"Reducing Insolvency by Asset Mix of Life Insurance Company"

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Summary

Financial liberalization generally increases the volatility of the yields life insurance companies will earn on their assets under management. Such insurers must reduce this high volatility in an efficient manner. Corporate asset investment managers must therefore forecast the yield on an investment, its volatility, and the correlation between different asset investments. The inherent complexity of the financial market makes it extremely difficult to make such projections. However, through the management of a life insurance company, we can understand how these three indicators are related to one another by using the OMNI model, which this paper is intended to delineate.

"Konkurs-reduzierende Wirkung von Asset Mix (Vermögen-Mischung) von Versicherungsgesellschaften"

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Zusammenfassung

Mit zunehmender Liberalisierung der Finanzmärkte wird im allgemeinen die Volatilität der Renditen aus Kapitalanlagen von ebensversicherungsgesellschaften höher. Für die Lebensversicherungsgesellschaften wird es erforderlich, die erhöhte Volatilität wirkungsvoll zu reduzieren. Daher muss der Kapitalanleger Renditen aus Investitionen, deren Volatilität sowie die Korrelation zwischen den Investionen von verschiedenen Finanzmitteln voraussehen. Die Finanzmärkte sind sehr kompliziert, und es ist daher äußerst schwierig, die Entwicklung dieser Faktoren vorauszusagen. Aber wir können für den Betrieb von Lebensversicherungsgesellschaften die Relation zwischen diesen drei Faktoren ermitteln. Dies wird durch das im vorliegenden Beitrag vorgestellte OMNI-Modell ermöglicht.

Chapter 1 Risk Management Strategy for Insurance Companies

1. Preface

An insurance company manager must first determine Gross premium rate of a certain level and a net premium rate of a fixed level. Accordingly, the manager makes insurance payments, operates business, and conducts simulation to determine an attainable profit level. The level of an operating insurance premium in open competition is thus determined based on these results.

When calculating the profit ratio based on a constant gross premium rate, we know that the correlation between individual calculation base rates and product mix among individual products affect the profit rate. Because considering each calculation base rate independently is usually insufficient, the correlation between individual calculation base rates must be considered. Moreover, the profit ratio differs depending on whether such consideration is made for a single product or multiple products.

An insurance manager must first determine the gross premium rate for a single product, consider whether product mix can offer business operation at a lower gross premium rate, then determine the operating insurance premium rate.

This method of determining the premium rate supports the concept of an attainable profit rate by assuming that the company itself assumes the market risk. Thus, this method differs from the method of determining price on a cost basis, whereby an extra premium is added to the net premium (excluding the safety premium).

When using the first method, we must identify what is most important to the insurance company manager and the issues that require our attention.

A fund manager must project the return on investment, volatility, and correlation between individual investment media for each fund. The complexity of financial markets complicates such prediction. The management of insurance companies can provide insight to the relationship between these three elements. The OMNI (Omori = Matsumura = Nakagami = Ide) model described here allows us to attain the stated goals.

At the 26th ICA Conference in Birmingham, We suggested that introducing owner's equity could implement risk-taking management and ensure competitive power.

At the conference held in Birmingham, someone stated that low-risk, high-return phenomenon never occurs in an efficient market. At the annual conference of the Institute of Actuaries of Japan held in Tokyo, I responded by examining the simulation of a probability model for general interest rates and indicated the dividing point at which the low-risk, high-return phenomenon occurs. I concluded that the CIR (SR) model is a sufficient probability model for interest rates used in simulation.

At the 27th ICA Conference held in Mexico, We will measure, evaluate, and analyze the individual effects of mixed assets and mixed products by applying the OMNI model.

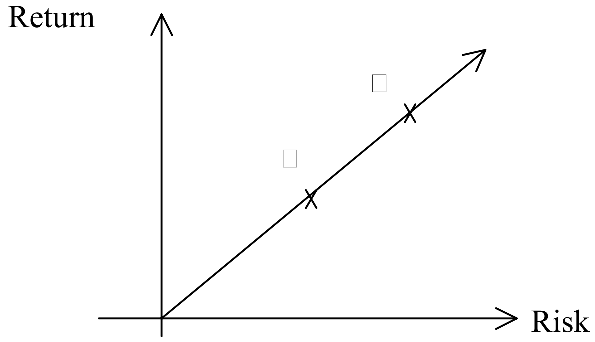
2 Risk-return relationship and probability of Insolvency

The administrative deregulation directed towards insurance companies allows the companies to act freely in financial markets, while being required to guarantee insurance benefits. Moreover, such benefits must be made with premiums as low as possible. In other words, benefits must be guaranteed when the same gross premiums are paid and dividends must be increased as much as possible. Thus, an actuary must determine product price by considering risk and return of the product in an open and transparent financial market.

One method considers the correlation between the risk and return of products. The drawback of this method is that all points with the same return level per unit risk (on the straight line extending from the origin shown in Figure 1.1) are treated at the same level, even though the probability of bankruptcy varies.

As an alternative, I propose a method in which product price is determined by reflecting the bankruptcy of insurance benefit.

Figure 1.1 Risk-Return plane



This paper focuses on the accumulation of shortage (as opposed to guaranteed price) at insurance maturity. Whether the bankruptcy of insurance benefit occurs before maturity is not a concern. We define the insolvency of insurance benefit as bankruptcy and the insolvency rate as the probability of bankruptcy.

Each point in Figure 1.1 (Risk-Return line) represents the risk and return of an insurance product at maturity. Point 1 represents product investment with low risk and low return; Point 2 represents high risk and high return by assuming the same premium.

As a matter of course, the probability of insolvency differs between products having the characteristics of 1 or 2.

However, the figure does not indicate the probability of insolvency. Thus, we must indicate the probability of insolvency for individual products.

3. Insolvency-Return plane

Markowitz developed the modern portfolio theory in financial markets. The OMNI model also considers the profit resulting from mortality and the difference between expected and actual expenses, as the OMNI model is applied to life insurance products.

Insurance policies are different from financial securities. Customers purchase an insurance policy based on an expected rate of return from the insurance company, and an insurance company settles the insurance policy at maturity of the insurance.

