"STRATEGY, STRUCTURE AND SELECTION"

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Summary

'Risk-budgeting' in the investment decision-making process encourages explicit separation of the sources of risk. One decomposition of risk that is popular among financial advisors and investment consultants is strategy, structure and selection. Strategy measures the extent of 'mismatch' between assets and liabilities; structuring is broadly the allocation of investments to 'passive' and 'active' strategies and selection is the choice of individual securities, or the choice of investment manager.

Investors are typically encouraged to make decisions on these sources of risk in a roughly hierarchical fashion: choose strategy first; then find a structure to implement the strategy and choose the specific instruments (or asset managers). The analysis in this paper focuses on the consequences of following the hierarchical approach compared with a full, joint optimisation across strategy, structure and selection. It demonstrates that unless the structure and selection steps are designed carefully, there can be a loss of efficiency as measured by the investor's expected utility. Furthermore, some implementations of the hierarchical approach can lead to inappropriate preference being given to assets depending on their systematic risk.

Keywords

Risk-budgeting; investment strategy; optimal portfolio construction; investment management structures; expected utility; appraisal ratio; information ratio

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Introduction

A simple model of the investment process is to maximise the expected utility associated with

$$R_{fund} = \delta_1 R_M + \delta_2 R_A + (1 - \delta_1 - \delta_2) r_f \qquad (1)$$

by selecting values for the δ's (and we will assume that they should all be positive). R_m is the return on the market, R_A is the return on an active portfolio and r_f is a risk-free return, which we will interpret as being the rate of increase in 'liability'.

Common practice is to decompose this process into a hierarchy of stages (see, e.g. Urwin et al., 2001), characterised as (1) strategy review, (2) structure review and (3) manager selection. Each stage typically depends on the outcome of the stages undertaken before.

Equation (1) can be reparameterised to make the stages clear:

$$R_{fund} = k\left[\varphi R_M + (1 - \varphi) R_A\right] + (1 - k) r_f \qquad (2)$$

So that a decision about k determines strategy (proportion in risky portfolio versus matching portfolio), a decision about φ determines structure (proportion passive) and the determination of the properties of R_A is the selection process. In reality, the issues surrounding strategy, structure and selection are far more significant than as described above. This paper abstracts just the most important aspects of these activities as they relate to risk and return in a theoretical world.

The advantages of the hierarchical structure are difficult to quantify, but can be summarised as:

(a) investors may be comfortable at moving one thing at a time, or changing things in a series of steps, but are not typically comfortable about 'big bang' alterations;

(b) some parts of the process may need amending more often than others: for example, investors may want to review their individual security (or investment manager) selections frequently and do not want to the strategy to have to change as a result (a form of mental accounting);

(c) measurability and accountability for the consequences of each of the steps are more clearly identified in an hierarchical, non-integrated process;

(d) it is arguably easier to explain to clients the decisions as being the result of drilling down to greater levels of detail.

Hodgson et al. (2000) focus on a generalisation of the choice of φ, i.e. the investment structure of the fund. They include 'behavioural finance' variables in an attempt to model irrational behaviour among investors.

Exley, Mehta & Smith (1997) (henceforth, EMS) argue along the lines of Modigliani & Miller that the choice of k is (to first order) an irrelevance for defined benefit pension funds.

They argue that investment consultancy should focus more on second order effects (which are not considered in this paper) and come to the conclusion that a matching strategy (i.e. $k = 0$) exploits inefficiencies and pseudo-arbitrages to the best advantage of both shareholders and pension fund members. Consequently, the present analysis focuses on individual investment decision making, hence the use of the risk-free rate as the ‘liability’ rate. However, this type of analysis could be extended to deal with more general liabilities and has application to institutional investment decision-making when ‘fund-centric’ approaches are relevant.

In this analysis frequent use is made of an “empirical CAPM model”. This is because the distinction between systematic (market, undiversifiable) risk and diversifiable (unique, specific) risk is important, but the use of a single common risk factor makes for simpler algebra without loss of insight. The analysis is also restricted to a single period investment horizon for simplicity of exposition.

