"Financial Engineering Tools Applied to Deposit Insurance Pricing and Bank Rankings according to Credit Risk"

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Summary

Pricing deposit insurance premiums has been widely studied in several countries. Opposite to life insurance, the lack of information has been a problem to develop a model.

Since there is not a standard methodology to price premiums for deposit insurance, each country has been using different methods and in many cases, a fixed premium is charged to all banks.

Aiming to develop a methodology to classify banks according to risk and price premiums, we introduce some methodologies based on default intensity functions and copula functions.

The construction of default intensity functions is based on Duffie-Singleton Model to price bonds. We can look at a loan as a coupon bond, and assuming we have the market value of the loan we can obtain the default intensity functions to estimate probabilities. In this model, a martingale is constructed, which leads us to the solution of a stochastic differential equation which will depend of a default intensity function. Such function will be used to estimate our default probabilities, then we will use a statistical copula to estimate the joint default probability for all loans.

A copula function is given by:

$$F(t_1,t_2,...,t_n) = C(F_1(t_1), F_2(t_2),..., F_n(t_n))$$

Where the F ´s denote the marginal distributions. In our case they are the default distribution functions for each type of loan.

"Ingeniería Financiera Aplicada al Seguro de Depósito y a la clasificación de Bancos de acuerdo al Riesgo Crediticio"

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Resumen

La valuación de primas justas con base en riesgo para el seguro de depósitos ha sido un tema de estudio en diversos países. A diferencia de un seguro de vida, la falta de información ha sido uno de los factores que limita el cálculo de primas del seguro de depósito.

Debido a que no hay una metodología definida como en seguros de vida para valuar un seguro de depósito, cada país a desarrollado sus propios métodos y en muchos casos se ha optado por el esquema de cuota fija.

Con el fin de desarrollar alguna metodología que permita clasificar a los bancos para hacer un cobro de cuotas con base en el riesgo, se presenta en este estudio algunas metodologías que puedan ser útiles. Las herramientas de estos modelos son funciones intensidad de default y cópulas estadísticas.

La construcción de funciones intensidad de default se basan en el modelo Duffie-Singleton para bonos corporativos. Viendo un crédito bancario como un bono y asumiendo que la cantidad que se tiene del crédito es el valor del mercado del mismo, se puede encontrar una función intensidad de default que se utiliza para la construcción de una probabilidad de default en un horizonte de tiempo determinado. En este modelo se construye una martingala, posteriormente la solución de una ecuación diferencial estocástica nos lleva a una función que captura el riesgo a través del spread y dependerá de la llamada función intensidad de default, análoga a la fuerza de mortalidad. Dichas probabilidades de incumplimiento son a su vez utilizadas para calcular la probabilidad de incumplimiento de la cartera de crédito usando las funciones estadísticas llamadas cópulas, las cuales incorporan una estructura de correlación.

Una función cópula esta dada por:

$$F(t_1, t_2, ..., t_n) = C(F_1(t_1), F_2(t_2), ..., F_n(t_n))$$

En donde las $F\,'s$ denotan las marginales que en nuestro caso son las funciones de incumplimiento de cada tipo de crédito de un banco.

Financial Engineering Tools Applied to Deposit Insurance Pricing and Bank Rankings according to Credit Risk

Our purpose is to develop a credit risk model which will allow us to estimate default probabilities for credits. We apply this model to classify banks according to their credit risk. A model to price a premium for loan defaults is presented, stochastic scenarios are used in the pricing.

Model Applications:

1.- The model will let us estimate default probabilities for each type of credit.

2.- Each default probability for each credit will be used to estimate joint defaults. A dependence structure is incorporated using statistical copulas.

3.- In Mexico, credit is a big percentage of bank activity, therefore we try to use credit risk as a variable which can give us an early warning of bankruptcy.

4.- Default intensity functions can be used to price premiums actuarially.

Executive Summary

In this paper we try to estimate default probabilities for bank loans. As shown in diagram 1, the first step is to construct the default probabilities for each type of credit. After that we use copula functions to estimate joint default probabilities. Finally we describe how to price a premium for defaultable loans.

