Risk Process Construction for Health Insurance

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Summary

The aim of the paper is to discuss some problems of the constructions of risk processes models for health insurance. The risk process is an important part of actuarial modelling. For health insurance, it is the most difficult thing to model. If it is believed that some details may be ignored to support the predictive power of the model, a Markov model could be constructed (even in a bit sophisticated way). The Markov property may not be common for health care application. But if we can construct a Markov model it makes the analysis the situation much easier.

Some examples demonstrate the extremely high degree of stability of the probability of transitions. It is very important for practical use because the actuary or public health planner could pay much more attention to costs of diagnostics and treatment than to risk process estimation.

Nevertheless, the probabilities of transition may fluctuate. It must be taken into account, but the type of models discussed allows doing it simpler.

There are at least four directions for further development of the model.

Zusammenfassung

Das Ziel der Arbeit ist eine Discussion der Probleme der Modellkonstruktion des Risikoprozesses fuer der Krankenversicherung. Das Risikoprozess ist der wichtigsten und swerigsten Teil der versicherungsmathematischen Modellirung. Markowsche Prozesse sind ein gewaltig Insrument der Analyse der Krankenversicherung. Aber die Markowsche Eigenschaft fordert eine Vereinfachung der modellierte Situation (sogar auf dem unnatuerlichen Weg).

Die numerischen Beispiele demonstrieren eine Stabilitaet der Wahrscheinlichkeiten. Das ist sehr wichtige Aspekt der praktischen Ausnuetzung der Markowschen Modelle. Obwohl die Wahrsacheinlichkeiten koennen oszilllieren, die Beispiele demonsrieren gute Eigenschaften der vorlegte Behandlung. Fier Direktionen der Weiterentwicklung sind diskusiert.

Introduction

The aim of the paper is to discuss some problems of the constructions of risk processes models for health insurance. There are different approaches, but the most popular one is Markov processes.

The classical actuarial model for health insurance is used for the analysis of transitions between basic states 'healthy', 'sick' and 'dead' (Waters, 1984). The possible expenditures can be connected (depended on the design of an insurance product) not only to the transition itself, but also to the duration of stay in state 'sick'. The basic problem is how to estimate the probabilities/forces of transition. Haberman and Pitacco offered the more advanced approach of health insurance modelling based on Markov processes (Haberman and Pitacco, 1999). Such a model allows specifying actuarial estimations, but it may be used for the adoption of underwriting and claims policy and for the risk management.

Not only could be used such a model for premiums estimation and reserves evaluation for the appropriate insurance, but also for the risk process investigation which is important for underwriting procedures, claims processes and risk management for health insurance. Moreover, it can be used outside of health insurance field, namely for the standartization and the rationing of medical services, for the diagnostic and treatment process management, for the resources planning of the health care services and the solution of malpractice problems.

A special type of such kind of a model is considered in the paper, namely the model of diagnostic and treatment process which allows to represent state 'sick' from the basic model as a process itself. The paper pays special attention to the construction of risk process, but not to applications because the adequacy of the risk process is a very important aspect of the effectiveness of actuarial modelling for health insurance.

Constructing a Markov model for a diagnostic and treatment process

The classical model is based on a series of implicit assumptions causing the simple enough set of transitions. Those assumptions were accepted for the model simplification, in particular because of improving an informational support. One of them is the assumption about the lack of any distinctions between different sicknesses. It makes the model much less adequate, but the necessity of the assumption is connected to the deficit of statistics (there is often only claim inception rates and loss distribution). The refusal of the assumption that divides state ‘sick’ into some states modelling the appropriate diagnostic and treatment process allows specifying pattern of the risk process related.

Nodes of the graph correspond to medical services or types of treatment. Expenditures and possible outcomes requiring transitions to any next nodes characterize each of types of treatment. Those transitions are described by edges of the graph. Each of them matches up with the probability of the appropriate outcome. Certainly, for each separate sick person, the outcome of his/her treatment could be univalent, however it is possible for group of those people to connect different probable outcomes with the appropriate fractions of that group. If it is not precisely known before the inception of disease what kind of treatment will be provided to sick person, the diagnostic and treatment process can be modelled with a random process of transition. In such a model, the diagnostic and treatment process for a particular sick person is represented with a path in the graph given from initial node to one of absorbing nodes and the graph as such is a representation of the random process of the diagnostics and treatment.