In discussing the return from insurance, return is defined as the interest rate at which the policyholder’s equity equals 0 at maturity. The gross and return are calculated based on certain conditions. In fact, the risk-return relationship differs from that in the modern portfolio theory. The insolvency-return plane is intended to incorporate the concept into actual insurance company management by replacing risk with insolvency.

The figure showing insolvency-return plane is a specialized form of the figure showing risk-return plane.

At the 26th ICA Conference in Birmingham, one technique applied (based on the modern portfolio theory) takes the capital investment of each product and its correlation into account.

At this conference, simulations were conducted separately for assets mix and product mix, followed by an evaluation and analysis of the results.

Figure 1.2 and 1.3 show the general conditions.

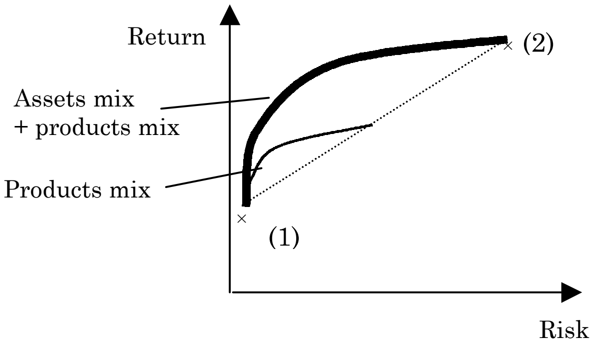


Figure 1.2 Risk-Return plane

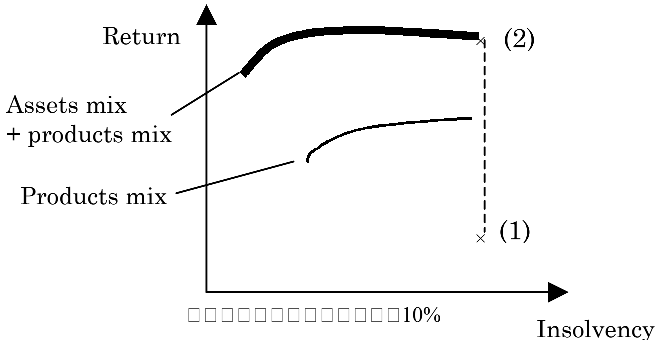


Figure 1.3 Insolvency-Return plane

4 Effects of assets mix and products mix on reducing risk

Assume that products having different structures of profit source are considered, such as endowment insurance and term insurance. Because customer needs differ for individual products, discussion should generally not be made solely along the risk-return lines. However, the risk-return lines shown in Figure 1.2 are used for discussion here. Point (1) represents risk and return for term insurance; Point (2) represents that for endowment insurance.

The efficient frontiers are indicated as two solid lines in Figure 1.2 showing the correlation between the yield on investments for endowment insurance and term insurance. The upper solid line represents the efficient frontier for assets mix and products mix combined; the lower solid line represents that for products mix only. We can see the reduction of risk in the fact that any solid line drawn according to the share of two products is above the dotted line (representing a correlation of 1.0).

The same reduction effect is reflected in the probability of Insolvency. Figure 1.3 illustrates this condition. The probability of Insolvency for both endowment insurance (2) and term insurance (1) is 10%. The efficient frontier is indicated by the solid line in Figure 1.3 for an optimal mix of products between endowment insurance and term insurance. When adding the effect of assets mix, the region expands in the direction of further reduced insolvency. The differences between Figures 1.2 and 1.3 are as follows: In Figure 1.2, the point at which risk is lowest at the fixed return level and the point which yields the highest return at a certain risk level can be selected. In contrast, Figure 1.3 illustrates the degree of reduction in insolvency, and the point where insolvency is lowest at a fixed return can be selected.

Chapter 2 Determination of Probability Model for General Interest Rates

1 Low-risk, high-return and high-risk, high-return

At the 26th ICA Conference in Birmingham, We presented an example of using the OMNI model intended to create a high-risk, high-return condition, while occasionally creating a low-risk, high-return condition. Some people remarked that this was too good to be true and hard to believe.

At the 26 ICA Conference in Birmingham, 100 paths were used in the simulation. Did the selection of interest paths cause the condition? Was the number of paths insufficient? Another question posed was whether the CIR (SR) model had any inherent problems. As discussion continued, a question arose regarding the relationship between this simulation and the individual coefficients of an estimated general type of interest rate model. To answer all these questions, in the paper introduced to the annual conference of the Institute of Actuaries of Japan, I modified the probability model for general interest rates as shown in equation (2.1) below.

$$dr = a(b\alpha - r)dt + r^{\gamma}\beta\sigma dZ \ldots\ldots (2.1)$$

The simulation was conducted with variable coefficients (e.g., drift term, γcoefficient, standard deviation σ) in the probability model for general interest rates. There were 1000 paths used in the simulation. Tables 2.1 and 2.2 show the results. Table 2.1 represents the CIR (SR) model; Table 2.2 represents the Brennan & Schwarz model.

When the probability of Insolvency is 10%, gross premium P is lower although α is higher in cases No.1 to No.4 which have variable α in both Tables 2.1 and 2.2 (based on the assumption that R/Σ is constant). The result is a low-risk, high-return condition.

When R/Σ remains constant, risk changes only to the same degree, whereas the interest rate increases α times higher and average interest rate R for interest rate r increases, which is considered the reason for above condition. Although average R increases, the change in risk is insufficient to offset the end result. Consequently, the gross premium with a 10% probability of Insolvency is lower in each situation.

The amount that falls below the payment at maturity also increases along with increased risk, however. Thus, a higher capital ratio is required. When the net worth of each insurance company is increased, each company can assume risk beyond the condition in which R/Σ is constant, and shift to the money markets in which the low-risk, high-return phenomenon does not occur because such phenomenon is short-lived.

