A Systematic Comparison of Two Approaches To Measuring Credit Risk: CreditMetrics versus CreditRisk+

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Summary

The objective of this paper is to compare two approaches to modelling Credit-Value-at-Risk: CreditMetrics and CreditRisk+. This is important for regulators and for risk managers who are concerned with allocating capital efficiently. The few studies already available on this subject focus narrowly on the risk of default. This paper incorporates both the risk of default and the risk which arises from changes in credit ratings (migration risk).

The paper builds on the work done by Koyluoglu and Hickman(1998), but we make a significant extension by assessing the impact of migration risk on credit-risk. We make very careful comparison of Credit-Value-at-Risk for the two models using Monte Carlo techniques on standardised portfolios of bonds.

The conclusion is that for regulators, the model which is used matters very little. This is because regulators are concerned with extreme values and loss distributions of both models capture information only about defaults at very high confidence levels. However, for internal purposes, where rating migrations matter more than default, CreditMetrics can generate higher estimates of risk.

1 Introduction

In the last decade, Credit Risk Modelling has become a topic of active research. The progress in the area is the result of several factors; such as the success of Credit Derivatives, and the concern of banking authorities and risk managers to quantify capital adequacy requirements and economic capital.

In the academic literature and within the banking industry, there are two credit risk models which have become popular: CreditMetrics of J.P. Morgan (1997) and CreditRisk+ of Credit Suisse Financial Products(1997). At first sight, both models are very different as they lie on different definitions of credit risk. On the one hand CreditRisk+ is a Default Model. Under this approach credit risk is the risk that security's borrower defaults on their promised obligations. Therefore, only borrowers' defaults can cause losses in the portfolio. On the other hand, CreditMetrics is a Rating Model. This approach defines credit risk as the risk that the security holder does not materialise the expected value of the security due to the deterioration of the borrower's credit quality. Therefore in CreditMetrics, not only default can cause losses but also downgrading in the credit quality of borrowers.

The objective of this paper is to compare Credit-Value-at-Risk (CVaR) as estimated by CreditMetrics and CreditRisk+ in fixed income portfolios. For regulators and internal risk managers this is important in order to calculate capital adequacy requirements and allocate capital efficiently. Few studies have examined the differences between these two models, see for example: Gordy(2000), Koyluoglu and Hickman(1998) and Finger (1999). They conclude that CreditMetrics and CreditRisk+ are conceptually very similar. However, these papers examine only the default component of credit risk and fail to incorporate changes in credit ratings as other source of credit losses.

In this paper we extend the analysis carried out by Koyluoglu and Hickman(1998). They formulate CreditRisk+ and a restricted version of CreditMetrics (which considers that only default can cause losses in the portfolio) under a common mathematical framework. This framework allows comparing the default distributions of both models under equivalent parameters. We extend this analysis in two aspects: First by comparing CreditRisk+ and the full version of CreditMetrics that considers Migration Risk. Second, by setting up a common mathematical framework to compare the loss distributions. Loss distributions are the main output of any credit risk portfolio model, as they allow estimating the CVaR and examining the impact on capital requirements.

We use Monte Carlo techniques to implement the new mathematical formulation of both models in two simulated bond portfolios. One portfolio has high credit quality and the other low credit quality. CVaR is calculated under different values of the parameters in order to carry out sensitivity analysis. The analysis is restricted to one-year time horizon.

We attribute the differences in CVaR between the models to three sources: 1) the omission of migration risk in CreditRisk+ model; 2) the shape of the default distributions of both models; and 3) the definition of "credit exposure" in CreditRisk+. We conclude that for both types of portfolios (low and high quality portfolios), most of the differences in CVaR between the models are due to the underlying assumptions of the distribution of default. However, for high quality portfolios and low confidence intervals of CVaR, the omission of migration risk is also significant to explain the differences between the models.

This paper contributes to the existing literature because it provides a comparison between a Default Model and a Credit Rating Model. We also assess the impact of migration risk on CVaR and identify portfolios for which migration risk is relevant to determine the differences of CVaR between the models.

For practitioners, the conclusions of this paper have important implications: 1) For the calculation of capital requirements, to choose between CreditMetrics or CreditRisk+ seems to be irrelevant. In the extreme tails of the loss distribution, only information about default is captured by any of the two models. 2) For internal purposes such as estimation of reserves, where rating migrations matter more than default, CreditMetrics could be a better approach.

The paper is structured as follows: Section 2 briefly describes the conceptual framework of CreditMetrics and CreditRisk+. Section 3 presents a revision of the literature on the comparison between these two models. In section 4 we extend the analysis of Koyluoglu and Hickman and derive a common mathematical framework for both models. Such formulation allows us to parameterise the models in a convenient way so we can perform a valid comparison between their loss distributions. In section 5, we implement the models in two types of simulated portfolios. In section 6, we analyse the sources of the differences of CVaR between the models. Section 7 is left for conclusions.

2 CreditMetrics and CreditRisk+: Description of the Models

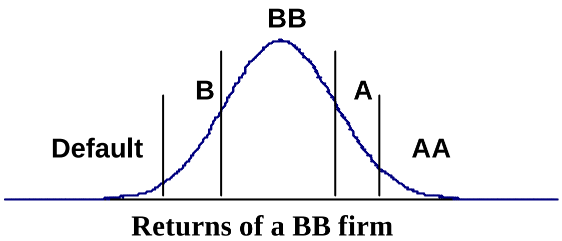
In this section we briefly describe the structure of each model.

2.1. CreditMetrics

The fundamentals of CreditMetrics lie in the credit pricing framework of Merton(1974). Merton models the debt value of a firm as the difference between the firm value and a call option on the value of its assets. In this setting the value of the assets drive the default process of the firm. Default can occur only at the debt maturity and when the firm value falls below a specific threshold (the value of firm's liabilities).

In CreditMetrics, Merton's model is extended by assuming that the assets returns of the firm determine not only its probability of default but also the probability of migrating to any other credit rating. Returns are assumed normally distributed, therefore a change in the credit quality of the firm occurs when its returns fall within certain thresholds in the normal distribution. See Figure 2.1.

Figure 2.1. Credit quality thresholds



For each possible credit rating at the end of the time horizon (usually one year), the price of the debt is calculated by using forward rates to discount the remaining cash flows. Forward rates are estimated assuming that the term structures of interest rate and credit spreads are deterministic. Because CreditMetrics generates estimates for the mark to market value of the debt at the end of the time- horizon, this model is also known as a Mark-to-Market Model (MTM).

In order to integrate all the individual exposures in the portfolio, JPMorgan(1977) models the correlation between the credit behaviour of the borrowers using their asset correlations as a proxy. The distribution of the portfolio value can then be constructed. This allows estimating the expected and unexpected value of the portfolio. CVaR is calculated as the difference between these two figures.

2.2 CreditRisk+

In CreditRisk+ default does not depend on firm´s fundamentals but is modelled as an exogenous variable, which follows a Poisson distribution with stochastic intensity parameter. This intensity parameter represents the default rate or the probability of default over a small time interval. This is assumed to be driven by an unknown “economic factor” which follows a Gamma distribution, hence the default rate is also Gamma distributed. In the event of default, the debt holder incurs a loss equal to the amount of debt less the recovery rate. Contrasting CreditMetrics, the value of the debt is not modelled in CreditRisk+.

To deal with portfolios and correlations between borrowers, these are classified into sub- portfolios. Each sub- portfolio is affected by a specific economic factor. Such classification allows assuming independence between borrowers in the same sub- portfolio. This facilitates the mathematics of the model. In addition, within each sub- portfolio, borrowers are classified into bands according to their credit exposure. In each band, the size of each credit exposure is adjusted, so each band is characterised by a common exposure V_i.

The stochastic default rate in a sub- portfolio affected by an economic factor X_k is:

$$p_i(X) = \frac{\varepsilon_i}{V_i}\frac{X_k}{\overline{p}_k} \sim \text{Gamma}(\alpha, \beta) \qquad (2.1)$$

where $p_i(x)$ represents the default rate or conditional probability of default, V_i stands for the adjusted credit exposure of the debt, ε_i for the expected loss and $\overline{p}_k$ for the unconditional rate of default in sub-portfolio k (i.e., the mean of the variable $p_i(x)$).

The parameters α and β are defined as $\alpha = \frac{\overline{p}_k^2}{\sigma_k^2}$, $\beta = \frac{\sigma_k^2}{\overline{p}_k}$, where σ_k is the unconditional default volatility in sub-portfolio k[1].

To estimate the loss distribution of the portfolio due to default, the model works in three steps. In the first step the probability generating function (pgf) of losses for one obligor is calculated. In the second step, the individuals in each sub-portfolio are aggregated to calculate the pgf of each sub-portfolio. Finally, in the third step, all the sub- portfolios are also aggregated. The loss distribution of the portfolio is then obtained from the pgf of the portfolio. The advantage of the model is that it generates closed form solutions that facilitates its implementation.

3 Previous research on the comparison of CreditMetrics and CreditRisk+

There is only a small literature on the comparison of credit risk models. Gordy(2000) substitutes the distributional assumptions of CreditMetrics into the mathematical structure of CreditRisk+ and in the other way around. He shows that the structures of both models are similar when CreditMetrics is restricted to measure only default risk. Gordy also carries out an empirical exercise and simulates portfolios by assuming a single systematic economic factor and four credit-types of assets. Models are calibrated so that they yield the same unconditional expected default rate and default correlation. The conclusions of the paper are: 1) there are not dramatic differences in CVaR between the restricted form of CreditMetrics and CreditRisk+; 2) on average both models behave

[1] CreditRisk+ suggests obtaining the unconditional rate of default and volatility of default ($\overline{p}_k$ and σ_k respectively) from rating agencies.

similarly for low values of default volatility; and 3) CreditRisk+ is more responsive to the credit quality of the portfolio.

Finger (1999) compares CreditRisk+ and also a restricted version of CreditMetrics using two types of bond portfolios: low and high credit quality portfolios. Within each portfolio, issuers are assumed homogeneous[2] and their credit quality driven by one-single economic factor. The bigger discrepancies between the models exist when the portfolio is composed of high quality bonds. The extreme tails of the default distributions generated by the models are very different. Finger concludes that when the asset correlation coefficient of CreditMetrics and the default volatility of CreditRisk+ are parameterised in a consistent way, both models produce similar distributions of default. However, in practice discrepancies can arise due to inconsistent parameters between the models, or technical implementations.

Koyluoglu and Hickman (1998) analyse the theoretical similarities between CreditMetrics and CreditRisk+. They assume a simple framework: the Vasicek representation of asset returns, fixed recovery rates and homogeneous portfolios. The mathematical structure of the models is reformulated in three common steps. 1) The estimation of the Distribution of Default. In both models default rates are driven directly or indirectly by stochastic economic factors. Therefore, default rates are also random variables and their probability distributions depend on stochastic movements of the economic factors. 2) The estimation of the Conditional Distribution of the Number of Defaults in the Portfolio. This distribution is estimated as if individual borrowers defaulted independently. This is because all the joint behaviour of borrowers has already been considered when calculating the conditional default rates. 3) The estimation of the Unconditional Distribution of Number of Defaults in the Portfolio. This distribution is obtained by aggregating homogeneous sub-portfolios[3] across all possible realisations of the economic factor.

