On the Pricing of Top & Drop Excess of Loss Covers

Jean-François Walhin

Secura Belgian Re

Université Catholique de Louvain

SECURA Belgian Re

Agenda

- Definition
- Numerical examples
- Modelization
- Exact solution
- False solution
- Approximate solution
- Conclusion

SECURA
Belgian Re

Top & Drop covers : definition

- Top layer covered
- Cover may drop to a lower layer

Numerical example 1

- Layer 1 : 300 XS 100
- Layer 2 : 400 XS 400
- Assume the cedent fears
  1) Big losses > 800
  2) Accumulation of losses >20 and < 100

SECURA
Belgian Re

Numerical example 1

- 200 XS 800
- OR
- 200 XS 200 aggregate for claims > 20 fgu with max 100 per claim
- No reinstatement

Numerical example 2

- Layer 1 : 200 XS 200 with one reinstatement
- Layer 2 : 400 XS 400
- Assume the cedent fears
  1) Big losses > 800
  2) Accumulation of claims in layer 1

Numerical example 2

- 200 XS 800
- OR
- 200 XS 200
- with an annual aggregate deductible = 400
- Unlimited free reinstatements

Modelization

- X : large claims
- N : number of large claims
- Y : small claims
- M : number of small claims
- Mutual independence

Example 1

$$X_i^{\text{Re}-top} = \min(200, \max(0, X_i - 800))$$

$$X_i^{\text{Re}-drop} = \min(100, X_i I_{X_i \geq 20})$$

$$Y_i^{\text{Re}-drop} = \min(100, Y_i I_{Y_i \geq 20})$$

Example 1

$$S = X_1^{\mathrm{Re}-top} + \ldots + X_N^{\mathrm{Re}-top}$$

$$T = X_1^{\mathrm{Re}-drop} + \ldots + X_N^{\mathrm{Re}-drop}$$

$$U = Y_1^{\mathrm{Re}-drop} + \ldots + Y_M^{\mathrm{Re}-drop}$$

$$Cover = \min(200, S + \max(0, T + U - 200))$$

SECURA
Belgian Re

Example 2

$$X_i^{\mathrm{Re}-top} = \min(200, \max(0, X_i - 800))$$

$$X_i^{\mathrm{Re}-drop} = \min(200, \max(0, X_i - 200))$$

$$Y_i^{\mathrm{Re}-drop} = \min(200, \max(0, Y_i - 200))$$

Example 2

$$
\begin{aligned}
S &= X_1^{\mathrm{Re}-top} + \ldots + X_N^{\mathrm{Re}-top} \\
T &= X_1^{\mathrm{Re}-drop} + \ldots + X_N^{\mathrm{Re}-drop} \\
U &= Y_1^{\mathrm{Re}-drop} + \ldots + Y_M^{\mathrm{Re}-drop} \\
Cover &= \max(0, S + T + U - 400)
\end{aligned}
$$

SECURA
Belgian Re

Multivariate Panjer's algorithm

- Let N belong to Panjer's family of distributions
- Let X and Y be possibly dependent
- Let $S=X_1+\ldots+X_N$
- Let $T=Y_1+\ldots+Y_N$
- Then a multivariate version of Panjer's algorithm exists

SECURA Belgian Re

Numerical example

- Severity : limited Pareto
- Frequency : Poisson
- Large claims : lambda = 0.3, A=400, B=1000, alpha=0.9
- Small claims : lambda=2.5, A=20, B=400, alpha = 1.4

Exact pure premiums

- Example 1 : 20.519
- Example 2 : 2.252

Assumption of independence

- **Example 1 : 21.131 > 20.519 (conservative)**
- **Example 2 : 1.153 < 2.252 (not conservative)**

SECURA Belgian Re

Fréchet space and theorem

- $R(F_1,F_2)$: space of all random vectors (distribution F_{12}) with fixed marginals F_1 and F_2
- Let $F^{min}=\max[F_1+F_2-1,0]$
- Let $F^{max}=\min[F_1,F_2]$
- Then $F^{min}<=F_{12}<=F^{max}$

Correlation order

- $(X_1,X_2) <_c (Y_1,Y_2)$ iif $F_{X1,X2} <= F_{Y1,Y2}$

Fréchet bounds

- Using some lemmas and Fréchet theorem we arrive at
- Example 1 : 19.469 < E[cover] < 21.279
- Example 2 : 0.952 < E[cover] < 5.471

Assumption of independence

- **Using some lemmas we arrive at**
  Example 1 : 21.131 (better upper bound)
  Example 2 : 1.153 (better lower bound)

Summary

- Example 1 : 19.469 < 20.519 (exact) < 21.131 (indep) < 21.279
- Example 2 : 0.952 < 1.153 (indep) < 2.252 (exact) < 5.471

Extension to dimension higher than 2

- Correlation order extends to supermodular order.
- Example 2 may be treated within that framework.
- However, example 1 may not : a specific framework was necessary in dimension 2.

Conclusion

- **Danger of falsely assumed independence.**
- **Existing bounds may be crude.**
- **Exact model is time-consuming but provides an exact solution.**

SECURA Belgian Re