Table 2.1 Endowment insurance (15-year term) proceeds at maturity
(Interest rate model: CIR (SR) model) (R/Σ = constant)

№	condition of interest rate model				P	proceeds at maturity			
	parameter		theoretical value	Simulation value		average (1)	MAX(2)	MIN(3)	(1-(3))/(1)
1	a = 1.0 b = 3.0 σ = 1.0 γ = 0.5	α = **1.0000** β = 1.4142	R = 3.0000 Σ = 1.7320 **R/Σ = 1.732**	R = 3.0027 Σ = 1.7582 **R/Σ = 1.707**	**0.063496 (e = 14.85%)**	1.07827	1.37725	0.94054	5.514%
		α = **1.1000** β = 1.4832	R = 3.3000 Σ = 1.9052 **R/Σ = 1.732**	R = 3.3030 Σ = 1.9340 **R/Σ = 1.707**	**0.062447 (e = 13.42%)**	1.08683	1.42201	0.93494	5.986%
		α = **1.2000** β = 1.5491	R = 3.6000 Σ = 2.0784 **R/Σ = 1.732**	R = 3.6033 Σ = 2.1098 **R/Σ = 1.707**	**0.061403 (e = 11.95%)**	1.09533	1.46787	0.92920	6.463%
2	a = 0.5 b = 3.0 σ = 1.0 γ = 0.5	α = **1.0000** β = 1.0000	R = 3.0000 Σ = 1.7320 **R/Σ = 1.732**	R = 3.0166 Σ = 1.7772 **R/Σ = 1.697**	**0.064619 (e = 16.33%)**	1.10019	1.50130	0.94324	5.159%
		α = **1.1000** β = 1.0488	R = 3.3000 Σ = 1.9052 **R/Σ = 1.732**	R = 3.3181 Σ = 1.9549 **R/Σ = 1.697**	**0.063758 (e = 15.20%)**	1.11201	1.58301	0.93838	5.541%
		α = **1.2000** β = 1.0954	R = 3.6000 Σ = 2.0784 **R/Σ = 1.732**	R = 3.6196 Σ = 2.1325 **R/Σ = 1.697**	**0.062868 (e = 14.00%)**	1.12335	1.62630	0.93295	5.968%
3	a = 0.3 b = 3.0 σ = 1.0 γ = 0.5	α = **1.0000** β = 0.7746	R = 3.0000 Σ = 1.7320 **R/Σ = 1.732**	R = 3.0249 Σ = 1.7930 **R/Σ = 1.687**	**0.065615 (e = 17.60%)**	1.11908	1.66168	0.97170	5.209%
		α = **1.1000** β = 0.8124	R = 3.2966 Σ = 1.9032 **R/Σ = 1.732**	R = 3.3240 Σ = 1.9702 **R/Σ = 1.687**	**0.064828 (e = 16.60%)**	1.13103	1.73947	0.93709	5.954%
		α = **1.2000** β = 0.8485	R = 3.5933 Σ = 2.0744 **R/Σ = 1.732**	R = 3.6232 Σ = 2.1475 **R/Σ = 1.687**	**0.064060 (e = 15.60%)**	1.14338	1.82102	0.93265	5.890%
4	a = 0.1 b = 3.0 σ = 1.0 γ = 0.5	α = **1.0000** β = 0.4587	R = 3.0000 Σ = 1.7320 **R/Σ = 1.732**	R = 3.0231 Σ = 1.7887 **R/Σ = 1.690**	**0.06642 (e = 18.87%)**	1.13818	1.80242	0.93498	5.712%
		α = **1.1000** β = 0.4794	R = 3.2330 Σ = 1.8666 **R/Σ = 1.732**	R = 3.2583 Σ = 1.9279 **R/Σ = 1.690**	**0.066238 (e = 18.38%)**	1.14841	1.87735	0.93192	5.928%
		α = **1.2000** β = 0.4992	R = 3.4661 Σ = 2.0011 **R/Σ = 1.732**	R = 3.4394 Σ = 2.0670 **R/Σ = 1.690**	**0.065814 (e = 17.85%)**	1.15837	1.95400	0.92860	6.163%

Table 2.2 Endowment insurance (15-year term) proceeds at maturity
(Interest rate model: Brennan & Schwarz model = 1000) (_R/Σ = constant)

№	condition of interest rate model				P	proceeds at maturity			
	parameter		theoretical value	simulation value		average (1)	MAX(2)	MIN(3)	(1-(3))/(1)
1	a = 1.0 b = 3.0 σ = 1.0 γ = 1.0	α = 1.0000 β = 0.7200	R = 3.0000 Σ = 1.7746 R/Σ = 1.691	R = 3.0068 Σ = 1.7365 **R/Σ = 1.731**	0.062927 (e = 14.08%)	1.06798	1.42442	0.95417	4.291%
		α = 1.1000 β = 0.7200	R = 3.3000 Σ = 1.9520 R/Σ = 1.691	R = 3.3075 Σ = 1.9102 **R/Σ = 1.731**	0.061802 (e = 12.52%)	1.07490	1.47292	0.94951	4.697%
		α = 1.2000 β = 0.7200	R = 3.6000 Σ = 1.1295 R/Σ = 1.691	R = 3.6000 Σ = 2.0784 **R/Σ = 1.732**	0.060696 (e = 10.92%)	1.08191	1.52268	0.94486	5.096%
2	a = 0.5 b = 3.0 σ = 1.0 γ = 1.0	α = 1.0000 β = 0.5010	R = 3.0000 Σ = 1.7367 R/Σ = 1.727	R = 3.0224 Σ = 1.7434 **R/Σ = 1.733**	0.063664 (e = 15.08%)	1.08294	1.64633	0.95967	3.724%
		α = 1.1000 β = 0.5010	R = 3.2998 Σ = 1.9100 R/Σ = 1.728	R = 3.3245 Σ = 1.9176 **R/Σ = 1.733**	0.062657 (e = 13.71%)	1.09165	1.71723	0.95564	4.063%
		α = 1.2000 β = 0.5010	R = 3.5997 Σ = 2.0834 R/Σ = 1.728	R = 3.6266 Σ = 2.0918 **R/Σ = 1.733**	0.061663 (e = 12.32%)	1.10049	1.79107	0.95161	4.397%
3	a = 0.3 b = 3.0 σ = 1.0 γ = 1.0	α = 1.0000 β = 0.3887	R = 3.0000 Σ = 1.7394 R/Σ = 1.725	R = 3.0293 Σ = 1.7486 **R/Σ = 1.732**	0.064288 (e = 15.90%)	1.09508	1.91004	0.95784	3.850%
		α = 1.1000 β = 0.3891	R = 3.2967 Σ = 1.9108 R/Σ = 1.725	R = 3.3290 Σ = 1.9220 **R/Σ = 1.732**	0.063384 (e = 14.70%)	1.10421	1.99717	0.95412	4.155%
		α = 1.2000 β = 0.3893	R = 3.5933 Σ = 2.0812 R/Σ = 1.727	R = 3.6287 Σ = 2.0946 **R/Σ = 1.732**	0.062469 (e = 19.45%)	1.11308	2.08623	0.94982	4.508%
4	a = 0.1 b = 3.0 σ = 1.0 γ = 1.0	α = 1.0000 β = 0.2368	R = 3.0000 Σ = 1.7611 R/Σ = 1.703	R = 3.0262 Σ = 1.7465 **R/Σ = 1.732**	0.065250 (e = 17.14%)	1.11301	2.14649	0.95271	4.248%
		α = 1.1000 β = 0.2399	R = 3.2331 Σ = 1.8985 R/Σ = 1.703	R = 3.2618 Σ = 1.8832 **R/Σ = 1.732**	0.064712 (e = 16.45%)	1.12024	2.23636	0.95075	4.396%
		α = 1.2000 β = 0.2425	R = 3.4661 Σ = 2.0346 R/Σ = 1.704	R = 3.4975 Σ = 2.0188 **R/Σ = 1.732**	0.064136 (e = 15.70%)	1.12680	2.32418	0.94825	4.693%

