IBNR-Methode auf Basis der Prämie

Svetlana Dudkina
Estland

Zusammenfassung

Das Problem der Reservierung der IBNR-Schäden ist von einer wichtigen Bedeutung, wenn es um eine Sachversicherung geht. Mehrere bekannte Methoden basieren auf dem Abwicklungsdreiecke. Nachteile dieser Methoden sind:

- funktioniert nicht beim Beginn einer Geschäft;
- langsame Reaktion auf Veränderungen in Versicherungsbedingungen;
- langsame Reaktion auf Veränderungen in der Grösse des Portfolio;
- es ist nicht möglich die Deckungsreserve einer einzelnen Police und eines Teils des Portfolio zu kalkulieren.

Das Ziel der vorliegenden Unterlage ist die Wichtigkeit der Einschätzung einer Gefahrenprämie zu zeigen (entsprechend den Limiten und Selbstbehalten von der Police) und die IBNR-Methode auf Basis der Prämie zu empfehlen. Die wichtigste Zufallsvariable dieser Methode ist die Nachmeldefrist, die getrennt für jeden Geschäftszweig eingeschätzt werden soll. Die Methode ist mit einem Beispiel illustriert.

"Policy-based IBNR calculation method"

Svetlana Dudkina
Estonia

Summary

The problem of reserving incurred but not reported claims is of great importance in non-life insurance. Several of well-known methods are based on the run-off triangles [1]. The drawbacks of those methods are:

not working when starting the business;
slow reaction on changes in the insurance conditions;
slow reaction on changes of the size of portfolio;
not possible to calculate the reserve for a single policy and segment of the portfolio.

The idea of this paper is to show the significance of rating risk premiums (according to limits and deductibles of the policy) and describe a premium-based IBNR calculation method. The main random variable of this method is the reporting period, which should be estimated separately for every line of business.

"Policy-based IBNR calculation method"

Svetlana Dudkina
Estonia

The problem

The goal of the insurance system is to build a foundation towards compensation for losses resulting from certain perils under specified conditions. It is very important to have this foundation on an appropriate level. There are two factors influencing the size of that foundation:

1) premiums rating;

2) size of technical reserves (in immediate connection with building up this foundation).

This foundation can be divided into two parts: liabilities from claims already incurred and claims that will occur in future regarding the time point under consideration. The first part of this foundation can be estimated as unexpired risk reserve (more general comparing to unearned premium reserve). The second part of this foundation is built up from earned premiums decreased by claims paid and should cover both reported but not paid yet claims (case reserve) and claims incurred but not reported (IBNR). As the case reserve is usually estimated case by case for every claim separately, there are no mathematical problems when estimating the size of this reserve. We will pay our attention on how to estimate a size of liabilities from claims already incurred but not reported, i.e. a size of IBNR .

We need a method that could be flexible towards three points:

(i) changes in portfolio like taking new (refusing some) recoverable risks in a cense of changing insurance conditions;

(ii) different deductibles applied in different contracts in the same business line;

(iii) character of business in a cense of changing perspectives of business (changes in sale sizes during the year (seasonal) or years (global trend)).

One of the possible solutions to attain all the goals defined previously is to calculate IBNR for every contract separately (using maximum of information we need like insurance period risk premium) and aggregate it over the business line or the portfolio.

Reporting period

To build up a model we will introduce a random variable, which describes speed of claims reporting. Reporting period is a time interval between the time-moment, when insurance case was incurred and the time-moment when it was reported to the company. Usually length of reporting period is measured in days. It is clear from construction that it is a non-negative random variable.

Assumptions

The first and most important assumption is that our risk premium is supposed to be on adequate level. It means that premium is calculated for every policy using risk analysis and deductible applying to a given risk. To simplify our model we will also assume that:

1) Premium is calculated using pure risk premium principle. Namely

$$P = E[X]E[N],$$

where P - premium for the contract under consideration;
$E[X]$ - average claim size for the contract under consideration;
$E[N]$ - average number of claims for the contract under consideration.

2) Claims process is a Poisson process. It means that the number of claims in an interval of length t is Poisson distributed with mean λt, if the process is with rate $\lambda > 0$ (see [2], for example).

3) Distribution of claim size has not seasonal factors.

In these assumptions we can estimate an average claim size for every optional period.

Model

Our model for IBNR estimation includes such variables as risk premium and insurance period and parameter(s) (different in different business lines) - reporting period.

Let us suppose that random variable L, which describes a length of reporting period is with distribution function F. As we mentioned before, L is non-negative and we have $F(0+) = 0$.

Reserves will be estimated not more often than daily, so values as $F(1), F(2), \ldots$ will be of our interest.

