"Standard Claims Development Curves for Short Tail General Reinsurance Business"

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Summary

This paper looks at desirable properties for claims development curves. A number of different claims curves are then tested against these properties. The paper looks to see what are the factors that impact the "best" curve to use.The paper examines the parameters needed in these curves. Are more or less parameters needed? Which parameters vary by class of business and which by individual risk? How much of a difference does risk attaching and loss occurring make?

"Standard Schadensverlaufskurven für allgemeine Rückversicherungs-verträge mit kurzem Schadenverlauf"

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Zusammenfassung

Diese Abhandlung untersucht die gewünschten Merkmale für Schadens-abwicklungskurven. Verschiedene Schadenskurven werden dann gegen diese Merkmale getestet. Das Dokument prüft, welches die Faktoren sind, die die zu benutzende „beste" Kurve beeinflussen.

Das Dokument untersucht die für diese Kurven benötigten Parameter. Werden mehr oder weniger Parameter gebraucht? Welche Parameter unterscheiden sich nach Rückversicherungsklasse und welche unterscheiden sich nach individuellem Risiko? Welchen Unterschied macht eine Risks Attaching bzw. Losses Occurring Police?

1. The Problem

Our employer was a newly established A&H reinsurer transacting mostly single year facultative treaties. We had no historical data on its business other than occasional historical experience supplied at time of underwriting.
We needed to model how the claims might be expected to develop.

We were aware that the "S" shaped curve is a depiction of how claims or loss ratios may develop for London market business.

It may be described (assuming that development is to unity) by the formula: -

$$\text{Development}(t) = 1 - \exp[-(t/b)\text{^}c]$$

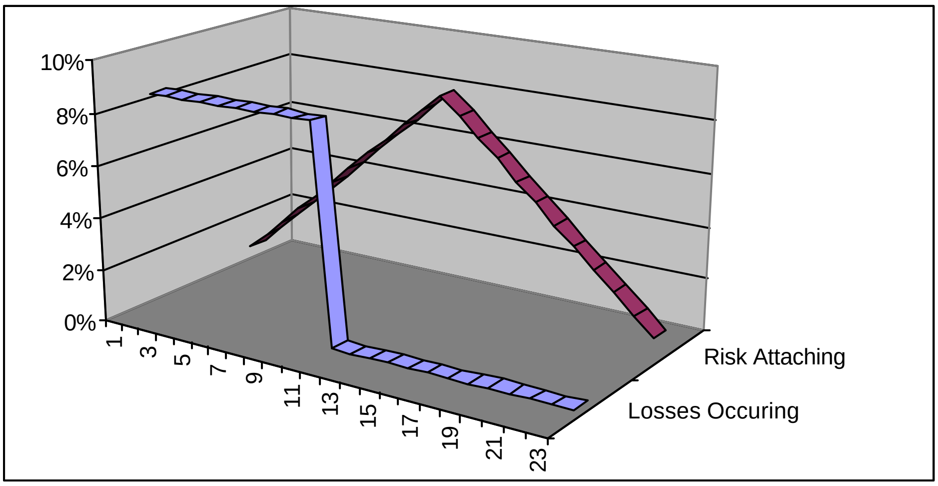
This curve is known in actuarial literature (see appendix) as Craighead's or as a Weibull distribution in statistical terms.

This paper describes both our theoretical consideration of how such a curve might arise for London market A&H business and our practical adaptation for use in connection with credibility measures for reserving on this business.

2. How such a curve might evolve.

We felt it was necessary that we establish a theoretical derivation of how such a curve might develop.

Therefore we considered two possibilities of how the premiums under a single treaty may be earned. Firstly a simple loss occurring ("LO") treaty in which the exposure is even over the contract period (for simplicity taken as 1 year). Secondly a risk attaching ("RA") treaty in which the risks attach evenly over the treaty year and themselves last for a year. The earning pattern of the risk is shown graphically as: -



We can then hypothesise that claims might follow from the earning of premiums in the following ways:-

(a) A simple delay
(b) A straight line of period n
(c) A geometric decay
(d) A triangle
(e) A normal distribution

Obviously more complicated forms could be considered, but given that this is only a model of reality, such complexity might be spurious.

Simple Delay
This merely shifts the premium-earning pattern in time. This would be a straight line for the loss occurring pattern and apparently produce an “S” shaped curve for the risk attaching. We say “apparently” as the curve is actually the joining of two quadratic curves. The first quadratic is the formula for triangular summation i.e. 0.5*n*(n+1); the second is its inverse.

Straight Line with Period n.
If n is 12 then for the LO case we have the same curve for simple delay for the RA. By experimentation we found that the S shape is produced provided the period is between about 8 and 20 months.

For RA we found that any n below about 20 gave an S shape.

Geometric Decay
We could suppose that claims follow the earnings pattern in a geometric decay where the claims paid in each period in respect of a given period’s earnings reduces by a constant percentage each period.

For LO we found that to give an “S” shaped curve the decay rate had to be between 10% and 20%.

For RA we found that reduction rates above 10% give reasonable “S” shaped curves. Below this the curve has too long a tail.

Triangular Claims
This alternative supposes that claims build up for each period of exposure linearly to a maximum and then fall away at the same rate.

For LO we found that provided the period to the maximum of the triangle was at least 5 we obtained an “S” shaped curve

For RA all periods of the triangle gave an “S” shaped curve.

Normal Distribution
For this we assumed that the distribution of the claims payment from the earnings period is normally distributed.

We found that for both LO and RA an “S” shaped curve was produced provided the ratio of Mean chosen to standard deviation chosen was not much above 3.

These experiments convinced us that there was sufficient justification for selecting “S” shaped curves as our basic pattern.

