"Risk caused by the catastrophic downturns of the national economy"

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Summary

When a country experiences deep economic depression the losses in credit (surety) insurance may reach catastrophic dimensions for several years. During that time the number of claims can be extraordinary large and, what is more important, the proportion of excessive claims can be much higher than usual. This kind of phenomenon is difficult to capture with the standard methods of risk theory.

In this paper we study credit risk modelling in the economic cycle point of view. First we discuss the usefulness of economic forecasts to actuaries. Then we propose a model that utilises a modification of a well-known Markov process description of an economic business cycle. The states of the used Markov process represent economic expansion, recession and deep depression. The Markov model fits well into the accumulation process of claims in consecutive years. When a simple description of the states of the Markov process is found, this approach leads to parsimonious modelling. The proposed actuarial model is used for simulating purposes in order to study the effect of the economic cycle on the needed pure premium and initial risk reserve. The choice of parameters for the model is motivated by empirical observations.

"Une solution à l´évaluation de risque pour les périodes de catastrophes"

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Résumé

Lorsqu´un pays traverse une grave crise économique les pertes des assurances de crédit peuvent atteindre une ampleur catastrophique' pour plusieurs années. Le nombre des demandes d´indemnité peut être particulièrement haut pendant cette période et, ce qui importe le plus, la proportion des demandes excessives peut être nettement plus importante qu en temps normal. Le phénome'ne de ce genre est difficile à percevoir à travers des methodes standardisées de la théorie des risques.

Lors de cette étude, nous etuduerons les modèles sur les risques de crédit du point de vue des cycles économiques. D´abord nous discuterons l´utilité des pronostics économiques pour les actuaires. Ensuite nous mettions en avant un modèle qui modifie la description bien-connue du processus de Markov pour le cycle économique. Les situations économiques présentées par le processus de Markov sont expansion, récession et depression profonde. Le modèle de Markov est bien approprié pour l´accumulation des demandes en années successives. Après la de'couverte d´une description simple des situations du processus de Markov, l´approche conduit à un modéle avec peu de parametres. Le modèle d´assurance proposé est utilisé pour simuler les effets des cycles économiques sur les réserves pures de risques de primes et les re'serves initiales nécessaires. Le choix des parametres du modèle est basé sur des observations empiriques.

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1. Introduction

During the years 1991-1993 Finland suffered from a depression that in many ways was at least as severe as the Great Depression in the 1930s. Naturally that period at the beginning of 1990s was harmful to all sectors of the economy and society as a whole. In a few years unemployment rates rose from 3.5 percent to almost 20 percent. However, the injuries suffered in the insurance sector were only moderate, at least compared with the problems of the banking sector at the same time. An important exception was credit insurance related to the Finnish statutory earnings-related pension scheme of the private sector. The administration of the pension scheme is decentralised to numerous insurance companies**,** pension funds and foundations. The central body for the Finnish earnings-related pension scheme is the Central Pension Security Institute (CPSI).

Special credit insurance was administrated by the CPSI. The business was started in 1962 and continued successfully until Finland was hit by the deep depression in the 1990s. Then the losses took catastrophic dimensions and the credit insurance activity of the CPSI was closed. Claims paid by the CPSI are shown in Figure 1. In the same period the loss ratio started from a level of 2 per mil and reached a maximum of 28.5 per mil in 1992. Afterwards the CPSI's run-off portfolio was transferred to the new company named "Garantia". The CPSI is still responsible for any further deficit from earlier contracts. A detailed description of the case of the CPSI can be found in Romppainen (1996), (see also Figures A-D in Appendix). In order to promote the capital supply, the CPSI had a legal obligation to grant credit insurance to pension funds and foundations for which insurance was obligatory. Hence, it employed fairly liberal risk selection and tariffs. This probably had an influence on the magnitude of the losses. Hence, the data reported by Romppainen (1996) cannot be expected, as such, to be applicable in other environments. The risks would be smaller in conventional credit insurance, which operates solely on a commercial basis. However, Romppainen's report reveals certain important features of the credit risk that are presumably widely applicable:

- Credit risk is usually provoked by a recession in the national economy. An obvious consequence of a recession is that the claim frequency greatly increases.

- During a deep depression the claim size distribution is significantly changed. This can be explained as follows. At first the number of bankruptcies of small and median-sized companies increases. When a recession continues, the effects will also be felt in big companies and recession converts into a deep depression.

- A deep depression may last for several years. Then the accumulation of the total loss from several consecutive years greatly increases the credit risk.