The outcome of the particular diagnostic and treatment manipulation not only affects the transition to the next one, but also the probability distribution of present and future transitions. Hence, the Markov property for any process of diagnostics and treatment is not fulfilled. The Markov property is strong enough condition – it is not even fulfilled for the classical actuarial model if we suppose that the probabilities/forces of the transition from state 'sick' depend on the duration of stay in that state. Moreover, the graph may contain a directed circuit, that the infinite reiteration of the appropriate transitions is theoretically assumed, although the number of reiteration of treatment manipulations is finite in reality.

Nevertheless, the use of Markov processes (especially Markov chains) would allow constructing the model more effectively from mathematical point of view. So, we need to explain how the Markov property could be achieved.

A possible approach consists of an aggregation of the set of initial states into the set of aggregated states. In other words, some initial states concerning to a particular sequence of diagnostic and treatment manipulations could be united in aggregated one, which describes those sequence as a unit. The necessary and sufficient conditions for such an aggregation are quite strong (Kemeny and Snell, 1960). But the idea is to use the degree of aggregation as a tool of the construction of the model for which Markov property assumption would be adequate enough.

The procedure seems to be too sophisticated, but it is strongly believed to allow reaching an acceptable compromise between the adequacy of model, the sources of statistical data and the Markov process approach for modelling. The basic problem is to find the degree of aggregation in order to achieve the Markov property, but not to decrease the adequacy of the model.

For example, if we are interested in the sequence of transition from a hospital into another one and the duration of stay in it, but not in any specific features of treatment in the particular hospital, it is meaningful to consider a treatment in that particular hospital as a state.

But the procedure, unfortunately, does not guarantee an automatic achievement of the Markov property. So, another approach may be used. It consists of a separation of the appropriate state into a few ones so that oriented graph associated with the random process or the appropriate part of it represents an oriented tree under the deletion of the edges, which are incident to the absorbing nodes.

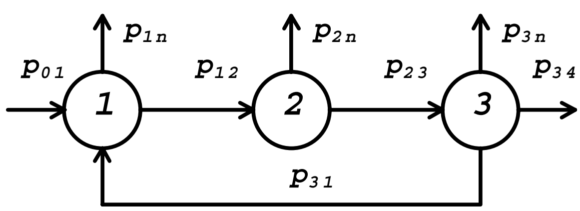
This principle could be explained with the following examples.

The first example deals with a graph with a directed circuit. The nodes formed the directed circuit and the edges of the graph which are incident to those nodes are represented in Fig. 1, Case A. Such a directed circuit describes a reiteration of the particular medical manipulations or of the treatment in the same type of hospital. There are two possibilities:

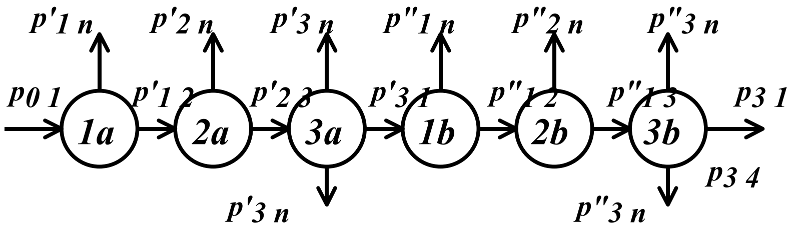
1) The directed circuit could take place at different stages of the treatment. In that case the number of the reiterations is finite.

2) The directed circuit could take place at the treatment of chronic diseases, so the number of the reiterations for a defined period, for example, a year, is finite as well.

Hence, that directed circuit may be represented as the appropriate number of the reiteration of medical services included in the directed circuit (see Fig. 1, Case B). In this case, the graph transformed is more adequate as it takes into account a number of reiterations of the specified group of services. It allows, if necessary, to specify estimators for the probabilities of transition (it is symbolically represented with primes in Fig. 1, Case B).