The above common set up of the models allows Koyluoglu and Hickman examining their similarities in a structural way. They perform comparisons between the distributions of default rather than perform comparisons between the loss distributions, arguing that for very large portfolios both

[2] A portfolio is homogeneous if borrowers have similar credit ratings and size exposures.

distributions are very similar. Empirically the differences between the default distributions seem not to be significant when parameters have been set up consistently. They conclude that CreditMetrics and CreditRisk+ are conceptually based on the same philosophy. But the differences between the models can arise due to aggregation techniques and the estimation of parameters.

It is important to point out that some of the assumptions used by Koyluoglu and Hickman(1998) or Finger(1999) might be considered unrealistic, however it must be said that they are commonly used by practitioners. For instance, practitioners often assume that the determinants of credit losses are independent. The recovery rate in most cases is a deterministic parameter. Borrowers within same specific risk sectors are assumed statistically the same. Practitioners also often assume that parameters of the models are stable. Though empirically none of these assumptions seems to be true, the lack of data generally limits the formulation of more sophisticated assumptions.

The above analyses contrast with the project on Credit Risk Modelling led by the Institute of International Finance and The International Swaps and Derivatives Association (IIF/ISDA 2000), which was carried out by practitioners. The objective of this project was to understand the performance of credit risk models[4] used in 25 banks from 10 countries with different sizes and specialities. In this study, risk managers were given standard portfolios and inputs and asked to report CVaR. Practitioners were also asked to run models through a variety of implementations and scenarios. The objective was to determine whether models used by practitioners were directionally consistent (models outputs moved in the same direction), when given similar key inputs. In the end, the exercise led to different outcomes and to no very clear conclusions about what are the sizes and sources of the differences between the models, as implemented. We should point out that no efforts were made to parameterise the models such they yielded consistent results. This might explain the wide range in the results.

The IIF/ISDA study concludes that there is consistency within the results of the same type of model. For instance, CreditMetrics results resemble other mark- to- market approaches. Credit Risk+ gives

[3] Each sub-portfolio is affected by one economic factor.
[4] The examined models were: CreditMetrics (J.P.Morgan), CreditPortfolioView (McKinsey), CreditRisk+ (Credit Suisse Financial Products), Portfolio Manager (KMV) and 11 proprietary internal models.

the highest estimates of CVaR (in relation to internal models and CreditMetrics). The explanation lies in the correlation assumption: whereas CreditMetrics and other mark-to-market models were run with equivalent correlation coefficients, CreditRisk+ was fed with a more conservative parameter. Hence it is concluded that the calculations of the correlation coefficient play an important role in the generation of discrepancies. Finally IIF/ISDA conclude that differences between models must be attributed to model inputs, pre-processing of data, valuation and different implementations.

In summary, the literature shows that CreditMetrics and CreditRisk+ are conceptually very similar, provided that they are fed with proper parameters.

4. A Common Framework for CreditMetrics and CreditRisk+.

This section is the core of the paper. Here we reformulate the mathematical structure of CreditMetrics and CreditRisk+ by extending Koyluoglu and Hickman (1998)'s approach. We extend their approach in two directions: 1) by constructing a common framework between CreditRisk+ and the full version of CreditMetrics, which considers migration risk; 2) by comparing not only the default distributions generated by the models but also their loss distributions and CVaR.

The output of any credit risk model is the distribution of losses or the distribution of the portfolio value. In this section, we describe how we can reformulate both models to estimate their distributions of losses. To estimate the distributions of losses we proceed in four steps, as follows: First we determine the rates of default for both models and also the rates of migration to any other no-default states for CreditMetrics. All rates are conditional on the behaviour of the economic factor. Given the default and migration rates for each realisation of the economic factor, we estimate the Conditional Distributions of Default and Migration. Second, we find the Conditional Distributions of the Number of Events. These distributions describe the number of defaults and number of changes in the credit quality of the borrowers in the portfolio. In the third step, we determine the Unconditional Distributions of the Number of Events. Finally we explain how to get the Loss Distributions of the portfolio.

We should point out that the reformulation of the models that we propose do not modify their fundamentals or distributional assumptions but only their implementation. Such way to construct the loss distributions is the key to carrying out a structural comparison between the models and identifying the sources in the discrepancies of CVaR.

In order to keep calculations simple, we assume a portfolio formed by N bonds equally rated, with the same exposure size[5], the same time to maturity, and affected by a single economic factor.

The rating system consists of three credit states or ratings: A, B and D. Where A represents the highest credit quality, B is an intermediate state and D represents the default state. For each borrower in the portfolio, with a given initial credit state, the transition probabilities or unconditional rates of migration (including default) are $\overline{p}_A$, $\overline{p}_B$ and $\overline{p}_D$. Where $\overline{p}_J$ (J=A, B and D) represents the probability of migrating to state J at the end of the time horizon.

At the end of the time horizon, the credit quality of each borrower in the portfolio is determined by one single- economic factor. Therefore, given a realisation of the economic factor, the probabilities of a borrower migrating to states A, B or D, are $p_A|_x$, $p_B|_x$, and $p_D|_x$, respectively. This definition is consistent with the Mark-To-Market version of CreditMetrics. For CreditRisk+ and the restricted version of CreditMetrics, in which only default matters, we define the rate of the no default state as $p_{ND}|_x = 1 - p_D|_x$.

In section 4.1, following Koyluoglu and Hickman, we derive functional forms for the rates of default of CreditMetrics and CreditRisk+. We also set up the conditions to generate consistent parameters for the default distributions of both models. In Section 4.2, we estimate the functional forms for the migration rates of CreditMetrics. Finally, in Section 4.3 we explain how to construct the loss distributions for both models.

[5] In order to simplify terminology, we refer to “exposure” as the difference between the value of the debt at the time of default and the recovery rate.

4.1. Estimation of the Default Rates and Parameterisation of the Models.

In CreditMetrics, default is driven by the asset- return process of the firm. Likewise, returns are driven by economic factors that are assumed normally distributed. Firm's returns can then be expressed in terms of a set of k orthogonal normal variables that represent economic factors:

$$r_i = b_{i,1}X_1 + b_{i,2}X_2 + ... + \sqrt{1-\sum_k b_{i,k}{}^2}\varepsilon_i \qquad (4.1)$$

Where $b_{i,k}$ are the factor- loadings, X_k represents the k-th economic factor and e_i are movements specific to each obligor. Given that X_k and e_i are both standard normal, r_i is also a normal standard random variable.

Koyluoglu and Hickman(1998) argue that when the portfolio is composed either of one single borrower, or of borrowers who are affected by a single economic factor and have similar size exposures and credit ratings, the systematic factors in equation (4.1) can be represented by a single normally distributed variable X:

$$r_i = \sqrt{\rho}X + \sqrt{1-\rho}\varepsilon_i \qquad (4.2)$$

where $\rho = \sum_k b_k^2$ is the correlation of the borrowers´ assets in the portfolio.

Using the above representation of the returns, Vasicek (1987) derives a functional form for the default process in terms of the driving variable. Therefore, given a realisation of the economic factor X, and a correlation between the borrowers of the portfolio, the conditional probability of default or default rate for CreditMetrics is:

$$p_D^{CM}\Big|_{X} = \Phi\left[\frac{c_1 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right] \qquad (4.3)$$

where X~N(0,1), $\Phi(c_1) = \overline{p}_D^{CM}$ is the unconditional rate of default and $\Phi(s)$ is the cumulative density function of the normal distribution. See appendix A for a full derivation of equation (4.3). Remember that the unconditional rate of default is an input in the model and forms part of the vector of transition probabilities, which is associated to the initial credit quality of the borrower. Observe that in this derivation the probability of default is confined to the [0,1] interval.

On the other hand, CreditRisk+ assumes that default is driven linearly by an economic factor which is Gamma distributed (see formula 2.1). As a result the conditional probability of default or default rate is also Gamma distributed. In order to make models comparable, Koyluoglu and Hickman (1998) suggest modifying the distribution of the economic or background factor in CreditRisk+. They substitute the gamma distribution of the economic factor for a normal distribution. In order to preserve the gamma distribution of the default rate, which is one of the fundamental assumptions of CreditRisk+, they transform the functional form between the economic factor and the default rate. The transformation function consists of all points (χ, ξ) that satisfy:

$$\int_0^{\xi} \Gamma(p_D^{CR}; \alpha, \beta)dp = \int_{\chi}^{\infty} \varphi(x)dx \qquad (4.4)$$

where $\Gamma(z;\alpha,\beta)$ is the Gamma density function with parameters α and β and $\varphi(x)$ is the Normal density function.

The conditional rate of default resulting from the transformation (4.4) is given by:

$$p_D^{CR}\Big|_x = \Psi^{-1}(1 - \Phi(x); \alpha, \beta) \qquad (4.5)$$

where $\Psi(z;\alpha,\beta)$ is the Gamma cumulative distribution.

Note that in order to define completely the distribution of $p_D^{CR}|_x$ in (4.5), the parameters α and β have to be specified. As was mentioned in the previous section, these parameters are functions of the unconditional mean and standard deviation of the default rate ($\overline{p}_D^{CR}$ and σ_D^{CR} respectively), which are inputs of the model. Likewise, the transformation for $p_D^{CM}|_x$ in (4.3) needs the unconditional rate of default $\overline{p}_D^{CM}$ and the asset correlation coefficient ρ to be completely specified.

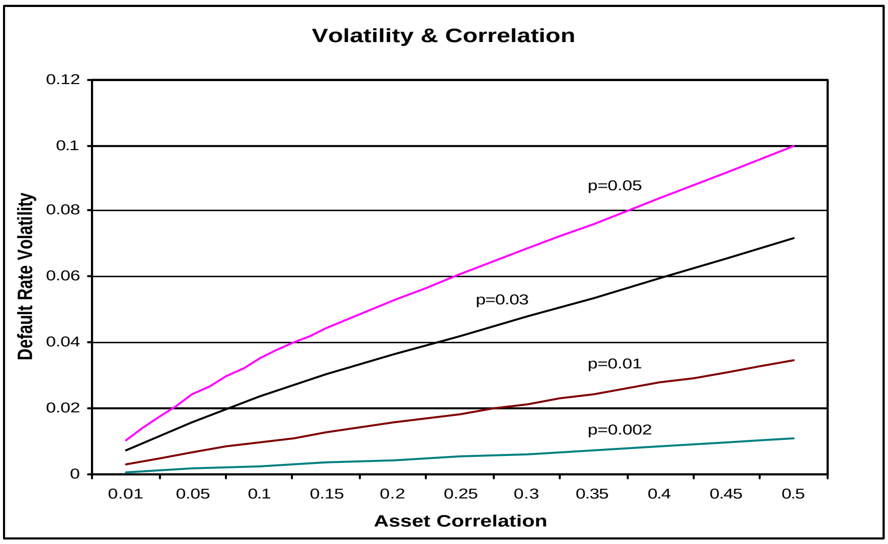
In order to produce consistent default rates between the models (equations (4.3) and (4.5)), Koyluoglu and Hickman argue that their means and standard deviations must be the same across the models. Therefore, we set the unconditional default rates $\overline{p}_D^{CM} \equiv \overline{p}_D^{CR}$.

