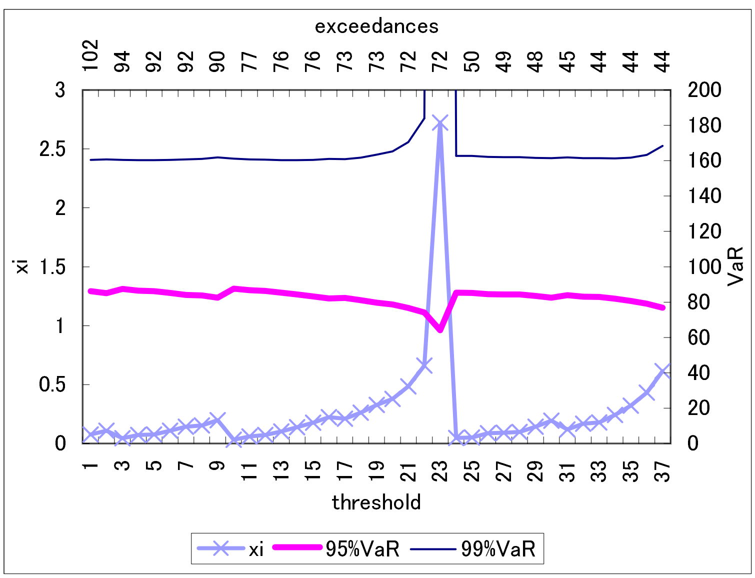

Figure 16: The change of the estimated value of shape parameter ξ, 95% and 99%VaR when the threshold is changed（BL1:MLM）

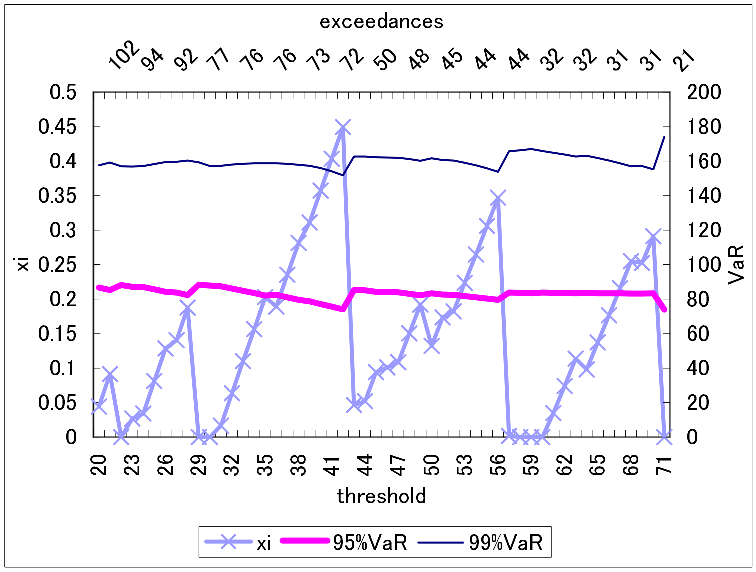

Figure 17: The change of the estimated value of shape parameter ξ, 95% and 99%VaR when the threshold is changed（BL1:MOPWM)

3.4 Simulation results

At last, we present some simple simulation results of the aggregated severity for business line 1(BL1) during a certain period. Here, the logarithmic normal distribution(LN) and the general Pareto distribution(GPD) are taken up as a model of the severity distribution. The consequence of the least square method excluded because the risk is too overvalued. Hence the maximum likelihood method(MLM) and the method of moments(MOM) (method of probability-weighted moments(MOPWM) for GPD) are tested by the simulation. The average parameter of the likelihood is assumed to be actual accident number 400 of BL1 for the observation period. The random numbers are generated following Poisson distribution.

Figure 18 - 21 are a histogram for the aggregated severity after 10,000 trials of simulation. The value of 95%VaR (the 500th largest value of the simulation result) and 99%VaR (the 100th largest value of the simulation result) are specified as well as the aggregated loss of actual data is specified with the position in the histogram.

It follows from the simulation results based on MLM estimation (Figure 18 and 20) that the risk is very overvalued for both LN and GPD when compared with the value of the actual aggregated loss. On the other hand, the simulation results based on MOM or PWME estimation (Figure 19 and 21) imply that the aggregated actual loss locates near the median of the aggregated severities obtained from simulation.

