”Some Considerations on Health Insurance Premium Rates”

Erling Falk, Reidunn Falk

Norway

Synopsis

This paper deals with health insurance, which purpose is to secure the financing of some health services. The cover is only available to people who have not reached retirement age, and will only be effective in relation to claims originating in that period. The insurance is based on the natural premium system. An aggregate premium rate is constructed, and some main risk factors are described.

Based on official statistics on surgical operations, together with reasonable assumptions concerning the various components that are integral parts of the premium, calculations have been done to describe the tariff profile for two different product alternatives.

Key words
Health insurance, Norwegian perspective, select claims intensity, aggregated rating.

"Considérations concernant les taux de primes en assurance santé"

Erling Falk, Reidunn Falk

Norway

Résumé

Cet article traite de l'assurance santé, qui a pour but de financer certaines dépenses de santé. La garantie est cependant réservée aux personnes n'ayant pas encore atteint l'âge de la retraite. L'assurance est fondée sur le principe de la prime naturelle. Un taux de prime accumulée est calculé, et certains des principaux risques sont mis en évidence.

Sur la base des statistiques en matière d'opérations chirurgicales, ainsi que des hypothèses raisonnables sur les différentes composantes inhérentes à la prime, des calculs ont été menés dans le but de décrire le profil de la tarification pour deux catégories de produits.

Mots-clés
Assurance santé, le cas norvégien, intensité sélective d'indemnisation, taux de prime accumulée.

Some Considerations on Health Insurance Premium Rates

Erling Falk, Reidunn Falk

1. Introduction

Health insurance, characterised by an insurance company covering specifically agreed expenses for examinations, tests, medical treatment, operations etc. is in its early infancy in Norway. Up to only a few years ago the health service, at lest that part of it that was aimed at more serious treatment and surgery, was completely in the hands of the public authorities. Such services could not be bought, because there were none being sold. Now, however, it would appear that there is a fairly general political consensus in the country that a private supplement to the public hospital services is the right thing. As an ever better functioning private health service is gradually being built, including a complete hospital and surgical service, there is a great probability that a health insurance model that will succeed in the marked, will also be developed. The need is manifested in that those suffering from problems or illness that can be treated, but where treatment is not immediately available from the public health service, want to ensure that they do not have to take a place at the back of the "health queue", but that examinations and treatment can be provided without any loss of time.

Hence, we are still at the modelling stage, where the task is provide the market with an effective health insurance product that will appear as a natural supplement to the traditional public health service as it functions today.

We will consider in more detail such a product's benefits, tariff scales and risk factors.

We will limit product assessments in relation to Long Term Care (LTC) and other health services in relation to the elderly segment. Even though such products can be modelled as risk products, financed more or less by current risk premium in the risk period (Olivieri & Pitacco (1999)), it is equally natural to consider this an insurance need that should be fully financed by the end of the working life (see for example Hellmann (2000)). All markets will have a range of suitable life insurance products for this purpose - both linked products and traditional guaranteed products. In a savings-accentuated insurance for this purpose the inclusion of disability insurance will be of special interest. One realistic model in this connection might be survival benefit in the form of a lump sum that can act as a single premium that wholly or partly finances an insurance that will cover the health risk through the retirement period (Olivieri & Pitacco (1999, 2001)).

We therefore consider an insurance that secures financing of health services during the economically active period and that is offered for sale during the whole period or at least most of it. It is not unusual for a certain age group, near to the age when the insurance is terminated, to be excluded for taking out the insurance, because some times one can doubt whether the underwriting procedure adequately captures the true risk inherent in such age groups. In Norway, the general retirement age in the National Insurance Scheme is still 67 years, but the social partners have negotiated a fairly general early retirement possibility on reaching the age of 62 years.

The real idea of such insurance must be to cover the cost of hospital treatment above and beyond what is offered by the public service. First and foremost it will be a matter of covering the cost of surgery and post-operative treatment. All payments for claims must be made in accordance with documented medical needs, and must specifically be within the scope of the insurance terms. The terms can naturally have a more or less liberal scope as regards cover. One example of such an assessment of the degree of liberalisation that has already been to the fore is the question of having the insurance cover the cost of very specific, very expensive operations that are only available from special foreign medical circles. In addition to such cases involving significant travel and subsistence expenses, there is also the probability that the level of successful results will be uncomfortably low.

In Section 2 we will look more closely at the product assessment and in Section 3 analyse the product alternative that appears most likely on the basis of a simple Markov model. In Section 4 we will consider in more detail how a select claims intensity, defined on the basis of expected claims, can be decomposed. Section 5 contains tariff assessments for the product alternative described, while Section 7 includes a numerical assessment, where the results are commented on. Section 6 describes another product alternative.

2. Product evaluations

The insurance will be in effect in the period that the policyholder is economically active. It can be offered for sale throughout the entire period or the period can be reduced so that cover is taken out at the latest by a specific agreed-upon age. Since the intention of the insurance is limited to covering a need for health insurance before the policyholder reaches the age of retirement, considerations will be isolated from death risk, general disability insurance as such and insurance involving survival benefits.

One must consider the question of what is to be done in the event that the insured enters into a state of defined disability or sickness during the term of the contract. Naturally, there is no reason for not maintaining the cover in this state, with an unchanged financing schedule, even though there is a great deal of tradition for providing a waiver of premium under disability as additional cover.