Using expression (4.3), Koyluoglu and Hickman find that the default rate volatility for CreditMetrics can be expressed as a function of $\overline{p}_D^{CM}$ and ρ:

$$\sigma_{CM}^2 = \int_{-\infty}^{+\infty} \left(\Phi\left[\frac{\Phi^{-1}(\overline{p}_D^{CM}) - \sqrt{\rho}x}{\sqrt{1-\rho}} \right] - \overline{p}_D^{CM} \right)^2 \varphi(x)dx \qquad (4.6)$$

Set the variance for CreditRisk+ and CreditMetrics as $\sigma_{CR}^2 \equiv \sigma_{CM}^2$. From equation 4.6, observe that given a specific value of the unconditional default rates $\overline{p}_D^{CM} \equiv \overline{p}_D^{CR}$, we have implicitly set up a relationship between the asset correlation parameter of CreditMetrics (ρ) and the variance of CreditRisk+ (σ_{CR}). This relationship is plotted in Figure 4.1.

Figure 4.1. Consistent parameters for CreditMetrics and CreditRisk+



Note: p is the unconditional probability of default.

From the plot, the higher the assets correlation the higher the volatility of the default rate. Intuitively, if asset returns are highly correlated then default of one borrower is likely to be followed by the default of another. In other words, as borrowers are correlated through the effect of the same economic factor, their default rates will move together, causing high volatility levels.

It is worth mentioning that although unconditional default rates (means) and variances of the distributions of default of both models are restricted to be the same, this is not the case for higher moments (Skewness and Kurtosis). This will be investigated later in the paper.

4.2 Estimation of Migration Rates

So far we have derived the functional forms for the rates of default for CreditMetrics and CreditRisk+. Those rates constitute the link between the models. We have parameterised those functions such that default rates are consistent between the both models. Here, consistency means that means and standard deviations of the default distributions are the same across the models.

We still need to define the migration rates to no- default states for both models. Those are not necessarily the same between the models since each model assumes different definitions of credit risk.

For CreditRisk+ the no- default rate given a realisation of the economic factor is given by:

$$p_{ND}^{CR}\big|_x \equiv 1\text{-}\,p_D^{CR}\big|_x \qquad (4.7)$$

For the mark-to-market version of CreditMetrics, we need to calculate three migration rates (or conditional probabilities of migration): $p_A^{CM}\big|_x$, $p_B^{CM}\big|_x$ and $p_D^{CM}\big|_x$. The latter has already been derived in the previous section.

Under the above assumptions, it is straightforward the derivation of the migration rates to ratings A and B (see Appendix A for the full derivation):

$$p_B^{CM}\Big|_x = \Phi\left[\frac{c_2-\sqrt{\rho}x}{\sqrt{1-\rho}}\right]-\Phi\left[\frac{c_1-\sqrt{\rho}x}{\sqrt{1-\rho}}\right] \qquad (4.8)$$

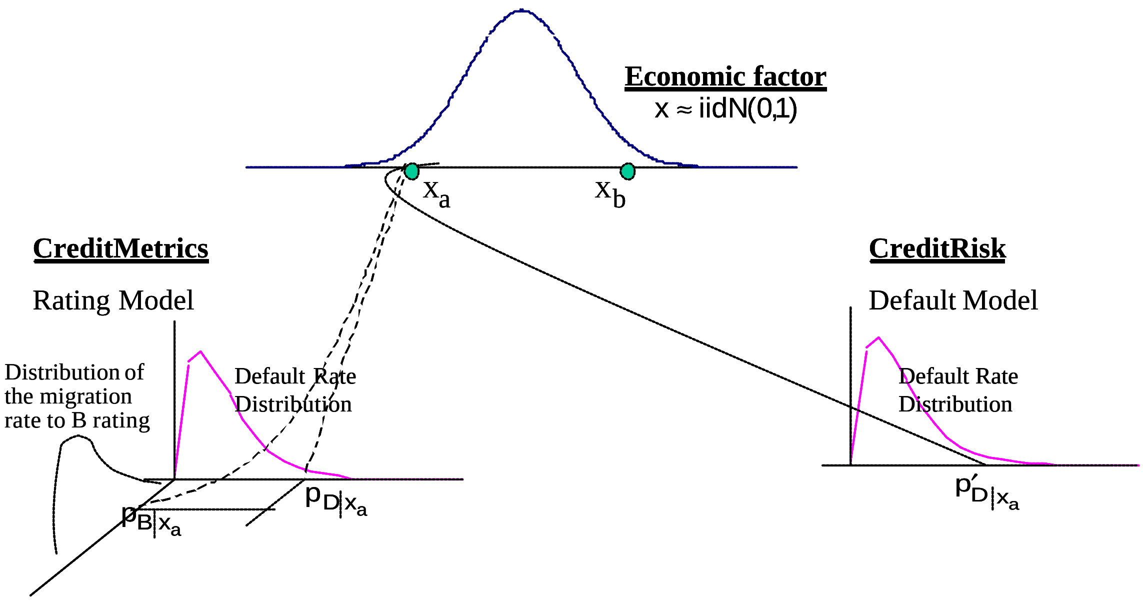
$$\text{and } p_A^{CM}\Big|_x = 1-p_B^{CM}\Big|_x - p_D^{CM}\Big|_x \qquad (4.9)$$

where $c_2 = \Phi^{-1}(\overline{p}_B + \overline{p}_D)$, $\overline{p}_B$ is the unconditional migration rate to credit rating B and $\overline{p}_D$ is the unconditional default rate.

The effect of realisations of the economic factor on migration rates and default rates are illustrated in Figure (4.3). The realisations of the economy are modelled by a normal random variable, which is the distribution at the top of the diagram. The five transformations that we have derived (equations 4.3, 4.5, 4.7, 4.8 and 4.9) map realisations of the economic factor into the default rates.

Assume a borrower with intermediate credit quality B at the beginning of the period. In figure 4.3, assume that an observation from the left- tail of the normal distribution X_a has occurred. This represents a realisation of the economic factor. Actually a small value X_a indicates an economy in recession. An economy in recession leads to more defaults in the portfolio, or equivalently, to high values of the borrower's default rate. In CreditMetrics and CreditRisk+ such default rates are represented by $p_D\big|X_a$ and $p'_D\big|X_a$ respectively. In figure 4.3, the realisation of X_a generates a high value of $p_D\big|X_a$ in CreditMetrics, which means a point at the right- tail of the Default Rate Distribution. Similar effect has X_a on CreditRisk+.

Figure 4.3. Effect of the economic factor on migration rates.



The realisation of the economy, X_a, also effects the migration rates to states A and B. In the diagram for CreditMetrics, only the distributions for the default rate and migration rate to rating B are plotted. When the economy is in recession (i.e. for small values of X_a), the probability that a borrower suffers a downgrading in his credit rating increases. Therefore, the probability that a B-rated borrower keeps the same rating at the end of the period is small as is expected that his credit quality deteriorates. Therefore, a low value of the economic factor implies low values of migration rates to ratings A or B, i.e., $p_A\big|_{X_a}$ and $p_B\big|_{X_a}$ respectively. In the figure, $p_B\big|_{X_a}$ is located close to zero, representing a low value for this variable. The reverse is true for a high value of the economic factor X_b.

To generate, the distributions of the migration rates in figure 4.3, we need several realisations of the economic factor. According to CreditRisk+, the distribution of the default rate follows a Gamma distribution, whereas that for CreditMetrics this distribution is:

$$f(p) = \frac{dP}{dp} = \frac{\sqrt{1-\rho}\,\varphi\left(\dfrac{C_1 - \sqrt{1-\rho}\,\Phi^{-1}(p)}{\sqrt{p}}\right)}{\sqrt{p}\,\varphi\left(\Phi^{-1}(p)\right)}$$

Where $\varphi(z)$ is the standardised normal density function. See appendix A for a full derivation of this equation.

4.3 Estimation of the Loss Distribution

To determine the Loss Distribution for both models we follow an actuarial approach. First we determine what will be the credit quality of the borrowers at the end of the time horizon. We can then determine the number of borrowers within each credit class (A, B or D). Finally we combine the number of individuals in each credit class with the size of the exposures in order to estimate the Distribution of Losses of the portfolio.

In the previous sections we estimated the default rate and migration rates. Given a realisation of the economy we estimate the probability that the borrower migrates to any credit rating at the end of the time period. The second step, in the construction of the distribution of losses is <u>the calculation of the number of borrowers that fall in each credit rating category at the end of the period</u>. We should point out that given a realisation of the economic factor, borrowers in the portfolio should be assumed independent. All the correlation between borrowers has been captured through their relationship with the economic factor.

On the one hand, in CreditRisk+, given that all individuals in the portfolio are independent and homogenous[6], and a fixed value of the rate of default, the number of defaults follows a Binomial process. Asymptotic properties of the Binomial Distribution state that when the rate of default $p_D^{CR}|x$ is small and the number of bonds (N) in the portfolio is large, then the Number of Defaults in the portfolio ($N_D^{CR}|_x$) is approximately Poisson distributed with parameter $\lambda = (p_D^{CR}|_x) \cdot N$:

$$N_D^{CR}\Big|_x \approx \text{Poisson}(\lambda) \qquad (4.10)$$

Therefore, the number of no-defaults in the portfolio is $N_{ND}^{CR}\Big|_x = N - N_D^{CR}\Big|_x$.

On the other hand, for computational simplicity, CreditMetrics uses Monte Carlo techniques to simulate the changes in the credit quality of the borrowers in the portfolio. Implicitly the model assumes a Multinominal distribution to describe the number of individuals that fall in each credit rating. At the end of the period, an individual can migrate to any of the three credit states [A, B, D] with probabilities $[\, p_A|_x \,, p_B|_x \,, p_D|_x \,]$ respectively. If we sample N individuals independently and $[N_D^{CM}, \quad N_B^{CM}, \; N_A^{CM}]$ are the number of defaults, the number of B-rated borrowers and the number of A-rated borrowers in the portfolio, then this vector will follow a Multinomial Distribution.

$$\begin{bmatrix} N_B^{CM}\big|_x \\ N_D^{CM}\big|_x \end{bmatrix} \approx \text{Multinomial}(p_B^{CM}|_x, p_D^{CM}|_x, N) \qquad (4.11)$$

And $N_A^{CM} = N - N_B^{CM} - N_D^{CM}$. Where N is the size of the portfolio.

Observe that in the restricted version of CreditMetrics (in which only default risk matters), the Multinomial distribution becomes a Binomial distribution with parameters: $p_D^{CM}|x$ and N. As the limit distribution of a Binomial distribution is a Poisson, the restricted version of CreditMetrics and CreditRisk+ asymptotically yield similar shapes of the default distribution.

At the end of the period, the number of individuals in each state is conditioned on the random migration rate or default rate. Hence that <u>the third step</u> in the calculation of the distribution of losses

[6] Recall that because the portfolio is homogeneous, all individuals in the portfolio have the same unconditional default rate.

involves the aggregation of the conditional distributions of the number of borrowers in each credit rating class. We aggregate all conditional distributions, each is the result of an specific migration or default rate, which likewise depend on the realisations of the economy.

