Investment Guarantees in Equity-Linked Insurance: The Canadian Approach

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This paper describes some of the theory and principles underlying the proposals of the Canadian Institute of Actuaries Task Force on Segregated Fund Investment Guarantees. The Task Force proposes a methodology for determining the policy liabilities and the total balance sheet provision for the liability arising from the guarantees. The basic principle is stochastic simulation of the liabilities. A method to calibrate investment models is presented. This allows companies freedom in model choice but constrains the investment model output to ensure high risk areas of the distribution are suitably represented. The simulation of liabilities is described and complicating issues briefly discussed. A comparison of quantile and conditional tail expectation risk measures for a sample contract is shown.

1 Introduction

In many jurisdictions, including Canada, insurers have designed combined insurance and investment contracts to compete with the mutual fund industry for stewardship of policyholders investments by adding guarantees to a pure investment contract. This has the benefit of allowing the contract to be eligible for treatment as insurance, but also, importantly, is a highly popular option with policyholders aware of the potential downside risk of pure equity investment. These contracts are known as segregated fund contracts in Canada and they have become very popular, with around C$45 billion of assets held in segregated funds in August 2000. Segregated fund contracts are essentially the same as unit-linked contracts in the UK, and are similar to Variable Annuity contracts in the US.

The guarantees offered on segregated fund contracts vary; for eligibility under the insurance regulations a minimum guarantee of 75% of deposits, payable on death or at contract maturity is required, and many companies offer only this minimum. Others have increased the guarantee to 100% of deposits on death, and some offer 100% of deposits on death or maturity. In some cases the guarantee can be reset from time to time at a higher level. There are two common forms of reset.

- Automatic resets: at the end of the official contract term the contract can be renewed with guarantee at the higher of the initial level and the fund value at renewal. The official term of the contract is usually 10 years (the minimum for insurance eligibility), but the true term is usually substantially longer as there is an expectation that contracts will be renewed a number of times. This is sometimes referred to as the rollover option.

- Voluntary resets: some contracts offer the policyholder the option of electing to reset the guarantee to the current fund level from time to time. In a typical contract voluntary reset moves the maturity date to 10 years from the reset date.

Segregated fund contracts are usually established as recurring single premium contracts, with premiums deposited in specified 'segregated funds' which are designed to resemble closely mutual funds. These are often external to the contract issuer. A percentage of the assets is deducted monthly in management charges, of which a portion goes to the segregated fund management company, leaving the remainder available to the insurer to cover the guarantee cost, expenses and profit.

From 1997 there was a growing recognition that traditional valuation methods did not translate well to the equity-linked class, and that new techniques were required.

In October 1999 the Canadian Institute of Actuaries (CIA) appointed a Task Force with a mandate to develop recommended approaches for assessing the valuation liability for segregated fund contracts. Early in discussions, the Task Force concluded that valuation issues should be considered in conjunction with solvency requirements, and the mandate was extended to cover the total balance sheet requirement for segregated fund contracts as well as the assessment of policy liabilities.

In this paper we summarize the methodology and conclusions of the Segregated Fund Task Force (SFTF) of the CIA. The full report of the Task Force (SFTF, 2001) is published by the Canadian Institute of Actuaries and is available from the CIA website address given in the bibliography. The report is written as a guide to practitioners, and as such is relatively detailed around the implementation of the Task Force recommendations, but does not contain very much detail about the technical details behind some of those recommendations. The report is not the right place to describe those details. The objective of this paper is to summarize the conclusions of the Task Force, together with the technical background to those conclusions. The paper is therefore designed both for Canadian practitioners interested in the theory behind the Task Force recommendations, and for others possibly interested in adapting the Canadian approach to their own products.

2   A short history of maturity guarantees

In the late 1970s the UK insurance industry faced a very similar problem to that faced by the Canadian industry some 20 years later. Unit-linked contracts offered mutual fund type investment, with guarantees of 100% of deposits for maturing contracts. Actuaries realized that these guarantees could not be ignored, particularly with the then recent memory of the prolonged market depression of 1972-1973, associated with the oil crisis.

The Institute of Actuaries and the Faculty of Actuaries formed a working party to consider the valuation and solvency issues associated with maturity guarantees. The Maturity Guarantees Working Party (MGWP) (1980) concluded that stochastic simulation of the assets and liabilities was essential for assessing the potential costs of the guarantees. A predecessor of the Wilkie model was published in conjunction with the MGWP paper as a basis for stochastic simulation. The precise form of the asset model was a controversial issue at the time the paper was presented to the UK profession.

Meanwhile, much work in academic circles dealt with the relation between maturity guarantees and options. Brennan and Schwartz (1976) and Boyle and Schwartz (1977)

both used a financial economics approach to the valuation and risk management of guarantees and many others have followed in their footsteps. Still few actuaries dealing with maturity guarantees in practice have adopted financial engineering techniques. In 1980 the Maturity Guarantees Working Party referred to the Black and Scholes (1973) formula as 'unproven'.

3 Preliminary principles

3.1 Canadian valuation and solvency principles – very briefly

For traditional insurance business in Canada the policy liabilities are first assessed on a gross premium basis without implicit or explicit margins in the valuation basis. This is increased by a *Provision for adverse deviation* or PfAD to allow for parameter uncertainty and reasonable random adverse experience. The best estimate valuation plus the PfAD comprises the GAAP policy liability. For some risks, particularly investment related risks, the PfAD is determined by projecting cashflows under a variety of adverse investment scenarios and determining the initial assets required in each case.