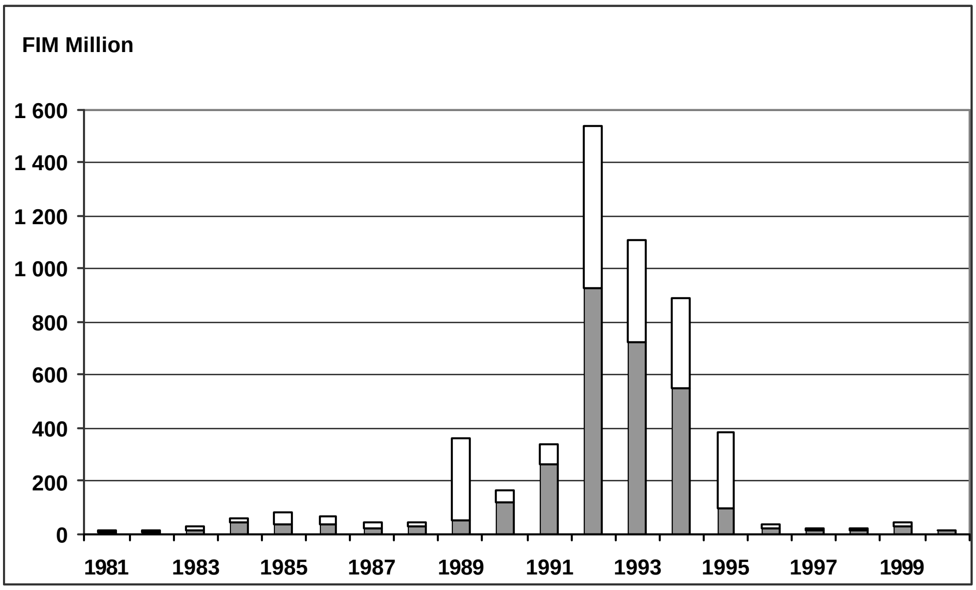

Figure 1. Claims paid from credit insurance by the Central Pension Security Institute of Finland between 1981 and 2000. The lower dark part of the bar describes the final loss by August 2001. (Inflation-adjusted series; Source: Romp painen Y., Garantia, Finland; 1 EURO $\approx$ 5,95 FIM;).

Pentikainen (1996) argues that the aforementioned credits risk cannot be well fitted into the conventional insurance models, outlined, for example, in Daykin, Pentikainen and Pesonen (1994). He proposes a special submodel that is linked with a standard risk theory model.

Cairns (2000) points out that uncertainty in actuarial modelling arises from three sources: uncertainty due to the stochastic nature of a given model, uncertainty in the values of the parameters in a given model, uncertainty in the model underlying what we are able to observe and determining the quantity of interest. In credit insurance the main problem seems to fall into the third category. We approach it by quantifying the risk caused by the movement of the national economy. This task is worked out by incorporating a business cycle description into the risk model.

2. National economic business cycle

A deep downturn in the national economy is the most severe source of risk for credit insurance. Hence, an early and reliably forecast for business cycle turning points, dates when the economy switches from expansion to recession and vice versa, would be very valuable in the risk assessment. However, the quality of published macro economic forecasts is not encouraging. For instance, two recent articles indicate that the value of the forecast made by major economic institutes is minor for actuaries. Fintzen and Stekler (1999) showed that in the US the recessions have generally not been predicted prior to their occurrence. For example, the 1974 and 1981 peaks were not recognized even as they occurred. While missing the actual turning points, the forecasts did indicate that the economy would be slowing down. Oller and Barot (2000) found that the accuracy of real annual output (GDP) growth forecasts made for 13 European countries was low. They found that the root mean squared error of one-year-ahead forecasts of GDP growth was 1.9 percent. Despite the size of the errors all forecasters were superior to the naive forecast: average growth, latest growth and random growth direction. Those investigations show that risk assessment cannot be built on economic forecasts substantially. Models for economic cycles can, however, be useful when modelling the phenomenon itself.

Business cycle modelling is a permanent source of interest in macroeconomic research. Numerous methods for modelling economic downturns, with varying degrees of technical sophistication, have been developed over the years. Naturally, one important problem is the selection of a suitable model. A model should give a simple and parsimonious description of the phenomenon in question, but at the same time the model should be able to describe the essential features of the economic cycles. A good general view on modelling is given in Boldin (1994).

We will utilize the Markov chain description for the US post war business cycle that was produced by a well-known Markov switching model introduced by Hamilton (1989). In the Hamilton method all the dating decisions or, more correctly, the probability that a particular time period is in recession, are based on the observed data. The method assumes that there are two distinct regimes (state) in quarterly GNP– one for expansion and one for recession – that are governed by a Markov chain. The stochastic nature of the GNP growth depends on the prevailing regime. Here we are interested in the 2-state Markov chain describing the business cycle, not the stochastic specification of GNP.