In CreditRisk+, the number of defaults is conditioned on the random default rate. Therefore the unconditional distribution of the number of defaults is given by the following convolution integral:

$$P_{CR}(N_D^{CR}) = \int_P \text{Poisson}(N, p_{D|x})\Gamma(\alpha,\beta)dp \qquad (4.12)$$

Likewise, for CreditMetrics the conditional distribution of the number of individuals in each credit rating is conditioned on a normally distributed economic factor. The unconditional distribution of the number of events is given by:

$$P_{CM}(N_D^{CM}, N_B^{CM}) = \int_x \text{Mult}(N, P_{D|x}, P_{B|x})\varphi(x)dx \qquad (4.13)$$

The fourth and final step is the calculation of the Distribution of Losses of the portfolio. To calculate it, the information about the number of events in each state and the size of the exposures should be combined.

A distinct feature of CreditRisk+ is that it produces a distribution of losses, whereas CreditMetrics produces a distribution of the portfolio value. For comparison of CVaR figures, it is more convenient to transform the distribution of portfolio value of CreditMetrics into a distribution of losses. This transformation will be explained in the next section.

For CreditRisk+ the expression for the distribution of losses should resemble equation (4.12). Only the random variable N_D^{CR} needs to be scaled by the size of the exposures under default. For CreditMetrics, the expression for the distribution of losses is more complex, so it will be estimated by using Monte Carlo techniques in the next section.

5. Implementation of the Models and Estimation of CVaR

In this section we implement CreditMetrics and CreditRisk+ and find the loss distributions following the common steps specified in previous sections. In Section 5.1 we construct the default distributions of both models. In Section 5.2 we generate the distribution of losses and calculate CVaR.

We implement both models on two types of simulated bond portfolios: a Low- Credit Quality portfolio (LQ) and a High- Credit Quality portfolio (HQ). The LQ portfolio consists of bonds, whose issuers or borrowers have credit quality "B", whereas the HQ portfolio consists of bonds whose borrowers have "A" credit quality. The migration rates for credit ratings A and B appear in Table 5.1. Each bond has 2- year to maturity and a face value of $1. Each portfolio has been designed with 10,000 borrowers.

Table 5.1. Unconditional Transition Probabilities

Transition Probabilities		
Rating	**Low Quality**	**High Quality**
A	0.88	89.45
B	92.06	9.55
D (default)	7.06	1.00

To gain some insights into the performance of the models we run several tests to estimate the sensitivity of CVaR to different values of the correlation parameter (ρ=0.05, 0.15, 0.25, 0.35, 0.45) and confidence levels ((1-α)%=90, 93, 95, 97, 99, 99.9%). The time horizon for CVaR is one year.

5.1 Generation of the Distributions of Default and Migration

We use Monte Carlo techniques to simulate realisations of the economic factor and construct the default distribution and migration distributions for CreditMetrics. The mean and volatility of the default distribution of CreditMetrics are then used as input parameters to construct the default distribution for CreditRisk+.

Simulations for the generation of the Default Distribution and Migration Distributions for CreditMetrics

The inputs to generate the default distribution of CreditMetrics are the unconditional default rate and migration rates, i.e., $\overline{p}_D$, $\overline{p}_B$ and $\overline{p}_A$, and the correlation between the borrowers in the portfolio ρ.

The Monte Carlo simulations involve the following steps for each model:

1. Simulate a standard Normal variable, which represents the economic factor of the portfolio. In order to get reliable results we need a large number of simulations. Faure quasi-random sequence numbers are used to generate random numbers in the interval [0,1]. Application of the inverse of the normal distribution provides us with a standard normal distribution[7].

2. For each realisation of the economic factor calculate the default rate according to equation (4.3).

3. Repeat the process (1 and 2) 1,000,000 times (the number of realisations of the economic factor).

4. Calculate the mean, standard deviation, skewness, and kurtosis of the sample.

5. Calculate α and β for CreditRisk+ using the estimated mean and standard deviation of CreditMetrics.

This procedure is repeated for all the combinations of the vector of transition probabilities [$\overline{p}_A$, $\overline{p}_B$, $\overline{p}_D$] and the correlation coefficient. See Table 5.1 for the values of the vector of transition probabilities. Use following correlation coefficients ρ=0.05, 0.15, 0.25, 0.35, 0.45.

Simulations for the generation of the Default Distribution under CreditRisk+

The inputs to generate the default distribution for CreditRisk are α and β as estimated by using CreditMetrics.

1. As for CreditMetrics, simulate a standard Normal variable, which represents the economic factor.

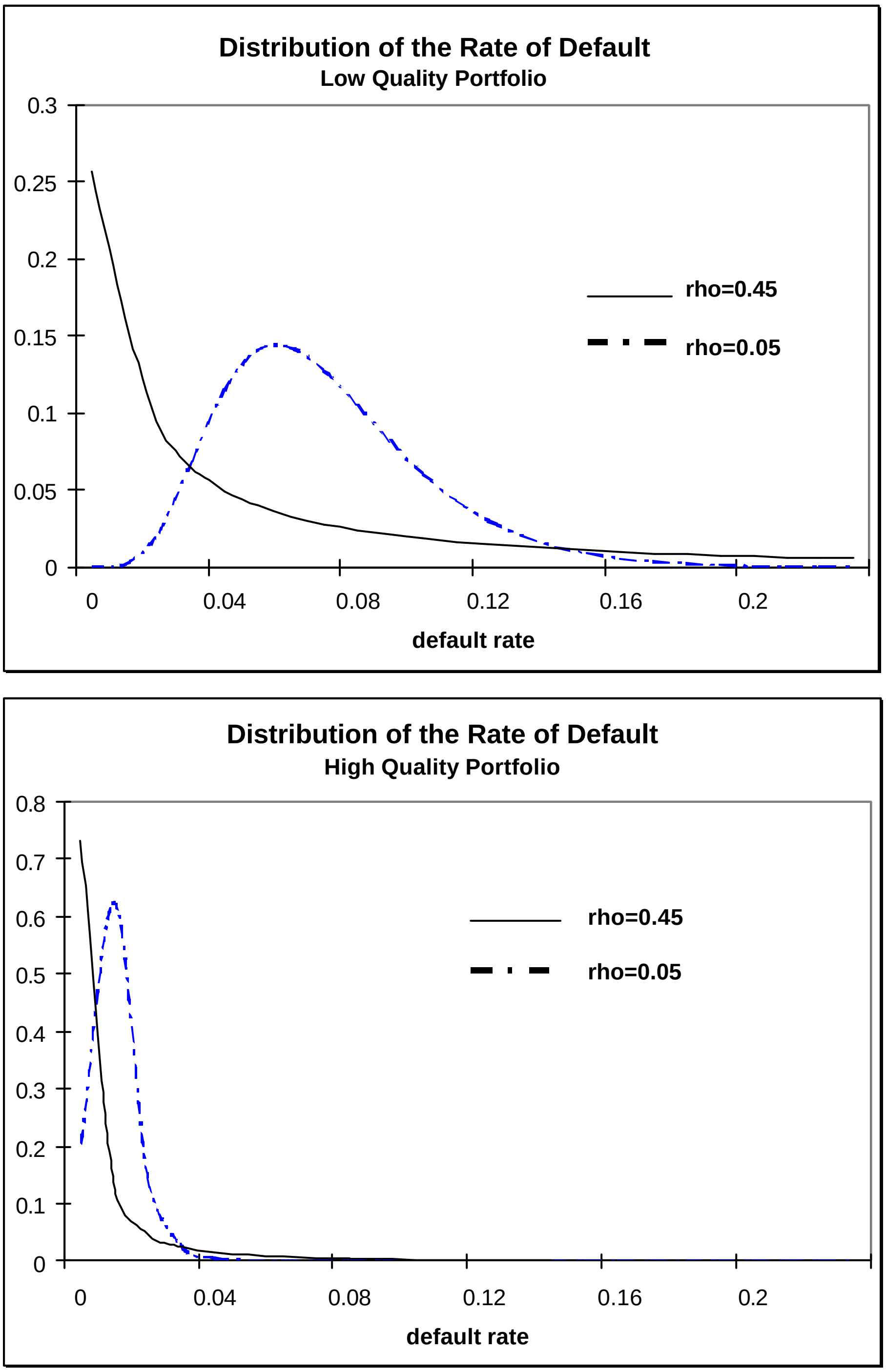
2. For a realisation of the economic factor calculate the default rate according to equation (4.5).[8]

3. Repeat the process (1 and 2) 1,000,000 times (number of realisations of the economic factor).

[7] Moro (1995) derivation is used to approximate the inverse cumulative Normal distribution.
[8] The calculation of this equation may not be straightforward. Instead, solve equation [4.4] using numerical integration. For a given value of χ, find ξ, such that both integrals are equal. To approximate the cumulative distribution of a variable X~ $\Gamma(\alpha,\beta)$, it is more convenient to approximate the integral when Y~ $\Gamma(\alpha,1)$ and use the fact that $X = \beta Y \sim \Gamma(\alpha,\beta)$.

4. Calculate skewness and kurtosis of the empirical distribution of the default rate.

Figure 5.1. Default Distributions for CreditMetrics. Low and High quality portfolios.



From the simulations for CreditMetrics, we plot in Figure 5.1 some default distributions for different values of the correlation parameter. Observe that high correlations (rho=0.45) are associated with long and fat tails. Note that the distributions of the HQ portfolios are shifted to the left, which indicates higher probability of getting low default rates than for LQ portfolios.

In Table 5.2 we report some descriptive statistics for the distributions of both types of portfolios and different levels of asset correlation (0.05 to 0.45).

Table 5.2 Statistics of the Distributions of Default for CreditMetrics and CreditRisk+

STATISTICS OF THE DISTRIBUTION OF DEFAULT						
			CreditMetrics		CreditRisk+	
Correlation	**Mean**	**Stand Dev.**	**Skewness**	**Kurtosis**	**Skewness**	**Kurtosis**
LOW QUALITY PORTFOLIO (LQ)						
0.05	7.06%	3.10%	0.997	4.531	0.878	4.150
0.15	7.06%	5.66%	1.733	7.394	1.600	6.821
0.25	7.06%	7.67%	2.211	9.735	2.167	10.009
0.35	7.06%	9.51%	2.556	11.472	2.686	13.756
0.45	7.06%	11.29%	2.811	12.652	3.188	18.138
HIGH QUALITY PORTFOLIO (HQ)						
0.05	1.00%	0.64%	1.728	8.183	1.273	5.419
0.15	1.00%	1.26%	3.524	24.714	2.509	12.391
0.25	1.00%	1.84%	5.081	46.651	3.663	22.985
0.35	1.00%	2.45%	6.424	68.945	4.870	38.205
0.45	1.00%	3.10%	7.522	87.575	6.171	59.430

Note: Mean and Standard Deviation are the same for both models.

From Table 5.2 we should make the following remarks:

1. The low quality (LQ) portfolio exhibits significantly higher levels of standard deviation than the high quality (HQ) portfolio. If we consider the standard deviation as an indicator of the portfolio risk, then higher standard deviations should be associated with the LQ portfolio. As mentioned earlier, the higher the correlation, the higher the standard deviation. In addition, the standard deviation of the HQ portfolio is more sensitive to changes in the correlation coefficient than the standard deviation of the LQ portfolio. For instance, a change of 28% in the asset correlation (from ρ=0.35 to ρ=0.45), implies a change of 18% in the standard deviation of the LQ portfolio (from 9.51% to 11.29%) and a change of 26% in the standard deviation of the HQ portfolio (from 2.45% to 3.10%).