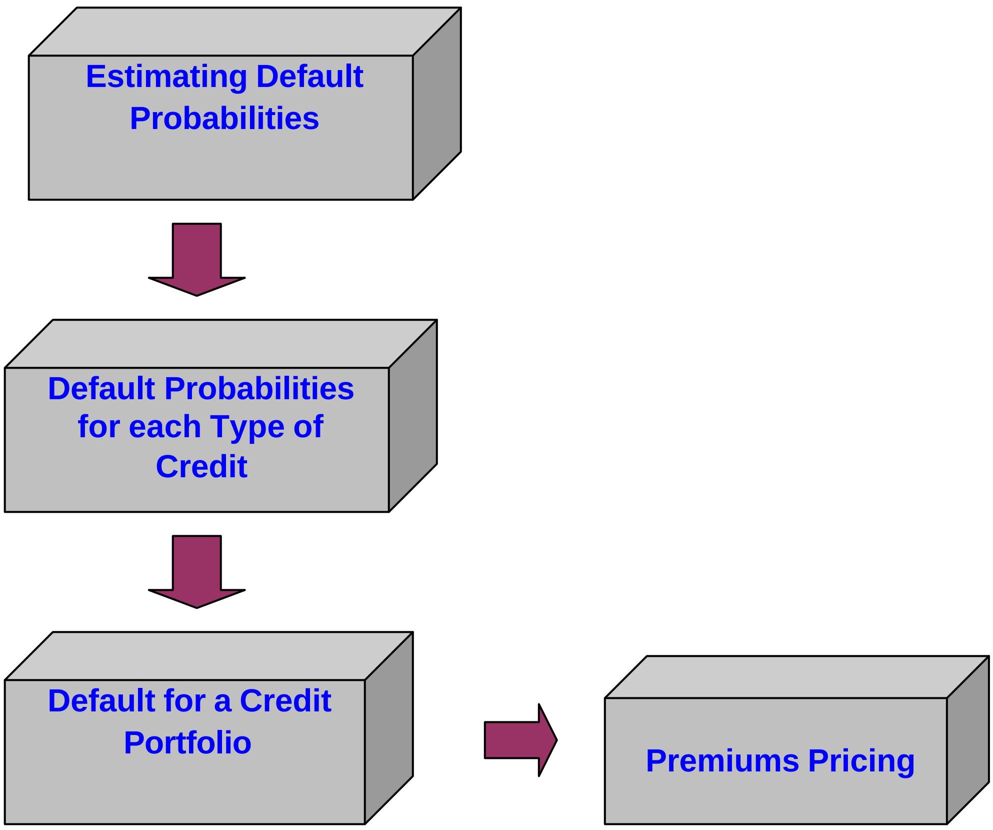


Diagram 1

As mentioned before, the starting point is to estimate default probabilities for bank loans in Mexico. The methodology is based on the construction of a so called "Default Intensity Function", which will allow us to construct the default probabilities.

Duffie and Singleton developed a model for bonds. We apply this model for bank credits. Then we use copula functions to construct a joint distribution that could be used to estimate joint default probabilities.

It is assumed that the market is efficient and historic information is not used because of the Markov property.

This paper is divided in the following sections:

- The first part focuses on the description of the model.
- In section 2 we give an example applied to México.
- In section 3 we describe how we can obtain joint default probabilities for a portfolio of credits.
- We give an example in section 4.
- In section 5 we give a description on how to estimate the premiums.

1. Model Settings

Modeling Deposit Insurance is a topic where applications of risk management can become practical. One of the most important tasks in credit risk management is the estimation of default probabilities. We use a stochastic model to evaluate probabilities of default, a time horizon of one year is used because this period reflects the typical interval over which:

a) New capital could be raised.
b) Loss mitigating action could be taken to eliminate future risk from the portfolio.
c) New obligor information could be revealed.
d) Default rate may be published.
e) Internal budgeting, capital planning and accounting statements are prepared.
f) Credits are normally reviewed for renewal.