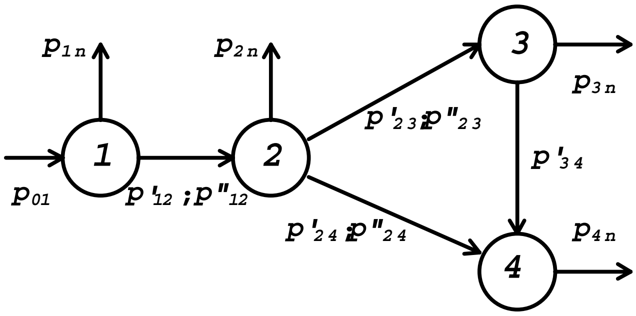
Case A



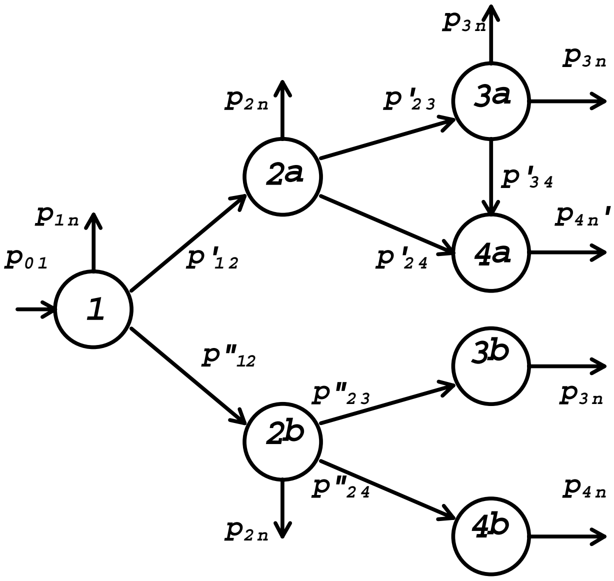
Case B

Fig. 1. Transformation of the graph with a directed circuit

Another example illustrates the influence of the particular medical service to the probability distribution on consequent steps (see Fig. 2, Case A). The realisation of every possible outcome could be thought as a random event for whole group of patients, so it is meaningful to divide this sequence into "branches" each of which has its own probabilities of transition (see Fig. 2, Case B).



Case A



Case B

Fig. 2. Transformation of the graph where there is the influence of outcomes (designated with one or two primes) on the probability distribution of consequent steps

Thus, the transformation of any oriented graph will ensure the transformation of random process associated and, eventually, the achievement of Markov property for the diagnostic and treatment process.

An example of constructing the model

An example of constructing the Markov chain of the diagnostic and treatment process is represented by the results of the research of treatment process for acute cranial and cerebral trauma which was realized in co-operation with the experts from St.Petersburg Medical Academy and Russian Neuro-Surgical Research Center named after Polenov (Kudryavtsev et al., 1997).

The outcomes of the trial research are shown as a graph on Fig. 3. The diagnostic and treatment process is aggregated very much in order to achieve the Markov property. It consists of 14 states representing treatment in particular hospitals or medical services provided by the particular institutions (states 2 to 11) and of the outcomes of treatment (states 12 to 14). State 1 represent the inception of trauma.