In addition solvency capital based on risk-based capital principles is required. The additional solvency capital is the *Minimum Continuing Capital and Surplus Requirement* or MCCSR.

An insurer must also undertake deterministic dynamic solvency testing to assess, *inter alia* whether any further solvency capital is required.

The Task Force in its expanded mandate determined that it would address the basic policy liability, the PAD and the MCCSR in a holistic approach.

3.2 Policy Liabilities

Traditional valuation is based on expected values. This relies on risks being diversifiable, so that when a sufficient number of risks is insured, the law of large numbers applies to ensure that the total liability will be close to the expected value, and the central limit theorem gives the distribution of the sum so that the likely variation from the expected value is known.

The key difference with maturity guarantees is that much of the risk is undiversifiable. If equities fall to a 10-year low, payments will become due on a whole cohort of

contracts. While this is an unlikely event, if it does occur the sums required to meet the liabilities could be very substantial. The low frequency and high severity combination leads to many comparing the financial guarantee liability with catastrophe insurance cover.

The expected value for such contracts is not generally a very meaningful measure. Liabilities will not be close to expected values as they are for diversifiable risks; for most contracts and cohorts the liability will be zero, for a few the liability will be substantial. The Task Force concluded that expected values could not form the basis of policy valuation for segregated fund guarantees. To value and manage financial guarantees it is necessary to understand the full distribution of the liability. It cannot be captured in one or two moments of the distribution. For this reason we also rejected deterministic scenarios as the basis for valuation or solvency requirements. While deterministic stress testing can be useful in some circumstances, in the context of segregated fund investment guarantees deterministic scenarios do not quantify risk and are very inefficient at differentiating more high risk policy design from the more benign. Interpretation is difficult; deterministic scenario testing tends to systematically over or under estimate the true risk, and for any specified deterministic scenario set there is generally no agreement whether it is overly cautious or incautious. Indeed, a scenario set may be too cautious under one policy design and too incautious under another. For setting solvency requirements, deterministic scenarios do not provide probabilistic information on solvency, and so may be subject to misinterpretation – for example that a policy has negligible risk because the risk was not captured in a small set of deterministic scenarios. This adds to the risk that a minimum solvency standard becomes a target solvency standard.

The method recommended by the Task Force is to use stochastic simulation to determine the policy liabilities and expense ratio income. The key stochastic variable is the return on the segregated fund, discussed in the following section. Other stochastic variables include mortality and lapses. The effect of allowing stochastic mortality is tiny compared with the variability inherent through the investment model so it is usual to use a deterministic approach to mortality. Lapses (including partial withdrawals) are affected by the investment performance of the segregated fund and could be simulated with a link to the investment model. However, in the absence of a convincing model, it was decided to adopt a simple deterministic approach to lapses.

3.3 Financial engineering or actuarial risk management

The asset side of asset liability modeling must reflect the risk management strategy adopted by insurers. There are two approaches for the assets held against financial guarantees. The financial engineering approach is to adopt a dynamic hedging strategy, using the guarantee premium portion of the MER to fund, for example, a long position in bonds and a short position in stocks, which (under certain conditions) will exactly meet the guarantee at the end of the contract. If the guarantee has zero cost the assets will have zero value. The traditional actuarial approach is to hold the assets in risk free bonds in a more passive strategy. If the guarantee is in the money at maturity (has non-zero cost) then the bonds are available to meet the cost. If not then the bonds are released back into the company surplus.

Very few companies in Canada adopt the financial engineering approach to risk management for guarantees. In determining factors for minimum solvency capital therefore, the Task Force assumed that actuarial risk management is being used in respect of the guarantee risk (see Section 10). However, the Task Force acknowledged that risk mitigation practices should be recognized especially with respect to the solvency requirement. The underlying principle of the Task Force recommendations is that the net costs of investment guarantees should be stochastically simulated using a realistic probability distribution for the segregated fund asset appreciation. This principle applies equally whether the risk is being hedged or not.

A company need not choose between full dynamic hedging and full actuarial risk management; it is common for companies which do use financial engineering to adopt a hybrid risk management strategy. Consistency of treatment of the two approaches means that the principles of the Task Force report can be readily adopted for all such combination risk management strategies.

4 Investment Models

Stochastic models of equity returns abound; from the simplest univariate lognormal models to the multi-variate Wilkie type approach (Wilkie, 1995). The Task Force decided not to mandate a specific model or models. However it is important that any model used to value financial guarantees which are often deeply out of the money accurately captures the risk of the guarantee moving into the money.

The Task Force set out a calibration for the investment models which emphasizes the left tail of the asset return distribution over three different time periods; 1 year, 5 years and 10 years. An insurer may use any stochastic asset model that, when

Accumulation Period n	2.5th percentile of A_n (maximum)	5th percentile of A_n (maximum)	10th percentile of A_n (maximum)
1 Year	0.76	0.82	0.90
5 Years	0.75	0.85	1.05
10 Years	0.85	1.05	1.35

Table 1: Calibration criteria for accumulation factors

fitted to the baseline data (TSE 300 total return index, 1956-99) generates left tail probabilities at least as large as those in the Calibration Table, reproduced here as Table 1. The model must also generate a mean 1-year accumulation factor close to the true mean of the data (in the range 1.10 to 1.12), and the standard deviation of the 1-year accumulation factor must be at least 17.5%.