Hamilton (1989) assumed that the business cycle **bc**(t) (observed quarterly) is a dichotomous Markov chain, where states (regimes) 1 and 2 can be interpreted as follows:

$$\mathbf{bc}(t) = \begin{cases} 1, \text{ if the economy is in expansion at time t} \\ 2, \text{ if the economy is in recession at time t} \end{cases} \quad (1)$$

We will adopt this business cycle description as a starting point. The transition probabilities defining **bc**(t) are

$$p(ij) = \text{prob}\{\mathbf{bc}(t) = i \mid \mathbf{bc}(t\text{-}1) = j\}. \quad (2)$$

Hamilton's estimate for the matrix of transition probabilities is

$$\mathbf{P} = \begin{pmatrix} 0.905 & 0.245 \\ 0.095 & 0.755 \end{pmatrix} \quad (3)$$

The stationary distribution $\underline{v}$ of the Markov chain **bc**(t) satisfies the equation

$$\mathbf{P}\,\underline{v} = \underline{v}. \tag{4}$$

For **P** defined in (3) the stationary distribution is

$$\underline{v} = \begin{pmatrix} 0.72 \\ 0.28 \end{pmatrix}. \tag{5}$$

This means that in the long run the economy of the United States will be 72 percent of the time in a state of expansion and 28 percent of the time in recession. Similarly, the expected number of recessions in a century (= 400 Q) can be calculated from (3) and (5); the number is 27. Most of those recessions are short and mild.

3. A credit risk model using a Markov business cycle description

Credit risk is clearly dependent on the development of the national economy. According to Crook (1998) cycles in real net credit extended lead those in the GDP of the UK by an average of three quarters of a year. On the other hand, incurred credits lag GDP. To conclude, credit losses follow the national economic business cycle to some extent, but in a lagging phase. Hence, it is important to include a business cycle description in an actuarial model for credit insurance. For instance, Rantala (1984) analysed the country-wide bankruptcy rates particularly taking into account the great depression of the 1930s. He modelled the business cycle by means of a second order autoregressive model.

A mild and short downturn in the national economy increases the losses suffered by credit insurers just moderately. In credit risk assessment catastrophic downturns in the national economy are crucial. The Finnish experience from the beginning of the 1990s and the Asian crisis in the late 1990s are good examples of this. Those periods are difficult to model by an ordinary autoregressive time series model. As such the two-state Markov model of Hamilton is inadequate. Deep recessions constitute the greatest risk in credit insurance and an extra state is needed for it. To conclude, separate states are needed for expansion, ordinary recession and deep recession in a suitable “credit loss -cycle” model.

3.1 A business cycle model for credit insurance

We modify the model defined in (1-3) by taking two states (regimes) in the Markov chain to describe two types of economic downturns, one for ordinary recession and one for a deep depression:

$$\mathbf{bc}(t) = \begin{cases} 1, & \text{expansion} \\ 2, & \text{recession} \quad \text{(ordinary)} \\ 3, & \text{depression} \quad \text{(deep)} \end{cases} \tag{6}$$

Our purpose is to define the transition probabilities between the states such that only one extra parameter is needed for the extra state. In a way we change the error caused by the selection of an incorrect two-state model to the problem of the selection of the value of the extra parameter b for state 3. Furthermore, we assume that the modified model preserves Hamilton's business cycle and obeys Romppainen's (1996) observation that deep depression starts as an ordinary recession. Hence, the constraints for the 3-state model are:

a) Economic expansion appears as often as in the Hamilton (1989) model, which is defined by formulas (1-3). In addition, the lengths of the expansion periods are assumed to be equal to that model;

b) The system can move to a deep depression only through an ordinary recession.

The structure of the resulting model is described in Figure 2. The probability of transition from an ordinary recession to a deep depression cannot be obtained on the basis of the results of Hamilton (1989). For this reason we have to add an extra parameter b in our model. Parameter b must satisfy the inequality $0 \le b \le 1$. It determines the probability that an ordinary recession will change into a depression in the following way

$$p = \frac{0.755 \text{ x } b}{0.245 + 0.755 \text{ x } b} \tag{7}$$

Formula (7) implies that probability p must satisfy the inequality $0 \le p \le 0.755$ in order that b would satisfy the inequality $0 \le b \le 1$.

Modelling the credit risk based on the 3-state business cycle can be done as wished. In Figure 2, the states are characterized by the expected number of claims $E[\mathbf{k}]$. Additionally the claim size distribution $S(\mathbf{Z})$ has exceptionally heavy tail in deep depression (state 3) periods as observed in Romppainen (1996) and Pentikainen (1996).