The value of R/Σ (which stabilizes efficient financial markets) is unknown, but if the low-risk, high-return phenomenon never occurs in the market, the degree of increased risk is estimated to be more than that of R in the market. Tables 2.3 and 2.4 represent risk and return based on a constant gross premium and β value using the equation of the probability model for general interest rates(2.1). When the gross premium is constant, β becomes even higher than the β in the situation in which R/Σ is constant.

Table 2.3 [CIR (SR) model] and Table 2.4 [Brennan & Schwarz model] show β and R/Σ at $\alpha = 1.0$ in each pattern in No.1 to No.4 with a constant gross premium. Tables 2.3 and 2.4 allow us to compare investment pattern A and other patterns for β value and Σ corresponding to different α. For example, the low-risk, high-return phenomenon is assumed to occur if β of investment pattern B is below 1.778 in No. 1 in Table 2.3. If this phenomenon actually occurs, it is considered a good investment.

However, β is different for No. 1 to No.4 in Tables 2.3 and 2.4. The reason why is that β depends on individual coefficients (α, γ) in the probability model for general interest rates. Therefore, we must first determine these coefficients, then find the dividing point at which the low-risk, high-return phenomenon occurs.

As stated earlier, assume that the low-risk, high-return condition is short term, and never lasts for long term in efficient markets.

Based on above concept, we first determine the value of α and γ, then determine the premium rate with a constant probability of Insolvency. Then we measure the insolvency of an insurance company by using the interest rate scenario with the highest insurance premium rate. This can be one of the methods used to evaluate the management basis of an insurance company.

The following shows an example.

Table 2.3 Endowment insurance (15-year term) proceeds at maturity
(Interest rate model: CIR (SR) model) (P = constant in each case No.)

№	condition of interest rate model				P	proceeds at maturity			
	parameter		theoretical value	simulation value		average (1)	MAX (2)	MIN(3)	(1-(3))/(1)
1	a = 1.0 b = 3.0 σ = 1.0 γ = 0.5	α = 1.0000 β = **1.4142** (Investment pattern A)	R = 3.0000 Σ = 1.7320 R/Σ = 1.732	R = 3.0027 Σ = 1.7582 **R/Σ = 1.707**	**0.063450** **(e = 14.85%)**	1.07827	1.37725	0.94054	5.514%
		α = 1.1000 β = **1.7780** (Investment pattern B)	R = 3.3000 Σ = 2.2838 R/Σ = 1.444	R = 3.3030 Σ = 2.4225 **R/Σ = 1.353**	**0.063450** **(e = 14.85%)**	1.10598	1.48613	0.92598	6.692%
		α = 1.2000 β = **2.5300** (Investment patternC)	R = 3.6000 Σ = 3.3943 R/Σ = 1.060	R = 3.4860 Σ = 3.1829 **R/Σ = 1.095**	**0.063450** **(e = 14.85%)**	1.14211	2.02162	0.93307	6.083%
2	a = 0.5 b = 3.0 σ = 1.0 γ = 0.5	α = 1.0000 β = **1.0000** (Investment pattern A)	R = 3.0000 Σ = 1.7320 R/Σ = 1.732	R = 3.0166 Σ = 1.7772 **R/Σ = 1.697**	**0.064619** **(e = 16.33%)**	1.10019	1.50130	0.94324	5.159%
		α = 1.1000 β = **1.2300** (Investment pattern B)	R = 3.2998 Σ = 2.2342 R/Σ = 1.476	R = 3.3114 Σ = 2.2848 **R/Σ = 1.449**	**0.064619** **(e = 16.33%)**	1.12946	1.71684	0.93504	5.751%
		α = 1.2000 β = **1.4470** (Investment pattern C)	R = 3.5996 Σ = 2.7452 R/Σ = 1.311	R = 3.6414 Σ = 2.9269 **R/Σ = 1.244**	**0.064619** **(e = 16.33%)**	1.15828	1.97542	0.93285	5.797%
3	a = 0.3 b = 3.0 σ = 1.0 γ = 0.5	α = 1.0000 β = **0.7746** (Investment pattern A)	R = 3.0000 Σ = 1.7320 R/Σ = 1.732	R = 3.0249 Σ = 1.7930 **R/Σ = 1.687**	**0.065615** **(e = 17.60%)**	1.11908	1.66168	0.97170	5.209%
		α = 1.1000 β = **0.9130** (Investment pattern B)	R = 3.2966 Σ = 2.1389 R/Σ = 1.541	R = 3.3240 Σ = 2.2076 **R/Σ = 1.508**	**0.065615** **(e = 17.60%)**	1.14679	1.89525	0.93833	5.377%
		α = 1.2000 β = **1.0835** (Investment pattern C)	R = 3.5933 Σ = 2.6489 R/Σ = 1.356	R = 3.6510 Σ = 2.7516 **R/Σ = 1.326**	**0.065615** **(e = 17.60%)**	1.16858	2.243132	0.92837	6.086%
4	a = 0.1 b = 3.0 σ = 1.0 γ = 0.5	α = 1.0000 β = **0.4587** (Investment pattern A)	R = 3.0000 Σ = 1.7320 R/Σ = 1.732	R = 3.0231 Σ = 1.7887 **R/Σ = 1.690**	**0.06642** **(e = 18.87%)**	1.13818	1.80242	0.93498	5.712%
		α = 1.1000 β = **0.5090** (Investment pattern B)	R = 3.2330 Σ = 1.9817 R/Σ = 1.631	R = 3.2618 Σ = 2.0447 **R/Σ = 1.595**	**0.06642** **(e = 18.87%)**	1.15655	1.98661	0.9329	5.854%
		α = 1.2000 β = **0.5548** (Investment pattern C)	R = 3.4661 Σ = 2.2236 R/Σ = 1.558	R = 3.5004 Σ = 2.2925 **R/Σ = 1.526**	**0.06642** **(e = 18.87%)**	1.17536	2.14008	0.93073	5.893%