The table is based on the accumulation factors generated by a stochastic model. For example, if a model is used to generate rates of return for successive months, I_t, then the accumulation factor over n months is

$$A_n = \prod_{t=1}^{n}(1 + I_t)$$

The calibration process is as follows:

1. Calculate maximum likelihood estimates for the parameters using the baseline data. (Other estimation methods may be used if justified.) Let $\hat{\Theta}_{MLE}$ denote the maximum likelihood estimated parameter vector.

2. Check whether the accumulation factors generated are less than the table values for each cell. That is, the probability that the 1-year accumulation factor is less than 0.76 (a 24% drop in value) must be at least 2.5%. For some models the factors can be checked analytically, for others simulation may be used.

3. If the model does not pass the calibration test, adjust the parameters so that it does, while still satisfying the mean and variance constraints for the 1-year factor. Let $\hat{\Theta}_{calib}$ denote the adjusted parameter vector. The adjustment vector is

$$\Delta = \hat{\Theta}_{calib} - \hat{\Theta}_{MLE}$$

Accumulation Period	Percentile	Empirical Range	RSLN	SAV	Stable
1-Year	2.27%	(0.61, 0.82)	0.74	0.76	0.77
	4.55%	(0.76, 0.85)	0.82	0.81	0.83
	9.09%	(0.85, 0.92)	0.89	0.87	0.90
5 Years	11.11%	(0.98, 1.41)	1.05	0.99	1.04
10 Years	20.0%	(1.60, 2.59)	1.88	1.60	1.73

Table 2: Empirical and model estimates of percentiles of accumulation factors.

4. The model may then be fitted to other data sets. The fitted parameters should then be adjusted by adding the vector Δ to the parameter vector fitted to the new data set.

The data is too sparse to provide percentiles for five and ten year periods. There are only eight non-overlapping periods of 5-year returns and four non-overlapping periods of 10-year returns. There are several non-overlapping periods to choose from as we can choose different starting months. These series are not independent but provide slightly different empirical estimates of the underlying distribution percentiles. The calibration table was constructed by extrapolating from the data using three different heavy tailed models that appeared to fit the data, and then compared with the available percentiles of the 1956-1999 TSE data. The three models are:

- a regime switching model with 2 regimes, and lognormal processes within regimes (RSLN);

- a stochastic volatility model with autoregressive volatility (SAV)

- a stable distribution.

The details are described in an appendix to the CIA report (SFTF, 2000). Table 2 is reproduced from that appendix. The first column gives the accumulation factor under consideration. The second column gives the percentile available from the data. For the five and ten year figures these are the minima of the data. The third gives the range of accumulation factors at that percentile, taking different starting points. The final three columns give the percentiles generated by the three models using maximum likelihood fitted parameters. These show that the models are consistent with the observed empirical percentiles.

Many insurers use a lognormal model for monthly, quarterly or annual returns. Calibration to this table implies an adjustment of around +3% to the volatility from

the maximum likelihood to the calibrated figures. In Figure 1 the effect of calibration on the probability density function for the 10-Year accumulation factor is plotted. The top figure compares the Regime Switching Lognormal model with the lognormal model with maximum likelihood parameter estimates, without calibration. The important area of the graph for most guarantees is the left tail, where it can be seen that the lognormal density is substantially thinner than the RSLN density. On the lower graph the same RSLN density is plotted against the calibrated lognormal density. The left tails of the two density functions show a much closer fit.

5 Liability Modeling

5.1 Preliminaries

Once the investment model has been selected and fitted, the rest of the liability model can be constructed. S_t denotes the stock index at t that determines the fund returns; it is generated using the investment model. G_t denotes the guarantee at t; if it is fixed the subscript is omitted. F_t denotes the segregated fund at t given that the contract is still in force; m denotes the rate of management charge (the Management Expense Ratio or MER) deducted from the fund, expressed as a continuous rate per month. Then

$$F_t = F_{t-1} \frac{S_t}{S_{t-1}} e^{-m}$$

Part of the management charge is available to fund the guarantee; let m_r denote this part. Then the management charge income for the guarantee cost is a random process M_t:

$$M_t = F_{t-1} \frac{S_t}{S_{t-1}} \left(e^{m_r} - 1 \right)$$

To allow for exits, use a double decrement model for lapses and deaths, where q_{x+t}^d denotes the mortality rate at $x + t$ (in months) and q_{x+t}^w represents the lapse rate. p_{x+t}^τ denotes the dependent survival probability.