The Markov matrix $\mathbf{P}$ = p(ij) (i,j = 1,2,3) of the transition probabilities which corresponds to the proposed model is

$$\mathbf{P} = \begin{pmatrix} 0.905 & 0.245 & 0.245 \\ 0.095 & 0.755x(1-b) & 0 \\ 0 & 0.755xb & 0.755 \end{pmatrix} \tag{8}$$

Note that elements p(22) and p(32) depend on probability p (that an ordinary recession turns into a deep depression) through parameter b. The only parameter of the business cycle model defined by (6) and (8) is the probability p (or as well b). Next we address the problem of the selection of the parameter by reviewing historical statistics.

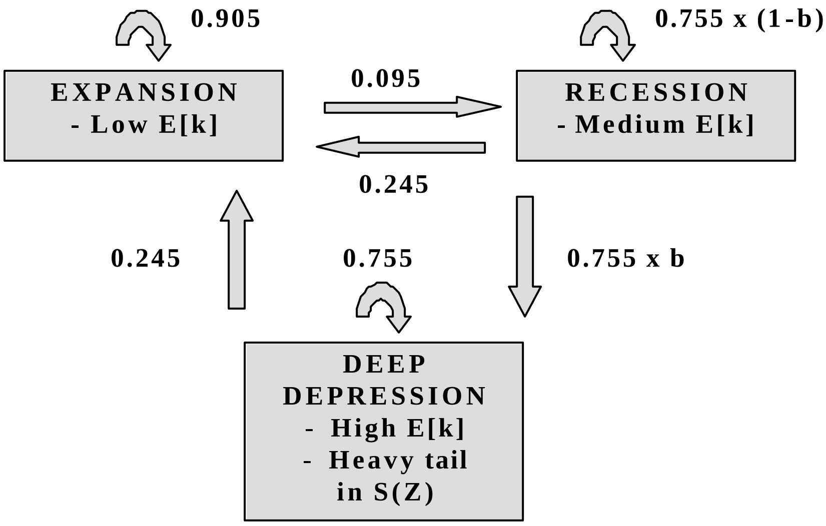

Figure 2. The transition probabilities of the 3-state business cycle model. Parameter b adjusts the probability that an ordinary recession turns into a deep depression.

A typical economic recession is associated with a 0-2 percent drop in GDP. Its duration is usually from 2 - 4 quarters. On the contrary, a deep depression may last for several years and a dramatic drop in GDP may result. During that period the losses of a credit insurer may reach much higher levels than during an ordinary (mild and short) recession. The proposed Markov model allows both long and short deep depression periods (time stayed in state 3).

Country	**Depression Years**	**Drop in GDP**
Finland	1991-93 (3)	12 %
UK	1980-81 (2)	4 %
Sweden	1991-93 (3)	5 %
Switzerland	1974-76 (3)	9 %

Table I Certain long-lasting deep depression periods in European countries. (Source: Oller and Barot (2000)).

Clearly the approximation of the density of the appearance of catastrophic depressions is an important issue. We judge it from past statistics. History knows several economic depressions, which can be called catastrophic. These include the Great Depression in the 1930s, World Wars I and II, and the oil crisis in the 1970s. In recent years one of the events causing excessive credit losses was the Asian crisis in the 1990s. Table I indicates some examples of deep depressions in a number of countries. These examples show that an economic depression may last for several years. In such cases losses from credit insurance can take catastrophic dimensions. It is difficult to make accurate estimates of the occurrences of deep depressions among all recessions on the basis of previous observations. However, the above considerations indicate that the proportion of deep depression regimes has been historically between 3 and 15 percent of the years. Table II shows the values of parameter p that are related to that range. The last column is the stationary distribution ($\mathbf{P}\underline{v} = \underline{v}$) of the Markov chain **bc**(t). Probability parameter p = 0.1 is related to a process that is 3 percent of the time in a deep depression. Respectively, if p = 0.5 the process is 14 percent of the time in a deep depression.

In Table II for each choice of probability p the first component of stationary distribution $\underline{v}$ is 0.72 and the sum of the second and third components is 0.28. This is compatible with the stationary distribution (5) of the Hamilton model as the constraint a) for the 3-state Markov model calls for.

p	$\underline{\mathbf{v}}$
0.1	(0.72 0.25 0.03)
0.2	(0.72 0.22 0.06)
0.25	(0.72 0.21 0.07)
0.33	(0.72 0.19 0.09)
0.5	(0.72 0.14 0.14)

Table II In the first column there is the selection for the probability of transition from a recession to a deep depression. The respective stationary distributions are presented in the last column.

The expectation for the length of (ordinary) recession periods is dependent on probability parameter p. The conditional expectations with different values for p are given in Table III.