The empirical (return generating) model used is:

$$R_i - r_f = \beta_i \left(R_m - r_f\right) + \varepsilon_i \,. \tag{3}$$

The conventional CAPM is obtained by taking expectations of the above with the last term constrained to be independent and with zero expectation. In this note, we allow for non-zero expectations in the residual term that might be motivated, for example, by the existence of patents, monopolies and other frictions that are not subject to market risk. The expectation is referred to as alpha and corresponds with Jensen’s alpha as used in performance measurement.

Unconstrained optimisation

We can re-write equation (1) as:

$$R_{fund} - r_{f=} = \delta_1 \left(R_M - r_f\right) + \delta_2 \left(R_A - r_f\right) \tag{4}$$

Then, using the return-generating formula specified earlier,

$$R_{fund} - r_{f=} = \left(\delta_1 + \delta_2 \beta_A\right)\left(R_M - r_f\right) + \delta_2 \left(\alpha_A + \varepsilon_A\right), \tag{5}$$

where now the ε-term has zero mean. We assume that the portfolio A and the parameters δ_1 and δ_2 are chosen so as to maximise expected end of period utility of return. We assume that investors have power utility functions, or at least that the optimal investment arrangement can be determined by maximising:

$$E[U(R_{fund})] - r_f = E[U(R_{fund} - r_f)] = E[R_{fund} - r_f] - \frac{\tau}{2} Var[R_{fund} - r_f], \tag{6}$$

where τ represents the investor’s level of risk-aversion. The use of the above expected utility function as a method of specifying ‘value added’ is in keeping with many authors, e.g. Grinold & Kahn (1999).

Portfolio A is defined by $x_1, x_2, \ldots, x_n$ and η. The parameter η represents the proportion of wealth allocated to A that is invested in risky securities. The parameter x_i represents the proportion of that wealth invested in the risky securities that is invested in security i with $\sum_{i=1}^{n} x_i = 1$. This parameterisation is of course over-elaborate for the unconstrained case, but is useful for dealing with the hierarchical approach later, where it would be usual for the active, ‘risky’ portfolio to include some investment in the risk-free asset.

The return on portfolio A can therefore be written as:

$$R_A = (1-\eta)r_f + \eta\sum_{i=1}^{n} x_i R_i \text{ , with} \tag{7}$$

$$\begin{aligned} \beta_A &= \eta\sum x_i\beta_i \\ \alpha_A &= \eta\sum x_i\alpha_i \;. \\ \varepsilon_A &= \eta\sum x_i\varepsilon_i \end{aligned}$$

The expected utility is then

$$E[U(R_{fund} - r_f)] = \left\{\delta_1^*(E[R_m] - r_f) - \frac{\tau}{2}\left(\delta_1^*\right)^2 Var[R_m]\right\} + \left\{\sum y_i\alpha_i - \frac{\tau}{2}\sum y_i^2\sigma_i^2\right\} \tag{8}$$

where $y_i = \delta_2\eta x_i$ and $\delta_1^* = \left(\delta_1 + \sum y_i\beta_i\right)$.

The necessary optimality conditions are that:

$$y_i = \frac{\alpha_i}{\sigma_i^2}\tau^{-1} \text{ and } \delta_1^* = \frac{E[R_m] - r_f}{Var[R_m]}\tau^{-1}.$$

This analysis indicates that the investor should select risky securities in proportion to their so-called appraisal ratio, α_i/σ_i^2 . The proportion invested in passive market instruments, δ_1, is then just the difference between δ_1^* and the beta of the risky portfolio. In a sense, therefore, the natural hierarchy is to first select the best portfolio of individual risky securities and then structure the fund so that the overall exposure to systematic risk maximises expected utility. The structure of the fund is therefore just a way of cementing together optimal systematic risk allocation and optimal security selection. Because of the over-parameterisation of the problem, there are an infinite number of ways of achieving the same level of utility.