Table 2.4 Endowment insurance (15-year term) proceeds at maturity
(Interest rate model: Brennan & Schwartz model) (P = constant in each case No.)

№	interest rate model				P	proceeds at maturity			
	parameter		theoretical value	simulation value		average (1)	MAX(2)	MIN(3)	(1-(3))/(1)
1	a = 1.0 b = 3.0 σ = 1.0 γ = 1.0	α = 1.0000 β = **0.7200** (Investment pattern A)	R = 3.0000 Σ = 1.7746 R/Σ = 1.691	R = 3.0068 Σ = 1.7365 **R/Σ = 1.731**	**0.062927** **(e = 14.08%)**	1.06798	1.42442	0.95417	4.291%
		α = 1.1000 β = **0.9500** (Investment pattern B)	R = 3.3000 Σ = 2.9925 R/Σ = 1.103	R = 3.3046 Σ = 2.7137 **R/Σ = 1.217**	**0.062927** **(e = 14.08%)**	1.09528	1.98169	0.94300	5.204%
		α = 1.2000 β = **1.1440** (Investment pattern C)	R = 3.6000 Σ = 4.9533 R/Σ = 0.727	R = 3.5909 Σ = 3.8193 **R/Σ = 0.940**	**0.062927** **(e = 14.08%)**	1.12337	1.94108	0.93550	5.742%
2	a = 0.5 b = 3.0 σ = 1.0 γ = 1.0	α = 1.0000 β = **0.5010** (Investment pattern A)	R = 3.0000 Σ = 1.7367 R/Σ = 1.727	R = 3.0224 Σ = 1.7434 **R/Σ = 1.733**	**0.063664** **(e = 15.075%)**	1.08294	1.64633	0.95967	3.724%
		α = 1.1000 β = **0.6305** (Investment pattern B)	R = 3.2998 Σ = 2.6798 R/Σ = 1.231	R = 3.3340 Σ = 2.6014 **R/Σ = 1.281**	**0.063664** **(e = 15.075%)**	1.11156	2.17976	0.95435	3.724%
		α = 1.2000 β = **0.7480** (Investment pattern C)	R = 3.5997 Σ = 4.0506 R/Σ = 0.889	R = 3.6426 Σ = 3.6722 **R/Σ = 0.991**	**0.063664** **(e = 15.075%)**	1.14202	2.98645	0.94869	4.107%
3	a = 0.3 b = 3.0 σ = 1.0 γ = 1.0	α = 1.0000 β = **0.3887** (Investment pattern A)	R = 3.0000 Σ = 1.7394 R/Σ = 1.725	R = 3.0293 Σ = 1.7486 **R/Σ = 1.732**	**0.064288** **(e = 15.9%)**	1.09508	1.91004	0.95784	3.850%
		α = 1.1000 β = **0.4780** (Investment pattern B)	R = 3.2967 Σ = 2.5741 R/Σ = 1.281	R = 3.3386 Σ = 2.4934 **R/Σ = 1.338**	**0.064288** **(e = 15.9%)**	1.12299	2.67313	0.95382	4.112%
		α = 1.2000 β = **0.5595** (Investment pattern C)	R = 3.5933 Σ = 3.7017 R/Σ = 0.971	R = 3.6450 Σ = 3.3698 **R/Σ = 1.081**	**0.064288** **(e = 15.9%)**	1.15316	3.93002	0.95146	4.209%
4	a = 0.1 b = 3.0 σ = 1.0 γ = 1.0	α = 1.0000 β = **0.2368** (Investment pattern A)	R = 3.0000 Σ = 1.7611 R/Σ = 1.703	R = 3.0262 Σ = 1.7465 **R/Σ = 1.732**	**0.065250** **(e = 17.14%)**	1.11301	2.14649	0.95271	4.248%
		α = 1.1000 β = **0.2690** (Investment pattern B)	R = 3.2331 Σ = 2.2091 R/Σ = 1.464	R = 3.2661 Σ = 2.1520 **R/Σ = 1.517**	**0.065250** **(e = 17.14%)**	1.13158	2.65928	0.95020	4.401%
		α = 1.2000 β = **0.2983** (Investment pattern C)	R = 3.4661 Σ = 2.6995 R/Σ = 1.284	R = 3.5055 Σ = 2.5765 **R/Σ = 1.360**	**0.065250** **(e = 17.14%)**	1.15103	3.34741	0.94879	4.449%

2 Determination of coefficient in general probability model for interest rates used in interest rate scenario

Suppose that average interest rate R and its standard deviation Σ for a certain period in efficient financial markets were calculated as R = 3.0% and Σ = 1.732%, respectively. The value range of “a” is between 1.0 and 0.5, and γ was actually 0.5 and 1.0. The γ of 0.5 represents the CIR (SR) model and 1.0 represents the Brennan & Schwartz model.