P	0.1	0.2	0.25	0.33	0.5
Time	3.7	3.3	3.0	2.7	2.0

Table III The conditional expectations for the length of (ordinary) recession given the probability of transition from a recession to a deep depression.

The expectation for the length of (deep) depression periods is 4.08 quarters (It is independent of p). A long lasting depression is the greatest risk for a credit insurer. Hence, it is not enough to know the expectation. Figure 3 presents the inverse cumulative function for the length of depression. There is a 25 percent probability that depression lasts over one year (the expected value). Furthermore, the probability that depression lasts over two (three) years is 8 (3) percent.

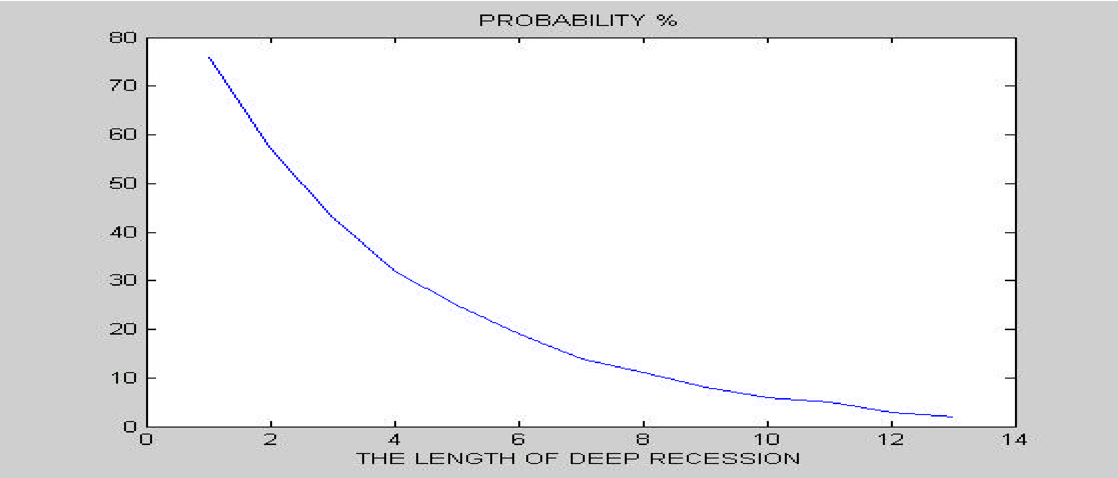

Figure 3. The probability that the length of a deep depression is at least a given number (in x-axis) of quarters.

3.2 The characterization of states

Until now we have concentrated on the business cycle dynamics, but for the sake of the risk analysis it is essential to obtain a suitable description of the claim process in each state. The aforementioned 3-state business cycle model allows us to use a standard risk theoretic submodel in each state. We just let both the number of claims **k** and the claim size $\mathbf{Z}_i$ of *i*th claim occurring during prevailing quarter t to be dependent on **bc**(t). Then the aggregate claim amount in quarter t can be expressed as

$$\mathbf{X}(\,t\;|\,\mathbf{bc}(t) = s) \;=\; \sum_{\boldsymbol{i}=1}^{\boldsymbol{k}(\boldsymbol{s})} \mathbf{Z}_i(s) \tag{9}$$

Now we may assume that in each state s the aggregate claim amount $\mathbf{X}(t\,|\,\mathbf{bc}(t) = s)$ has its own (standard) distribution. For example we may assume that in each state s, the aggregate claim follows a compound (mixed) Poisson process. In that case the roles of the mixing variable and the other characteristics of the states should be considered carefully.

The observations made in Romppainen (1996) (see also Appendix: Figures A-D) result in a model, whose structure is illustrated in Figure 2. A crucial feature of the credit risk is that during a deep depression the claim size distribution is changed. The changed properties of the compound (mixed) Poisson distribution during a deep depression (in state 3) can be summarized:

- The Poisson parameter n(3) (the expectation of the number of claims) is considerably higher;

- The claim size distribution $S(\mathbf{Z}_i(3))$ of individual claims has exceptionally heavy tail.

4. A parsimonious simulation model

The most important characteristic of the state in the proposed model is the conditional expectation of the aggregate claim variable. A parsimonious and simple model for studying the effects of the business cycle can be obtained when the states are characterized by expectations. Then the only source of randomness in the model is the business cycle. The needed conditional expectations are

$$E[\mathbf{X(t)} \mid \mathbf{bc}(t) = 1] = 1$$
$$E[\mathbf{X(t)} \mid \mathbf{bc}(t) = c] = u(c), \quad c = 2, 3. \qquad (10)$$

Since merely the relative sizes of the expectations are important, just two parameters u(1) and u(2) are needed in addition to parameter p, that is, the probability of a (ordinary) recession turning into a deep depression. The underlying assumption of this straightforward approach is that the variation between different states (regimes in the economy) is much more important than the variation within a particular state. Of course, this model is a crude approximation of reality. Hence, it constitutes only the first step in the modelling process that should lead to a more realistic description of the risk.