One of the main problems during this first research was the quality of information. Not only were incomplete data sets found, but also wrong information provided by the bank, therefore some assumptions had to be done to be able to run the model. Nevertheless, it is expected that the new format R04 from the CNBV will help to improve quality of information.

Our second step after calculating probabilities of default is to use them to build a joint distribution. Concepts of dependence and statistical copulas will be used. This will allow us to construct a premium for a deposit insurance.

Let the set A be defined as follows: A = All the credits with common maturity, payable quarterly, semi-annually and annually. Let T be a continuous random variable which measures the length of time to default. Therefore we have the following:

$$F(t) = P(T \le t) \text{ is the probability that credit A defaults before } t_i$$
$$S(t) = 1 - F(t) \text{ is the probability that credit i survives up to } t_i$$

Since we are interested in estimating the default probabilities in a time horizon we introduce the following notation:

$$def_{x_1}^{t_i} = P(T - x_i \le t_i | T > x_i) \text{ probability that credit i defaults in the following } t_i \text{ years given that it survived up to time } x_i$$

$$sur_{x_i}^{t_i} = P(T - x_i > t_i | T > x_i) \text{ probability of not defaulting in the following } t_i \text{ years given that the credit survived up to time } x_i$$

These probabilities can be constructed using the so-called default intensity function, widely used in biostatistics and actuarial science to model survival probabilities. The default intensity function is defined as follows:

$$P(x_i < T \le x_i + \Delta x_i | T > x_i) = \frac{F(x_i + \Delta x_i) - F(x_i)}{1 - F(x_i)} = \frac{f(x_i)\Delta x_i}{1 - F(x_i)}$$

This function can be seen as a force of increment. The relationship between the default intensity function and the probabilities is the following:

$$sur_{x_i}^{t_i} = e^{-\int_0^{t_i} h(x_i+s)ds} = e^{-\int_{x_i}^{x_i+t_i} h(s)ds}$$

By complement we can obtain the default probabilities. As it can be easily seen, our task is reduced to model the default intensity function.

Estimation of Default Probabilities

Let us assume that we have loans which are paid monthly, quarterly, and annually, this assumption can be broken without affecting the model as long as we have specific information on each bank. We introduce the following notation:

T_{ijk} – Total paid quantity (Interets plus Capital)
$i = 1,2,3,...,12P$ if $k = m(monthly)$ in year j
$i = 1,2,...,4P$ if $k = q(quarterly)$
$i = 1,2,...,P$ if k = a(annually)
C_{hj} – Denotes payments of all credits for period h and year j.

We are interested in estimating the default intensity function for credits, to simplify we value the payments of interest and outstanding semi annually. We use the following notation:

C_{hj} – Denotes payments in period h and year j.

$$C_{1j} = \sum_{i=1}^{5} T_{ijm} \prod_{h=i}^{5} (1 + i_{m(h+1)}) + T_{6jm} + T_{1jt} \prod_{h=3}^{5} (1 + i_{m(h+1)}) + T_{2jt}$$

$$C_{2j} = \sum_{i=7}^{11} T_{ijm} \prod_{h=i}^{11} (1 + i_{m(h+1)}) + T_{12jm} + T_{3jt} \prod_{h=9}^{11} (1 + i_{m(h+1)}) + T_{4jt} + T_{ija}$$

This way we are looking at credits as if they were bonds, it is assumed that the market value of the loans is given by the book value of loans, big assumption?, yes any suggestions are welcomed. Using Duffie-Singleton approach we have:

$$CreditPV = \sum_{k=1}^{n} \sum_{i=1}^{2} C_{ik} e^{-\int_0^{5i+k-1} [r(s)+(1-R(s)h(s)]ds} \quad ...(1)$$

A brief development for equation (1) is given in the appendix, for further details about the model applied to bonds see Duffie-Singleton [4].

We can solve equation (1) for h(s) obtaining the default intensity function which will be used to construct our probabilities.