In practice, investment is rarely done this way. Investors typically decide first that they wish to allocate money to risky assets and then decide where to invest. Indeed, institutional advisors seem to advocate specifically that the strategy is decided first, then structuring and finally, only, selection (Urwin *et al.*, 2001).

Strategy

The reparameterised version of the investment process that is useful for discussing the hierarchical approach can be rewritten so as to emphasise the systematic and unsystematic components:

$$R_{fund} - r_f = k\left[\{\varphi + (1-\varphi)\beta_A\}\left(R_m - r_f\right) + (1-\varphi)(\alpha_A + \varepsilon_A)\right] \tag{9}$$

The expected utility is then

$$\begin{aligned} E[U(R_{fund} - r_f)] = \; & \left\{k\delta(E[R_m] - r_f) - \frac{\tau}{2}k^2\left(\delta\right)^2 Var[R_m]\right\} + \\ & + \\ & \left\{k(1-\varphi)\alpha_A - \frac{\tau}{2}k^2(1-\varphi)^2\sigma_{\varepsilon_A}^2\right\} \end{aligned} \;, \tag{10}$$

where $\delta = \{\varphi + (1-\varphi)\beta_A\}$. The choice of *k* clearly requires some assumptions to be made about the construction of portfolio A and/or the structure of the fund, i.e. φ.

The conventional approach is first simply to assume that the second term in braces is non-negative (and then ignore it for the purposes of selecting k) and, second, to constrain β_A to be unity. The first assumption effectively places a minimum level on portfolio A's appraisal ratio in order to ensure that the selection process later on will not reduce the utility, viz.:

$$\frac{\alpha_A}{\sigma^2_{\varepsilon_A}} \geq \frac{\tau}{2}(1-\varphi)k\,. \tag{11}$$

This minimum level is fairly intuitive: it increases with the proportion invested in A and with the total proportion invested in risky securities. It also increases with the investor's level of risk aversion. However, what it does quite clearly show is that an appraisal ratio that is merely non-negative is not necessarily adequate for all structures.

The second constraint about unit beta is effectively requiring that the active portfolio be 'style' neutral, i.e. it should have unit exposure to all the market risk factors. With these assumptions and constraints, the first order condition for maximising the expected utility is:

$$k = \frac{E[R_m] - r_f}{\sigma^2_m}\tau^{-1}, \tag{12}$$

i.e. the exposure to the non-matching assets is inversely proportional to the level of risk aversion, with a proportionality constant equal to the ratio of expected excess return divided by the variance of the market returns (an 'appraisal ratio' for the market as a whole). We denote the value of *k* determined in this way by *K*.

Structure

The next stage is to consider structure. There is less uniformity of practice in distinguishing structuring and selection. However, after choosing *k* to be *K*, the expected utility can then be maximised over φ, or equivalently (1 – φ).

Keeping the constraint that portfolio A should have unit beta, the expected utility after deciding strategy is given by:

$$E[U(R_{fund} - r_f)] = \left\{K(E[R_m] - r_f) - \frac{\tau}{2}K^2 Var[R_m]\right\} + \left\{K(1-\varphi)\alpha_A - \frac{\tau}{2}K^2(1-\varphi)^2\sigma^2_{\varepsilon_A}\right\}. \tag{13}$$

The first term in braces does not depend on (1 – φ) and so the optimisation can focus solely on the second term. The first order condition for maximising the expected utility is therefore:

$$(1-\varphi) = \frac{\alpha_A}{\sigma^2_{\varepsilon_A}}(K\tau)^{-1} = \frac{\alpha_A}{\sigma^2_{\varepsilon_A}}\frac{\sigma^2_m}{E[R_m] - r_f} \tag{14}$$

The structure therefore depends inevitably (and unsurprisingly) on the appraisal ratio of the active portfolio, A.