To explain our model let us look at one theoretical example. Assume that we are going to calculate IBNR for an insurance contract with the insurance period $[\tau_1, \tau_2]$ and the risk premium P. Suppose that all the assumptions we have done above hold. Now if date of statement is $\tau_1 + 1$, we shall estimate a size of claim that incurred but has not been reported in the following this way:

$$\frac{P}{\tau_2 - \tau_1 + 1} \cdot (1 - F(1)),$$

where $\frac{P}{\tau_2 - \tau_1 + 1}$ is an average claim per day and multiplier $(1 - F(1))$ is a probability of not reporting during the first day after the insurance case has incurred.

Now if the date of statement is $\tau_1 + 2$, we should consider two days when the contract has been in force. Estimated IBNR for the first day of contract is now $\frac{P}{\tau_2 - \tau_1 + 1} \cdot (1 - F(2))$, and for the second $\frac{P}{\tau_2 - \tau_1 + 1} \cdot (1 - F(1))$. So the total IBNR for the contract under consideration is now

$$\frac{P}{\tau_2 - \tau_1 + 1} \cdot ((1 - F(1)) + (1 - F(2))).$$

Proceeding our calculations we will get a general formula:

$$IBNR_{Exp}(t) = \begin{cases} 0, & t < \tau_1 \\ \frac{P}{\tau_2 - \tau_1 + 1} \cdot \sum_{i=1}^{t-\tau_1+1} (1 - F(i)), & \tau_1 \le t \le \tau_2 \\ \frac{P}{\tau_2 - \tau_1 + 1} \cdot \sum_{i=t-\tau_2}^{t-\tau_1+1} (1 - F(i)), & t > \tau_2 \end{cases} \qquad (1)$$

where

P	- risk premium of the given contract;
τ_1	- first day of insurance of the given contract;
τ_2	- last day of insurance of the given contract;
t	- date of statement;
F	- distribution function of the reporting period;
IBNR(t)	- Incurred But Not Reported claims' reserve as at *t*.

In fact we have proven the following result:

PROPOSITION. *If claims process is a Poisson process, claim size distribution has no seasonal factor and premium calculation principle is pure risk premium principle then incurred but not reported claims reserve (IBNR) can be estimated using the model (1).*

There are two distributions which are of interest in practice when describing a character of the reporting period: exponential and lognormal.

Corollary 1. *If a random variable* L*, describing the reporting period is exponentially distributed (*$F_L(l) = 1 - \exp(-\eta l),\ \eta \geq 0$*) with parameter* $\eta > 0$*, then in assumptions of our proposition we will get the following model for reserve of incurred but not reported claims:*

$$IBNR_{Exp}(t;\eta) = \begin{cases} 0, & t < \tau_1 \\ \dfrac{P}{\tau_2 - \tau_1 + 1} \cdot \sum\limits_{i=1}^{t-\tau_1+1} \exp(-\eta i), & \tau_1 \leq t \leq \tau_2 \\ \dfrac{P}{\tau_2 - \tau_1 + 1} \cdot \sum\limits_{i=t-\tau_2}^{t-\tau_1+1} \exp(-\eta i), & t > \tau_2 \end{cases}.$$

Corollary 2. *If a random variable* L*, describing the reporting period is lognormally distributed (*$F_L(l) = \Phi\left(\dfrac{\ln(l) - \mu}{\sigma}\right),\ l > 0$*) with parameters* μ *and* σ*, then in assumptions of our proposition we will get the following model for reserve of incurred but not reported claims:*