2. Application: Default Probabilities for each Type of Credit of a Portfolio

In this section we apply the model developed in section 1. Two Mexican banks were taken for our analysis. Unfortunately we did not have information on the spreads and therefore, assumptions were made. The CNBV (Banking regulatory Institution in Mexico) has asked banks a new report (R04) in which we expect to find the missing information we have in our model.

Bank 1

Type of Loan	Loan
Comercial Loans	2,424,091,694.33
State Government Loans	95,460,575.33
Consumer Loans	526,582,200.83
Mortgage Loans	703,939,964.00
Government Loans	797,296,084.17

Table 1. Source: SAF

Bank 2

Type of Loan	Loan
Comercial Loans	2,199,176,197.50
Financial Institutions	16,131,848.50
Mortgage Loans	128,216,922.67
Government Loans	8,668,614.17

Table2. Source: SAF

The following table shows the assumptions on the recovery rate and the spread for each type of credit. Average recovery is about 17%, excluding government loans for which recovery is 100%.

Bank 1

Type of Loan	Spread*	Recovery
Comercial Loans	5	10%
State Governement Loans	1.5	50%
Consumer Loans	17	10%
Mortgage Loans	10	30%
Government Loans	0.8	100%

Table 2. Assumptions

Bank 2

Type of Credit	Spread*	Recovery
Comercial Loans	5	10%
Financial Institutions	1	15%
Government Loans	0.8	100%
Mortgage Loans	10	30%
Government Loans	0.8	0%

Table 4. Assumptions

The default intensity and probabilities obtained for both banks are given below:

Bank 1

Type of Credit	Default Intensity	1 Year Default Probability
Comercial Loans	4.540%	4.4383%
State Government Loans	1.5812%	1.5688%
Consumer Loans	12.08%	11.38%
Mortgage Loans	8.74%	8.37%
Government Loans	1.0098%	1.0047%

Table 5. Results

Bank 2

Type of Loan	Default Intensity	1 Year Default Probability
Comercial Loans	11.660%	11.060%
Financial Institutions	3.299%	3.245%
Mortgage Loans	14.992%	13.922%
Government Loans	0.0000%	0.0000%

Table 6. Results

These probabilities are used to estimate the loss for each type of loan. Then we use the bank´s equity to construct a ratio, since a loss is given this will affect equity. The lower the ratio, the better the bank is financially to confront a possible loss.

Bank 1

Type of Loan	Probability	Total Loan	Expected Loss
Comercial Loans	11.6600%	2,424,091,694.33	282,649,091.56
State Government Loans	6.4676%	95,460,575.33	6,173,960.44
Consumer Loans	18.23%	526,582,200.83	96,000,937.74
Mortgage Loans	14.99%	703,939,964.00	105,532,919.55
Government Loans	0.000%	797,296,084.17	0.00
		Total Expected Loss	490,356,909.30
Expected Loss % equity rules 2003			8.59%