The gross premium is calculated with a 10% probability of Insolvency. Figure 2.1 shows the results.

Figure 2.1 Endowment insurance (15-year term) at maturity

a	α	γ = 0.5		γ = 1.0	
		β	P	β	P
1.0	1.0000	1.4142	0.063496 (e = 14.85%)	0.7200	0.062927 (e = 14.08%)
0.5	1.0000	1.0000	0.064619 (e = 16.33%)	0.5010	0.063664 (e = 15.08%)

According to the prudent-human rule, the case which produces a higher gross premium is suitable for the interest rate scenario used for simulation. In one sense, the case with a = 0.5 and γ = 0.5 of No.2 in the CIR (SR) model (shown in the equation below) is most suitable.

$dr = 0.5(3.0\%-r)dt+r^{0.5}0.4587dZ$

Based on these coefficients, the premium is calculated in a form whereby the low-risk, high-return phenomenon does not occur and the effect of product portfolio and investment portfolio is measured.

Chapter 3 Effects of Mixed Assets and Mixed Products on Reducing Risk

1 Base rate of calculating gross premium for endowment insurance, ten times term endowment insurance, and term insurance

The effects of mixed products on reducing the probability of insolvency is discussed here using an example of three products with different profit source structure, endowment insurance, ten times term endowment insurance (special endowment insurance), and term insurance.

The gross premiums of these three products are calculated using the OMNI model described in the Appendix. The process for yield on investment for each of the three products is described in the equation below.

$$dr_t = a(b\times\alpha - r_t)dt + \sqrt{r_t}\cdot\beta\cdot\sigma_r dB_r \quad \text{----------------- (3.1)}$$
$$(a = 0.5,\ \ b = 3.0,\ \ \sigma_r = 1.0)$$

Assume the correlation shown in Table 3.1 for the process for yield on investment for the three products.

The net premium rate is calculated based on an expected interest rate of 2.7%, mortality based on the l996 standard life table for insurance (in case of death) (q_{30+t}, t = 1to15), and a policy term of 15 years from age 30.

The gross insurance premium rate is determined by adding the expected operating expense rate to the net premium rate. In this case, the yield on investment is determined by equation (3.1), actual mortality by the process of mortality based on binomial distribution, and actual operating expense rate by the process of operating expense rate based on the process of inflation rate.

The gross premium rate is determined at a level of 10% probability for the asset share to be below the amount of required accumulation at maturity. Therefore, each product has a gross premium rate with a 10% probability of insolvency when only considering each product.

Table 3.1 Correlation table

	endowment	Special endowment	term	Inflation
Endowment	1			
Special endowment	ρ	1		
term	ρ	1	1	
Inflation	1	ρ	ρ	1

Table 3.2 shows the amounts of scheduled payment at maturity and at death for gross premium 1.

Table 3.2 Sums payable at maturity and at death for gross premium = 1

	estimated operating expense rate	gross premium rate	sum payable at maturity	sum payable at death
Endowment	e=12.97%	P=1.0	S=16.097	S=16.097
Special endowment	e=12.47%	P=1.0	S=13.3860	S=133.860
Term	e=12.95%	P=1.0	S=0.0	S=691.936

2 Scenario generating method

1) Conditions for an investment scenario and inflation rate model

Equation (3.2) (general probability model for interest rate) are used as an investment scenario model and inflation rate model. (meaning unclear)

$$dr_t = a(b \times \alpha - r_t)dt + r_t^{\gamma} \cdot \beta \cdot \sigma_r dB_r \text{ --------(3.2)}$$

a: adjustment factor, b: average regression level, σ:volatility, γ:sensitivity level of standard deviation for interest rate, α:adjustment factor for average regression level, β:adjustment factor for volatility, dB: an increment in Brownian movement

Parameters of the base model, investment model 1 (investment 1), are as follows: adjustment factor 0.5, regression level 3.0%, and standard deviation 1.0. Parameters of the inflation rate are adjustment factor 0.5, regression level 1.2%, and standard deviation 1.0. Assume the correlation coefficient (ρ) between investment models and inflation rate as base rate 0.5.

For investment scenarios (investment 2 and investment 3) other than the base model, adjust the average regression level (bα) of the model with α so that it produces a higher-risk, higher-return condition than the base investment scenario (investment 1). Then adjust the standard deviation ($\sigma\beta$) with β for investment 2 (α = 1.1) and investment 3 (α = 1.2) so that the probability of insolvency is 10% for product 1 (endowment insurance) as the basis.

Table3.3 Parameters

	a	b	α	σ	β	σ	r(0)
investment 1	0.5	3.0	1.0	1.0	1.0	0.5	3.0
investment 2	0.5	3.0	1.1	1.0	1.367	0.5	3.3
investment 3	0.5	3.0	1.2	1.0	1.6545	0.5	3.6
Inflation	0.5	1.2		1.0		1.0	1.2

(β for investment 2 and 3 is determined by the relationship between the scenario and gross premium.)

Table3.4 Correlation coefficients

	investment 1	investment 2	investment 3	inflation rate
investment 1	1			
investment 2	ρ	1		
investment 3	ρ	1	1	
inflation rate	1	ρ	ρ	1

(base rate of ρ = 0.5)

2) Mortality scenario

A mortality scenario is made based on the number of people living in each age group in the life table (for insurance against death) and the number of deaths determined through stratified sampling with uniform random numbers generated in accordance with binomial distribution for the number of deaths.