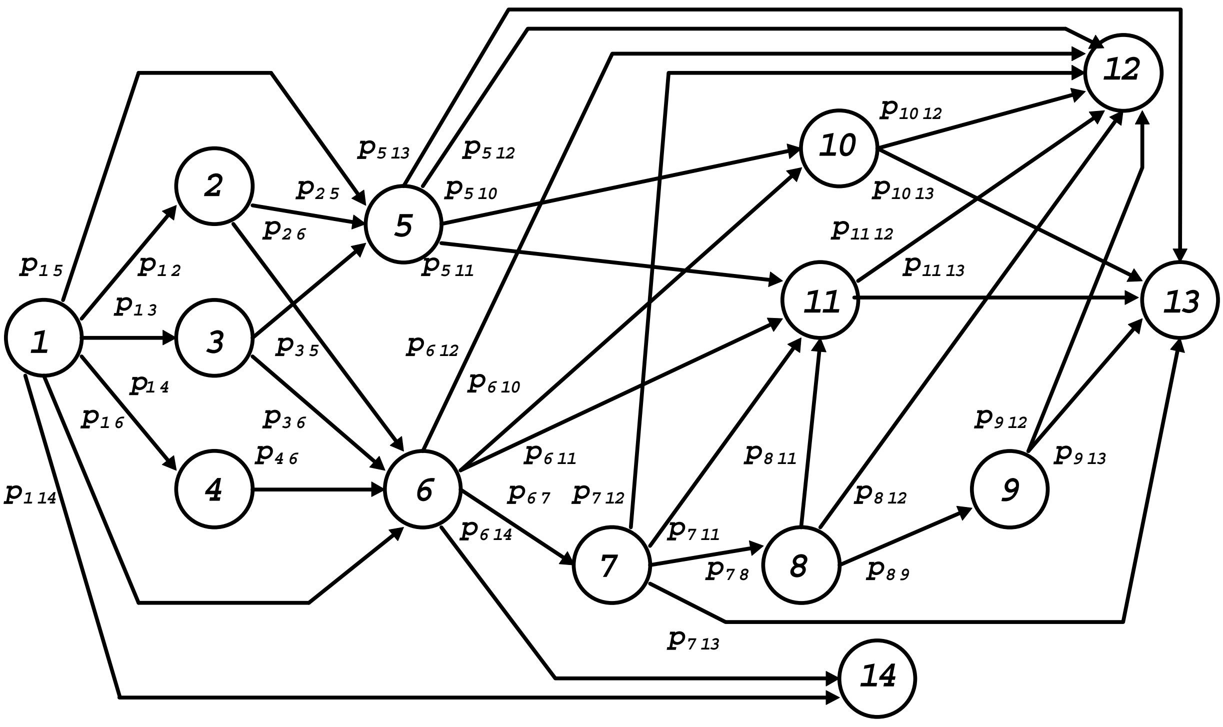


Fig. 3. The diagnostic and treatment process for acute cranial and cerebral trauma
1 - the inception of trauma; 2 - first aid provided by special traumatological out-patient hospital; 3 - first aid provided by in-patient hospital; 4 - first aid provided by ambulance; 5 - out-patient treatment; 6 - in-patient treatment: the first hospitalization; 7 - out-patient treatment; 8 - in-patient treatment: repeated hospitalization; 9 - out-patient treatment; 10 - rehabilitating in-patient treatment; 11 - rehabilitating out-patient treatment; 12 - practically healthy; 13 - permanently disabled; 14 - dead.

The initial graph that was obtained after primary processing of information had a directed circuit related with repeated hospitalization. The outcomes of treatment are linked with the number of hospitalization during the treatment period because of the gravity of trauma. It required to separate between the first and repeated hospitalizations in the graph transformed

(Fig. 3), that caused to have some states representing identical medical services. For example, states 5, 7 and 9 represent treatment in special traumatological and general out-patient hospitals whereas states 7 and 9 speak for out-patient medical services after in-patient treatment and, hence, are more or less similar to each other on volume of the aid.

That process can be considered as Markov one with the following matrix of the probabilities of transition:

$$\left\| \begin{array}{ccccccccccc|ccc}
0 & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{1\,14} \\
0 & 0 & 0 & 0 & p_{25} & p_{26} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & p_{35} & p_{36} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & p_{46} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{5\,10} & p_{5\,11} & p_{5\,12} & p_{5\,13} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & p_{67} & 0 & 0 & p_{6\,10} & p_{6\,11} & p_{6\,12} & p_{6\,13} & p_{6\,14} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{78} & 0 & 0 & p_{7\,11} & p_{7\,12} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{89} & 0 & p_{8\,11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{9\,12} & p_{9\,13} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{10\,12} & p_{10\,13} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{11\,12} & p_{11\,13} & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array} \right\|$$

It is possible to use such a matrix for evaluating some specific features of the diagnostic and treatment process (for example, the probability that a patient will get into the particular type of hospitals). Moreover, the comparison of real and ideal probabilities will allow making conclusions about the quality of treatment and, wider, about the quality of functioning of the public health care system as a whole. It is possible to analyse more local problems, for example, how the diagnostic errors may influence on the treatment process.