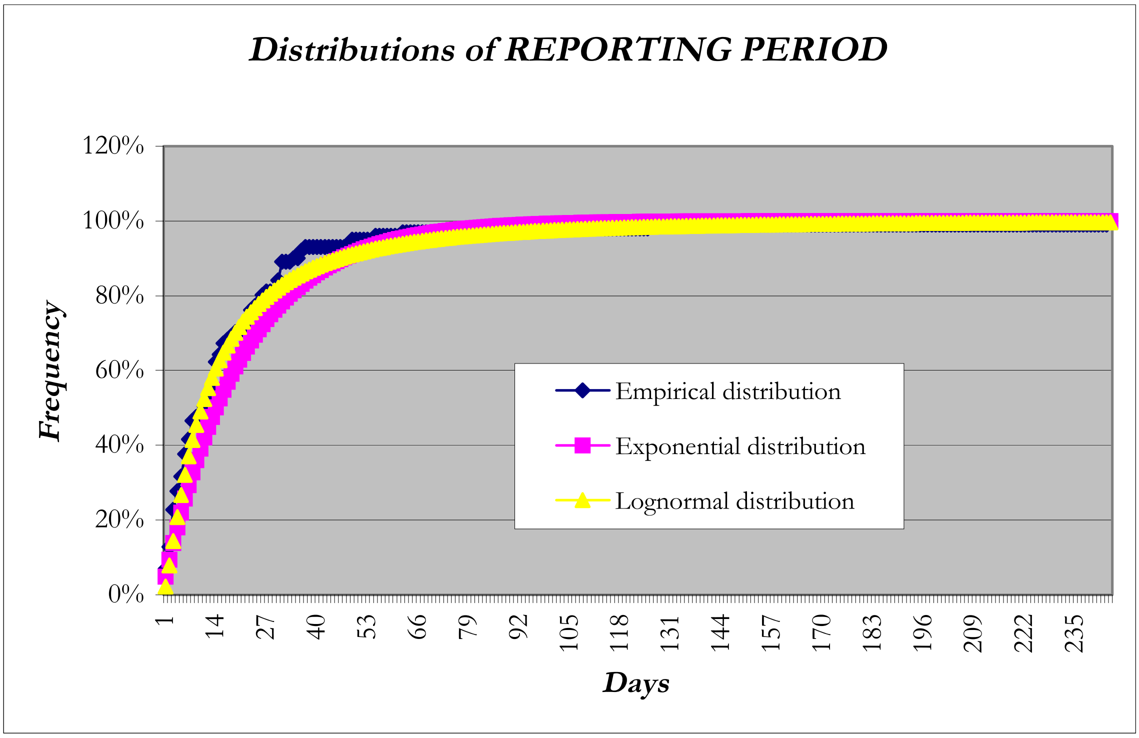
$$IBNR_{LogNorm}(t;\mu,\sigma) = \begin{cases} 0, & t < \tau_1 \\ \dfrac{P}{\tau_2 - \tau_1 + 1} \cdot \sum\limits_{i=1}^{t-\tau_1+1} \left(1 - \Phi\left(\dfrac{\ln(i) - \mu}{\sigma}\right)\right), & \tau_1 \leq t \leq \tau_2 \\ \dfrac{P}{\tau_2 - \tau_1 + 1} \cdot \sum\limits_{i=t-\tau_2}^{t-\tau_1+1} \left(1 - \Phi\left(\dfrac{\ln(i) - \mu}{\sigma}\right)\right), & t > \tau_2 \end{cases}.$$

where Φ is the distribution function of the standard normal distribution $N(0, 1)$.

Example

We will calculate IBNR using exponential and lognormal distributions of reporting period to get a picture of changing a size of reserve. Let our contract be with insurance period 01.01.01 – 31.12.01 and risk premium of 100 monetary units.

We have approximated the distribution of reporting period using exponential distribution with parameter $\eta = 0{,}0496$ and lognormal distribution with parameters $\mu = 2{,}3309$ and $\sigma = \sqrt{1{,}3445}$.



The simplified model of reported claims is the following:

Claims Reported, theoretical = *Premiums Written (Pure Risk)*
- *Unearned Premiums reserve*
- *IBNR*

The next table gives us a numerical example, where IBNR for the contract under consideration was estimated in two ways:

1) variable L was assumed to be exponentially distributed with parameter $\eta = 0{,}0496$ ($IBNR_{Exp}$);

2) variable L was assumed to be lognormally distributed with parameters $\mu = 2{,}3309$ and $\sigma = \sqrt{1{,}3445}$ ($IBNR_{LogNorm}$).

Accounting Period	*[01.01.01; 01.04.01)*	*[01.01.01; 01.07.01)*	*[01.01.01; 01.10.01)*	*[01.01.01; 01.01.02)*	*[01.01.01; 01.04.02)*	*[01.01.01; 01.07.02)*	*[01.01.01; 01.10.02)*	*[01.01.01; 01.01.03)*
Premiums Written (Pure Risk)	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00
Unearned Premiums Reserve	75,27	50,27	25,00	0,00	0,00	0,00	0,00	0,00
$IBNR_{Exp}$	5,33	5,38	5,38	5,38	0,06	0,00	0,00	0,00
$IBNR_{LogNorm}$	5,42	6,15	6,42	6,56	1,23	0,54	0,29	0,18
Claims Reported, theoretical (if $IBNR_{Exp}$*)*	19,40	44,34	69,62	94,62	99,94	100,00	100,00	100,00
Claims Reported, theoretical (if $IBNR_{LogNorm}$*)*	19,31	43,58	68,58	93,44	98,77	99,46	99,71	99,82

As we can see from the graph and the table, lognormal model gives us more conservative estimation for incurred but not reported claims.

References

1. Straub, E. *Non-Life Insurance Mathematics*, Springer-Verlag, Berlin, 1988
2. Stuart A. Klugman, Harry H. Panjer, Gordon E. Willmot *Loss Models From Data to Decisions*, Wiley, 1998.