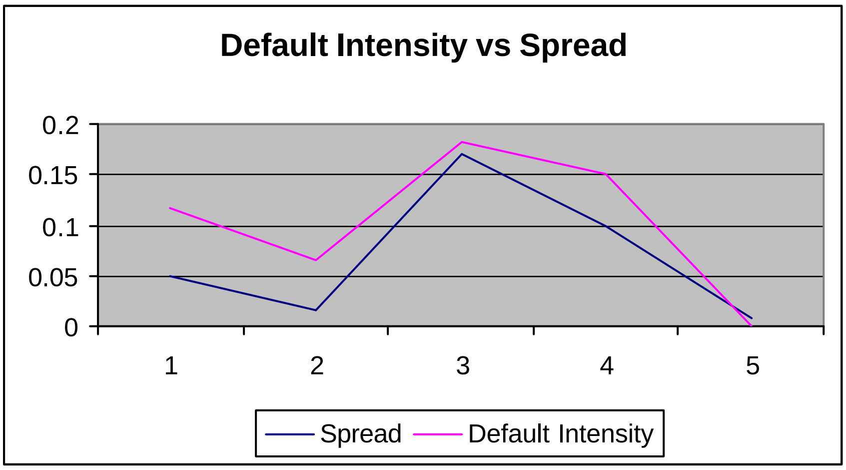
Table 7. Expected Loss

Bank 2

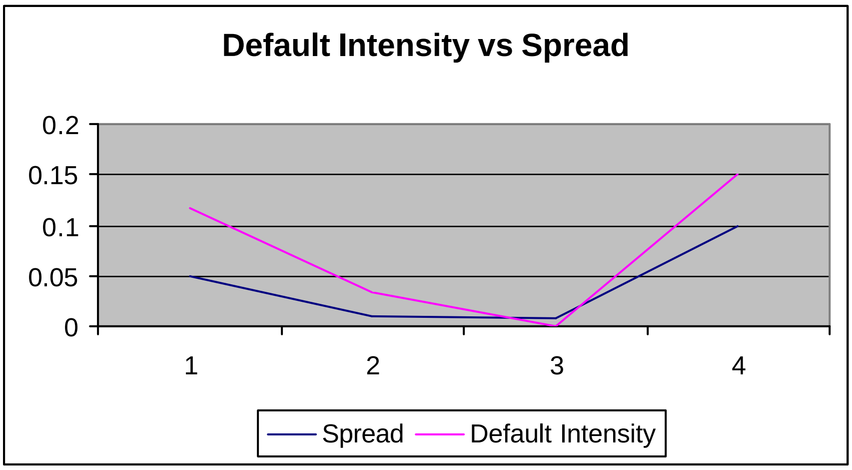
Type of Loan	Probability	Total Loan	Expected Loss
Comercial Loans	11.6600%	2,199,176,197.50	256,423,944.63
Financial Institutions Loans	3.299%	16,131,848.50	532,189.68
Mortgage Loans	14.992%	128,216,922.67	19,222,024.61
Government Loans	0.000%	8,668,614.17	0.00
		Total Expected Loss	276,178,158.92
Expected Loss % equity rules 2003			1.60%

Table 8. Expected Loss

As the spread increases so does the force of default and therefore the probability of default. This sounds natural since a larger spread means higher risk which should be reflected in the default probability. If we obtain these ratios for all banks we will have a measure to compare banks according to their credit risk. Graphs for the spread and default intensities are given for both banks:



Graph 1



Graph 2

Assuming a constant force of default we can easily verify that the distribution of default probabilities for the first bank are given as follows:

$$f(x) = .1166e^{-.1166x}\, I_{[0,1]}(x) + .8899\, I_{Otherwise}(x) \text{ for Commercial Loans.}$$

$$f(x) = .06468e^{-.06468x}\, I_{[0,1]}(x) + .9373\, I_{Otherwise}(x) \text{ for State Government Loans.}$$

$$f(x) = .1823e^{-.1823x}\, I_{[0,1]}(x) + .8333\, I_{Otherwise}(x) \text{ for Consumer Loans.}$$

$$f(x) = .1499e^{-.1499x}\, I_{[0,1]}(x) + .8608\, I_{Otherwise}(x) \text{ for Mortgage Loans.}$$

$$f(x) = 0\, I_{[0,1]}(x) + 1\, I_{Otherwise}(x) \text{ for Government Loans.}$$

Whereas for the second bank we have:

$$f(x) = .1166 e^{-.1166x}\, I_{[0,1]}(x) + .8899\, I_{Otherwise}(x) \text{ for Commercial Loans.}$$
$$f(x) = .03299 e^{-.03299x}\, I_{[0,1]}(x) + .9675\, I_{Otherwise}(x) \text{ for Financial Institutio ns.}$$
$$f(x) = .1499 e^{-.1499x}\, I_{[0,1]}(x) + .8608\, I_{Otherwise}(x) \text{ for Mortgage Loans.}$$
$$f(x) = 0\, I_{[0,1]}(x) + 1\, I_{Otherwise}(x) \text{ for Government Loans.}$$

This model allow us to construct default probabilities for one year, the extension to more years is straightforward, however for practical reasons we consider only one year for the reasons mentioned at the beginning of this section.

3. Joint Default Probabilities

In this section we estimate joint default probabilities using copula functions. A traditional problem in applied probability is the construction of joint distributions. When there is no independence among variables going from the marginal distributions to the joint distribution is not an easy task.