2. For both models, skewness and kurtosis of HQ portfolios are more sensitive to changes in the correlation parameter than those of LQ portfolios. For example, in CreditMetrics a change of 28% in the correlation level (from ρ=0.35 to ρ=0.45) produces a change of 10% in the kurtosis of the LQ portfolio (from 11.47% to 12.65%). Whereas an equivalent change in the correlation in the HQ portfolio produces a change of 27% (from 68.95% to 87.58%).

CreditRisk+ is more sensitive to changes in the correlation parameter than CreditMetrics. For instance, for an equivalent change of 28% in the correlation coefficient (from ρ=0.35 to ρ=0.45), the change in kurtosis for CreditRisk+ for LQ portfolio is 32% (from 13.76% to 18.14%), whereas for CreditMetrics this figure was 10%.

Finally, the discrepancies between those coefficients for both models increase for higher levels of the asset correlation parameter.

3. For HQ portfolios, CreditMetrics distributions are dramatically more leptokurtic[9] than those for CreditRisk+. Only for LQ portfolios and high correlations CreditRisk+ produces higher estimates than CreditMetrics. Therefore, for HQ portfolios, and for LQ portfolios which are poorly correlated, CreditMetrics forecasts larger credit losses and capital requirements due to default than CreditRisk+. This implies that more capital would be required by regulators if migrations to other non-default states were considered.

Looking at the distributions of default, we conclude that although the two models can be parameterised to yield the same mean and standard deviation of default distribution the differences between higher moments suggest that CVaR figures (of credit losses) may differ significantly. This difference is even more pronounced for high-quality portfolios than for low-quality portfolios.

5.2 Generation of the Distributions of Losses

To calculate the distribution of losses, for each realisation of the economic factor[10] we compute the number of individuals in the portfolio that fall into each credit state at the end of the period. These numbers can be found through inverting the integrals in equations (4.12) and (4.13) by using Monte Carlo techniques. The number of individuals in each rating class is combined with the size of the exposures to produce an estimate for the losses in the portfolio. Before describing the Monte Carlo

[9] We say that a distribution is leptoukortic, if its kurtosis coefficient is larger than 3.
[10] Recall that a realisation of the economic factor generates a set of conditional probabilities of migration, which are needed to estimate the probabilities of number of borrowers in each credit rating at the end of the period.

steps to calculate the loss distributions, we explain how the loss function is defined for each of the models.

The main output of CreditMetrics is the distribution of the value of the portfolio. CreditRisk+ produces a distribution for the losses of the portfolio. To facilitate the comparison between the models, the distribution generated by CreditMetrics is transformed into a loss distribution. In the traditional version of CreditMetrics, the mark-to-market value of a bond at the end of the time horizon is calculated by discounting the remaining cashflows and using the term-structure associated with the credit rating of the obligor at the end of period.

Let $P_{t+1,J}$ be the mark-to-market price of the bond associated to credit rating J (J=A, B, D). Therefore the mark- to market value of a bond (P_{t+1}) at the end of the time horizon is:

$$P_{t+1} = \sum_{J} P_{t+1,J} I_{(J)}\ , \quad J=A,\ B,\ D$$

Where $I_{(J)}$ is an indicator function with value 1 when borrower has been rated with credit quality J at the end of the period, and 0 otherwise.

Define the credit loss of a bond for the mark-to-market version of CreditMetrics as the difference between the expected value of the bond and its value at the end of the time horizon[11]:

Mark to Market Version of CreditMetrics

$$L^{CM3}_{t+1} = \sum_{J} \left(E_t(P_{t+1}) - P_{t+1,J}\right) I_{(J)} \quad J=A,\ B,\ D \qquad (5.1)$$

where $E_t(P_{t+1})$ is the expected value of the bond, which is equal to $\sum_{J=A,B,D} p_J P_{t+1,J}$. And p_J is the probability of migration to rating J (=A, B, D). Also remember that the value of the bond in case of default is defined as the recovery rate, which in this case is assumed fixed.

Likewise, for the restricted or default version of CreditMetrics, we define losses as:

Restricted Version of CreditMetrics

$$L_{t+1}^{CM2} = \sum_{J} \left(E_t(P_{t+1}) - P_{t+1,J}\right) I_{(J)} \qquad J=D,\ ND \qquad (5.2)$$

where **ND** is the no-default state and $P_{t+1,ND} = \left(\frac{\sum_{J=A,B} p_J P_{t+1,J}}{\sum_{J=A,B} p_J} \right)$ is the value of a non-defaulted bond.

For CreditRisk+, credit losses occur only when default occurs:

$$L_{t+1}^{CR+} = (\text{Exposure}) I_{(D)}$$

where different definitions of "Exposure" have been used in the literature and among practitioners[12].

In order to carry out comparisons between the models, we consider three versions of CreditRisk+. They differ only in the definition of exposure:

Non-default-Value Version of CreditRisk+ (CR+1)

$$L_{t+1}^{CR+1} = \left(P_{t+1,ND} - P_{t+1,D}\right) I_{(D)} \qquad (5.3)$$

Book-Value Version of CreditRisk+ (CR+2)

$$L_{t+1}^{CR+2} = \left(BV_t - P_{t+1,D}\right) I_{(D)} \qquad (5.4)$$

Expected-Value Version of CreditRisk+ (CR+3)

$$L_{t+1}^{CR+3} = \left(E_t(P_{t+1}) - P_{t+1,D}\right) I_{(D)} \qquad (5.5)$$

Where "BV" is the book value of the bond. It is important to recall that the version two of CreditRisk+ (the Book-Value version) corresponds to the original framework.

Once we have defined the loss function of a bond for both models, we can describe how to use Monte Carlo to generate the distribution of losses for each model.

[11] This definition of losses is consistent with the definition of mark-to-market value of a bond, in the sense that the CVaR of both distributions (portfolio value and losses) would yield the same results.

Simulations for the generation of the Distribution of Losses under CreditMetrics

As inputs of this process we use: a) The above generated samples of the rates of migration and default generated by using CreditMetrics, under a specific set of transition probabilities (type of portfolio) and a specific value of the correlation coefficient ρ.

1. For each set of transition rates, Calculate the conditional number of individuals that fall in each rating category. These variables follow a multinomial distribution. Therefore, we apply the Kemp and Kemp (1987) algorithm to simulate multinomial random numbers:

   1.1 Simulate the number of defaults $N_D^{CM}\big|_x$ as a binomial with parameters (N=10,000, $p_D^{CM}\big|_x$).

   1.2 Then simulate N_B^{CM} as a binomial (N-$N_D^{CM}\big|_x$, $\dfrac{p_B^{CM}\big|_x}{1-p_D^{CM}\big|_x}$).

   1.3 Calculate $N_A^{CM}\big|_x$ =N-$N_B^{CM}\big|_x$ -$N_D^{CM}\big|_x$

2. Obtain the portfolio loss by adding the individual losses of each bond according to the following equation:

$$PL_{t+1}^{CM3}\big|_x = \sum_J (E_t(P_{t+1}) - P_{t+1,J})N_J^{CM}\big|_x \text{ ……J=A,B,D} \qquad (5.6)$$

   Also calculate the portfolio loss for the restricted version of CreditMetrics:

$$PL_{t+1}^{CM2}\big|_x = \sum_J (E_t(P_{t+1}) - P_{t+1,J})N_J^{CM}\big|_x \text{ …..J=D, ND} \qquad (5.7)$$

3. In order to obtain the unconditional distribution of losses, repeat steps 1 and 2 for each set of transition rates [$p_D^{CM}\big|_x$, $p_B^{CM}\big|_x$, $p_A^{CM}\big|_x$].

4. Calculate descriptive statistics, including CVaR at the confidence levels: 90%, 93%, 95%, 97%, 99% and 99.9%.

5. Repeat 1-4 for all possible samples, i.e., combinations of different portfolios qualities (LQ and HQ) and levels of the correlation coefficient ρ (=0.05, 0.15, 0.25, 0.35, 0.45).

[12] Recall that "exposure" has been defined as the amount owed minus the recovery rate (which is the actual

Simulations for the generation of the Distribution of Losses under CreditRisk+

For CreditRisk+, the number of defaults is simulated in a similar way, except for steps 1 and 2, which are substituted by:

1. Calculate the conditional number of defaults in the portfolio $N_D^{CR}\big|_x$ by simulating a Poisson variable with parameter $\lambda = N^* p_D^{CR}\big|_x$.

2. Obtain the portfolio loss using ND, and the size of the exposures under each scenario.

$$PL_{t+1}^{CR+1}\Big|_x = (P_{t+1,ND} - P_{t+1,D})N_D^{CR}\Big|_x$$

Also calculate the portfolio loss for the other two versions of CreditRisk+, using equations (5.4) and (5.5).

Table 5.3 shows some descriptive statistics for the MTM version of CreditMetrics and the Book Value of CreditRisk+. The latter model produces similar levels of skewness and kurtosis for any version.

Table 5.3 Statistics of the Loss Distributions for CreditMetrics and CreditRisk+

STATISTICS OF THE DISTRIBUTION OF LOSSES				
	CreditMetrics		CreditRisk+	
Correlation	Skewness	Kurtosis	Skewness	Kurtosis
LOW QUALITY PORTFOLIO				
0.05	0.991	4.513	0.879	4.155
0.15	1.729	7.377	1.600	6.822
0.25	2.207	9.713	2.167	10.009
0.35	2.553	11.455	2.685	13.737
0.45	2.810	12.640	3.188	18.140
HIGH QUALITY PORTFOLIO				
0.05	1.669	7.849	1.272	5.409
0.15	3.401	23.214	2.513	12.428
0.25	4.904	43.697	3.664	22.990
0.35	6.209	64.742	4.871	38.245
0.45	7.286	82.665	6.173	59.486

In general, the properties of the distribution of losses inherit the properties of the default distribution. Therefore we can anticipate that this factor will be determinant in the CVaR of the portfolio. Comparing table 5.2 with 5.3, we can observe that the largest differences between skewness and kurtosis for both distributions are those for the High Quality portfolio, under the version of CreditMetrics. This is consistent with the belief that the tail of the distribution of losses, under

value of the debt under default).

CreditMetrics, incorporates information not only about default but also about downgrading in the portfolio.

6. Analysis of the Differences in CVaR between the Models

Above, the distributions of losses and CVaR for both models and for each of their versions were generated. We attribute the discrepancies in CVaR to three factors: a) the omission of migration risk in CreditRisk+; b) the shape of the tails of the default distributions of both models; and c) the definition of credit exposure in CreditRisk+. In section 6.1, we examine the individual impact of these three factors in the discrepancies of CVaR. In section 6.2, we put together these three factors and analyse their global impact on CVaR.

6.1. Effect of Individual Factors that explain the differences in CVaR

a) The effect of Migration Risk

Consider the two versions of CreditMetrics: the mark-to-market version and its restricted version. In the generation of the distribution of losses of both versions, inputs and underlying distributions are the same. The difference between the two versions lies in the definition of credit loss. The restricted version of CreditMetrics aggregates information from the two non-default states (A and B), leaving out the effect of migration risk. In contrast, the full version of CreditMetrics considers both states individually (equations (5.1) and (5.2) respectively), taking into account migration risk.