Then for each cohort, simulate a value for S_t, calculate F_t and M_t. If the guarantee may be reset depending on the fund value, G_t will also need to be updated.
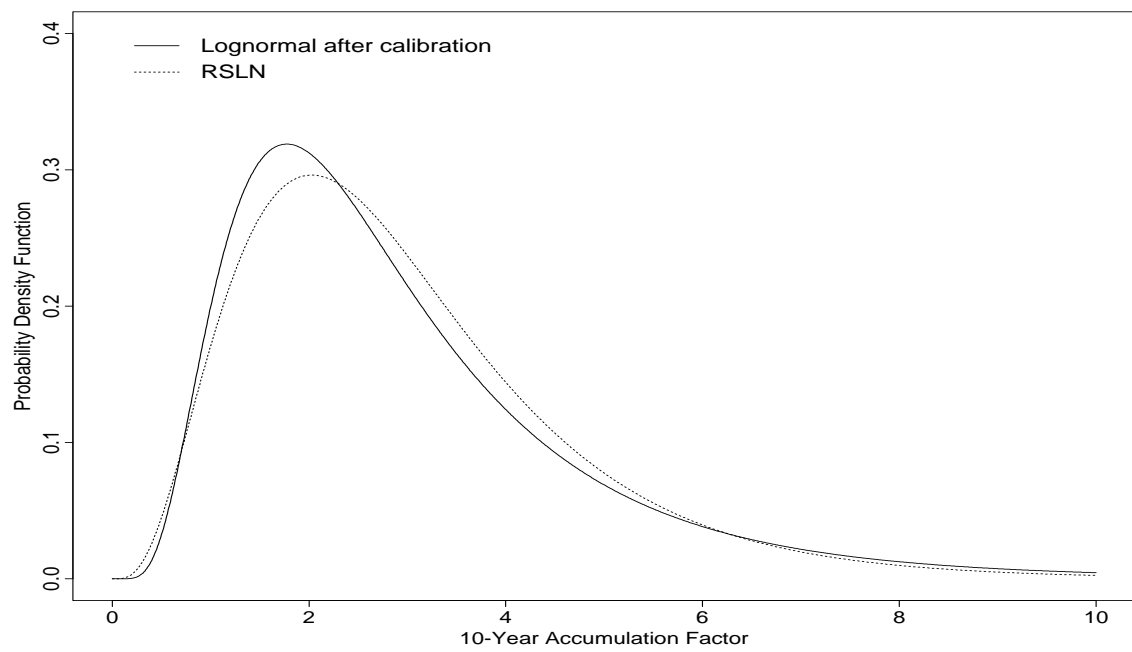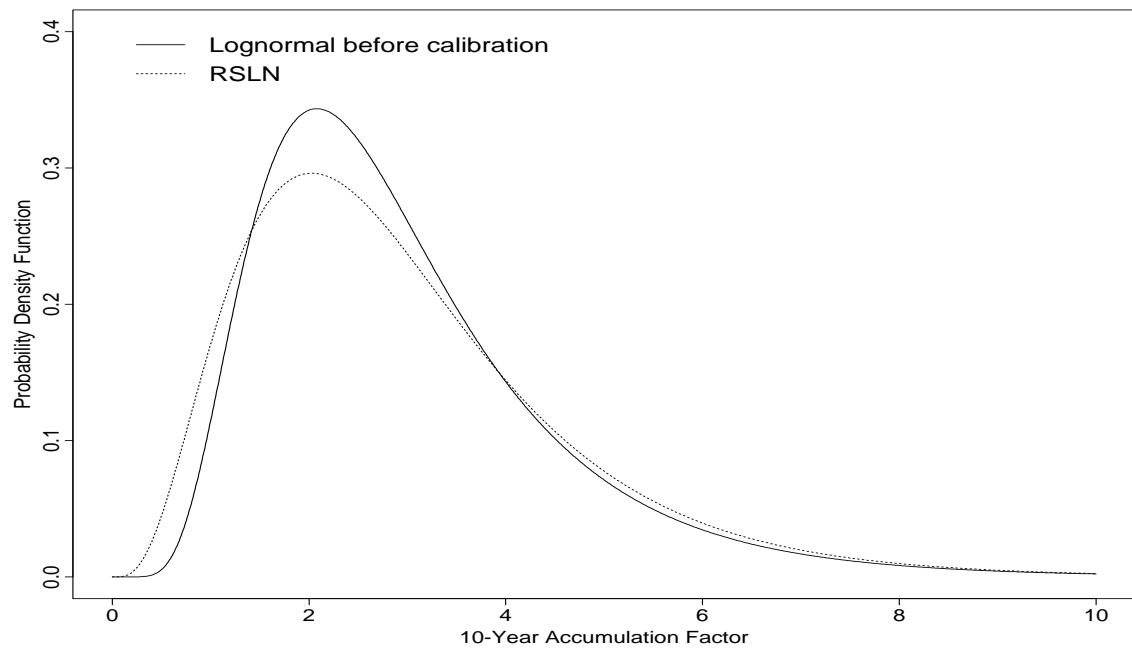
Figure 1: Ten-Year Accumulation Factor PDFs – Lognormal and RSLN

For a contract without the rollover option (that is, with a single maturity date at n) the total simulated liability cashflow under the contract at $t = 0, 1, 2, ..., n-1$ is

$$C_t = {}_{t-1}p_x^\tau \, q_{x+t-1}^d \max(G_t - F_t, 0) - {}_tp_x^\tau M_t \qquad (1)$$

and at maturity the cashflow is

$$C_n = {}_np_x^\tau \max(G - F_n, 0)$$

Factors which may complicate this process include changing MERs, rollovers and voluntary resets, and fund transfers.

5.2   MER adjustment

Although companies may retain the right to adjust MERs from current levels, the actuary should not assume a change unless there is a clear reason for doing so. It is reasonable to expect substantial competitive pressure not to increase MERs.

5.3   Rollovers and resets

In many cases the policy maturity date is adjustable at the policyholder's option; there may be a guaranteed renewal option at the maturity date for example. This means at the original maturity date, perhaps ten years from the policy inception, the policyholder may renew the contract. If at that time the guarantee is in-the-money, then the segregated fund would be increased to the guarantee level and the contract renewed with the same guarantee. If the guarantee is out-of-the-money, the contract is renewed with the guarantee increased to the fund level at that time. This is sometimes referred to as a rollover option. The actuary should assume that at least a proportion of policyholders will exercise the rollover option, both in the 'in-the-money' and 'out-of-the-money' cases.