The Markov process constructed is one with absorbing states. So, we may use some well-known formulas for modelling, e.g. ones based on fundamental matrix approach (see Kemeny and Snell, 1960). In particular, one can calculate matrices $\boldsymbol{B=NR}$ and $\boldsymbol{H} = (\boldsymbol{N} - \boldsymbol{E}) \cdot \boldsymbol{N}_{dg}^{-1}$ which include probabilities of getting into one state beginning from another one. Actually, only the first rows of them are interesting for modelling because of the initial distributions of entries $\beta = (1, 0, \ldots, 0)$.

However, the most interesting results are achieved when the costs of treatment connected to every state are installed into the model. It allows estimating expected cost of treatment and its variation related with the oscillating character of treatment. If, e.g., the costs $\boldsymbol{S_i}$ are connected to each aspect of the aid $\boldsymbol{i}$ (in the example $i = 2 \div 11$), and there is permanent disability benefit S_{13} and death benefit S_{14} the combined cost of treatment per one sick person can be estimated by

$$S = \sum_{i=2}^{11} h_{1i} \cdot S_i + b_{1\,2} \cdot S_{13} + b_{1\,3} \cdot S_{14}, \qquad (*)$$

where $\boldsymbol{h_{1i}}$ is the probabilities of providing the appropriate aid which are estimated on the base of the probabilities of transition from one state to another one, $\boldsymbol{b_{1j}}$ is the probabilities of getting into absorbing state $\boldsymbol{j}$.

Thus, if the diagnostic and treatment process could be represented with a Markov chain with absorbing states, the model simplifies the analyses of specific features of that process. Although the construction of such a model is faced with a number of difficulties, its practical use is important for the standardartization of treatment paths and for the organization of managed health care.

Stability of probability estimators

The problem of informational support for the model including the stability of the estimators of the probability of transition and possible approaches to make collecting information cheaper (in particular, use macrodata and expert information) has been recently investigated.

The following example illustrates some arguments in favour of the stability of the probabilities of transition. The stability was tested for more simple graph in comparison to Fig. 3 (see Fig. 4) and on the base of data for children. The graph drawn on Fig. 4 could be seen as a part of more complex graph (something like one shown on Fig. 3)

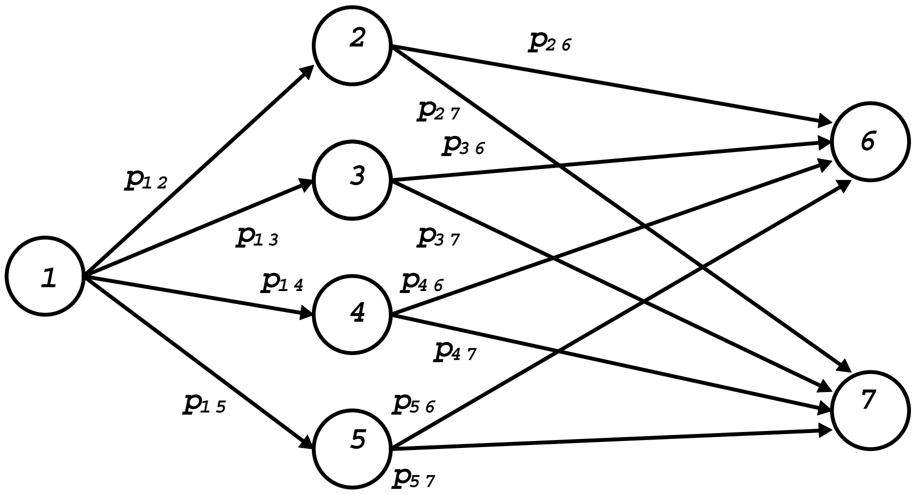


Fig. 4. The simplified diagnostic and treatment process
for acute cranial and cerebral trauma

1 - the inception of trauma; 2 - first aid provided by special traumatological out-patient hospital; 3 - first aid provided by ambulance; 4 - first aid provided by out-patient children's hospital; 5 - first aid provided by other types of medical institutions; 6 - in-patient treatment; 7 - out-patient treatment.

A comparison were made on data taking from the research of specific features of the first aid at children's acute cranial and cerebral trauma for one district of St.Petersburg in 1987 and 1994. These years were chosen because the socio-economic situation had cardinally changed: if in 1987 it was rather stable, in 1994 it was the worst for some last decades, first of all, measured with mortality, morbidity and economic criteria.