Methods for dealing with this unavoidable overlap diverge in practice. One option (Option 1) is to proceed to maximise the contribution to utility from the assets being actively managed and then use implied ratio to specify the proportion invested in active management via equation (5). The second option (Option 2) is to set the proportion invested in active management based on unmodelled considerations, such as a 'reasonable' ratio. Some investors may be 'forced' to hold significant proportions of their wealth in individual securities and that may provide a minimum bound for (1 – φ).

Selection

Substituting the first order condition for maximum expected structural utility back into the expression for expected utility, we get:

$$E[U(R_{fund} - r_f)] = const. + \tfrac{1}{2}\tau^{-1}\frac{\alpha_A^2}{\sigma_{\varepsilon_A}^2} \tag{15}$$

We discuss the two options suggested earlier for maximising the expected utility.

Option 1: finding the maximum skill level and then fixing the proportion invested in the active portfolio

The implication here is that for any level of risk aversion, the expected utility of the risky part of the fund is maximised by finding the largest value of $\alpha_A^2 / \sigma_{\varepsilon_A}^2$. This problem was first explored by Treynor & Black (1973) and they showed that in the unconstrained case, the proportion invested in each risky security, *i*, should be in proportion to their $\alpha_i / \sigma_{\varepsilon_i}^2$ ratios.

At first sight, the constraint that the beta of the active portfolio should be unity might affect this result. However, as indicated earlier, it is realistic to assume that the active portfolio is allowed to contain investment in the risk-free asset. This is because many investors access active management via pooled vehicles that are permitted to hold some cash. This provides an additional degree of freedom that makes the constraint redundant and the proportion invested in the risky securities will be in proportion to their appraisal ratios.

The hierarchical approach using this option can therefore lead to exactly the same investment decision as the unconstrained optimisation.

It is interesting to note that maximising utility is NOT achieved by maximising the appraisal ratio of the active portfolio, but by alpha multiplied by the appraisal ratio. Under the assumption that the market enforces non-negative alphas for all securities, the maximisation is equivalent to maximising the information ratio (alpha divided by the residual standard deviation). However, the proportion invested in the active portfolio depends on the appraisal ratio. This is a subtle difference, but can have important implications.

For example, suppose that instead of maximising utility, the investor maximises the active portfolio's appraisal ratio (and hence the proportion invested in the active portfolio), subject to unit beta, i.e., the investor selects $x_1, x_2, \ldots, x_n$ and η so as to maximise:

$$\frac{\alpha_A}{\sigma_{\varepsilon_A}^2} = \frac{\eta\alpha_p}{\eta^2\sigma_{\varepsilon_p}^2} = \frac{\sum x_i\alpha_i}{\eta\sum x_i^2\sigma_{\varepsilon_i}^2} \tag{16}$$

subject to

1. $\eta\sum x_i\beta_i = 1$ and
2. $\sum x_i = 1$.

The first order conditions then are that:

$$x_i = \frac{1}{2}\frac{\sigma_{\varepsilon_i}^{-2}}{\sigma_{\varepsilon_p}^{-2}}\left[\frac{\alpha_i}{\alpha_p} + \frac{\beta_i}{\beta_p}\right]. \tag{17}$$

As in the utility-maximising case, the proportion invested in an asset is linearly and positively related to its appraisal ratio. However, it is also noteworthy that the higher the systematic risk of the instruments, the higher the weighting. This is a consequence of the way in which the overall beta is adjusted to be unity, i.e. by levering the active portfolio with an asset that has zero alpha, zero beta and zero residual risk.