A statistical copula can be used to construct a joint distribution using the marginal distributions. Moreover, we will be able to introduce a correlation matrix to estimate our probabilities.

Let us assume that for each portfolio´s credit i we have a survival random variable T_i with distribution function $F_i(t_i)$. The joint distribution function is given by:

$$F(t_1, t_2, ..., t_n) = C(F_1(t_1), F_2(t_2), ..., F_n(t_n))$$

If we use a normal copula we have:

$$F(t_1, t_2, ..., t_n) = \Phi_n\left(\Phi^{-1}\left(F_1(t_1)\right), ..., \Phi^{-1}\left(F_n(t_n)\right)\right)$$

Where Φ_n is a multivariate normal distribution with correlation ? and where ? denotes the cumulative distribution function of a multivariate normal random vector.

There are reasons why a normal copula is used instead of another kind of copula. The first one is that its properties make it easier to deal with. The normal distribution belongs to the elliptical family of distributions which can be defined only with the first and the second moment. In addition to this the computational part becomes much easier when we deal with normal variables than any other joint distribution[1]. The algorithm is in Visual Basic.

4. Application: Joint Default Probabilities

For this example we assumed the following correlation matrix.

[1] A Jacobi algorithm was used.

	Data			Iterations	100000
	Correlation Matrix			No. of Variables	5
	Comercial	**State Government**	**Consumer**	**Mortgage**	**Government**
Comercial Loans	1.00	0.50	0.40	0.82	0.20
State Government Loans		1.00	0.10	0.20	0.00
Consumer Loans			1.00	0.75	0.10
Mortgage Loans				1.00	0.15
Government Loans					1.00

	Dependence	**Independence**
Probability	0.00200000%	0.00000777%

As we can see using dependence among variables gives us a higher probability of default. Many models assume independence among variables, as it can be seen in our example, independence is not a reasonable assumption since the values of the probabilities change considerably.

5. A Credit Derivative for Loans

In this section we price a premium for loan defaults using the results in section 2. In order to do the pricing we consider an insurance for defaults for one year. The fair price will be given by the expectation under the density function for default. We consider the case for bank one:

$$E(v_i^t) = m\int_{t_i}^{t_n} v_i^t e^{-d_i t} dt$$

where v_i^t (discount function) and d_i depend on the path of the interest rates

In order to model the interest rates, a stochastic model was used based on the Cox-Ingerssoll-Ross model.

$$dr(t) = k(\Theta - r(t))dt + s\sqrt{r(t)}dW(t)$$

The parameters of the model were calibrated according to the information available up to the 26th of November of 2001. The detailed development of this model is not discussed on this paper, for further references see [2]. 1000 scenarios were generated using Monte Carlo Simulation and the loan insurance was priced using each of the scenarios and then they were averaged. The results are shown below:

	Comercial	**State Gvt.**	**Consumer**	**Mortgage**	**Government**
Premium	0.10963406	0.06238307	0.16601785	0.13868413	0

As it can be seen the highest premium is for consumer loans, this happens because it is the loan with highest likelihood of default. In summary what we have is that the higher the spread, the higher the instantaneous likelihood of default and therefore the probability of default increases since there is more risk reflected by the spread.

There are several points to consider here, first of all this premium is based on the fact that the bank does not have reserves to cover any sudden loss. A consideration should be done about this. This is just the pure premium based on the risk of default. It is important to mention that the current fee for banks in Mexico to protect the savings is .004 for each peso, which is much lower than our estimates.

5. Conclusions

A model to estimate default probabilities was constructed using a closed-form-model to obtain the default intensity function. This function was assumed constant over the period of evaluation. Default probabilities and a premium for bank loans defaults were estimated.

It is important to consider that reserves are not considered in the model. The assumption of assuming the default intensity function constant can be broken using a stochastic model. Some researchers suggest using a mean reverting process for the modelling, however there are problems when parameters are estimated and assumptions about them have to be done; however some applications of stochastic models for intensity functions are planned to be carried out.

