Reinsurance Bermudan Style

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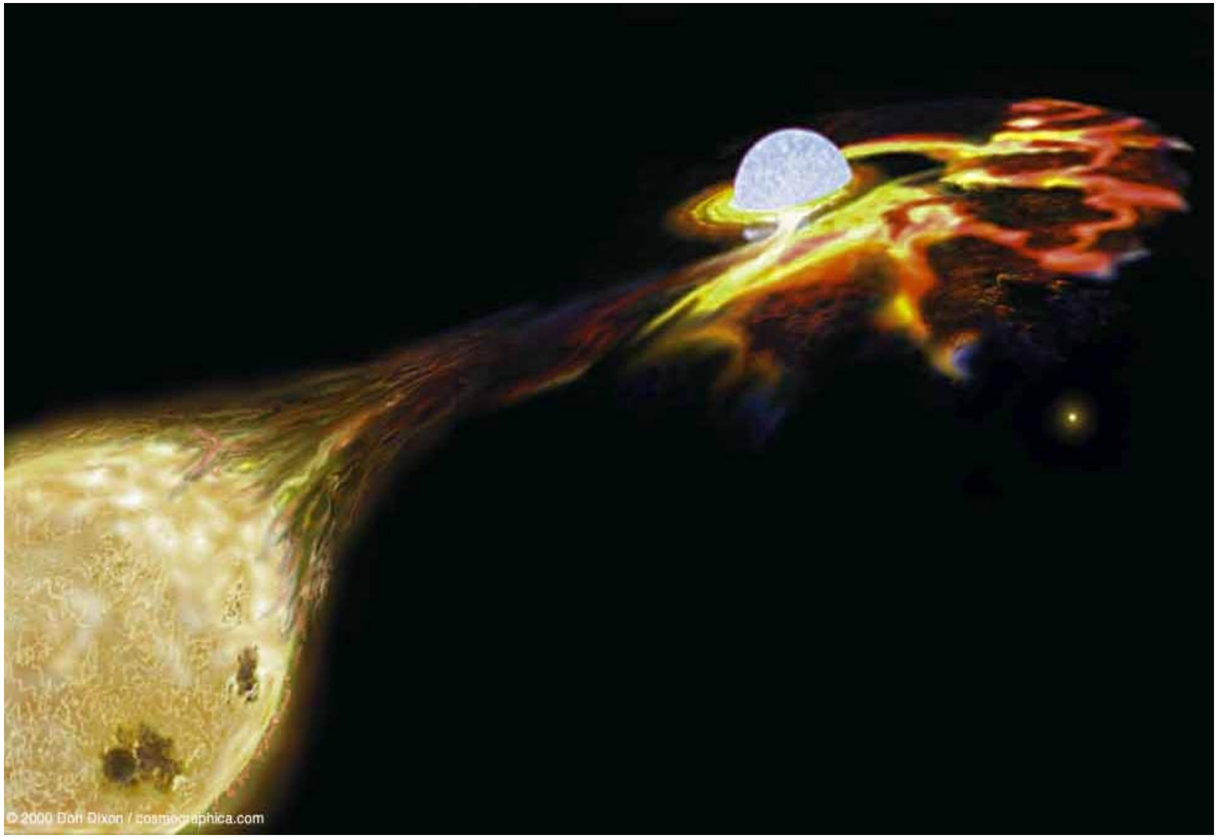
Introduction

Motivation: Catastrophes can be short ...

Introduction

... or long in duration



Introduction

There is a need for a reinsurance cover which is of:

• variable duration of coverage,

• variable date of coverage,

• intermediate between short term catastrophe (event) cover and a full 1-year cover.

Introduction

Bermudan Option Type Covers

Gives the right to sell (or buy) shares at specific times $t_1,\ldots, t_n <= T$ before the expiry date T for a fixed price.