Option 2: fixing the proportion invested in the active portfolio and then selecting securities

Under this option (which is perhaps a stricter interpretation of the hierarchical approach), the proportion invested passively is determined by unmodelled considerations. We denote this proportion by Ψ. The expression for expected utility is then:

$$E[U(R_{fund} - r_f)] = const. + \left\{K(1-\Psi)\alpha_A - \frac{\tau}{2}K^2(1-\Psi)^2\sigma_A^2\right\}. \tag{18}$$

Maximum expected utility is then obtained by maximising

$$E[U(R_{fund} - r_f)] = const. + \left\{\alpha_A - \frac{\Lambda}{2}\sigma_A^2\right\}. \tag{19}$$

subject to

1. $\eta\sum x_i\beta_i = 1$
2. $\sum x_i = 1$

The first order conditions for solving the above are:

$$x_j = \frac{\alpha_j}{\sigma_{\varepsilon_j}^2}(\Lambda\eta)^{-1} + \phi\frac{\beta_j}{\sigma_{\varepsilon_j}^2}(\Lambda\eta)^{-1}. \tag{20}$$

where ϕ is a Lagrange multiplier, which can be obtained from

$$\phi = -(\alpha_A - \Lambda\sigma_{\varepsilon_A}^2).$$

As with option 1, the proportion invested in security i is linearly positively related to its appraisal ratio. However, the relationship with beta depends on the optimal appraisal ratio for the active portfolio. The multiplier ϕ will be positive if the following holds:

$$\frac{\alpha_A}{\sigma_{\varepsilon_A}^2} < \tau(1-\Psi)^2K^2. \tag{21}$$

We recall from equation (11) that active management will only add to utility if the appraisal ratio is greater than half of the proportion of total wealth invested in the active portfolio multiplied by the level of risk aversion.

If the proportion invested in the active portfolio is small then the multiplier will be almost certainly be negative and so the proportion invested in each security will have a negative correlation with their beta coefficients. On the other hand, if the proportion invested in the active portfolio is closer to one, then the multiplier will usually be positive.

Some numerical illustrations

This section contains some illustrative examples that demonstrate the above results and provide some quantification of the impact of the decision-framework for different investors. For this purpose, we assume a very simple investment market comprising ten risky securities with non-zero alpha, a passive market instrument that tracks the market perfectly and a risk-free rate.

	α	β	σ_ε	AR	$\beta/(\sigma_\varepsilon)^2$
Passive	0.0%	1	0%		
Security 1	2.0%	1	30%	0.22	11.11
Security 2	0.5%	1	30%	0.06	11.11
Security 3	1.0%	1.5	30%	0.11	16.67
Security 4	1.0%	1.25	30%	0.11	13.89
Security 5	1.0%	1	30%	0.11	11.11
Security 6	1.0%	0.75	30%	0.11	8.33
Security 7	1.0%	0.5	30%	0.11	5.56
Security 8	1.0%	0.9	20%	0.25	22.50
Security 9	1.0%	0.9	30%	0.11	10.00
Security 10	1.0%	0.9	40%	0.06	5.63
Cash in active portfolio	0.0%	0	0%		

The market is assumed to have an expected rate of return of 10% over the time period, with standard deviation of 20%. The risk-free rate of return is assumed to be 5%.

Unconstrained

We consider two investors, one with a level of risk aversion of 2 (high risk investor) and one with a level of risk aversion of 10 (low risk investor). In order to avoid any problems with over-parameterisation, we have fixed the proportion of the active portfolio invested in cash to be 5%. The optimal investment portfolios for these investors and their expected end of period utilities are:

	AR	$\beta/(\sigma_\varepsilon)^2$	Proportion (high risk)	Proportion (low risk)
Passive (φ)			2.6%	2.6%
Security 1	0.22	11.11	17.7%	17.7%
Security 2	0.06	11.11	4.4%	4.4%
Security 3	0.11	16.67	8.8%	8.8%
Security 4	0.11	13.89	8.8%	8.8%
Security 5	0.11	11.11	8.8%	8.8%
Security 6	0.11	8.33	8.8%	8.8%
Security 7	0.11	5.56	8.8%	8.8%
Security 8	0.25	22.50	19.9%	19.9%
Security 9	0.11	10.00	8.8%	8.8%
Security 10	0.06	5.63	5.0%	5.0%
Cash in active portfolio			5.0%	5.0%
k			67.9%	13.6%
Utility			1.925%	0.385%