Research is still being carried out and the ideas in this paper are solely of the author and does not reflect the position of the IPAB.

Appendix

Let us use the following notation:

$V(t^-)$ is the value of a zero-coupon bond just before default.
$R(t)$ is the recovery rate given default.

Then we have the following:

$$V(t) = P(t)E\left[\frac{V(T)}{P(T)}\Pr\left[T < t \middle| t < t\right] + \int_t^T \frac{V(s)R(s)}{P(s)} f(s|t < t)ds\right] \text{...(1)}$$

We know that

$$\Pr\left[T < t \middle| t < t\right] = \frac{\Pr[T < t]}{\Pr[t < t]} = \frac{S(T)}{S(t)} \text{...(2)}$$

where $S(t)$ is the survival function and we have:

$$f\left(s \middle| t < t\right) = \frac{f(s)}{S(t)} \text{...(3)}$$

Combining (1), (2) y (3) we have:

$$V(t) = P(t)E\left[\frac{V(T)S(T)}{P(T)S(t)} + \int_t^T \frac{V(s)R(s)}{P(s)S(t)} f(s)ds\right]$$

Which can be written as:

$$\frac{V(t)S(t)}{P(t)} = E\left[\frac{V(T)S(T)}{P(T)} + \int_t^T \frac{V(s)R(s)}{P(s)} f(s)ds\right]$$

We define:

$$\frac{V(t)S(t)}{P(t)} = x(t) \text{...(4)}$$

Therefore

$$x(t) = E\left[x(T) + \int_t^T \frac{x(s)R(s)f(s)}{S(s)}ds\right] = E\left[x(T) + \int_t^T x(s)R(s)h(s)ds\right]$$

Arrenging terms:

$$x(t) = E\left[x(T) + \int_0^T x(s)R(s)h(s)ds - \int_0^t x(s)R(s)h(s)ds \right]$$

Which implies that:

$$M(t) = x(t) + \int_0^t x(s)R(s)h(s)ds$$

is a martingale whose stochastic differential can be written as:

$$dx(t) = -x(t)R(t)h(t)dt + dM(t)$$

and the solution is given by:

$$x(t) = e^{-\int_0^t R(s)h(s)ds} \left[x(0) + e^{\int_0^t R(s)h(s)ds} dM(t) \right] \text{...(5)}$$

From (4) and (5) we have the following:

$$\frac{V(t)e^{-\int_0^t h(s)ds}}{P(t)} e^{\int_0^t R(s)h(s)ds} = E\left[\frac{V(T)e^{-\int_0^T h(s)ds}}{P(T)} e^{\int_0^T R(s)h(s)ds} \left| \Im_t \right| \right]$$

Which leads us to:

$$\frac{V(t)}{P(t)} = E\left[\frac{V(T)}{P(T)} e^{-\int_t^T [1-R(s)]h(s)ds} \right]$$

Let us define

$$P(t) = e^{-\int_0^t r(s)ds} \text{ and } L(s) = 1 - R(s)$$

Then we have:

$$X(0) = E\left[V(T)e^{-\int_0^T [r(s)+L(s)h(s)ds]} \right] \text{...(6)}$$

Since the cash flows of a bank loan can be seen as the cash flow of bonds we can derive the following equation using the notation described in section 2

$$CreditPV = \sum_{k=1}^{n} \sum_{i=1}^{2} C_{ik} e^{-\int_0^{.5i+k-1} [r(s)+(1-R(s)h(s)] ds}$$

From this equation we can obtain our hazard rate function which we will use to calculate our default probabilities.

Sklar Theorem: Given a joint distribution function $F(x_1,...,x_n)$ for random variables $X_1,...,X_n$ with marginal cumulative distribution functions $F_1(x_1),...,F_n(x_n)$, F can be written as:

$$F(x_1,...,x_n) = C[F_1(x_1),...,F_n(x_n)]$$

where $C(u_1,...,u_n)$ is a joint distribution function with uniform marginals. If all F_i are continuous, then $C(u_1,...,u_n)$ is unique.

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