In some cases the policyholder may reset the guarantee at various times to the level of the fund at the reset date. Usually the maturity of the contract is automatically moved when the policyholder exercises the reset option, often to 10 years from the option date. There may be a limit to the number of times a policyholder may reset in each calendar year. The actuary should assume that this option is exercised, though it is not necessary to assume that policyholders behave with 100% efficiency

(even if we knew what 100% efficient behaviour looked like). An ad hoc method for simulating resets is to assume that a proportion of policyholders reset when their fund is some set multiple of the guarantee, for example 120%.

5.4 Fund transfers

Most contracts allow the policyholder to transfer the segregated fund assets between different funds at little or no cost. Again, the model should assume some policyholders do so, though it is not necessary to assume that all policyholders select against the company.

6 Asset-Liability Modeling

Using actuarial risk management the assets are assumed held in risk free bonds, and the simulated cashflows C_t can be discounted at a suitable rate of interest for the distribution of the present value of the liability cost.

Using financial engineering, the hedging strategy needs to be determined and the hedge cash flows modeled in parallel with the liability cash flows. The modeling should allow for

- basis risk between the underlying segregated fund assets and the hedge positions;

- uncertain future implied volatility if the hedge strategy involves derivatives trades;

- transactions costs and discrete hedging risk.

For example, in the simplest case where the hedge is a combination of bonds and short stocks, let H_t represent the hedge portfolio at t, let ϕ_t represent the units of stock in the hedge portfolio and let ξ_t represent the bond portion of the hedge portfolio. We assume the hedge is re-balanced at unit time intervals. Also, let τ denote the transaction cost related to stock transactions per \$ of stock traded (bonds are assumed to be transacted cost-free).

$$H_t = \phi_t S_t + \xi_t$$

Then the liability cashflow at t for this model, using C_t from (1) above, is

$$
\begin{aligned}
C_t^H \;=\; & C_t - \{_{t-1}p_x\,(\phi_{t-1}S_t + \xi_{t-1}e^r)\} + {}_t p_x H_t \\
& +\tau\,S_t\,[(_t p_x\,(|\phi_{t-1} - \phi_t|)) + (_{t-1}p_x - {}_t p_x)\,|\phi_{t-1}|]
\end{aligned}
\tag{2}
$$

where the term in $\{\}$ is the hedge brought forward from the previous re balancing and the term in $[]$ is the transactions costs, allowing for the survival or exit of the policyholder in the interval (t-1, t). C_0^H is the hedge portfolio H_0 and in the final month:

$$
\begin{aligned}
C_n^H \;=\; & {}_n p_x \max(G - F_n, 0) - \{_{t-1}p_x\,(\phi_{n-1}S_n + \xi_{n-1}e^r)\} \\
& +\tau\,S_n\,[(_n p_x\,(|\phi_{n-1} - \phi_n|)) + (_{n-1}p_x - {}_n p_x)\,|\phi_{n-1}|]
\end{aligned}
\tag{3}
$$

In this modeling, the risk neutral (Q) probability distribution would be used to determine the hedge portfolio (ϕ_t and ξ_t), but the modeling of S_t would use the investment model calibrated to Table 1, which is the fitted real world (P) measure.

The present value of the cashflows for the contract would be the sum of the discounted values of the cashflows. Stochastic simulation provides a distribution for the present value of the cashflows including the hedge cost.

7 Risk Measures

A Risk Measure is a mapping of a random variable to a real number. An example of a risk measure is the premium principle, which takes the distribution of possible future losses and maps it to an appropriate premium. In this section we consider risk measures for valuation and solvency.

Let L_0 represent the present value of the future liability for the contract. Under actuarial risk management this would be

$$
L_0 = \sum_{t=0}^{n} C_t e^{-rt}
\tag{4}
$$

where r is the risk free rate per month, continuously compounded. (Note that a stochastic approach to the interest rate or a term structure, or both, may be incorporated. A constant risk free rate is assumed here for simplicity.)

Using financial engineering risk management we discount the C_t^H cashflows instead of C_t.

In either case we have the simulated present value distribution at the risk free rate of the future liability. This is used as the basis for the valuation and the solvency capital for the contract.

7.1   Quantile Risk Measures

The p-quantile risk measure, for $0 \leq p \leq 1$ is V_p where

$$V_p = \inf \{V : Pr[L_0 \leq V] \geq p\} \tag{5}$$

This has a long actuarial tradition, and can be simply interpreted as the amount which, with together with interest at the risk free rate and with the subsequent management charge income will be sufficient to pay the liability when it arises with probability p.

The much discussed Value-at-Risk is an example of a quantile risk measure. Typically p =99% is used for the unhedged 10-day risk in a banking enterprise.

The quantile risk measure is very easy to apply when the liability is simulated. By ordering the simulations, the p-quantile risk measure is the (Np)th value of the ordered liability values, where N is the number of simulations (provided N is sufficiently large; the Npth value is actually an unbiased estimator of the $Np/(N+1)$-quantile).

The quantile risk measure has some disadvantages from a solvency or valuation viewpoint. First, it may be less than the mean liability. For example, a random variable X where

$$X = \begin{cases} 0 & \text{with probability} \ \ 0.99 \\ 1000 & \text{with probability} \ \ 0.01 \end{cases} \tag{6}$$

The 95% risk measure for this random variable X is $V_{0.95} = 0$, but the mean loss is E[X]=10.

This simple example illustrates a problem that is quite common for segregated fund guarantees, which often have a very small probability of a materially positive liability.

The second important problem with quantile risk measures is the high sample variability for different simulations, particularly if the number of simulations is not large and p is close to 1.0.

Problems with the quantile risk measure are discussed in more detail in Artzner et al (1999).