We consider the following occupational alternatives after disability occurs:

a) The insurance cover continues as before, with an unchanged financing schedule
b) The insurance cover continues as before, as a fully paid insurance, because of waiver of premium.
c) The insurance terminates with the payment of a fixed benefit, either in the form of a lump sum or in the form of a disability annuity that runs through to the agreed termination age (retirement age).
d) The insurance liability expires without any payment.

Because of a lack of background statistics, b) would not appear likely from a risk point of view, assuming that it is the actual heath expenses that are to be covered, while d) is assumed unlikely from a marketing point of view. We will consider products based on a) and c) when disability occurs in more detail.

It is assumed that financing will be by way of natural risk premiums that run throughout the period of cover. In principle, the company can then demand cover for the year's insurance costs (in arrears) offset in accordance with a reasonable distribution principle. However, for market-related reasons a greater degree of price stability is needed. In other words, tariffs are set in accordance with expected claims on the principle of equivalence and at the same time, from the very beginning, seek to maintain tariffs that are sustainable also after the portfolio has matured somewhat

3. The model, product alternative a).

After the conditions, i.e. terms and tariff, are set when the policy is taken out, cover continues with an unchanged payment schedule through to the agreed expiry date, irrespective of the state of health of the insured. The insurance company does not need, in fact, to request any reporting of possible changes in the state of health, the premium will be collected purely on the basis of the age (and gender) of the person insured.

The insured will make an random number of claims, with a random claims amount each year.

Here, we are mainly interested in looking more closely at how other circumstances than the stochastic illness risk affect the premium tariff on the assumption that the equivalence principle is to be applied. We have in mind such circumstances as selection, voluntary termination/lapsing and the dynamism in the portfolio as a result of new entrances.

We also make the following assumptions regarding risk:

1) Expected payments increase with age, however that it is the claims frequency that increases, while the anticipated size of the individual claim settlement is not related to age.

2) There are systematically higher payments in the state "Sick" than in the state "Healthy" in such a way that the size of the individual claim is irrespective of the state, but the claims frequency is higher.

It follows from this that the claims frequency is dependent on the duration in the portfolio, i.e. we are dealing with the phenomenon of selection, due to underwriting standards.

Let the random variable $Y_{[x]+t;h}$ be the total claims amount for an insured person whose age is $x+t$, issued at age x in the age range $(x+t,\ x+t+h)$, assuming that the insured is still alive and insured at the age of $x+t+h$.

Assume that with this insurance the company has offered cover for N different claims categories.

$C^{(i)}_{[x]+t;h}$ is the random number of claims of category i for an insured person at the age of $x+$t, issued at age x in the age range $(x+t,\ x+t+h)$. $p^{(i)}$ is the age-independent price of the benefit of the category i.

Hence

(1) $$Y_{[x]+t\,;\,h} = \sum_{i=1}^{N} C_{[x]+t\,;\,h}^{(i)} \cdot p^{(i)} .$$

Should the insurance be strictly priced according to the equivalence principle, i.e. the company should collect a premium equal to the expected payments for the individual insured persons, this would have resulted in the following natural premium for the period h, if we ignore interest rate discounts through the period:

(2) $$P_{[x]+t;h}^{N} = E(Y_{[x]+t;\,h}) = \sum_{i=1}^{N} p^{(i)} \cdot E(C_{[x]+t;\,h}^{(i)}) .$$

We define the claims intensity

(3) $$s_{\,[x]+t} = \lim_{h \to 0} \frac{E(Y_{[x]+t;\;h})}{h} ,$$

so that under the equivalence principle we would theoretically have a premium intensity as follows:

$$p_{\,[x]+t} = s_{\,[x]+t} \qquad\qquad 0 \le t \le x_u - x ,$$

where x_u is the termination age, which coincides with the age of retirement. After reaching the age of x_n, a person is not allowed to take out insurance cover. The lowest entry age for the health insurance is x_0 ,

However, it is such that at a rating system based on select mortality tables, combined with natural premiums, would not be practical, so long as healthy people can at any time lapse their policies and demand the ordinary issuing rate for their specific age, by continuously declaring their healthiness to the underwriters. At the same time, those who can no longer pass the underwriting test will remain in the system, paying the rates from the select table, which by then would be insufficient to cover the actual risk represented. Thus the basis for using the select tables explicitly for rating purposes is no longer present.

The rating problem is analysed by using a simple time-continuous Markov model with age-dependent forces of transition.

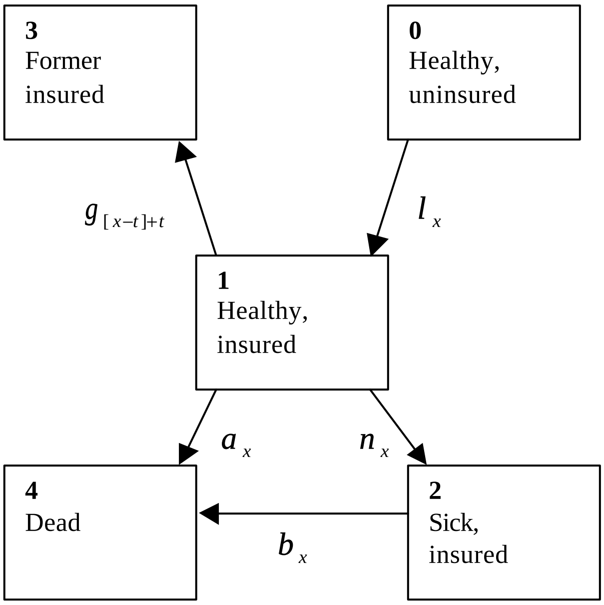

Figure 1: Markov model.