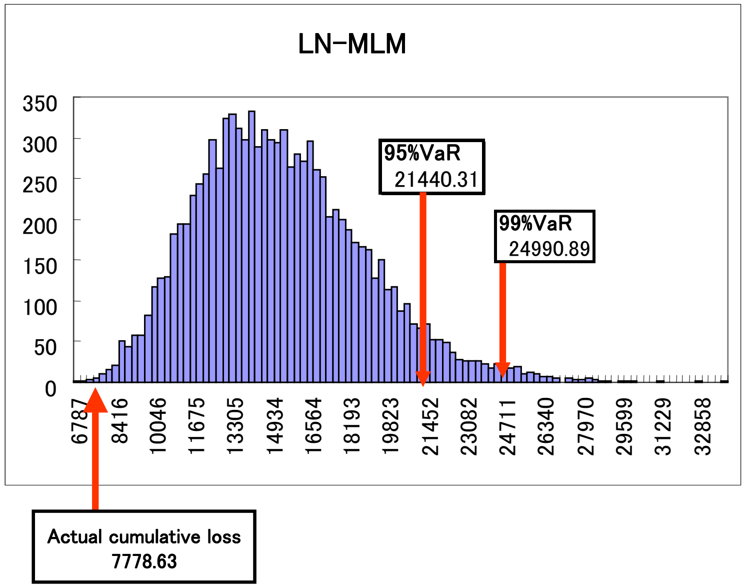

Figure 18: The histogram of the consequences by Monte Carlo simulation of the aggregated severity based on the estimation for a pair of LN and MLM