7.2   Conditional Tail Expectation

The $100p\%$ Conditional Tail Expectation or CTE(p) is a risk measure which has been shown to overcome the problems with the quantile risk measure. For $0 \leq p \leq 1$, define V_p as above. Then, in the simplest case where V_p does not lie in a probability mass, CTE(p) is

$$\text{CTE}_p = E[L_0|L_0 > V_p] \tag{7}$$

In the case where V_p falls in a probability mass, the CTE is calculated as follows.

Find $\beta' = \sup\{\beta : V_p = V_\beta\}$

$$\text{then} \quad CTE(p) = \frac{(1 - \beta')\, E[L_0|L_0 > V_p] + (\beta' - p)\, V_p}{1 - p} \tag{8}$$

The CTE is the expected loss **given** that the loss is in the upper $(1 - p)$ quantile of the distribution.

If $p = 0$, CTE$(p) = E[L_0]$. For any $p > 0$ the CTE must be greater than the mean, overcoming the first disadvantage of the quantile measure.

The CTE is easy to calculate using simulation output, as CTE(p) is the mean of the highest cost $100(1 - p)\%$ outcomes from the simulation. This is very simple to implement and to understand. By taking an average of the worst case projections, the estimate is more robust with respect to sampling error than the quantile method.

Actuaries accustomed to insolvency probabilities may want to interpret the CTE in terms of a quantile. In general the CTE at level p will approximate to the quantile
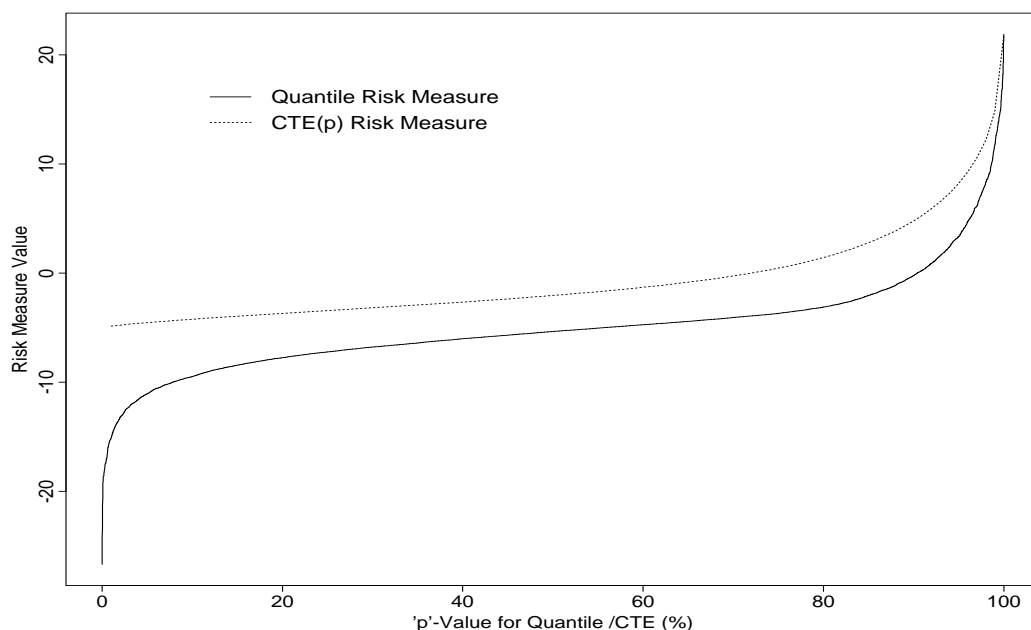
Figure 2: Quantile and CTE Risk Measures for Rollover Guarantee

risk measure at level $(1+p)/2$ – for example, a 90% CTE will approximate to a 95% quantile measure. If the loss is fat tailed (which is common for out-of-the-money guarantees) the CTE at level p will be greater then the quantile measure at $(1+p)/2$. At $p = 0$, the CTE is the mean of the losses, while the quantile risk measure at 0.5 is the median of the losses.

An illustration of the quantile and CTE risk measures is given in Figure 2. This shows the quantile and CTE for all values of p for a rollover type maturity guarantee. The sample contract has initial Fund \$100, management charge 3% per year of which 0.5% is available to fund the guarantee. There are 8 years to go to the next rollover and a maximum term to maturity of 28 years. Lapses of 8% per year are assumed. The policyholder is assumed to be 50 years old at the valuation date.

More information on risk measures, including quantile and CTE measures, is available in, for example, Wirch and Hardy (1999). The CTE measure is also known as Tail-VaR and is discussed in Artzner *et al* (1999)

8 Valuation

8.1 CTE

The CTE measure was selected by the Task Force as the basis for both the valuation of the guarantes and for the total balance sheet requirement including solvency capital. It has been shown to be relatively easily understood and adopted by the companies implementing valuation by stochastic simulation.

The choice of an appropriate value of p for the CTE used for policy liabilities needs to take into consideration market fluctuation and parameter and model uncertainty, under the terms of the Canadian rules requiring 'Provision for Adverse Deviation'.

8.2 Provision for market fluctuation

The provision for adverse deviation with respect to market risk should cover reasonable (not extreme) market fluctuation and parameter uncertainty. The Task Force suggested that 'reasonable fluctuation' would be captured by using a CTE at $p = 0.5$, that is using the mean of the worst 50% of losses. This gives a result generally somewhat greater than the 80th percentile of the distribution.