The concept underlying an analysis of selective insurance risk in this way has been adopted from Norberg (1988), and Falk (2001) has made a further application of such a model for purposes of tariffs.

The states are defined according to the criteria healthy/sick and insured/uninsured. Note that we assume only one level for each of the states "healthy" and "sick" even though there should be an approach to a multi-state model of a far wider range, defining sub-categories of sick individuals ("frailty at level I" - "frailty at level II" - etc.) to give a more adequate description of the health situation.

Moreover, the "sick, insured" state is defined as the state where individuals are risk objects to such a degree that they are no longer able to pass the underwriting test, if they (theoretically) should try. But they of course still have their insurance cover, based on earlier underwriting.

In state **1** the claims intensity for an x-year old is $s_x^{(1)}$, and in state **2** the claims intensity is $s_x^{(2)}$, where $s_x^{(1)} < s_x^{(2)}$, cf. assumption 2) above. This analysis concentrates on those people who sooner or later will take out insurance cover - before reaching the age of x_n. Their state will change in accordance with a time-continuous Markov chain with age-dependent forces of transition. As indicated in the figure we base ourselves on the following forces of transition:

- From the "healthy, uninsured" state to the "healthy, insured" state at the age of x: l_x
- From the "healthy, insured" state to the "sick, insured" state at the age of x: n_x
- From the "healthy, insured" state to the "dead" state at the age of x: a_x
- From the "sick, insured" state to the "dead" state at the age of x: b_x

- From the "healthy, insured" state to the "healthy, uninsured" state at the age of x: $g_{[x-t]+t}$

The last above-mentioned force of transition, the force of lapsing, indicates the occurrence of withdrawal from the portfolio before reaching the age x_u. Reasonably this intensity is a function of both the age of the insured person and the duration, but we are unable to make any assumptions concerning this, because of a lack of data. However, it can be argued that the experiences earned by death- and disability insurance do not fit if applied to health insurance.

In general it may be assumed that the lapsing frequency is affected by social, economic and other trends in society.

In the following calculations we regard g as a constant, choosing alternative values that seem intuitively reasonable.

Irrespective of the criteria used for the definition of the "sick" state, all people with deadly illnesses will be situated in this state. It is therefore clear that mortality here will exceed mortality in the healthy state and we have

$$b_x > a_x \qquad x > 0 .$$

We also make the following reasonable assumptions:

- The probability of returning to state **1** from state **3** is disregarded. A person who actually is performing this transition is looked upon as a new entry into the system (from state **0**)
- The probability of returning to state **1** from state **2** is negligible.
- The probability of lapsing from state **2** is negligible.

A key reference to multi-state modelling in life insurance is Hoem (1969, 1988). The following notation is adopted:

> Let $S(x)$ be the state of a randomly selected x-year old person, which sooner or later in his life will take out insurance, and denote the transition probabilities in the Markov process $\{S(x);\ x_0 \le x \le x_n\}$ by $P_{ij}(u,x) = P\{S(x) = j \mid S(u) = i\};\ u < x\ ;\ \ i, j = 0, ...,4$

Then the following relations hold:

(4) $$P_{11}(x, x+t) = e^{-\int_x^{x+t}(n_u + a_u + g)\,du},$$

(5) $$P_{12}(x, x+t) = \int_x^{x+t} e^{-\int_x^{z}(n_u + a_u + g)\,du} \cdot n_z \cdot e^{-\int_z^{x+t} b_u\,du}\, dz \qquad 0 < t < x_u - x .$$

The probability that a person who has been allowed to enter into the insurance portfolio (state **1**) at the age of x still being alive and a member of the portfolio at the age of $x+t$, will be

(6) $$_t\, p_{[x]} = P_{11}(x, x+t) + P_{12}(x, x+t)\,.$$

4. The select claims intensity

We will consider in more detail how the claims intensity $s_{[x]+t}$ given by (3) can be decomposed.

A randomly selected person who takes out insurance at the age of x, and who is still alive and insured at the age of $x+t$, is in either state **1** or state **2**. Since we will make no attempt to assess the probability distribution for claims arising in these two states, but instead primarily concentrate on other circumstances that affect the tariff fixing of health insurance, we assume deterministic claims costs equal to the expected values, cf. (2).

This means that the person in question has either claims intensity $s^{(1)}_{x+t}$ or claims intensity $s^{(2)}_{x+t}$. The respective probabilities of the outcome of the two are $\dfrac{P_{11}(x,x+t)}{_t\, p_{[x]}}$ and $\dfrac{P_{12}(x,x+t)}{_t\, p_{[x]}}$ respectively.

The deterministic assumptions mean that possible claims payments for the randomly selected person in the age interval $(x+t,\ x+t+dt)$ will be either $s^{(1)}_{x+t}dt$ or $s^{(2)}_{x+t}dt$ with the probabilities shown above.