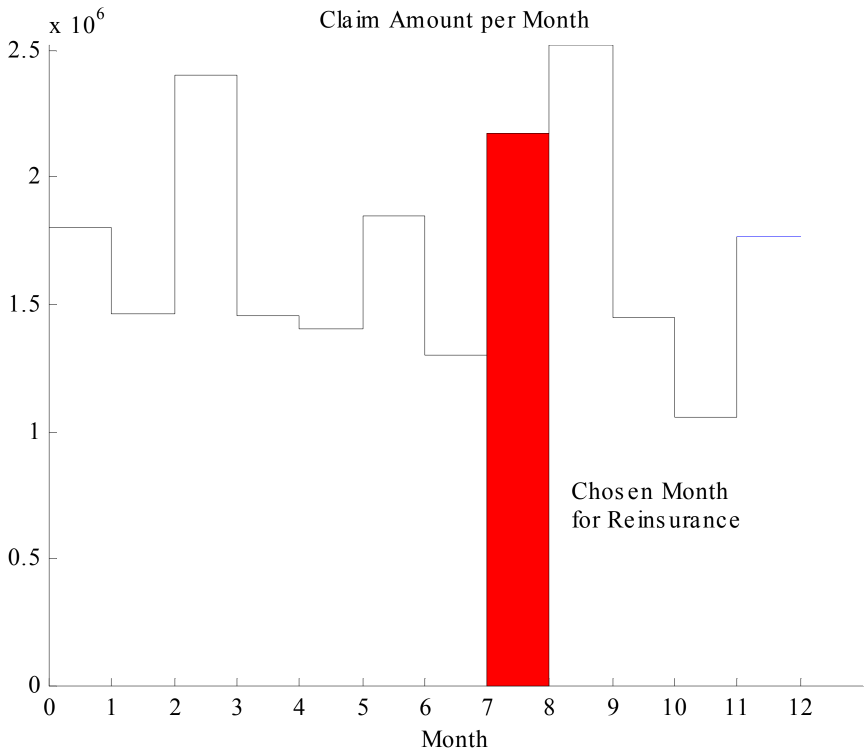
Bermudan Reinsurance Covers

Gives the right to invoke the cover for any time interval $[t_i, t_{i+1})$ before the expiry date T.

Introduction

Idea: Buyer can choose a duration *d* and a reinsurance coverage (Quota Share, Stop Loss, XL,...).

Then during the year he can choose at time *t* to invoke the cover for interval *[t-d,t)*.



Introduction

If duration of coverage *d* is short, the cover acts similar to a Catastrophe cover.

If duration of coverage *d* is long (e.g. $d>=1$ year), the cover is similar to a normal one - year reinsurance.

In contrast to a Catastrophe cover, the cover always pays since at $t = 31$ December one can always choose to exercise

Introduction

Why Harem Cover?

A Sultan can choose each year one lady from *n* girls, shown to him one after another. Once rejected, the Sultan can't choose the girl anymore.

What is the strategy of the Sultan to obtain with maximal probability the most beautiful lady?

Answer: Reject the first (n / e) girls and then take the one more beautiful than all before (or the last). ($e = 2.71828$)

Pricing

What is the optimal strategy of the buyer of the Harem cover?

Optimality: Maximize expected payout of the Harem cover.

Idea: Determine optimal stopping time (time of exercise).

Pricing

Simplify:

Assume $t = k \cdot d$, $k = 1,...,n$, time of exercise at end of each week or month.

If time of exercise $t = k \cdot d$, then reinsured interval is $[(k-1) \cdot d, k \cdot d)$.

Let D_k , $k = 1,...,n$ be independent r.v., reinsured claim amount during time $[(k-1) \cdot d, k \cdot d)$ and $D_k \sim F_{D_k}$.

Time of exercise $T^* = K \cdot d$. Choose T^* such that

$$T^* = \text{argmax}(\mathrm{E}[D_T])$$

Pricing

E_k : (random) payout of the Harem cover as seen before $t <$ $k \cdot d$ and $T >= k \cdot d$, i.e. when not yet exercised before $t = k \cdot d$.