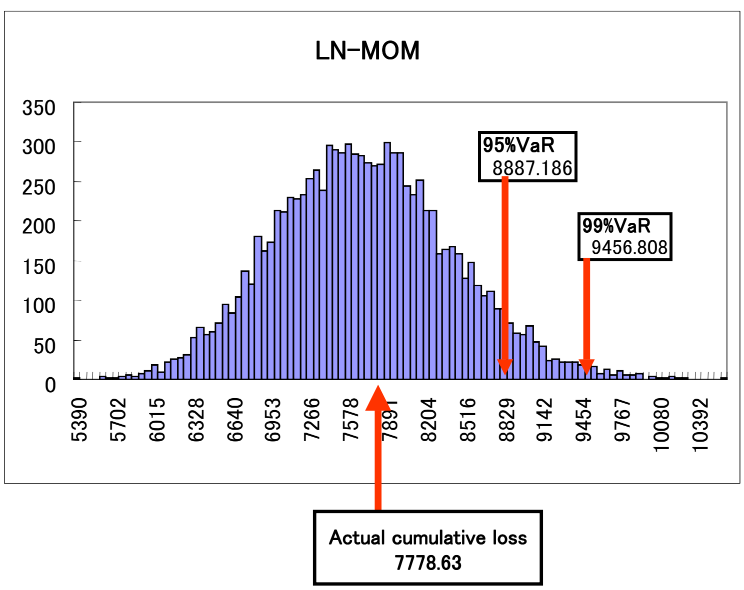

Figure 19: The histogram for a pair of LN and MOM

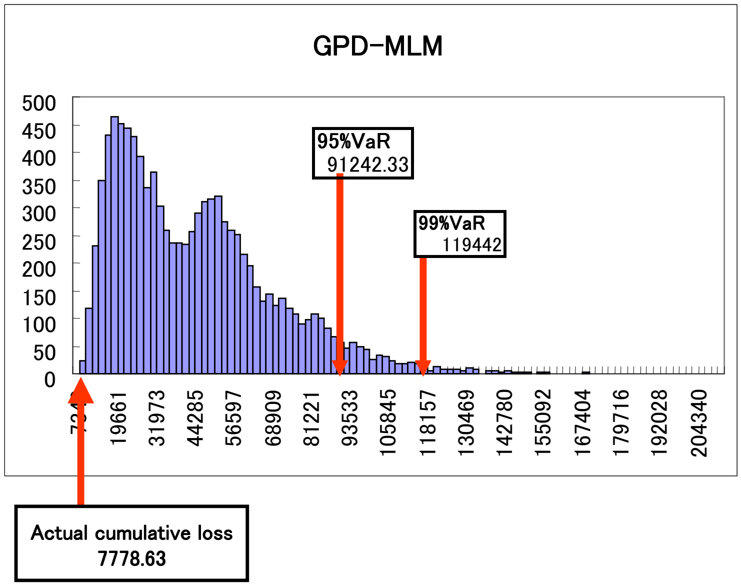

Figure 20: The histogram for a pair of GPD and MLM

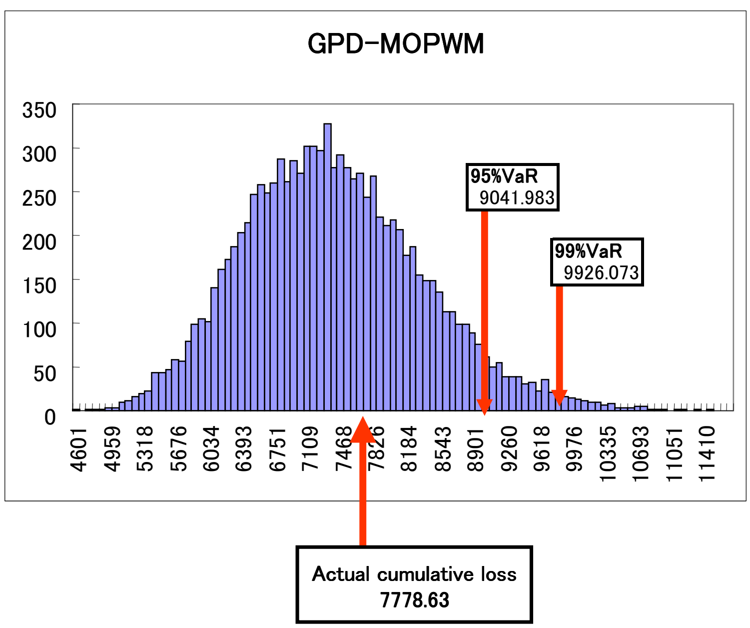

Figure 21: The histogram for a pair of GPD and MOPWM

3.5 Summary

We have shown some illustrations about the parameter estimation of the loss distribution model and the simulation of operational VaR in some business line based on the reporting data.

Though it is just a consideration based on the loss data used in this analysis, we summarize the result.

- The difference between the actual data and the value obtained from the least square method may be quite large around the tail of distribution function.
- Generally, the estimation consequence by the maximum likelihood method is not stable, probably because loss samples have concentrated on a narrow range. Of course, the number of data is crucial to this method.
- The estimation results by the method of moments(or the method of probability-weighted moments) are comparatively desirable.
- However, it is hard to conclude which distribution is suitable for the model of severity distribution only through this verification since the estimation results can be greatly different according to the estimation method. It can be better to always take account of a pair of distribution function and estimation method.
- The POT approach may be effective in the future when estimating the tail distribution more exactly.

We insist that such results may help us to see which distribution and which estimation are desirable, but it is a danger to decide the pair of the distribution function and method only from these results. Indeed, it is likely that the best pair depends upon the characteristics of sample data. Therefore we need to verify the model for a lot of different types of sample data.

4 Notes on the use of external data

In the last section, we present some illustrations of implementation of LDA for measuring the operational risk from the internal data. However, the value of VaR obtained from the simulation may be too low in comparison of the 12% - 20% figure of minimum regulatory capital that the Basel Committee thinks of as the suitable level of regulatory capital necessary to cover the bank's operational risk.

Unless the internal data contains a few cases of very large amount of operational loss, it seems natural to consider that any version of approach derived from LDA cannot improve such a situation essentially. Thus it is suggested the use of external data including the cases of huge loss that happened and were reported in the past or the stress scenario made artificially.

In this section we discuss the questions and remarks when not only the internal data also the external data is used to quantify the operational risk based on LDA.

The external data might belong to either of the following two types.

1. Large loss cases actually happened at another banks in the past.
2. Loss scenario made artificially through self-assessment and so on.

It seems that it is significant to supplement internal data as potential huge loss with the external loss samples if it is possible to identify what business line the loss belongs to and it is hard to deny that a similar accident may not happen from a point of the business circumstance around us.

On the other hand, it goes without saying that it is necessary to understand the amount of the risk qualitatively, through not only the accident data but also the internal audit and the assessment in the operational risk management. If the risk scenario suitable for an internal situation can be made and it can be transformed into the same format of internal data, it will be said that external data is effective to capture the potential operational risk.

However, there are the following questions about the use of external data.

- How should we evaluate consistently with internal data when scaling a loss of 10,000,000,000 yen occurred outside? Probably the valuation depends on different factors like the number of employees, the economic capital, the size of assets, the weights to each business and so on. However, it is difficult to show a reasonable scaling method. An idea is to convert the amount according to the risk exposure for the underlying business line.

- Even if the scaling method is established, it is subjective which external cases should be added for estimation of the severity distribution. Besides, it is hard to presume the likelihood that the similar event will happen. The supervisory authority should show the guideline of handling the actual external case to the financial institution objectively.

- When the stress scenario is added to the internal reported data, the transparency of the internal audit and assessment is indispensable. Moreover, we should pay attention to the time lag from the stage of auditing and assessment to the scenario making and the objectivity of loss scenario making itself.