The matrix of the probabilities of transition for 1987 is the following:

$$\begin{Vmatrix} 0{,}0000 & 0{,}2222 & 0{,}5937 & 0{,}0825 & 0{,}1016 & 0{,}0000 & 0{,}0000 \\ 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}4500 & 0{,}5500 \\ 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}6043 & 0{,}3957 \\ 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}7116 & 0{,}2884 \\ 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}0000 & 0{,}5156 & 0{,}4844 \end{Vmatrix}$$

$$\left\| \begin{matrix} \mathit{0,0000} & \mathit{0,0000} & \mathit{0,0000} & \mathit{0,0000} & \mathit{0,0000} & \mathit{1,0000} & \mathit{0,0000} \\ \mathit{0,0000} & \mathit{0,0000} & \mathit{0,0000} & \mathit{0,0000} & \mathit{0,0000} & \mathit{0,0000} & \mathit{1,0000} \end{matrix} \right\|$$

For 1994 the following probabilities of transition were estimated as following:

$$\left\| \begin{matrix} 0,0000 & 0,2620 & 0,5599 & 0,0808 & 0,0973 & 0,0000 & 0,0000 \\ 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,4800 & 0,5200 \\ 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,7781 & 0,2219 \\ 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,7778 & 0,2222 \\ 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,5077 & 0,4923 \\ 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,0000 & 1,0000 & 0,0000 \\ 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,0000 & 1,0000 \end{matrix} \right\|$$

Some rows of the matrices above were compared to each other with using chi-square test (Kanji, 1993). The results of comparing are shown on the following table.

Table 1. Comparison between probabilities of transition in 1987 and 1994 using chi-square test

Row	Empirical value	Critical value	Accept (+) or reject (–)?
1	2,82	7,81	+
2	26,46	3,84	–
3	0,28	3,84	+
4	0,61	3,84	+
5	0,01	3,84	+

The results of the comparisons give us some arguments that figures are rather close to each other, excluding only the first aid provided by special traumatological out-patient hospital, but the role of the latter in public health care system in Russia had been changed for that period.

Dispersion of probabilities as an additional source of uncertainty

Although the probabilities of transition are stable enough, they could demonstrate some fluctuations. The reasons may be the following: observational biases (selection), registration errors, seasonal changes etc. Such fluctuations in probabilities are the source of additional uncertainty for projections. For example, the total cost from formula (*) is sensitive enough to probabilities changing.

The assumption about multinomial distribution for each row of the matrix of the probabilities of transition could be too formal. Another (or an additional) approach is to estimate the variations of probabilities from data observed.

As an example, we can analyze a model based on 1987 data. The risk process studied is associated to the graph shown on Fig. 5 that is a bit more complicated version of situation shown on Fig. 4.

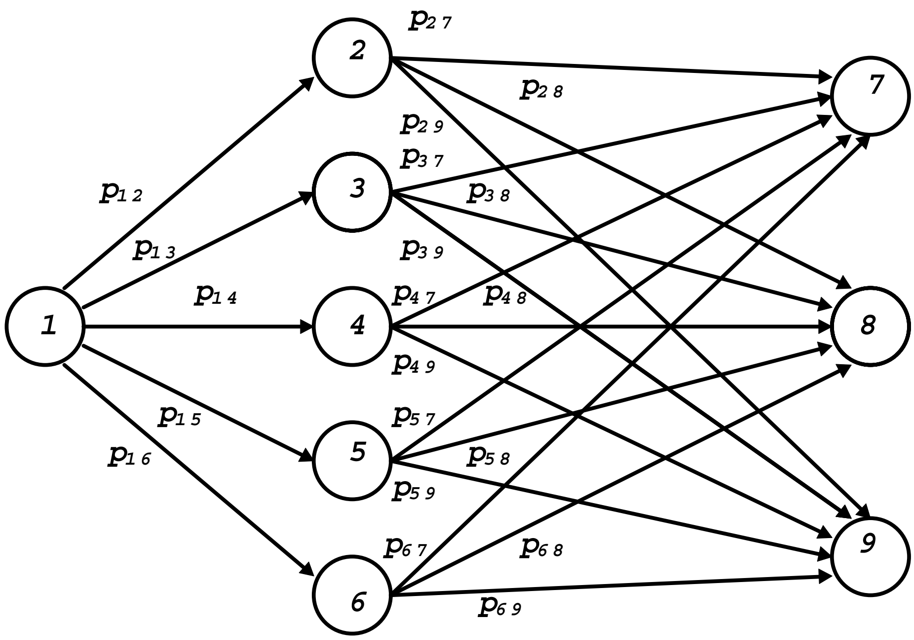


Fig. 5. The simplified diagnostic and treatment process
for acute cranial and cerebral trauma