8.3 Provision for parameter uncertainty

To explore the effect of parameter uncertainty on the results, the Task Force used a Bayesian approach, under which the parameters are treated as random variables. Using Markov Chain Monte Carlo techniques it is possible to use stochastic simulation to generate a sample from the joint posterior distribution of the parameters. The posterior distribution combines any prior information about the model parameters with information from the data indicating feasible combinations of values for the parameters. The process was applied to the regime switching lognormal model with two regimes. The predictive distribution of the original loss random variable can then be simulated by sampling a different set of parameter values for each individual projection. The predictive distribution allows both for uncertainty in the loss distribution and for uncertainty in the loss distribution parameters. The centre of the predictive distribution should lie close to the centre of the distribution using a single parameter estimate (which may be taken as the mean of the posterior distribution for the parameters). However, the added uncertainty should increase the

variability and therefore the CTE(p) measure for the loss distribution, for higher values of p. The method is described in detail in Hardy(2001).

Two illustrations are given in Figure 3. The contract used in the left hand figure is a renewable 10-year contract, with 8 years to the next renewal. If the policyholder survives, the contract will be renewed twice, and so has a maximum term of 28 years. Cash payments to the policyholder will be made at each renewal date and at the maturity date if the fund is less than the guarantee in effect at those dates. The guarantee is increased at each renewal to the current fund value if that is greater. The guarantee in force also applies on death. The starting ratio of the market value to the guarantee is assumed to be 1.0. In the right hand diagram the contract has no reset or rollover features, and is assumed to have 19 years to maturity. The guarantee is payable on death or maturity. In both cases the policyholder is assumed to be age 50 at the start of the projection and lapses of 8% per year are assumed.

The curves plotted represent the CTE risk measure for all possible values of p. The solid line is the CTE curve with no allowance for parameter uncertainty. The broken line is the CTE curve using the different values for the parameters in each simulation, and therefore allows for parameter uncertainty. The Task Force looked for the value of p on the solid line corresponding to CTE(0.5) on the broken line. After considering a range of contract types, it appeared that a CTE level of between 55% and 75% for the model with no allowance for parameter uncertainty corresponds to a CTE of around 50% for the model which allows for parameter uncertainty. The lower end of the range is appropriate for contracts which are at or in the money and quite close to a maturity/renewal date, where the high probability of a cash payment at first renewal is more important than parameter uncertainty. The higher end of the range applies to the low cost guarantees – for example a maturity benefit some way off, with no voluntary reset or rollover of guarantees, so that the probability of any payment being required is very slim.

9 Solvency capital requirements

9.1 Actuarial Risk Management

By using two values for the CTE level p, for example $p = 0.7$ for valuation and $p = 0.95$ for solvency capital, consistency between the valuation and solvency capital are ensured. One set of simulations can be used to generate both numbers and the possibility of negative solvency capital is excluded.

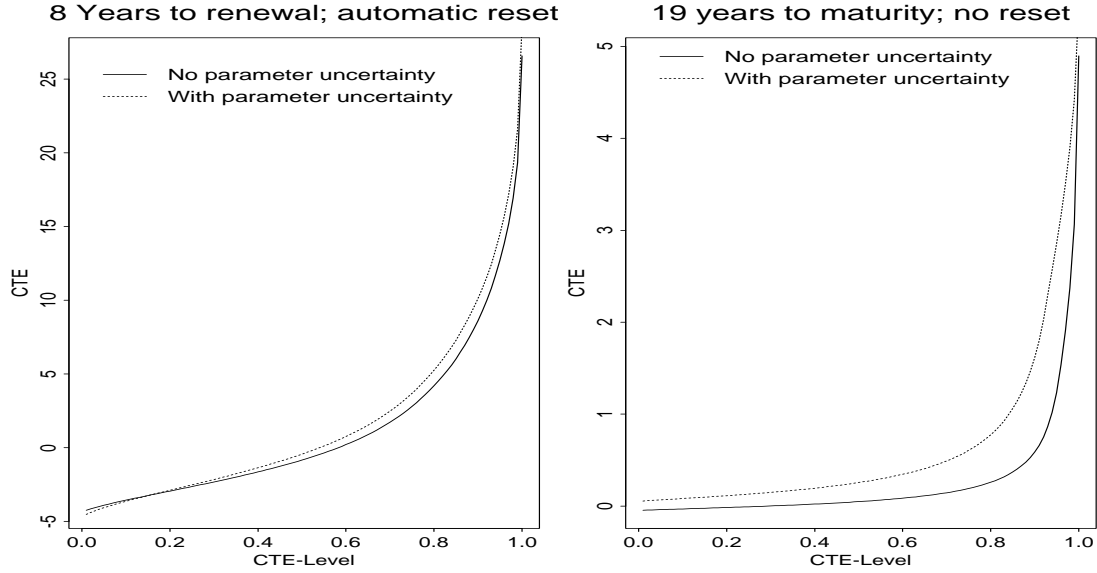For example we have projected the liability costs assuming actuarial reserving (no

Figure 3: CTE for rollover death/maturity guarantee and fixed death/maturity guarantee, with and without allowance for parameter uncertainty

hedging or reinsurance) for a rollover-type contract with 8 years to renewal and a maximum of two renewals before final maturity. We assume lapses at 8% per annum. The total expense charge is 3% per year, payable monthly, of which 0.5% is a premium for the guarantee and available to offset the guarantee cost. The segregated fund assets are assumed to be invested in a broad based fund with characteristics similar to the TSE 300. The initial fund value is $100 and the initial guarantee level is $85. The regime switching lognormal model was used to project the asset values.