Number of deaths at age X D(X): $\sum_{k=1,D(X)} {}_{L(X)}C_k \, q(x+t)^k (1-q(x+t))^{L(X)-k}$ = uniform random numbers (0 to1)

L(X): Number of people living, q(x+t): mortality at age X

3) Operating expense rate scenario

An operating expense scenario is generated with the operating expense rate scenario based on the inflation rate scenario generated in (3.1).

$$E(t) = P\, e(t)$$
$$= P\, e(0)\, (\Pi_{i=1,t}\,(1+f(i)))$$

E(t): Operating expense scenario (period t) ,P: gross premium,
e(t): operating expense rate scenario (period t)
e(0): beginning value of operating expense rate scenario
f(t):inflation rate scenario (period t)

3 Asset share simulation

Asset share for each insurance product is calculated based on scenarios generated in 2 (previous paragraph) . The scenario generates 1000 paths each to determine the asset share of such cash flow items as gross premium, sum payable, and operating expenses.

$$AS(t+1) = \{(P(t) - E(t) + AS(t))(1 + r(t)) - S(t+1)\,q(x+t)\}/(1 - q(x+t))$$

AS(t): asset share (period t), P(t): gross premium (period t), E(t): actual operating expenses (period t), S(t): sum payable at death(period t), q(x+t): actual mortality(at age x+t), r(t): yield on investment (period t)

The result of calculation is evaluated using the following risk evaluation indexes for return, risk, and the probability of insolvency.

1) Return and risk

First, profit (R) is derived from the difference between asset share (at maturity), which was determined by the scenarios for investment , mortality, and operating expenses, and liability reserve (at maturity). Then rate of return (irr) is calculated based on profit (R) and the value derived with the expected rate of interest and gross premium. This procedure is repeated 1000 times, and the average of return rate (irr) is defined as the return and standard deviation of return rate (irr) as the risk.

$\Sigma_{t=1,15} P(1+irr)^t = \Sigma_{t=1,15} P\,(1+i)^t + R$
i : expected rate of interest

2) Probability of insolvency

Probability of insolvency is defined as the probability that the asset share (at maturity) determined after 1000 simulations based on the scenarios for investment, mortality, and operating expense is below the liability reserve (at maturity).

Table3.5 Liability reserve

year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
endowment	0.0	0.88	1.79	2.72	3.67	4.65	5.66	6.70	7.76	8.86	9.98	11.14	12.32	13.55	14.80	16.10
special endowment	0.0	0.79	1.59	2.42	3.27	4.13	5.00	5.89	6.79	7.71	8.63	9.57	10.52	11.47	12.43	13.39
term	0.0	0.31	0.63	0.93	1.21	1.46	1.67	1.83	1.93	1.96	1.92	1.79	1.55	1.18	0.67	0.0

4 Effects of asset portfolio and product portfolio on reducing risk

1) Effects of asset portfolio and product portfolio on reducing risk

Probability of insolvency is reduced by combining investment models for a product having a gross premium at a level of 10% probability of insolvency for the investment scenario, mortality scenario, and operating expense scenario. The decrease in probability of insolvency is defined as the effect of an asset portfolio on reducing risk. For example, combining investment scenarios with correlation can reduce risk.

The effect of reducing risk is measured in the combinations of investment 1 2, and 3 for product 1 (endowment insurance). In this case, the correlation coefficient (ρ) between investment 1 and investment 2 or 3 is presumed to be 0.5.

Figures 3.1 and 3.2 show the efficient frontiers in the risk-return relationship and insolvency-return relationship. The probability of insolvency drops (to 4.6% at most) when 70% of investment is made with investment 1 and 10% with investment 2 , 20% with investment 3 as the effect of an asset portfolio on reducing risk.

Figure 3.1 Effect of asset portfolio on reducing risk (in risk-return relationship)

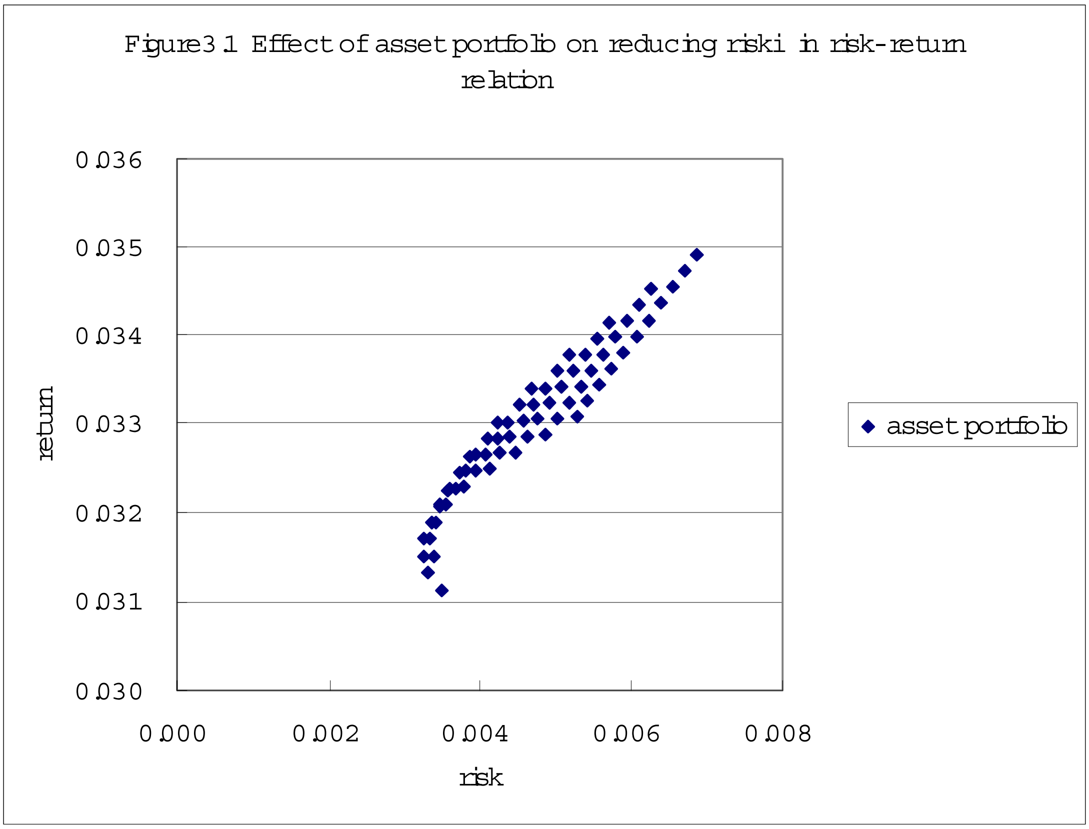


Figure 3.2 Effect of asset portfolio on reducing insolvency (in insolvency-return relationship)