Then we have

(7) $$s_{[x]+t}dt = E(Y_{[x]+t;\,dt}) = \sum_{j=1}^{2} s^{(j)}_{x+t} \cdot \frac{P_{1j}(x,x+t)}{_t\, p_{[x]}} \cdot dt\,,$$

wherefore $s_{[x]+t}$ can be written as follows

(8) $$s_{[x]+t} = s^{(1)}_{x+t} + \frac{s^{(2)}_{x+t} - s^{(1)}_{x+t}}{1 + r(x,t;g)}\,,$$

where $$r(x,t;g) = \frac{1}{\displaystyle\int_x^{x+t} n_z \cdot e^{\int_z^{x+t}(n_x + a_x - b_x + g)\,dx}\, dz}\,.$$

Hence the selective claims intensity can be described as the claims intensity of a healthy $(x+t)$-year old being given an additive component. Since $s^{(2)}_{x+t} > s^{(1)}_{x+t}$ per assumption and, at the same time, $r > 0$, the additive component must be positive for all $0 < t \le (x_u - x)$, and since $r(x,t;g)$ is a decreasing function of t, then $s_{[x]+t}$ must be an increasing function of t, in accordance with the expectations.

5. Tariff-related considerations

Above, we have decomposed the selective claims intensity and found an explicit expression for this, as a function of mortality for healthy and sick, defined in accordance with the model, the force of transition from "healthy" to "sick" and the force of lapsing, in addition to the claims intensity in accordance with the health insurance contract for "healthy" and "sick" as per deterministic assumptions. By applying the equivalence principle this claims flow will be met by a corresponding premium flow in accordance with $p_{[x]+t}$, defined in (3) so that the insurer can achieve balance.

As already pointed out, tariffs could not, for different reasons, be based in such selective premium tables, while at the same time the company uses a system with natural premiums. The most important reason is that healthy insured persons can at any time terminate their existing insurance cover and immediately take out new cover by declaring their state of health. In that case they will achieve a premium of $p_{[x+t]} = s^{(1)}_{x+t}$. This is a premium that is in conformity with that person's underlying claims risk, but is lower than the premium size in the tariff $p_{[x]+t}$, as shown above.

However, insured persons who are no longer "healthy" and who would thus not satisfy the health demands for take out insurance at the age of $x+t$, will of course maintain the contract with the agreed premium schedule. That person thus pays premium in accordance with the tariff, i.e. $p_{[x]+t} = s_{[x]+t}$, whilst the underlying risk is higher, namely $s^{(2)}_{x+t}$. The company thus finds itself in a deficit position, or an expected deficit position if one excludes the assumption of deterministic claims intensities. There are therefore no grounds for select premium tables.

One must therefore necessarily apply aggregated tables. In order to construct an aggregated table with the help of the decomposition of the actual tariff elements already made, another important assumption has to be made. We must assume that a dynamic portfolio will be developed, i.e. that it will successively be renewed. When individuals withdraw from the portfolio because of lapsing or death. It is assumed that new members enter in accordance with the model's force of transition l_x.

At the same time it is assumed having reasonably stable premium rates over time will be important to the market. A new supplier in the market risks quickly having its integrity doubted if he successively increases premium rates as the portfolio matures. This despite that fact that the company in fact has set tariffs in accordance with the equivalence principle in the start-up phase, i.e. the company has let premiums correspond to the true underlying "healthy" risk.

If the company, in its aggregated tariffs, is to take into account the need for assumed price stability, the expectations of dynamism as a consequence of the portfolio maturing must be taken into consideration.

Based on a randomly selected person who sooner or later takes out health insurance cover, i. e. being in state **0** according to the model, we can define the cumulative probability function for his entry age X as

$$P(X \leq x) = F(x),$$

where $F(x_0) = 0$ and $F(x_n) = 1$

We denote the corresponding probability density $\frac{dF(x)}{dx} = f(x)$ and, as indicated in the model, the force of entry from state **0** to state **1** as

$$l_x = \frac{f(x)}{1 - F(x)}.$$

In reality l_x will fluctuate, inasmuch as the tendency to take out health insurance has a considerable rub-off effect, in positive as well as negative directions.

The following equivalence gives a basis for determining an aggregated premium rate:

(10) $$p_x \cdot \int_{x_0}^{x} f(s) \cdot {}_{x-s}p_{[s]}\, ds = \int_{x_0}^{x} f(s) \cdot s_{[s]+x-s} \cdot {}_{x-s}p_{[s]}\, ds \qquad x_0 < x \leq x_n$$

$$p_x \cdot \int_{x_0}^{x_n} f(s) \cdot {}_{x-s}p_{[s]}\, ds = \int_{x_0}^{x_n} f(s) \cdot s_{[s]+x-s} \cdot {}_{x-s}p_{[s]}\, ds \qquad x_n < x \leq x_u\,.$$

The left-hand side indicates the premium flow for the group x-year olds, assuming an aggregated continuous premium rate p_x. The right-hand side indicates the claims flow for the group.

Should all the assumptions relating to transition forces coincide with the true underlying sizes, then (10) will give too high premiums for all x in the portfolio where the system has not worked long enough to all insurance durations being represented. This means that if the company has had the product operative for z years, then the above equivalence will give over estimated premiums for all x where $x - x_0 > z$. Therefore, the equations only provide a basis for determining a fair premium when the portfolio is mature.