In order to quantify the differences between both versions, we compute the ratio of CVaRs, for a given confidence level. This ratio (=CVaR of the restricted version of CreditMetrics divided by the CVaR of the mark-to-market version) will be referred as the “Effect of Migration Risk”. As the CVaR for the mark-to-market version is always higher than the CVaR for the restricted version, this ratio is bounded by one.

A summary of the effect of migration risk for a range of correlations and confidence levels of CVaR is reported in Table 6.1. Observe that for the LQ portfolio, most of the ratios are close to one. The omission of non-default credit states or migration risk is practically irrelevant. At high confidence levels, the coefficients are one. This indicates that for the LQ portfolio the information contained in the tails of the loss distributions for both versions is the same, provided that model inputs are the same except for the number of credit states. Therefore, it seems that for LQ portfolios, information about losses accumulated in the tails of the loss distribution comes mainly from defaults in the portfolio, and not from downgradings to other non- default states.

Table 6.1. Effect of Migration Risk on CreditMetrics.

Differences of CVaR: Default version of CreditMetrics vs MTM version of CreditMetrics						
	Confidence Levels					
Correlation	0.9	0.93	0.95	0.97	0.99	0.999
LOW QUALITY PORTFOLIO						
0.05	0.998	0.999	1.000	1.000	1.000	1.000
0.15	1.000	1.000	1.000	1.000	0.999	1.000
0.25	1.000	0.999	1.000	1.000	1.000	1.000
0.35	0.999	1.000	1.000	1.000	1.000	1.000
0.45	0.999	0.999	1.000	1.000	1.000	1.000
HIGH QUALITY PORTFOLIO						
0.05	0.954	0.957	0.960	0.962	0.967	0.970
0.15	0.953	0.955	0.961	0.965	0.969	0.979
0.25	0.944	0.952	0.958	0.964	0.973	0.983
0.35	0.932	0.946	0.954	0.963	0.976	0.987
0.45	0.908	0.934	0.946	0.961	0.977	0.991

The omission of migration risk in the restricted version is more relevant for the HQ portfolio. From Table 6.1, figures are significantly less then one for a given correlation coefficient. For a given low confidence level (90%, 93%, 95%), the higher the correlation levels the more the information that is omitted. For example, for $\rho=0.45$ and for CVaRs at 90% confidence, the ratio is 0.908, whereas this number is 0.954 when $\rho=0.05$. In percentage terms those numbers are equal to -9.2%(=0.908-1) and –4.6%(=0.954-1) respectively. Therefore, the default version of CreditMetrics omits up to 9.2% of information about downgrades. Intuitively, higher correlations are associated to higher levels of volatilities of the migration rates. Therefore, more downgrades and losses are expected to take place.

Also, for the HQ portfolio and high confidence levels, ratios are closer to one. This suggests that in the very extreme tails, distributions contain more information about losses generated by defaults than by downgrading events. At 99.9% confidence level, the largest omission of information is only for 3% (=0.97-1). For high correlation levels, ratios are even closer to one. This is because high volatility of the default rate causes more defaults in the portfolio. Therefore, both versions of credit risk should be more alike in the tails.

b) The effect of the Distribution of Default

Consider the restricted version of CreditMetrics and the first version of CreditRisk+ (its loss function is given by (5.3) and denoted by CR+1). The definitions of credit models for both models ((5.2) vs (5.3)) are algebraically the same except for an additive constant. This constant is irrelevant for CVaR calculations. As each model uses its own distributional assumptions, the differences in CVaR result from the discrepancies between their distributions of default. In order to quantify such discrepancies, for a given confidence level, we compute the ratio of CVaRs (=CVaR of CR+1 divided by the CVaR of the restricted version of CreditMetrics). This ratio will be referred as the "Effect of the Distribution of Default".

A summary of the discrepancies in CVaR due to the distributions of default for a range of correlations and confidence levels is reported in Table 6.2.

Note that for the LQ portfolio, most of the ratios are less than one. This means that the distribution of default produced by CreditMetrics is thicker and longer. As a consequence, CreditMetrics forecasts higher CVaR numbers and capital requirements than CreditRisk+ due to default. Only for high confidence intervals (99.9%) and high correlation levels (0.25-0.45), CreditRisk+ produces higher CVaRs than CreditMetrics. These conclusions are consistent with the results obtained in section 5.1.

Table 6.2. Effect of the Distributions of Default of CreditRisk+ and CreditMetrics.

Differences of CVaR: CR+1 vs Default version of CreditMetrics						
	Confidence Intervals					
Correlation	0.9	0.93	0.95	0.97	0.99	0.999
LOW QUALITY PORTFOLIO						
0.05	1.002	0.997	0.988	0.982	0.973	0.963
0.15	1.009	0.998	0.991	0.981	0.972	0.972
0.25	1.005	0.992	0.983	0.975	0.974	1.013
0.35	0.990	0.976	0.968	0.962	0.977	1.079
0.45	0.966	0.948	0.940	0.940	0.974	1.154
HIGH QUALITY PORTFOLIO						
0.05	1.042	1.020	1.001	0.972	0.929	0.854
0.15	1.159	1.115	1.080	1.034	0.951	0.827
0.25	1.239	1.193	1.153	1.094	0.991	0.838
0.35	1.263	1.236	1.196	1.138	1.018	0.866
0.45	1.217	1.231	1.215	1.157	1.036	0.892

For the HQ portfolio, differences in CVaR for both models are more dramatic. For high confidence intervals (99.9%), CreditRisk+ produces CVaR up to -17% (=0.827-1) lower than CreditRisk+. This is because the tail of the default distribution of CreditMetrics is thicker. For low confidence intervals, the opposite occurs: CreditRisk+ produces CVaRs up to 26.3% (=1.263-1) higher than CreditMetrics.

c) The effect of the Exposure

Finally, consider the differences between the three versions of CreditRisk+. Corresponding definitions of credit losses are given in equations (5.3), (5.4) and (5.5) respectively. We keep the same notation for the first version of CreditRisk+ (CR+1). We denote the second and third version by “CR+2” and “CR+3” respectively. The distribution of default for all three models is the same. Also, the loss functions are algebraically the same except for the definition of credit exposure. Hence the difference of CVaRs between any two versions should be a multiplicative constant. This constant is equal to the ratio of the exposures. Therefore, CVaR ratios between CR+2 and CR+1 or between CR+3 and CR+1 should be interpreted as the “Effect of the Exposure”. Contrary to other effects, the effect of the exposure is theoretically a fixed number[13] , as the exposures for each model are calculated exogenously, under CreditRisk+.

The discrepancies of CVaR due to differences in the definition of “exposure” are shown in Table 6.3.

Table 6.3 The effect of the Exposure in CreditRisk+.

Differences of CVaR: CR+2 and CR+3 vs CR+1		
	CR+2/CR+1	**CR+3/CR+1**
LOW QUALITY PORTFOLIO		
	1.007	0.929
HIGH QUALITY PORTFOLIO		
	1.001	0.990

6.2 Global effect of the factors that explain the differences in CVaR

In this section we examine the interaction of the three factors that explain the discrepancies in CVaR. Tables 6.4 and 6.5 illustrate the differences of CVaR between the Book-Value version of

[13] Some differences can arise due to calculation error.

CreditRisk+ (CR+2) and the mark-to-market version of CreditMetrics[14], for the low quality and high quality portfolios respectively. Models have been assessed through their performance on low and high quality portfolios, asset correlations and confidence levels.

In each table, the first line of each block represents the differences of CVaRs in percentage terms. The second line corresponds to the simple ratio of CVaR figures. The third line represents the effect of the exposure. The fourth line indicates the effect of the default distribution. Finally, the last line corresponds to the effect of migration risk. Observe that numbers in the second line are the arithmetic products of the numbers in the following three lines. This means that the variation in CVaR can be decomposed into those three factors.

Table 6.4. Differences of CVaR between CR+2 and CreditMetrics. Low Quality Portfolio.

Comparison of Distributions of Losses: CR+2 versus CreditMetrics (MTM) LOW QUALITY PORTFOLIO							
correlation	**quantiles**	**90%**	**93%**	**95%**	**97%**	**99%**	**99.90%**
	%Variation in CVaR	**0.72%**	**0.35%**	**-0.38%**	**-1.10%**	**-2.16%**	**-3.06%**
0.05	Variation in CVaR	1.007	1.003	0.996	0.989	0.978	0.969
	Effect of Exposure	1.008	1.007	1.008	1.007	1.006	1.007
	Effect of Dist.of Default	1.002	0.997	0.988	0.982	0.973	0.963
	Effect of Migration Risk	0.998	0.999	1.000	1.000	1.000	1.000
	%Variation in CVaR	**1.57%**	**0.52%**	**-0.24%**	**-1.25%**	**-2.23%**	**-2.06%**
0.15	Variation in CVaR	1.016	1.005	0.998	0.987	0.978	0.979
	Effect of Exposure	1.006	1.007	1.007	1.007	1.007	1.007
	Effect of Dist.of Default	1.009	0.998	0.991	0.981	0.972	0.972
	Effect of Migration Risk	1.000	1.000	1.000	1.000	0.999	1.000
	%Variation in CVaR	**1.15%**	**-0.14%**	**-1.02%**	**-1.83%**	**-1.96%**	**1.94%**
0.25	Variation in CVaR	1.011	0.999	0.990	0.982	0.980	1.019
	Effect of Exposure	1.007	1.007	1.007	1.007	1.007	1.006
	Effect of Dist.of Default	1.005	0.992	0.983	0.975	0.974	1.013
	Effect of Migration Risk	1.000	0.999	1.000	1.000	1.000	1.000
	%Variation in CVaR	**-0.38%**	**-1.70%**	**-2.60%**	**-3.19%**	**-1.65%**	**8.60%**
0.35	Variation in CVaR	0.996	0.983	0.974	0.968	0.983	1.086
	Effect of Exposure	1.007	1.007	1.007	1.007	1.007	1.007
	Effect of Dist.of Default	0.990	0.976	0.968	0.962	0.977	1.079
	Effect of Migration Risk	0.999	1.000	1.000	1.000	1.000	1.000
	%Variation in CVaR	**-2.82%**	**-4.54%**	**-5.43%**	**-5.43%**	**-1.99%**	**16.18%**
0.45	Variation in CVaR	0.972	0.955	0.946	0.946	0.980	1.162
	Effect of Exposure	1.007	1.007	1.007	1.007	1.007	1.007
	Effect of Dist.of Default	0.966	0.948	0.940	0.940	0.974	1.154
	Effect of Migration Risk	0.999	0.999	1.000	1.000	1.000	1.000

From the LQ portfolio in table 6.4, we can conclude the following:

Differences of CVaR vary between –5.43% and 16.18%. The most dramatic differences correspond to high correlations (0.35-0.45) and high confidence intervals (99%, 99.9%). Observe the joint effect

[14] Similar analysis can be done using the third version of CreditRisk+ (CR+3). Results are found in Appendix C.

of the three components that explain CVaR. The effect of omitting migration risk is practically nil on the overall difference. These numbers are close to one, therefore they do not make any contribution. The most relevant effect is that of the distribution of default.