A set of 10,000 projections of the fund up to the final maturity date (allowing deterministically for mortality) indicates that the probability that the premium income exceeds the guarantee outgo is around 91%. The density function from the simulated present values of net benefit is given in Figure 4. This shows the heavy probability that the net liability is negative, but also shows that there is a small probability of a substantial positive liability under the guarantee.

The CTE and quantile risk measures for this contract were shown in Figure 2 above. Some CTE figures are summarized in Table 3. The 95% quantile risk measure for this set of projections is 3.37% of the initial fund.

The insurer may adopt a policy liability basis for this contract of CTE(80%) and a total balance sheet provision (that is: valuation plus solvency capital) of CTE(95%), implying solvency capital of $6.78% of the initial fund would be required.
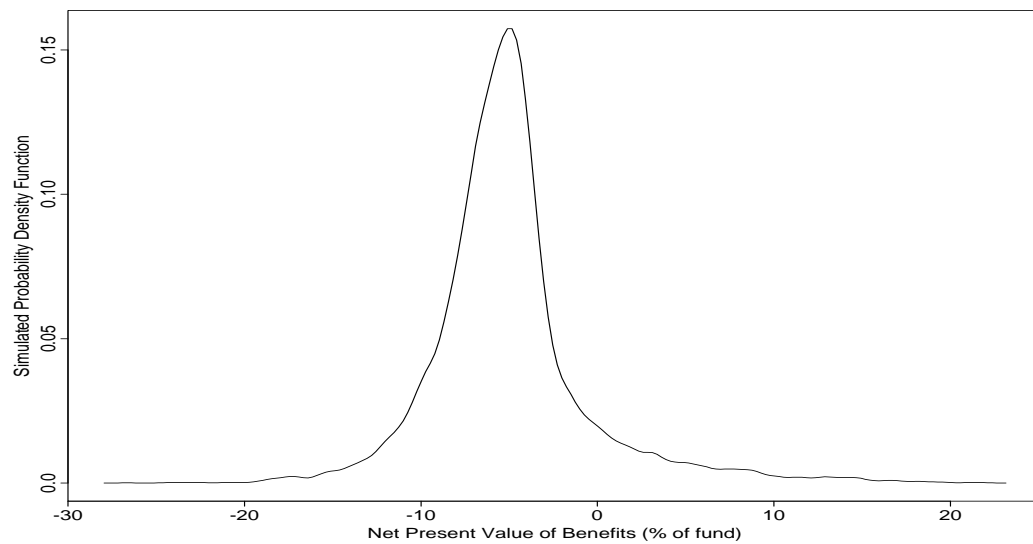
Figure 4: Simulated probability density function for net present value of rollover guarantee

p-value for CTE	CTE % of initial fund
70%	-0.26
75%	0.46
80%	1.43
85%	2.78
90%	4.79
95%	8.21

Table 3: CTE Values For Rollover Type Guarantee from Figure 4

9.2 Financial economics risk management

The same approach is proposed for financial economics risk management described in Section 6. The cash flows would be simulated and discounted and the CTE risk measure can be applied. However, because of additional uncertainty involved with the dynamic hedging strategy, additional margins may be required if the resulting valuation and solvency capital is smaller than that calculated assuming actuarial risk management.

10 The Factor Approach

The Task Force recognized a need for a more immediate answer to the question of how much capital is required to support the guarantees, bearing in mind that it will take some time for companies to develop systems for full stochastic simulation, and for the regulator to develop systems for auditing the company results. The Task Force therefore provided a set of tables for which insurers are required to use for the immediate future.

The tables were developed through extensive simulation of sample contracts using many different asset-liability models already in use in various companies and consultancies. The Tables given use a 90% CTE, though the regulator has since ruled that a 95% CTE would be more appropriate and revised tables have been prepared. The figures are split by contract type (death or maturity, rollover, reset or plain), with some (fairly rough) allowance for the 'moneyness' of the contract (fund value to guarantee value ratio), the remaining time to rollover, the management charge and guarantee premium level. Different factors apply to different underlying fund types, split into six categories from money market to aggressive equity. The tables are given in full in Appendix D of the Task Force report (SFTF, 2000). Insurers extract factors from each of four tables according to the properties of their contracts; some interpolation is permitted. The result is the total balance sheet requirement for the contract.

For the contract described in the section above, with CTE(90%) from Table 3 of $4.79% of the fund, the factor approach gives $5.02%. The difference arises because the table uses a combination of different investment models (all calibrated to Table 1) and because the allowance for the individual characteristics of the contract (such as management charge) are quite rough. The factor method clearly represents a substantial improvement over other deterministic methods that utilize a small set of stress testing deterministic investment scenarios.

It is proposed that the factors should be used until the regulator and the industry is confident that the insurers have stochastic models and systems that are capable of adequately predicting the risk profile of the business. There is expected to be a phase in period where the total capital requirements will be a weighted average of the results using factors and the results using internal modelling. A major issue at this time will be a move from seriatim valuation to whole portfolio valuation. Although different contracts are not independent, there is some diversification benefit from a portfolio with different maturity dates. (There is also benefit from diversification by fund, but some adjustment for this is included in the factor tables).

11   Conclusion

The stochastic methods proposed by the CIA Task Force have been generally welcomed in Canada. Implementation has persuaded some companies to withdraw or limit the guarantees from new products, as the MGWP did in the UK 20 years earlier. Other companies are taking up the challenge of adopting a more modern approach to risk management, implementing dynamic hedging strategies.

12   Acknowledgments

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