In addition, a component should have been included for the impact of expected changes in new entry scales, structured as a correction of *f(x)*. As (10) now appears, premium calculations in the system will be based on an assumption of no changes in new policy volume over time. After all, in a tariff-setting relation this should be a reasonable assumption.

In equation (10) we have decomposed the premium intensity for health insurance as per product alternative a). This intensity assumes a short-term premium system, i.e. the company guarantees the price for the period in question, but is otherwise free to change the price for the next period. This is characteristic in fact of a system with "natural premium payments". In order to secure reasonable price stability the company has to cope with the following risk factors:

- The randomness in claim frequencies for the relative *N* claims causes, for de respective states "healthy" and "sick". Even though the cost of each claim is hardly subject to randomness to the same degree, it is nevertheless the outcome of the year's total claims that are the basic insurance variable.
- The lack of persistency among the insured individuals, the tendency to lapse the contract.
- The tendency to change state from "healthy", state **1**, to "sick", state **2**.
- Mortality among the insured, especially mortality in state **2**. The risk lies in good life expectancy being observed in state **2**, simultaneous with the observation of a stronger claims frequency in this state.
- A possible shortfall in new entries into the portfolio. Successive decline in volume results in an increase in portfolio risk inasmuch as people with a long insurance period will be over-represented.

6. Numerical evaluation

We define the following technical basis for some (theoretical) numerical evaluations:

Age limits:
We set $x_0 = 20$, $x_n = 60$ and $x_u = 65$

Lapse rates:
We use $ln1.05$ and $ln1.1$ as alternatives for g .

Mortality and sickness:
The forces of mortality a_x og b_x are defined in accordance with Gompertz-Makeham. The same applies to the force of transition from "healthy" to "sick", n_x.

We make the following assumptions.

- $a_x = 0.0008 + 0.0000109 \cdot 10^{0.046 \cdot x}$
- $b_x = 0.00126 + 0.0000218 \cdot 10^{0,046 \cdot x}$
- $s_x = 0.0008 + 0.0000021 \cdot 10^{0.0716 \cdot x}$

Reference is made to Falk (2001) for the grounds for these selections, where i.a. data from SSB (2000) has been applied.

The force of entry

There is reason to believe that the pattern of entry (the distribution of the age of the applicants) concerning health insurance, will differ significantly from that concerning life insurance. Having said that, it must be stated that we have no data or any kind of observations in this respect. Even though a great deal of experience has been gained from this type of product internationally, as far as we are given to understand are also significant difference with regard to age distribution from country to country.

In our calculations we have assumed that the probability density as a function of age develops exponentially, and in such a manner that $\frac{f(x_n)}{f(x_0)} = g$. Hence

(11)
$$f(x) = \frac{g^{\frac{x}{x_n - x_0}}}{I(x_0, x_n g)},$$

where
$$I(x_0, x_n; g) = \int_{x_0}^{x_n} g^{\frac{t}{x_n - x_0}} dt .$$

The force of transition l_x can thus be expressed as

(12)
$$l_x = \frac{g^{\frac{x}{x_n - x_0}}}{I(x, x_n; g)} .$$

For g=1, i.e. that we e have a constant, age-independent probability density for new entries, then the probability density is degenerated to

$$f(x) = \frac{1}{x_n - x_0} .$$

In these calculations the alternatives g=0,5 and 2, have been applied, i.e. that the majority of new policies are sold to young period and to elderly people, respectively.

Claims intensity

The Norwegian register of Patients (NPR) has official data covering the number of surgical operations in Norway, by gender, age and category. These are selective operations, i.e. not emergency procedure, and thus the type of operation that it is most reasonable to include in the insurance cover. The data also contains average prices per claim cause, so that we in fact have an adequate basis for calculating tariffs assuming that it is the entire population that is to be included in the tariffs. This is not the case, but the data, combined with the equations that have been developed, give us some basis for numerical evaluations.

Fig. 2 shows the actual surgical costs for each age group measured against the population. We have here a visual expression of the claims intensity in the population.