We used the model defined by (6-8) and (10) in specifying the size of the initial risk reserve U_0 with respect to the pure risk premium P. In this connection, the pure risk premium is the unconditional expectation

$$P = E[\mathbf{X}(t)] \qquad (11)$$

and the total claim amount during time period [1,T] is the random sum

$$\sum_{t=1}^{T} E[\mathbf{X(t)} \mid \mathbf{bc}(t) = c] = \sum_{t=1}^{T} u(\mathbf{bc}(t)). \qquad (12)$$

In this framework, our task was to determine by use of a simulation study such an initial reserve U_0 that

$$\text{Prob}\{ U_0 + 100 \text{ x } P - (1/4) \sum_{t=1}^{400} u(\mathbf{bc}(t)) < 0 \} = 0.01 \qquad (13)$$

holds for given values of parameters u(c) and p (probability of a recession to change into a deep depression). We studied the same “suitable” values of parameter p as in Chapter 3. The risk premiums were computed from Table II. The simulation period, one hundred years (400 quarters), was repeated 5000 times and the model was implemented in the Matlab© program. The simulations are illustrated in Figure 4.

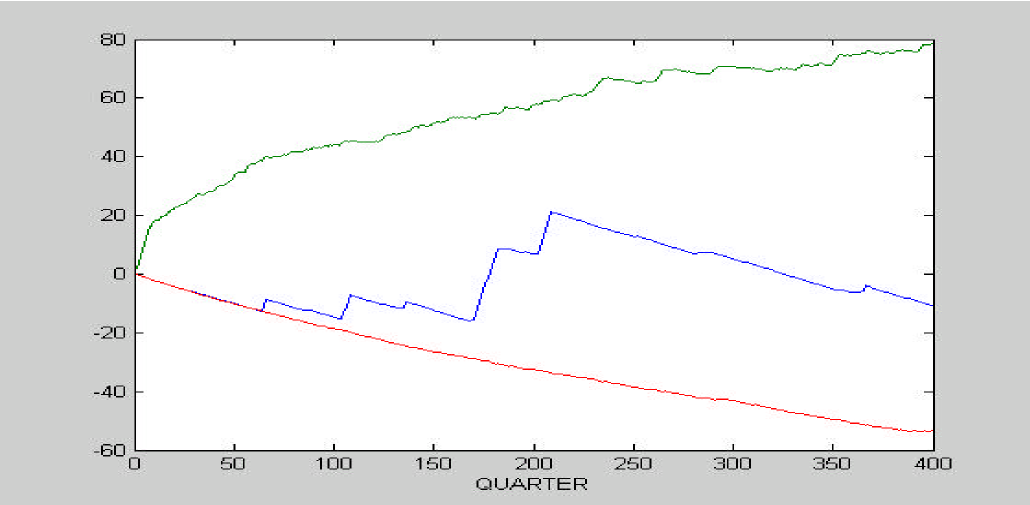

Figure 4. The middle curve is a simulated sample path of the difference between pure premiums and losses. Other curves are simulated 98 percent confidence intervals for the difference. The parameter values were p=0.25, u(2)=2 and u(3) =10.

Table IV reports the effect of the value of probability parameter p when the expectations are u(2) = 2 and u(3) = 10. U_0 is the risk reserve required to overcome 99 percent of the periods (formula (13)). A higher probability for a deep depression increases substantially the needed risk premium and risk reserve.

P	0.1	0.2	0.25	0.33	0.5
P	1.5	1.8	1.8	2.0	2.4
Uo	58	72	78	90	95

Table IV Simulation results for different selections of p with fixed expectations u(2) = 2 and u(3) = 10.

Table V shows that estimates omitting the consequences of deep depression periods may underestimate the risk premium badly and result in an unsafe risk reserve.

u(2)	2	1	3	6
u(3)	2	10	6	6
P	1.3	1.6	1.8	2.4
Uo	12	78	45	62

Table V Simulation results for different selections of u(2) and u(3) with fixed probability p=0.25.

5. Conclusions

Several observations can be made on the basis of this study.

It is almost self-evident that the claims from credit insurance are dependent on the economic cycle. For this reason, credit insurers should follow economic forecasts, however, keeping in mind that the accuracy of these forecasts has proved to be low. The economic forecasts also have a rather short time horizon. On the other hand, in modelling credit insurance it is useful to utilize models for economic cycles in order to make the descriptions of the timing and the length of the recessions more rational.