- It is necessary to think about the cumulative empirical probability if we estimate the parameters based on the least square method or the method of probability-weighted moments. When only the internal data is used, as seen before, the probability is given in proportion to the ascending order. Similarly for the external data, we need to give the value corresponding to the cumulative empirical probability. For example, if one assumes that some case may happen only once every 100 years and the average number of loss event is 400, then the estimated cumulative empirical probability might be

$$1 - \frac{1}{400 \times 100} = 0.999975(99.9975\%)$$

  If once a decade, it might be 99.975%. We need to study how to estimate the likelihood of the external data occurring.

- The maximum likelihood method and the method of moments might be avoided when the external data is added to the sample data for estimating the parameters of severity because even if the number of the external data added is only one, it might make a great influence on the estimates. Different from the other methods, a cumulative empirical probability is not necessary, so the inserted external data is equivalent to the other internal data in terms of the likelihood if not adjusted.

5 Conclusion

In this paper, we discuss how to implement LDA through the verification of the operational risk measurement based on the Cramér-Lundberg model. At this stage, it is difficult to judge whether our model based on LDA is helpful for operational risk management and control and we should remember that LDA is just one of many various approach to quantify the operational risk. Still, we are sure to keep on studying the LDA method in the future since it is expected that the number and the quality of significant loss data will increase.

Moreover, this approach motivates us to do research in the statistic theory about small and non-stationary samples, contrary to the large sample theory for the asymptotic characteristics.

Furthermore, the original model seems consistent with the valuation model in the insurance field. It seems preferable the application of insurance for hedging operational risk. Insurance product for operational risk will be more and more demanded in the future, so this approach should be also studied from a point of risk-mitigating cost by insurance. In the working paper published by the Risk Management Group of the Basel Committee [5], the issue of insurance as an operational risk mitigant is discussed. Now, the Group seems to intend to limit the total impact of insurance risk mitigation on the final capital amount, but implies that *works remains to be done to refine a potential treatment for insurance under the operational risk capital charges".*

References

[1] *Artzner, P., F. Delbaen, J.M. Eber and D. Heath (1999), "Coherent Measures of Risk, " Mathematical Finance,* **9***, 203-228.*

[2] *Basle (1998), "Operational Risk Management, " Basle Committee on Banking Supervision.*

[3] *Basle (2001), "The New Basel Capital Accord," Secretariat of the Basle Committee on Banking Supervision*

[4] *Basle (2001), "Operational Risk," Consultative Documnet, Basle Committee on Banking Supervision.*

[5] *Basle (2001), "Regulatory Treatment of Operational Risk," Working paper, Basle Committee on Banking Supervision.*

[6] *Danielson, J., de Vries, C. G.(1997), "Beyond the Sample: Extreme Quantile and Probability Estimation", Working paper.*

[7] *Embrechts, P., C. Klüppelberg and T. Mikosch (1997) Modelling Extreme Events for Insurance and Finance, Springer, Berlin.*

[8] *Frachot, A., Georges, P., Roncalli, T.(2001), "Loss Distribution Approach for Operational risk", Working paper.*

[9] *King, J. L.(2001), Operational risk: Measurement and Modelling, John Wiley and Sons.*

[10] *Lamberton, D. and B. Lapeyre (1997), Introduction au Calcul Stochastique Appliqué à la Finance, Ellipses.*

[11] *Leblanc, C., O. Renault, O. and O. Scaillet (2000), "A correction note on the first passage time of an Ornstein-Uhlenbeck process to a boundary, " Finance and Stochastics,* **4**, *109-111.*

[12] *McConnell, P. and K. Blacker (2000), "An Approach to Modelling Operational Risk in Banks, " Working paper.*

[13] *McNeil, A. (1999), "Exterme Value Theory for Risk Managers, " Working paper.*

[14] *Mori, T. and E. Harada (2000), "Internal Risk Based Approach, " Financial and Payment System Office Working Paper Series 00-No.2.*

[15] *Mori, T., J. Hiwatashi and K. Ide (2000a), "Measuring Operational Risk in Japanese Major banks, " Financial and Payment System Office Working Paper Series 00-No.1.*

[16] *Mori, T., J. Hiwatashi and K. Ide (2000b), "Challenges and Possible Solutions in Enhancing Operational Risk Measurement, " Financial and Payment System Office Working Paper Series 00-No.3.*

[17] *Smith, R. L., Goodman, D. J.(2000), "Bayesian Risk Analysis", Working paper.*

[18] *A Risk Special Report (1999), Operational Risk, Risk July.*

Notes

1 *This phrase 'direct or indirect' is deleted from the working paper published in September 2001 [5].*

2 *The likelihood means the number or the frequency of operational loss events during a given period. The severity stands for the amount of one loss event.*

3 *See Embrechts et al. [7] and McNeil [13] for Extreme value thoery.*