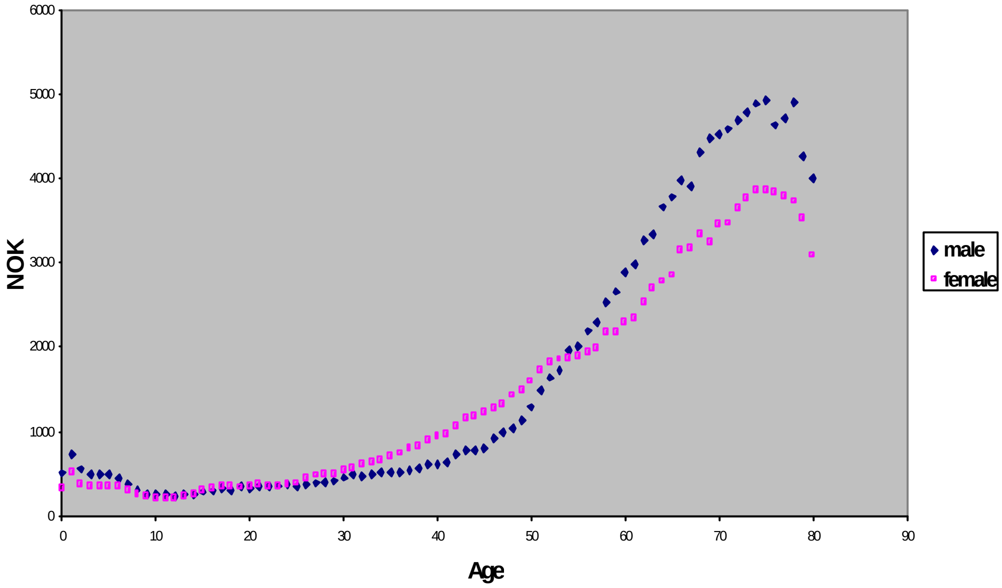

Figure 2. "The claims intensity in the population" (NPR 1994-1998)

It seems to be a substantial deviation between the male and female intensity curves. It should also be mentioned that there is a drop in the average surgery cost for the elderly measured against the population, scarcely because of the good health returning to them.

With regard to tariff considerations one must notice:

- The insurance is meant to have a wider cover than surgical costs.
- In a situation with an insurance cover, instead of relying on the public health service alone, we might experience a change in the behaviour regarding one's own health status.

Having said that, we observe that the claims intensity for the population approximately (at least virtually) develops exponentially in the actual age range (age 20 to 67). That is at least to such a degree that it should be reasonable to assume such a development in these theoretical calculations of tariffs.

We also assume that $s_x^{(1)}$ and $s_x^{(2)}$ can be represented by simple develop exponential Gompertz type functions, and that the following relation holds:

(13) $$s_{x+t}^{(2)} = k \cdot s_{x+t}^{(1)} \qquad 0 < t \le (x_u - x),$$

on the basis of

(14) $$s_x^{(1)} = s_1 \cdot m_1^{\frac{x-x_0}{x_u-x_0}}.$$

The claims intensity in the population, under the theoretical assumption that the entire populations participates in the insurance scheme in question, can be expressed as

(15) $$\bar{s}_x = s_x^{(1)} + \frac{s_x^{(2)} - s_x^{(1)}}{1 + r(0, x; 0)},$$

i.e. the "select" claims intensity for an x-year old who entered the portfolio at birth, under the assumption that withdrawal is not allowed.

The relationship between the population's claims intensities within the age range in question can be expressed as

$$\frac{\bar{s}_{x_u}}{\bar{s}_{x_0}} = m,$$

which by substituting from (13), (14) and (15) and some manipulating, leads to the expression

(16) $$m_1 = m \cdot \frac{1 + r(0, x_u; 0)}{1 + r(0, x_0; 0)} \cdot \frac{k + r(0, x_0; 0)}{k + r(0, x_u; 0)}.$$

Based on Norwegian experience from the years 1994-98 that are reported above, the value of m will be in the vicinity of 10. If we apply this to (16) we get $m_1 = 3{,}92$. This value has been applied in the calculations, together with $s_1 = 1$ and $k = 4$.

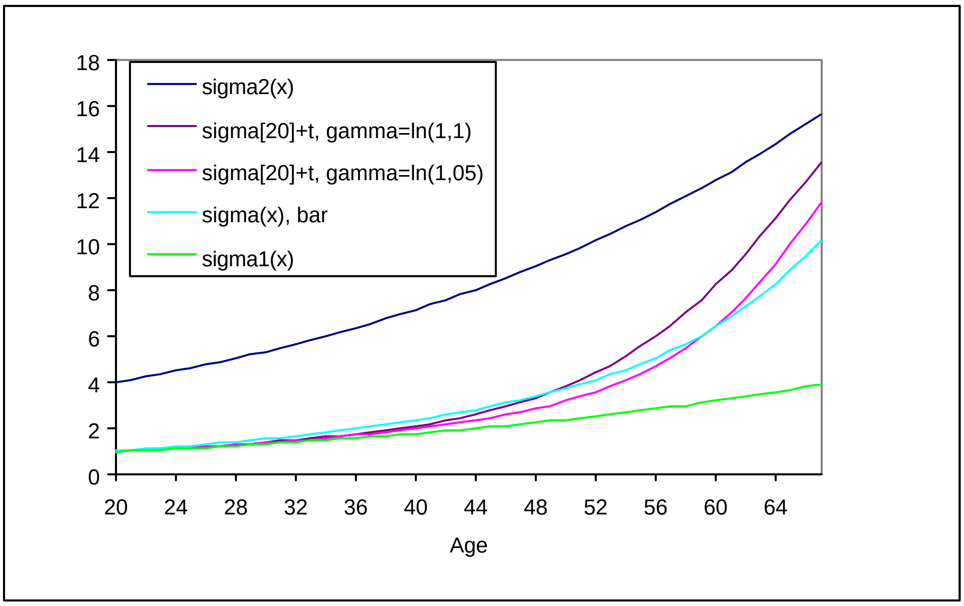

Figure 3. Select claims intensities

Figure 3 is a graphic representation of the select claims intensity $s_{[20]+t}$, with g = ln 1.05 and ln 1.1 respectively, and the claims intensity in the population, $\bar{s}_x$, given by (14), framed by $s_x^{(1)}$ and $s_x^{(2)}$.