In the modelling process one has to take into account the needs and the results of risk theory.

In a modelling process the model error is often the central problem. In credit insurance one has especially to take into account the accumulation of several bad years with high losses. This means that there are three different states in the business cycle. They are the states of economic expansion, recession and deep depression. In the case of credit insurance, the first two states, i.e. the regimes of expansion and recession, differ from each other mainly through different number of losses. In a deep depression state the distribution of the losses will also have thicker tail, as a result of which large claims will have considerably more weight than in an ordinary recession. One example of a deep depression is offered by the economic depression in Finland at the beginning of the 1990's.

In any statistical modelling process parsimony is an important target. Here a well-known business cycle model (with its estimated parameters) was adopted as a starting point. Then one parameter was needed to obtain a suitable description of "the credit loss cycle." Finally, two additional parameters were needed for a simplistic, but helpful simulation model.

The simulation results can be summarized as follows. First, if the effects of economic downturns are not considered properly, there is a danger that the premiums of the credit insurance will be set at a too low level. Second, in order to get through a long-lasting and deep depression period a credit insurer should have a fairly great risk reserve.

The modelling system applied in this study could be applied also to other types of insurance that depend on the state of the economic cycle. The proposed model probably has applications also in the banking sector.

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APENDIX

Certain characteristics of the size of claims paid from credit insurance
by the Central Pension Security Institute (Finland) in the years 1981-2000.
Solid line denotes the total amount of claims paid and dotted line the final loss after recoveries from reinsurers and realization of collaterals by August 2001.
(Inflation-adjusted series; Source: Romppainen Y., Garantia, Finland).

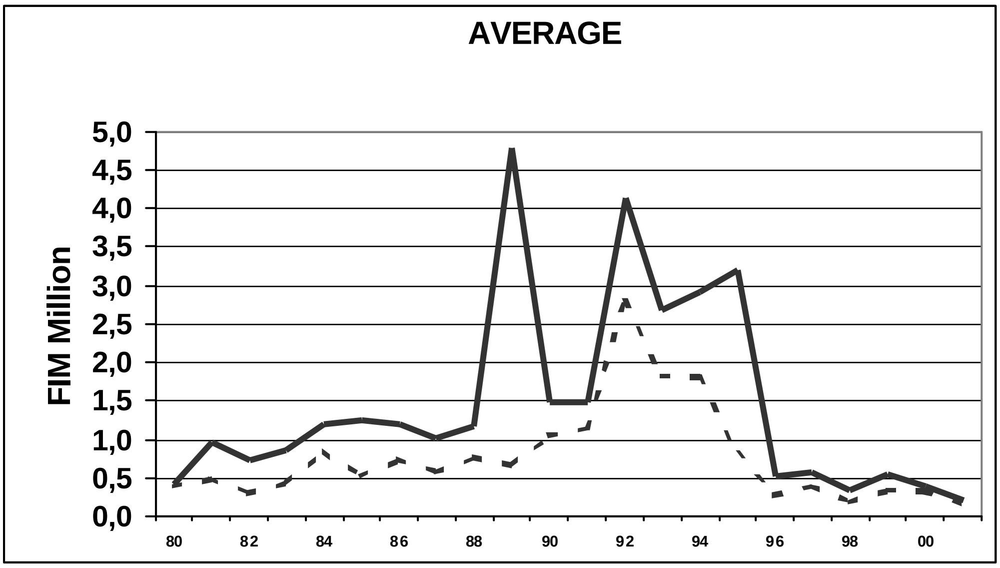

Figure A. Average claim size (total and final).

