# Number of IBNR Claims and Multivariate Compound Poisson Distribution

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# Plan of Paper

- 1- Introduction and notation
- 2- The pgf of the claims number
- 3- A Poisson model
- 4- A negative binomial model
- 5- References

# 1- Introduction

Claim incurred but not reported: IBNR

Jewell's model: homogeneous Poisson Process for number of claims incurred.

Generalizations:

1- delay probabilities varying over years

2- non-homogeneous Poisson process

3- $N \sim$ Compound Poisson distribution

# Assumptions of model

Discrete time period model with

Exposure period $\{1,2,\ldots,T\}$

Observation period $\{1,2,\ldots, k\}$, $k>T$

Reporting independent of incurral

Maximum possible value m for lag

# Notation

$N_i$ : # of claims incurred in accident period i

$R_{ij}$ : # of claims incurred in accident period i, reported j periods later (j=0,1,…, m)

$U_i$ : # of IBNR claims at end of obs. period

$R_i$: # of claims reported by end of obs. period

# 2- The pgf of the claims number

Pgf of $N_i$: $=\exp\{\lambda[P_i(z)-1]\}$

Prop. 1: Joint pgf of r.v. $R_{ik}$, $k=0,1,\ldots m-1$

Marginal pgf of $R_{ij}$

Expressions for mean, variance, covariance

Specific distributions for $N_i$: Poisson, NB

# Multi-period model

Assume $N_1, \ldots, N_T$ independent with compound Poisson distribution

Derive pgf of total # of claims in exposure period (Compound Poisson distribution)

# 3- A Poisson model

Expression for MLE's of parameters

Identifiability problem with:

- non-parametric distribution for reporting lag W
- certain continuous distributions for W

Use modified discrete distribution for reporting lag

# 4- A negative binomial model

- $N_i$ : Negative binomial (s,1-p)
- Joint distribution of ($R_{io}$,…, $R_{i,k-i}$):

Negative multinomial

Results: Marginal distribution of $R_{ij}$ ~ Negative binomial

Components $R_{ij}$ not independent of each other, as with Poisson assumption

# 4- A Negative binomial model

- Marginal distribution of $U_i$: NB
- Covariance between $U_i$ and $R_{ij}$ >0
- Total number of IBNR claims $U_1+\ldots+U_T$=
  Sum of ind. NB(s, -- )
  Representable as a C.P. distribution
- Conditional distribution of $U_i$ given
  $(R_{i0},\ldots,R_{i,\,k-i})$ also Negative binomial

# 5- References


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