We see that from a start where the selective intensities are characterised by very little distance from $s_x^{(1)}$, these intensities as well as the claims intensity in the population approach $s_x^{(2)}$-intensity when the insured is growing older. We also observe that the selective intensities are significantly higher than $\bar{s}_x$ for a greater part of the area. This is where the bad risk is over-represented in the portfolio because of the lapsing effect. There is a considerable gap between the two selective intensities for most of the area.

However, the select claims intensity $s_{[x_0]+t}$ with g = 0 (not included in figure 3), lies below $\bar{s}_x$ for all x. This is a result of the company, through the underwriting procedure, eliminates bad risk from the insurance portfolio, while similar risk naturally has an impact on the risk development for corresponding claims from the population. With $g = 0$, the effect from the underwriting procedure will last through the whole risk period.

We compare in figure 4 premium calculated on the basis of equation 10), for the selected values of g and g .

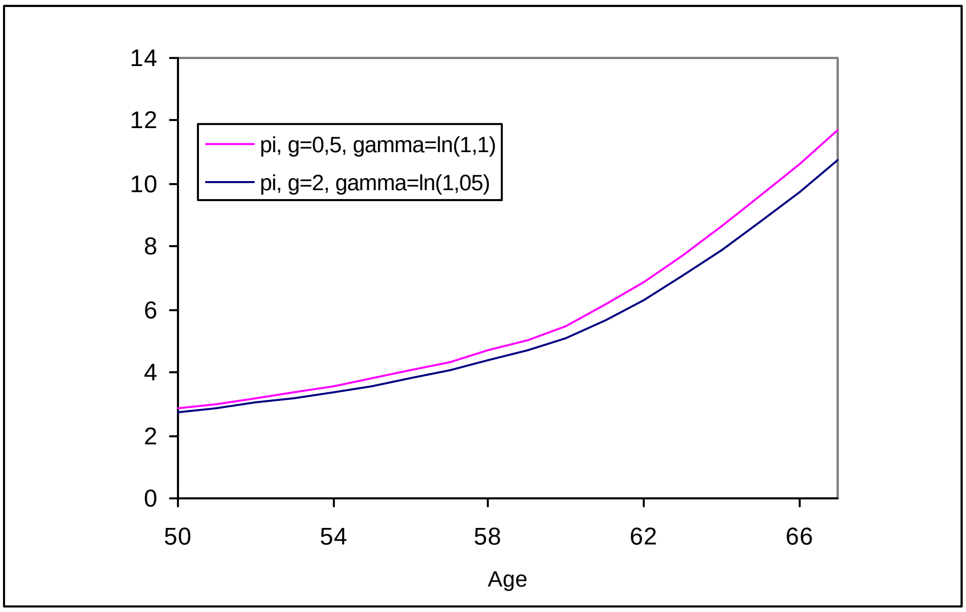

Figure 4. Comparing two tariffs

Even though figure 3 shows that there is a considerable gap between selective intensities calculated on 5% and 10% lapse rates, the assumption that there is dynamism in the portfolio has the affect that this gap to a great extent is wiped out. We nevertheless see that for the higher ages a tariff based on g=2 and g = ln 1.05 would without doubt be loss-bringing to the company if the company experiences a lapse rate in the range of 10% and a decline in new entries equal to g = 0.5.

Estimated premium intensity lies very close to claims intensity in the population, calculated according to the model at (15). The table below shows the premium values.

x	$p_x(g=0{,}05)$	$p_x(g=2)$	$\overline{s}_x$
20	1,000	1,000	1,048
25	1,165	1,164	1,227
30	1,359	1,358	1,439
35	1,592	1,588	1,692
40	1,880	1,870	2,003
45	2,260	2,234	2,409
50	2,811	2,752	2,989
55	3,724	3,590	3,931
60	5,384	5,092	5,626
65	9,025	8,764	8,724
66	9,977	9,735	9,566
67	10,991	10,771	10,484

As it appears, the calculations are based on $s_{x_0}^{(1)} = 1$ (i.e. $s_1 = 1$ in (14)). Working towards real premium rates one should rather originate from $\overline{s}_{x_0}$ and possibly get some help from statistical data, and then transform to aggregate premium rates as shown.

7. Product alternative c)

Persons who achieve state **2**, as this in practice might defined, receive a fixed benefit, either in the form of a lump sum payment or in the form of a disability annuity with fixed or indexed amounts, running to the expiry date agreed on in the insurance (retirement age). The intention is to provide cover for future expected health-related expenses without the insured paying any premium.

For the customer the most interesting product would be, naturally, that he/she is ensured cover for the actual health costs in this period, as in the active period, but still based on a waiver of premium. Such an offer implies a considerable risk for the company. In addition to the significant stochastic risk related to the development of claims, there is an inflation risk.

Basing the product on lump sum payment or annuities with a fixed amount, combined with the natural premium system, reduces the company's risk significantly, even if the annuity case involves some demographic risk.

By applying the earlier assumptions, the product could be offered with the following premium intensity

$$(17) \qquad p_x^1 = s_x^{(1)} + n_x \cdot \int_x^{x_u} P_{22}(x,x) \cdot s_x^{(2)} \cdot e^{(r-d)x}\, dx \ ,$$

where d is a constant force of interest, and r is the constant intensity which eventually can possibly be included, according to the intention of price-adjusted claims payments.