1 - the inception of trauma; 2 - first aid provided by special traumatological out-patient hospital; 3 - first aid provided by in-patient hospital; 4 - first aid provided by ambulance; 5 - first aid provided by out-patient children's hospital; 6 - first aid provided by other types of medical institutions; 7 - in-patient treatment; 8 - out-patient treatment; 9 - out-patient treatment in special traumatological out-patient hospital.

The matrix of the probabilities of transition is the following:

$$\begin{Vmatrix}
\mathbf{0{,}0000} & \mathbf{0{,}5902} & \mathbf{0{,}0281} & \mathbf{0{,}2848} & \mathbf{0{,}0421} & \mathbf{0{,}0548} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} \\
\mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0713} & \mathbf{0{,}0872} & \mathbf{0{,}8414} \\
\mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}3810} & \mathbf{0{,}5476} & \mathbf{0{,}0714} \\
\mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}5305} & \mathbf{0{,}3474} & \mathbf{0{,}1221} \\
\mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}5873} & \mathbf{0{,}2381} & \mathbf{0{,}1746} \\
\mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}2400} & \mathbf{0{,}1811} & \mathbf{0{,}5789} \\
\mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{1{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} \\
\mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{1{,}0000} & \mathbf{0{,}0000} \\
\mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{0{,}0000} & \mathbf{1{,}0000}
\end{Vmatrix}$$

For this section example, statistics were divided by 1-month period sets. Estimators for different months were different from each other. The variations calculated on the data are shown in the matrix on Table 2 for probabilities that are not equal to 0 or 1. It is obvious that the fluctuations observed are large enough to be taken in account.

Table 2. Variation estimates for probabilities which are not equal to 0 or 1

State	Row 1	Column 7	Column 8	Column 9
2	0,004090	0,001093	0,001671	0,002975
3	0,000268	0,068764	0,080669	0,030612
4	0,001340	0,009164	0,009042	0,002688

5	0,000108	0,078622	0,047940	0,019646
6	0,000693	0,040871	0,013091	0,043340

As an additional conclusion from the compariosons between examples of graphs constructed on the base of 1987 data one can see that numerical values of probabilities may depend on graph structures. Hence, the procedure of risk process constructing is not trivial. In order to be adequate enough, the model must be based on qualitative (interpretative) consideration.

Concluding Remarks

The risk process is an important part of actuarial modelling. For health insurance, it is the most difficult thing to model. If it is believed that some details may be ignored to support the predictive power of the model, a Markov model could be constructed (even in a bit sophisticated way). The Markov property may not be common for health care application. But if we can construct a Markov model it makes the analysis the situation much easier.

Some examples demonstrate the extremely high degree of stability of the probability of transitions. It is very important for practical use because the actuary or public health planner could pay much more attention to costs of diagnostics and treatment than to risk process estimation.

Nevertheless, the probabilities of transition may fluctuate. It must be taken into account, but the type of models discussed allows doing it simpler.

There are at least four directions for further development of the model.

The first approach is to research deeper the standartization of any diagnostic and treatment processes. Different kinds of the diagnostic and treatment processes should be tested and investigated. It is more biomedical research than economic or actuarial one.

Another approach is to investigate costs of treatment. The model allows to look into those costs as an relatively separate process which is important because of data collecting. The model could be involved in the managed care policy.

The third direction of the development of the model is to construct models based on controlled Markov processes. Such a modification could be used for the solution of the problem of the choice of diagnostic and treatment methods when it is necessary to find out whether the use of more expensive, but more effective medical services is profitable.

The last, but not least, approach is to consider a net of queuing systems, assuming, for example, that each node of the net is a server (a separate queuing system). The modification allows to analyze risk process on the generalized level (time and money spending could be investigate in one model) and makes the model more adequate.

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