

On the Pricing of Top & Drop Excess of Loss Covers

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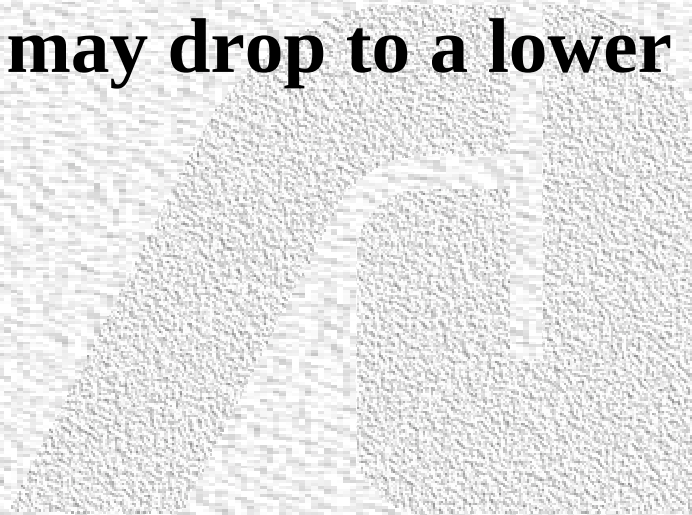
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Agenda

- **Definition**
- **Numerical examples**
- **Modelization**
- **Exact solution**
- **False solution**
- **Approximate solution**
- **Conclusion**

Top & Drop covers : definition

- Top layer covered
- Cover may drop to a lower layer



Numerical example 1

- Layer 1 : 300 XS 100
- Layer 2 : 400 XS 400
- Assume the cedent fears
 - 1) Big losses > 800
 - 2) Accumulation of losses > 20 and < 100

Numerical example 1

- 200 XS 800
- OR
- 200 XS 200 aggregate for claims > 20 fgw with max 100 per claim
- No reinstatement

Numerical example 2

- Layer 1 : 200 XS 200 with one reinstatement
- Layer 2 : 400 XS 400
- Assume the cedent fears
 - 1) Big losses > 800
 - 2) Accumulation of claims in layer 1

Numerical example 2

- 200 XS 800
- OR
- 200 XS 200
- with an annual aggregate deductible = 400
- Unlimited free reinstatements

Modelization

- **X : large claims**
- **N : number of large claims**
- **Y : small claims**
- **M : number of small claims**
- **Mutual independence**