The two product alternatives a) an c) have identical insurance intentions, i.e. financing health services within the insurance terms during the period of time agreed upon. Nevertheless, under the alternative c) a substantial part of the risk concerned is transferred from the insurer to the insured.

In accordance with the agreement, the insured amount given by the integral expression in (17) will either be paid out as a lump sum to the insured, who in that case will administer his own fund. Alternatively the insurer will pay an annuity, based on the claims intensity $s_x^{(2)}$, until the insured reaches the age x_u.

In both cases there are an understanding of a fund allocation sufficient to meet the need for financing health services according to the insurance terms.

The development of the premium intensity (17) through the insured period will deviate from p_x as appears in figure 5, where in addition to earlier assumptions $d = \ln 1.03$ has been applied and r is ln1.03 and 0 respectively.

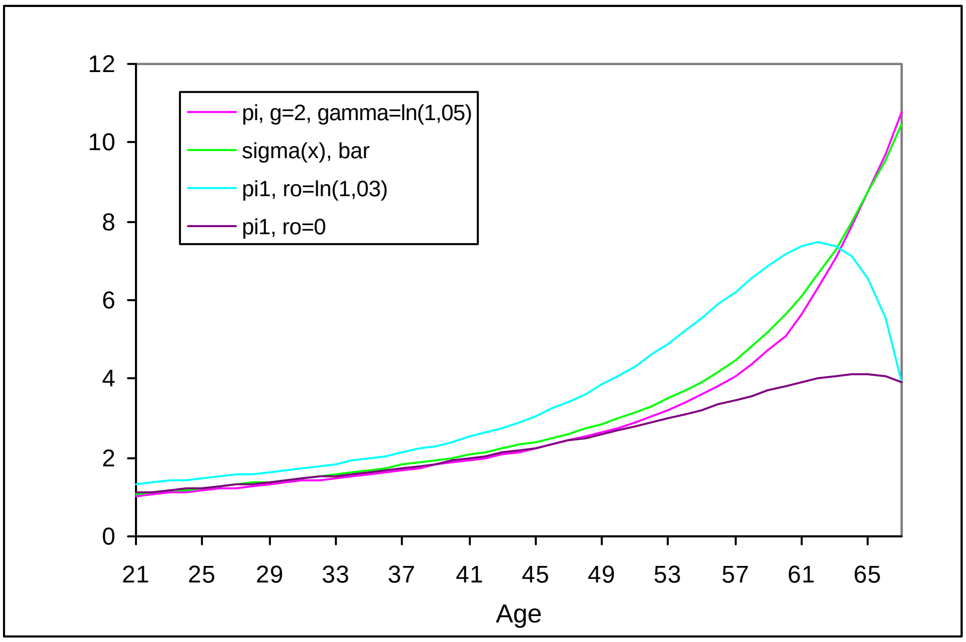

Figure 5. Premium rates - two alternative products

Comparing the nominal premiums for the two product alternatives, it shows that the fixed annuity assumption in alternative c) leads to lower premiums than the alternative a) premiums. This because parts of the payments are covered from the fund's return, and thus the insurer has gained a contribution from outside the system.

8. Concluding remarks

Based on a simple Markov model we have considered the problems related to setting premium rates for a voluntary health insurance based on a natural premium system. The forces of transition from the model, together with a deterministic defined claims intensity, is incorporated in a formula for determining an aggregated continuous premium rate.

Two different product alternatives have been considered, and some likely tariff profiles have been constructed, where official Norwegian statistics on surgical operations has been taken into account in addition to reasonable assumptions concerning the risk factors.

There will be considerable activities in the health insurance field in Norway in the very near future. Providing products that secures financing of health services as a natural supplement to the traditional public health service is the challenge to face. The community's adjustment to such a situation will involve several risk factors for the insurer, as shown. Because of lack of experience, especially in the Nordic markets, these risk factors have to receive much analytic attention as the products develops.

References

Hoem JM (1969), Markov chain models in life insurance, *Blätter der deutschen Gesellschaft für Versicherungsmathematik,* 9: 91-107.

Hoem JM (1988), The versatility of the Markov chain as a tool in the mathematics of life insurance, *Transactions of the 23rd International Congress of Actuaries,* Helsinki, R: 171 – 202.

Norberg R (1988), Select Mortality. Possible Explanations, *Transactions of the 23rd International Congress of Actuaries,* Helsinki, 3: 215 – 224.

Olivieri A and Pitacco E (1999), Funding sickness benefits for the elderly, *Proceedings of the XXX International ASTIN Colloquium,* Tokyo, 135-155.

SSB (2000), Statistisk Sentralbyraa: Statistisk aarbok 2000, table 83, *Official Statistics of Norway.*

Hellman A (2000), Long Term Care Insurance. A Finnish Perspective, *Proceedings of the XXXI International ASTIN Colloquium,* Porto Cervo, 315-339.

Olivieri A and Pitacco E (2001), Facing LTC Risks, *Proceedings of the 2001 International ASTIN Colloquium, Washington DC.*

Falk E (2001) Select Mortality - Aggregated Premium Rates, *Proceedings of the 2001 International ASTIN Colloquium, Washington DC.*