In most cases, the effect of the exposure reduces the discrepancies due to the effect of the default distribution. The negative effect of the distribution of default (numbers less than one) is offset partially by the positive effect of the exposure (numbers bigger than one). However, for a very large confidence interval, 99.9%, and correlation levels, ρ=0.25, 0.35, 0.45, both effects are positive, therefore differences in CVaR become larger.

Table 6.5. Differences of CVaR between CR+2 and CreditMetrics . High Quality Portfolio

Comparison of Distributions of Losses: CR+2 versus CreditMetrics (MTM) HIGH QUALITY PORTFOLIO							
correlation	**quantiles**	**90%**	**93%**	**95%**	**97%**	**99%**	**99.90%**
	%Variation in CVaR	**-0.05%**	**-2.06%**	**-3.91%**	**-5.84%**	**-10.06%**	**-17.11%**
0.05	Variation in CVaR	1.000	0.979	0.961	0.942	0.899	0.829
	Effect of Exposure	1.005	1.004	1.001	1.007	1.002	1.000
	Effect of Dist.of Default	1.042	1.020	1.001	0.972	0.929	0.854
	Effect of Migration Risk	0.954	0.957	0.960	0.962	0.967	0.970
	%Variation in CVaR	**10.46%**	**6.77%**	**3.94%**	**-0.13%**	**-7.70%**	**-18.96%**
0.15	Variation in CVaR	1.105	1.068	1.039	0.999	0.923	0.810
	Effect of Exposure	1.000	1.003	1.002	1.002	1.001	1.001
	Effect of Dist.of Default	1.159	1.115	1.080	1.034	0.951	0.827
	Effect of Migration Risk	0.953	0.955	0.961	0.965	0.969	0.979
	%Variation in CVaR	**17.08%**	**13.57%**	**10.44%**	**5.68%**	**-3.52%**	**-17.46%**
0.25	Variation in CVaR	1.171	1.136	1.104	1.057	0.965	0.825
	Effect of Exposure	1.001	1.000	1.000	1.002	1.001	1.001
	Effect of Dist.of Default	1.239	1.193	1.153	1.094	0.991	0.838
	Effect of Migration Risk	0.944	0.952	0.958	0.964	0.973	0.983
	%Variation in CVaR	**17.73%**	**16.95%**	**14.28%**	**9.69%**	**-0.51%**	**-14.47%**
0.35	Variation in CVaR	1.177	1.169	1.143	1.097	0.995	0.855
	Effect of Exposure	1.000	1.000	1.002	1.001	1.001	1.001
	Effect of Dist.of Default	1.263	1.236	1.196	1.138	1.018	0.866
	Effect of Migration Risk	0.932	0.946	0.954	0.963	0.976	0.987
	%Variation in CVaR	**11.22%**	**15.06%**	**15.13%**	**11.27%**	**1.24%**	**-11.52%**
0.45	Variation in CVaR	1.112	1.151	1.151	1.113	1.012	0.885
	Effect of Exposure	1.006	1.000	1.001	1.001	1.001	1.001
	Effect of Dist.of Default	1.217	1.231	1.215	1.157	1.036	0.892
	Effect of Migration Risk	0.908	0.934	0.946	0.961	0.977	0.991

From the HQ portfolio in table 6.5, we can conclude the following:

The differences of CVaR are wider than those for the LQ portfolio. They vary between –18.96% and 17.73%. Considering the overall effect of the three factors, we can say that CreditRisk+ generally gives higher estimates than CreditMetrics at low confidence levels and lower estimates than CreditMetrics at high confidence levels. These differences are mainly due to the discrepancies in the distribution of default. At low confidence intervals (90, 93, 95%) CreditRisk+ produces higher

CVaRs estimates due to default risk (numbers are much larger than one). This positive effect is slightly offset by the fact that CreditMetrics measures migration risk. For low confidence intervals, the effect of omitting rating changes is less than one. For high confidence intervals, the reverse occurs. Overall, CreditRisk+ underestimates CreditMetrics. This is due to important underestimation of the default risk respect to CreditMetrics. In addition, CreditRisk+ does not account for migration risk therefore differences become larger.

The difference between models due to the definition of exposure in CreditRisk+ is not important. This difference accounts for only 1%(=1.007-1) of the overall difference. Perhaps debt with longer maturity would produce a more important effect.

To summarise, we find that most of the discrepancies between the models are due to the differences in their probabilities of default. The omission of migration risk is relevant only for high-quality (HQ) portfolios and low confidence levels. Roughly speaking, if we assume that there are no discrepancies in the distributions of default, then the use of a two-credit- state model instead of a three-credit- state model will miss out up to 3% of information about downgrading at very extreme confidence levels (99.9%). This figure is 9.2% for lower confidence levels (90%).

7. Conclusions and Implications

Having assumed homogeneous portfolios and one-single economic factor, we carried out a structural comparison between CreditRisk+ and the mark-to-market version of CreditMetrics. We make an extension of Koyluoglu and Hickman (1998) framework and identified common steps to derive the distribution of losses. Likewise, we derived consistent parameters, which make both models comparable.

We find differences in Credit Value-at-Risk of up to 19% between the models. The model that forecasts higher values is not always the same, results depend on the quality of the portfolio, parameters values and confidence levels.

Three particular factors might explain the difference between models: a) the omission of migration risk in the model of CreditRisk+, b) the shape of the probability of default in both models, and c) the definition of credit exposure in CreditRisk+. In general, the differences in the shape of the default distributions generated by the two models explain most of the differences of CVaR. The omission of migration risk is significant only for high-quality portfolios and low confidence levels (lower than 90%). If we assumed that no discrepancies exist in the distribution of default, then CreditRisk+ would estimate lower CVaR than CreditMetrics, up to 10%.

For the low-quality (LQ) portfolio, both models behave similarly for all confidence intervals, except for very extreme levels (larger than 99%). In these cases CreditRisk+ estimates CVaRs up to 16% higher than CreditMetrics. This might be due to the properties of the gamma distribution, which is very sensitive to high unconditional default rates and high volatility levels. For high-quality portfolios (HQ), differences are more dramatic. CreditRisk+ again estimates higher CVaRs than CreditMetrics, except for high confidence intervals. In all these results the main driver of the differences between the models is the shape of the distribution of default of the models. These results are important since the use of high confidence levels in the measurement of credit risk is a common practice in the banking community. High confidence levels often compensate for the inability to test the reliability of the models. Hence capital requirements based on high confidence intervals seem to depend highly on which model is chosen.

The implications of the above results for risk management are quite clear. For low-quality portfolios we may forget about CreditMetrics, since migration risk accounts for very little in the overall CVaR of the portfolio. In this case, CreditRisk+ is a faster and less expensive approach for calculating capital requirements. On the other hand, if our purpose to use a credit risk model is to estimate reserves of capital, then CreditMetrics may provide more accuracy about the sources of losses.

In answering the question about which is better model to implement in banks, it is necessary to take in account the following considerations:

- The type of credit risk that is to be measured. In solving this paradigm, it might be helpful to think of the objective of the measurement. In the calculation of capital requirements, the main concern is the very extreme values of the distributions. Therefore, as was mentioned, default models should be sufficient to estimate the overall picture of the losses. However, if the model is needed to estimate reserves or provisions for credit risk, CreditMetrics would give more information about the size of the reserves needed for non- defaulted loans.
- The costs and reliability of the inputs. In particular CreditMetrics requires large amount of data. Whereas CreditRisk+ is cheaper and easier to implement.
- The overall environment of the risk management process, ie., how often the model is to be revised, the control processes, the ability of the managerial team to understand inputs and outputs of the model, etc. To interpret outputs from the two models demand good technical knowledge and skills, as they involve the same level of complexity. However, CreditMetrics seems to be more demanding in the administration and control of inputs.

Further research is needed in finding what types of portfolios or parameters produce bigger discrepancies. Stress- testing some parameters, such as the unconditional rate of default, the asset correlation coefficient, the recovery rate or interest rate could give more insights. Likewise, it would be useful to analyse the impact of specific grading systems on the overall performance of the models. For example, does the number of grades in the rating system matter? What is the impact of ranking borrowers in one credit class or in another? This analysis would provide a better understanding of the vulnerability of the models to other inputs. This is particularly important in credit risk, because the quality of data is usually the most important restriction in the implementation of models.

In this exercise the omission of migration risk in CreditRisk+ seemed to have been marginal. However, two broad credit states (default and no- default) might not represent accurately the riskiness of the portfolio for multi-year term loans.

Finally, there are some other caveats to discuss about the models. For instance, neither of these two models has been exactly designed to capture economic cycles. Although implicitly there are some economic factors that drive the credit quality of the borrowers, transition matrices, standard deviations or correlations are calculated from historical data across many credit cycles. In this sense, both models have been criticised, since it is empirically clear that parameters depend on business cycles. Hence credit losses might be overestimated in recession periods and underestimated in boom periods (Nickell and Perraudin (2000)). It remains to balance the benefits of the simplicity of the assumptions against the reliability of the outputs.

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APPENDIX A

Derivation of the Migration Rates for CreditMetrics

Assume a credit rating system composes of three states: A, B and D. Where A represents the highest credit quality, B represents and intermediate state and D represents the state of default. The transition probabilities or migration rates is determined by the vector $\left[\overline{p}_A,\ \overline{p}_B,\ \overline{p}_Q\right]$

Assume that the firms's returns follow a Vasicek's representation:

$$r_i = \sqrt{\rho}X + \sqrt{1-\rho}\varepsilon_i$$

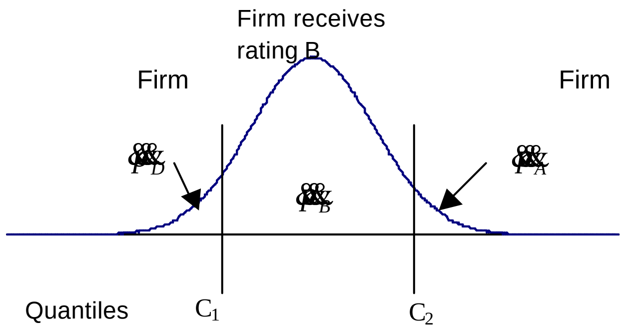
Where X and e_i represent the systematic and non- systematic factors of the firm respectively, and both are normally distributed. According to CreditMetrics, for very large portfolios, the economic factor drives the credit quality of the firms or borrowers in the portfolio.

In CreditMetrics, default occurs when the firm's returns fall below a threshold C_1 in the standard Normal. See Figure A.1.

Let

- $p_D^{CM}\big|_X$ be the default rate or conditional probability of default for CreditMetrics
- $C_1 = \Phi^{-1}(\overline{p}_D)$, where $\overline{p}_D$ is the unconditional default rate and $\Phi(s)$ is the normal cumulative density function.