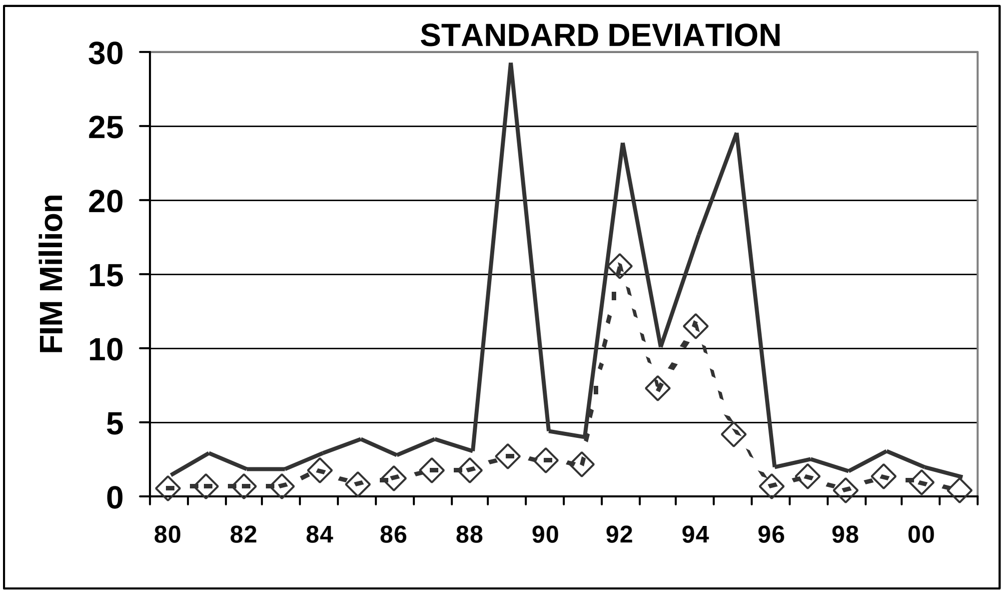

Figure B. Standard deviation of claim size (total and final).

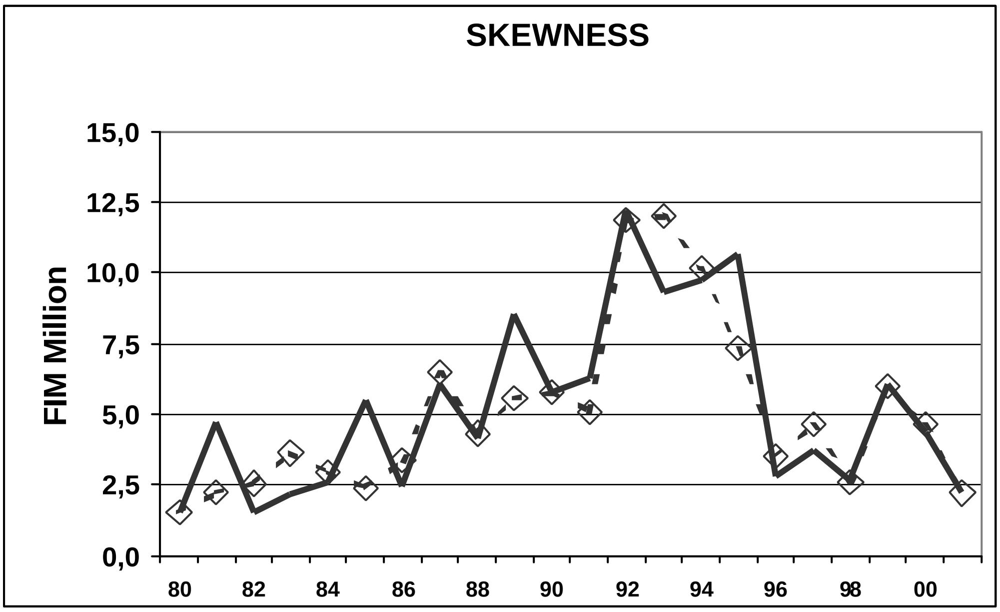

Figure C. Skewness of claim size (total and final).

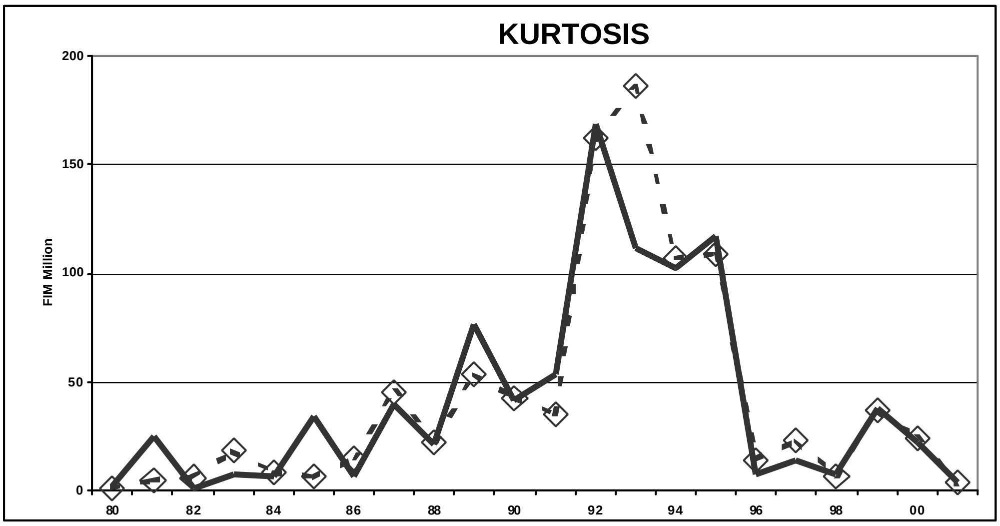

Figure D. Kurtosis of claim size (total and final).