Example 1

$$X_i^{\text{Re-top}} = \min(200, \max(0, X_i - 800))$$

$$X_i^{\text{Re-drop}} = \min(100, X_i I_{X_i \geq 20})$$

$$Y_i^{\text{Re-drop}} = \min(100, Y_i I_{Y_i \geq 20})$$

Example 1

$$S = X_1^{\text{Re-top}} + \dots + X_N^{\text{Re-top}}$$

$$T = X_1^{\text{Re-drop}} + \dots + X_N^{\text{Re-drop}}$$

$$U = Y_1^{\text{Re-drop}} + \dots + Y_M^{\text{Re-drop}}$$

$$\text{Cover} = \min(200, S + \max(0, T + U - 200))$$

Example 2

$$X_i^{\text{Re-top}} = \min(200, \max(0, X_i - 800))$$

$$X_i^{\text{Re-drop}} = \min(200, \max(0, X_i - 200))$$

$$Y_i^{\text{Re-drop}} = \min(200, \max(0, Y_i - 200))$$

Example 2

$$S = X_1^{\text{Re-top}} + \dots + X_N^{\text{Re-top}}$$

$$T = X_1^{\text{Re-drop}} + \dots + X_N^{\text{Re-drop}}$$

$$U = Y_1^{\text{Re-drop}} + \dots + Y_M^{\text{Re-drop}}$$

$$\text{Cover} = \max(0, S + T + U - 400)$$

Multivariate Panjer's algorithm

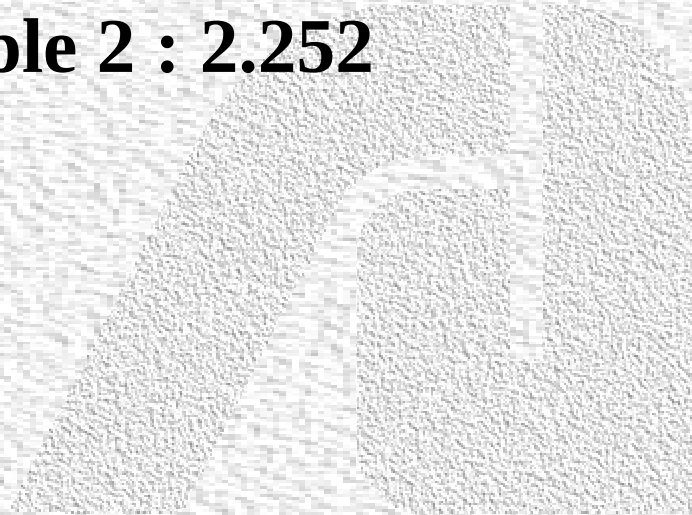
- Let N belong to Panjer's family of distributions
- Let X and Y be possibly dependent
- Let $S = X_1 + \dots + X_N$
- Let $T = Y_1 + \dots + Y_N$
- Then a multivariate version of Panjer's algorithm exists

Numerical example

- Severity : limited Pareto
- Frequency : Poisson
- Large claims : $\lambda = 0.3$, $A=400$,
 $B=1000$, $\alpha=0.9$
- Small claims : $\lambda=2.5$, $A=20$,
 $B=400$, $\alpha = 1.4$

Exact pure premiums

- **Example 1 : 20.519**
- **Example 2 : 2.252**



Assumption of independence

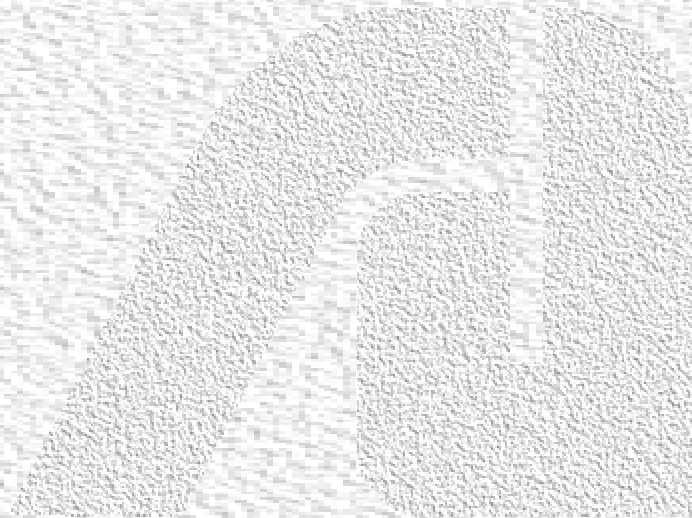
- Example 1 : $21.131 > 20.519$
(conservative)
- Example 2 : $1.153 < 2.252$ (not conservative)

Fréchet space and theorem

- $R(F_1, F_2)$: space of all random vectors (distribution F_{12}) with fixed marginals F_1 and F_2
- Let $F^{\min} = \max[F_1 + F_2 - 1, 0]$
- Let $F^{\max} = \min[F_1, F_2]$
- Then $F^{\min} \leq F_{12} \leq F^{\max}$

Correlation order

■ $(X_1, X_2) <_c (Y_1, Y_2)$ iif $F_{X_1, X_2} \leq F_{Y_1, Y_2}$



Fréchet bounds

- Using some lemmas and Fréchet theorem we arrive at
- Example 1 : $19.469 < E[\text{cover}] < 21.279$
- Example 2 : $0.952 < E[\text{cover}] < 5.471$

Assumption of independence

- Using some lemmas we arrive at
Example 1 : 21.131 (better upper bound)
Example 2 : 1.153 (better lower bound)

Summary

- **Example 1 : $19.469 < 20.519$ (exact) < 21.131 (indep) < 21.279**
- **Example 2 : $0.952 < 1.153$ (indep) < 2.252 (exact) < 5.471**

Extension to dimension higher than 2

- Correlation order extends to supermodular order.
- Example 2 may be treated within that framework.
- However, example 1 may not : a specific framework was necessary in dimension 2.

Conclusion

- **Danger of falsely assumed independence.**
- **Existing bounds may be crude.**
- **Exact model is time-consuming but provides an exact solution.**