Hierarchical: option 1

Under the hierarchical approach, the strategy is determined first using equation (12). The optimal investment decisions for both investors are then summarised as:

	AR	$\beta/(\sigma_\varepsilon)^2$	Proportion (high risk)	Proportion (low risk)
Passive (φ)			2.8%	2.8%
Security 1	0.22	11.11	17.7%	17.7%
Security 2	0.06	11.11	4.4%	4.4%
Security 3	0.11	16.67	8.8%	8.8%
Security 4	0.11	13.89	8.8%	8.8%
Security 5	0.11	11.11	8.8%	8.8%
Security 6	0.11	8.33	8.8%	8.8%
Security 7	0.11	5.56	8.8%	8.8%
Security 8	0.25	22.50	19.9%	19.9%
Security 9	0.11	10.00	8.8%	8.8%
Security 10	0.06	5.63	5.0%	5.0%
Cash in active portfolio			-3.5%	-3.5%
k			62.5%	12.5%
Utility			1.925%	0.385%

It is clear that although the structure of the fund is different, the maximum utility is unaffected by the approach and the allocation of wealth to the risky securities is also the same as in the unconstrained case. Of course, if the proportion of cash in the active portfolio cannot be changed, then this optimality will no longer hold.

Hierarchical: option 2

In this example, we have assumed that the structure is 50% passive and 50% active. The impact on the optimal portfolio is illustrated in the table below.

	AR	$\beta/(\sigma_\varepsilon)^2$	Proportion (high risk)	Proportion (low risk)
Passive (φ)			50.0%	50.0%
Security 1	0.22	11.11	24.5%	24.5%
Security 2	0.06	11.11	0.1%	0.1%
Security 3	0.11	16.67	4.2%	4.2%
Security 4	0.11	13.89	6.2%	6.2%
Security 5	0.11	11.11	8.2%	8.2%
Security 6	0.11	8.33	10.2%	10.2%
Security 7	0.11	5.56	12.2%	12.2%
Security 8	0.25	22.50	20.3%	20.3%
Security 9	0.11	10.00	9.0%	9.0%
Security 10	0.06	5.63	5.1%	5.1%
Cash in active portfolio			-9.3%	-9.3%
k			62.5%	12.5%
Utility			1.852%	0.370%

There are two notable effects: (1) the utility has dropped and (2) the allocation to risky securities is different and now takes into account the beta of the asset. In this case, the proportion allocated to each security drops as the beta increases. The decrease in utility is roughly equivalent to a decrease of 0.1% in market return (i.e. a drop from 10% to 9.9%) for an optimally allocated portfolio.

For an investor with greater risk tolerance (lower risk aversion), the proportions allocated to each risky security changes. For example, if the risk aversion parameter is dropped to 1.25 so that 100% of wealth is allocated away from the risk free asset and 100% of the investment is done on an active basis, then the allocation to the individual securities changes to:

	AR	$\beta/(\sigma_\varepsilon)^2$	Proportion
Security 1	0.22	11.11	17.5%
Security 2	0.06	11.11	4.6%
Security 3	0.11	16.67	9.0%
Security 4	0.11	13.89	8.9%
Security 5	0.11	11.11	8.9%
Security 6	0.11	8.33	8.8%
Security 7	0.11	5.56	8.7%
Security 8	0.25	22.50	19.9%
Security 9	0.11	10.00	8.8%
Security 10	0.06	5.63	5.0%

Concluding remarks

The analysis in this note suggests that a pragmatic, hierarchical process of deciding on investment structures need not affect optimality. One of the keys to ensuring this is that the structure of the fund (the proportion invested actively) should be decided only after the selection process has been specified.

If structure is fixed before selection then investors (a) will suffer a reduction in utility and (b) will allocate wealth across risky securities in a way that depends on their systematic risk. The management of companies can exploit this sub-optimality to the benefit of existing shareholders if they can manipulate their exposures to systematic risk without affecting their operational appraisal ratio, e.g. by changing the level of systematic risk in any defined benefit pension schemes that they support.

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