At time $t = k \cdot d$:

$$E_k = \begin{cases} D_k & \text{if exercised}, T^* = kd, \\ E_{k+1} & \text{if not exercised}, T^* > kd. \end{cases}$$

Pricing

When will the cover be exercised at time $k \cdot d$, i.e. $T = k \cdot d$?

$$D_k > \mathrm{E}[E_{k+1}].$$

If payout D_k at time $t = k \cdot d$ is bigger than expected future benefit.

$$\mathrm{E}[E_k] = P(\, D_k > \mathrm{E}[E_{k+1}]\,) \cdot \mathrm{E}[D_k \mid D_k > \mathrm{E}[E_{k+1}]\,]$$
$$+ P(\, D_k <= \mathrm{E}[E_{k+1}]\,) \cdot \mathrm{E}[E_{k+1}]\ .$$

Pricing

Let e_k = E[E_k]. Then

$$\begin{aligned} e_k &= \int_{e_{k+1}}^{\infty} x \cdot f_{D_k}(x)dx + F_{D_k}(e_{k+1})e_{k+1}, \\ e_n &= \mathrm{E}[D_n]. \end{aligned}$$

Calculate recursively e_k, $k=n, n-1,\ldots, 1$.

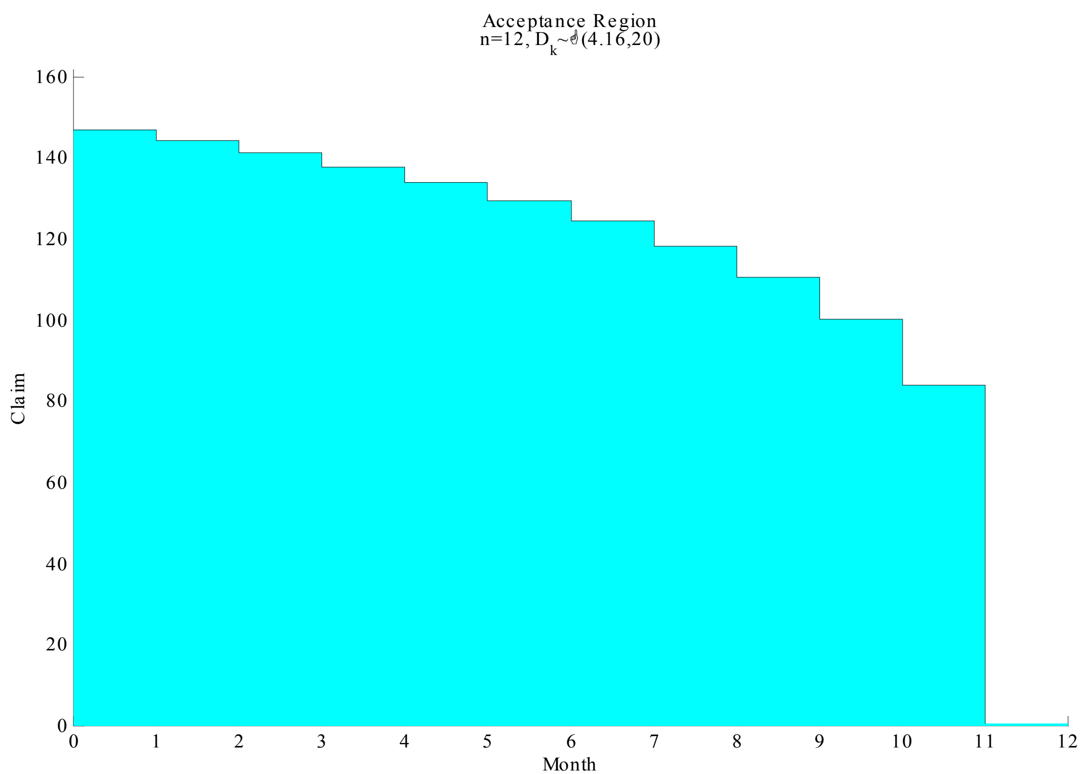
Alternatively

$$e_k = e_{k+1} + \int_{e_{k+1}}^{\infty} \overline{F}_{D_k}(x)dx \; .$$

Exercise of Option

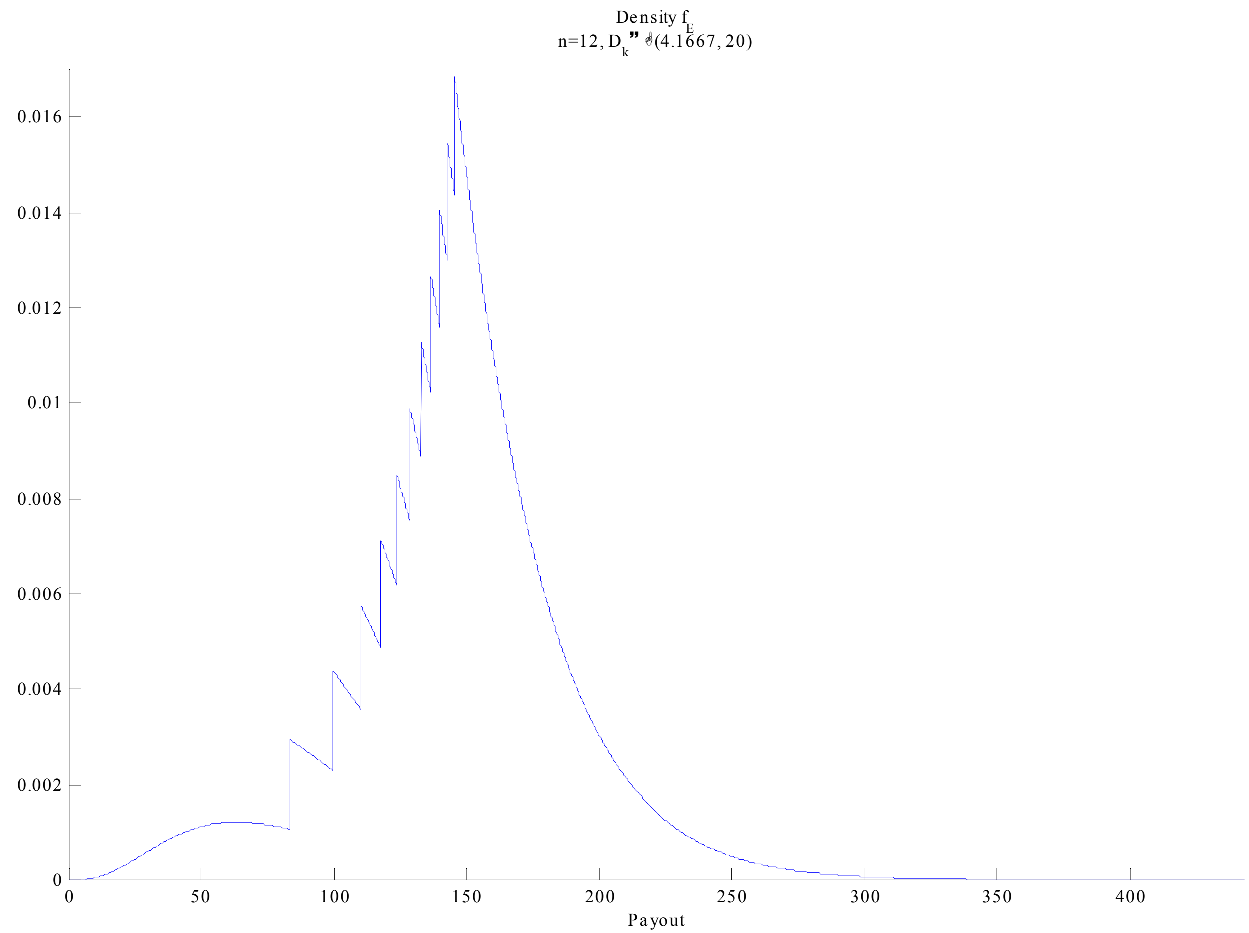
When to exercise?

At time $t = k \cdot d$, if $D_k > e_{k+1}$ exercise, else continue. Example $n = 12$, coverage for one month: (expected yearly loss=1000, loss per month = Gamma(4.166,20).



Density of Payout

Density of Payout for one monthly interval during one year:



Average Loss per Month: 83.3

Price of 1-Month Cover: 147

Extensions

Possible Extensions:

- Reinstatements: After exercise, option to reinstate cover.
- Extended lookback: At time $k \cdot d$, interval $[(k-l) \cdot d, (k-l+1) \cdot d)$, $l>0$ can be chosen.
- Time-Continuous