Figure A.1 Distribution of the Credit Quality of the firm



Then the default rate is the probability that asset returns fall below a specific quantile C_1, given a specific value of the economic factor X. This rate can be calculated as follows:

$$p_D^{CM}\Big|_x = P(r \le C_1|x) = P\left(\sqrt{\rho}x + \sqrt{1-\rho}\varepsilon \le C_1|x\right) = P\left(\varepsilon \le \frac{C_1 - \sqrt{\rho}x}{\sqrt{1-\rho}}|x)\right) = \Phi\left(\frac{C_1 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right)$$

The density function for the default rate can be estimated as follows:

$$P(p_D^{CM} \le p) = P\left(\Phi\left(\frac{C_1 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right) \le p\right) = P\left(x \le \frac{C_1 - \sqrt{1-\rho}\Phi^{-1}(p)}{\sqrt{\rho}}\right) = \Phi\left(\frac{C_1 - \sqrt{1-\rho}\Phi^{-1}(p)}{\sqrt{p}}\right)$$

$$f(p) = \frac{dP}{dp} = \frac{\sqrt{1-\rho}\varphi\left(\frac{C_1 - \sqrt{1-\rho}\Phi^{-1}(p)}{\sqrt{p}}\right)}{\sqrt{p}\varphi\left(\Phi^{-1}(p)\right)}$$

Where $\varphi(z)$ is the standardised normal density function.

According to Koyluoglu and Hickman(1998), the default rate volatility can be expressed in terms of ρ and $\overline{p}_D$ as follows:

$$\sigma^2 = \int_{-\infty}^{\infty}(p_D^{CM}\Big|_x - \overline{p}_D)^2\varphi(x)dx = \int_{-\infty}^{\infty}\left(\Phi\left[\frac{\phi^{-1}(\overline{p}_D^{CM} - \sqrt{\rho}x}{\sqrt{1-\rho}}\right] - \overline{p}_D\right)^2\varphi(x)dx$$

On the other hand, the migration rate to the credit state B can be derived in a similar way. By assuming that migration to rating B occurs when returns fall between the thresholds C_1 and C_2 (see figure A.1), where $C_2 = \Phi^{-1}(\overline{p}_D + \overline{p}_B)$, then the migration rate to state B given a specific value of the background factor is:

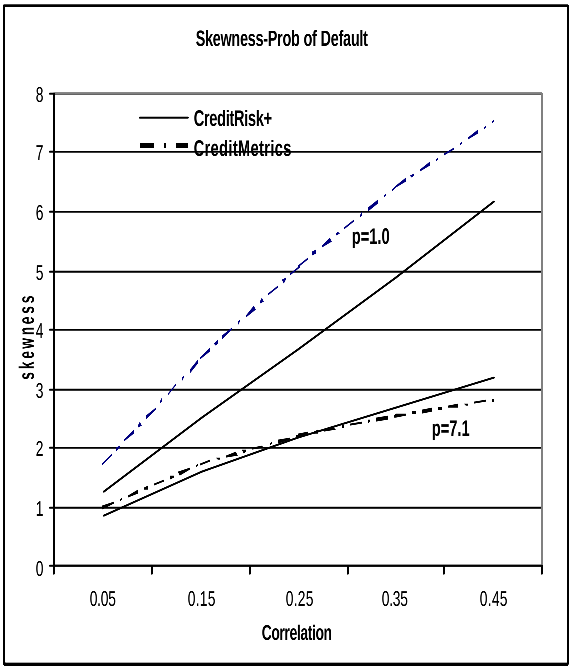
$$p_B^{CM}\Big|_x = P(C_1 \le r \le C_2|x) = P(r < C_2|x) - P(r < C_1|x) = \Phi\left(\frac{C_2 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{C_1 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right)$$

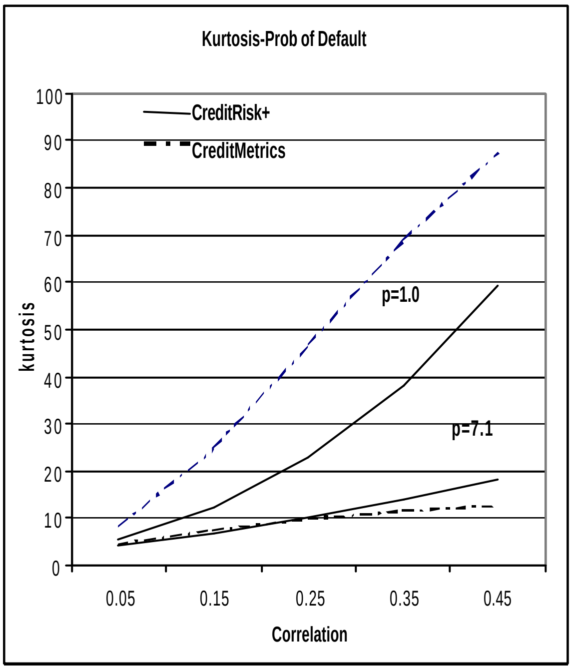
Finally, the migration rate to the credit state A can be derived using the fact that the sum of the transition probabilities should equal one. Therefore:

$$p_A^{CM}\Big|_x = 1 - p_B^{CM}\Big|_x - p_D^{CM}\Big|_x$$

APPENDIX B

Figure B.1. Skewness and Kurtosis of the Conditional Distributions of Default of CreditMetrics and CreditRisk+.





p is the unconditional probability of default. Skewness and Kurtosis are plotted against the obligors´s asset correlation level.

Figure B.2. Statistics of the distribution of migrating to rating B.

STATISTICS OF THE DISTRIBUTION OF MIGRATION				
	CreditMetrics			
Correlation	**Mean**	**Stand.Dev**	**Skewness**	**Kurtosis**
LOW QUALITY PORTFOLIO				
0.05	92.06%	2.65%	-1.418	5.785
0.15	92.06%	5.09%	-2.144	9.352
0.25	92.06%	7.17%	-2.482	11.289
0.35	92.06%	9.15%	-2.684	12.296
0.45	92.06%	11.10%	-2.820	12.759
HIGH QUALITY PORTFOLIO				
0.05	9.55%	3.53%	0.695	3.585
0.15	9.55%	6.29%	1.132	4.359
0.25	9.55%	8.35%	1.380	4.800
0.35	9.55%	10.18%	1.554	5.089
0.45	9.55%	11.93%	1.698	5.342

APPENDIX C

Differences of CVaR between CR+3 and CreditMetrics

C.1. Low Quality Portfolio.

Comparison of Distributions of Losses: CR+3 versus CreditMetrics (MTM)
LOW QUALITY PORTFOLIO

correlation	quantiles	90%	93%	95%	97%	99%	99.90%
	%Variation in CVaR	**-7.03%**	**-7.50%**	**-8.08%**	**-8.76%**	**-9.70%**	**-10.49%**
0.05	Variation in CVaR	0.930	0.925	0.919	0.912	0.903	0.895
	Effect of Exposure	0.930	0.929	0.930	0.929	0.929	0.930
	Effect of Dist.of Default	1.002	0.997	0.988	0.982	0.973	0.963
	Effect of Migration Risk	0.998	0.999	1.000	1.000	1.000	1.000
	%Variation in CVaR	**-6.30%**	**-7.22%**	**-7.94%**	**-8.87%**	**-9.73%**	**-9.60%**
0.15	Variation in CVaR	0.937	0.928	0.921	0.911	0.903	0.904
	Effect of Exposure	0.928	0.929	0.929	0.929	0.930	0.930
	Effect of Dist.of Default	1.009	0.998	0.991	0.981	0.972	0.972
	Effect of Migration Risk	1.000	1.000	1.000	1.000	0.999	1.000
	%Variation in CVaR	**-6.68%**	**-7.85%**	**-8.67%**	**-9.38%**	**-9.50%**	**-5.90%**
0.25	Variation in CVaR	0.933	0.921	0.913	0.906	0.905	0.941
	Effect of Exposure	0.929	0.929	0.929	0.929	0.929	0.929
	Effect of Dist.of Default	1.005	0.992	0.983	0.975	0.974	1.013
	Effect of Migration Risk	1.000	0.999	1.000	1.000	1.000	1.000
	%Variation in CVaR	**-8.06%**	**-9.29%**	**-10.08%**	**-10.65%**	**-9.21%**	**0.26%**
0.35	Variation in CVaR	0.919	0.907	0.899	0.893	0.908	1.003
	Effect of Exposure	0.929	0.929	0.929	0.929	0.929	0.930
	Effect of Dist.of Default	0.990	0.976	0.968	0.962	0.977	1.079
	Effect of Migration Risk	0.999	1.000	1.000	1.000	1.000	1.000
	%Variation in CVaR	**-10.27%**	**-11.91%**	**-12.71%**	**-12.69%**	**-9.52%**	**7.24%**
0.45	Variation in CVaR	0.897	0.881	0.873	0.873	0.905	1.072
	Effect of Exposure	0.930	0.929	0.929	0.929	0.929	0.929
	Effect of Dist.of Default	0.966	0.948	0.940	0.940	0.974	1.154
	Effect of Migration Risk	0.999	0.999	1.000	1.000	1.000	1.000

C.2. High Quality Portfolio.

Comparison of Distributions of Losses: CR+3 versus CreditMetrics (MTM)
HIGH QUALITY PORTFOLIO

correlation	quantiles	90%	93%	95%	97%	99%	99.90%
	%Variation in CVaR	**-1.40%**	**-3.36%**	**-4.75%**	**-7.09%**	**-11.09%**	**-18.08%**
0.05	Variation in CVaR	0.986	0.966	0.953	0.929	0.889	0.819
	Effect of Exposure	0.992	0.991	0.992	0.994	0.990	0.988
	Effect of Dist.of Default	1.042	1.020	1.001	0.972	0.929	0.854
	Effect of Migration Risk	0.954	0.957	0.960	0.962	0.967	0.970
	%Variation in CVaR	**9.11%**	**5.48%**	**2.73%**	**-1.17%**	**-8.71%**	**-19.91%**
0.15	Variation in CVaR	1.091	1.055	1.027	0.988	0.913	0.801
	Effect of Exposure	0.988	0.990	0.990	0.991	0.990	0.989
	Effect of Dist.of Default	1.159	1.115	1.080	1.034	0.951	0.827
	Effect of Migration Risk	0.953	0.955	0.961	0.965	0.969	0.979
	%Variation in CVaR	**15.67%**	**12.41%**	**9.29%**	**4.46%**	**-4.60%**	**-18.37%**
0.25	Variation in CVaR	1.157	1.124	1.093	1.045	0.954	0.816
	Effect of Exposure	0.989	0.990	0.990	0.990	0.990	0.990
	Effect of Dist.of Default	1.239	1.193	1.153	1.094	0.991	0.838
	Effect of Migration Risk	0.944	0.952	0.958	0.964	0.973	0.983
	%Variation in CVaR	**16.57%**	**15.54%**	**13.05%**	**8.54%**	**-1.60%**	**-15.35%**
0.35	Variation in CVaR	1.166	1.155	1.131	1.085	0.984	0.847
	Effect of Exposure	0.990	0.988	0.991	0.991	0.990	0.990
	Effect of Dist.of Default	1.263	1.236	1.196	1.138	1.018	0.866
	Effect of Migration Risk	0.932	0.946	0.954	0.963	0.976	0.987
	%Variation in CVaR	**9.85%**	**13.69%**	**13.93%**	**10.01%**	**0.13%**	**-12.45%**
0.45	Variation in CVaR	1.099	1.137	1.139	1.100	1.001	0.875
	Effect of Exposure	0.994	0.988	0.991	0.990	0.990	0.991
	Effect of Dist.of Default	1.217	1.231	1.215	1.157	1.036	0.892
	Effect of Migration Risk	0.908	0.934	0.946	0.961	0.977	0.991