3. Mathematical Considerations.

In the pure form, f(t) = 1 – exp[-(t/b)^c], when t=b f(t) simplifies to 1-exp(-1) or approximately 63%.

However by inserting an additional constant a giving
f(t) = 1 – exp[-a(t/b)^c]
we may choose the value of development at which t=b.

From discussions with our underwriters we came to the conclusion that if an underwriter said that a risk would be fully developed in X months this actually meant that about 95% would be developed by month X.
This led us to a value of 3 for a, and the formula
f(t) = 1 – exp[-3(t/b)^c] will pass through the 95% completion point at t=b. We were thus able to use underwriters estimates to set parameter b for each individual risk.

To simplify the formula we then expressed time as a proportion of this 95% point i.e. substituted a new variable r=t/b and the formula becomes
f(r) = 1 – exp[-3r^c]

We then considered a number of possible ways of setting the second parameter c. It is obvious that with only one parameter left to choose we could derive this from one (and only one) criterion.

(a) Inflection.

One possibility is to consider the point of inflection.

f(r) = 1 – exp[-3r^c]
df(r)/dr = exp[-3r^c](3cr^(c-1))
d2f(r)/dr2 = exp[-3r^c](-3cr^(c-1))(3cr^(c-1)) + exp[-3r^c](3c(c-1)r^(c-2))

at the point of inflection this is zero so after dividing by exp[-3r^c](-3cr^(c-2)) we get :

3cr^c =(c-1)
r^c =(c-1)/(3c)
log r =(log((c-1)/(3c)))/c

This then gives us the values of c which will produce the point of inflection at any required fraction of the 95% point.

r	c
0.1	1.22
0.2	1.43
0.3	1.67
0.4	1.97
0.5	2.37
0.6	2.95
0.7	3.91
0.8	5.77
0.9	11.28

By visual inspection it is clear that we would like the inflection some where between 25% and 75% which suggests that c should be in the range 1.5 to 4.5. It is also worth observing that at low values of c the curve is no longer “S” shaped.

(b) Gradient at 50%

If we consider that the curve has three parts, a slow start, an almost linear middle then a tail, then the slope of the linear part might be regarded as indication of the speed of development.

f(r) = 1 – exp[-3r^c]
df(r)/dr = exp[-3r^c](3cr^(c-1))

if we denote z as the value of r that sets f(r) to 0.5
0.5 = 1 – exp[-3z^c]
0.5 = exp[-3z^c]
0.693 = 3z^c (taking log of both sides)
0.231 = z^c
z = 0.231^(1/c)

therefore at this point gradient (G)
= exp[-3z^c](3cz^(c-1))
= 0.5*(3cz^(c-1))
= 1.5c * z^(c-1)
= 1.5c * 0.231^((c-1)/c)

This table gives the values of the gradient for the values of c we found acceptable above.

c	gradient
1.5	1.38
2.0	1.44
2.5	1.56
3.0	1.69
3.5	1.84
4.0	2.00
4.5	2.16

In practice we found this to be of little value in setting c. It does demonstrate that the higher the value of c the steeper the quasi-linear part but the insensitivity of the result to large changes in c gave us comfort that it might not matter too much in the centre of the curve if we could not set c accurately.

(c) Tail Considerations

We considered that a major item of importance was the length of the tail. In the early period of development the shape of the curve was not terribly important as we were using credibility method (very similar to Bornhuetter-Ferguson) so the expected net loss ratio pre-dominated. We have shown above that the central period was not very sensitive to the value of c.
It was at the tail that the curve was most critical.
We could clearly determine the curve by selecting a value y at which the curve had to be Y% developed (remembering that y is set in terms of multiples of the 95% term).

i.e. Y = 1 – exp[-3y^c]

re-expressing this in terms of the amount yet to be developed T (=1-Y)

logT =-3y^c
y^c =(-logT/3)
clogy =log(-logT/3)
c =log(-logT/3)/logy

It seemed that a suitable points to take might be the 99% and 99.9% developments.
99%

y	c
1.1	4.50
1.2	2.35
1.3	1.63
1.4	1.27
1.5	1.06
1.6	0.91
1.7	0.81

99.9%

y	c
1.1	8.75
1.2	4.57
1.3	3.18
1.4	2.48
1.5	2.06
1.6	1.77
1.7	1.57

This allowed us to set our second parameter in terms of the tail run off but lead to more issues. Firstly we had no evidence to the length of the tail. We suspected (and practice has confirmed) that the structure of different markets and practice of different counterparties would cause great variation in tails. We wondered whether we should be fitting different tails to different parts of the curve.
In practice when the time came, we found it preferable to model the tail by eye as seasonal patterns varied so much by market to make a formula approach invalid.

(d) Aesthetics

That still left us with the problem of fixing c. From (a) we knew that it had to be between 1.5 and 4.5, which suggested a number around 3. At this point we simply could not resist the aesthetic pleasure of using the value of pi!
This then gave us development values for the tail of

99%	1.15 * 95% point
99.9%	1.30 * 95% point
99.9%	1.43 * 95% point

The slope at the 50% point becomes 1.74.
The inflection point becomes 0.62.
All of these we found acceptable and so we stayed with pi.

4. Practice

We found the "S" shaped curves a suitable tool when we had no other data. As time has developed we have accumulated more experience and can now use this experience for triangulation. We now use both methods.

Areas where have found "S" shaped curves to break down in practice are where there is strong seasonality or where there is a mixture of claim types or risk types.

We found the pi value for "c" is suitable for our A&H business. This may not be the case for longer tail classes. Now that we have entered the tail of our business we can use more traditional tail modelling methods.

5. Reference

Financial Analysis of a Reinsurance Office, David Craighead, Published by Insurance and Reinsurance Research Group (1989)