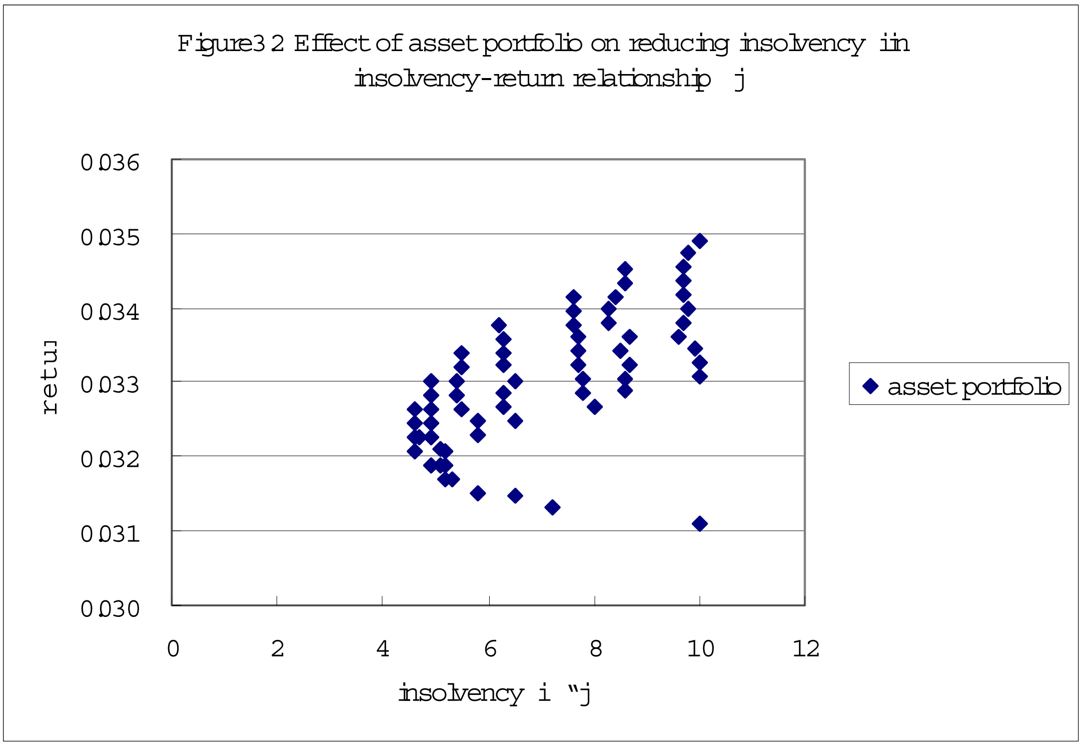


Table3.6 Effects of reducing risk and composition of assets

probability of insolvency (%) Case	4.6 (1)	4.6 (2)	4.6 (3)	10.0	10.0	10.0
return	0.03207	0.03225	0.03245	0.03111	0.03307	0.03491
risk	0.00347	0.00358	0.00373	0.00349	0.00529	0.00688
return/risk	9.242	9.008	8.700	8.914	6.251	5.074
investment 1	70	70	60	100		
investment 2	10		10		100	
investment 3	20	30	30			100

Considering each source of profit separately, the probability of insolvency (Def %) is 24.0% for profit resulting from the difference between actual and estimated return on investment alone. This is a 11.5% reduction in risk compared with the situation in which a 100% investment is made in investment 1. This suggests that the effect of an asset portfolio on reducing risk mainly results from the decrease in risk for profit resulting from investment.

Table3.7 Profit and loss by source of profit for asset portfolio

	investment 1 $\alpha = 1.0$ $\beta = 1.0$	investment 2 $\alpha = 1.1$ $\beta = 1.367$	investment 3 $\alpha = 1.2$ $\beta = 1.6545$	asset portfolio risk reduction effect
profit from all sources	return = 0.03111 risk = 0.00349 Def = 10%	return = 0.03307 risk = 0.00529 Def = 10%	return = 0.03491 risk = 0.00688 Def = 10%	return=0.03207 risk = 0.00347 Def = 4.6%
investment profit	return = 0.02849 risk = 0.00343 Def = 35.5%	return = 0.03045 risk = 0.00527 Def = 27.9%	return =0.03229 risk =0.00686 Def = 22.7%	return =0.02945 risk =0.00342 Def = 24.0%
mortality profit	return =0.02702 risk =0.00005 Def = 35.7%	return = 0.02702 risk = 0.00005 Def = 36.0%	return =0.02702 risk =0.00005 Def = 36.1%	return =0.02702 risk =0.00005 Def = 35.6%
expense profit	return =0.02964 risk =0.00029 Def = 0.0%	return =0.02969 risk =0.00030 Def = 0.0%	return =0.02974 risk =0.00033 Def = 0.0%	return =0.02967 risk =0.00029 Def = 0.0%

Conversely when discussing the effect of a product portfolio on reducing risk, the probability of insolvency decreases by combining products for a product having a gross premium determined at the level of 10% probability of insolvency in each scenario for investment, mortality, and operating expenses. For example, combining endowment insurance and term insurance decreases the probability of insolvency because the profit resulting from the investment for endowment insurance and mortality profit from term insurance cancel out the risk.

The effect of reducing risk is measured by combining a portfolio of savings-type products with a portfolio of guaranteed-type products, such as endowment insurance and term insurance. Figures 3.3 and 3.4 show the efficient frontiers for the risk-return relationship and insolvency-return relationship. By investing 70% in product 1 and 30% in product 3, the probability of bankruptcy decreases to 4.4%.

Figure 3.3 Effect of product portfolio on reducing risk (in risk-return relationship)

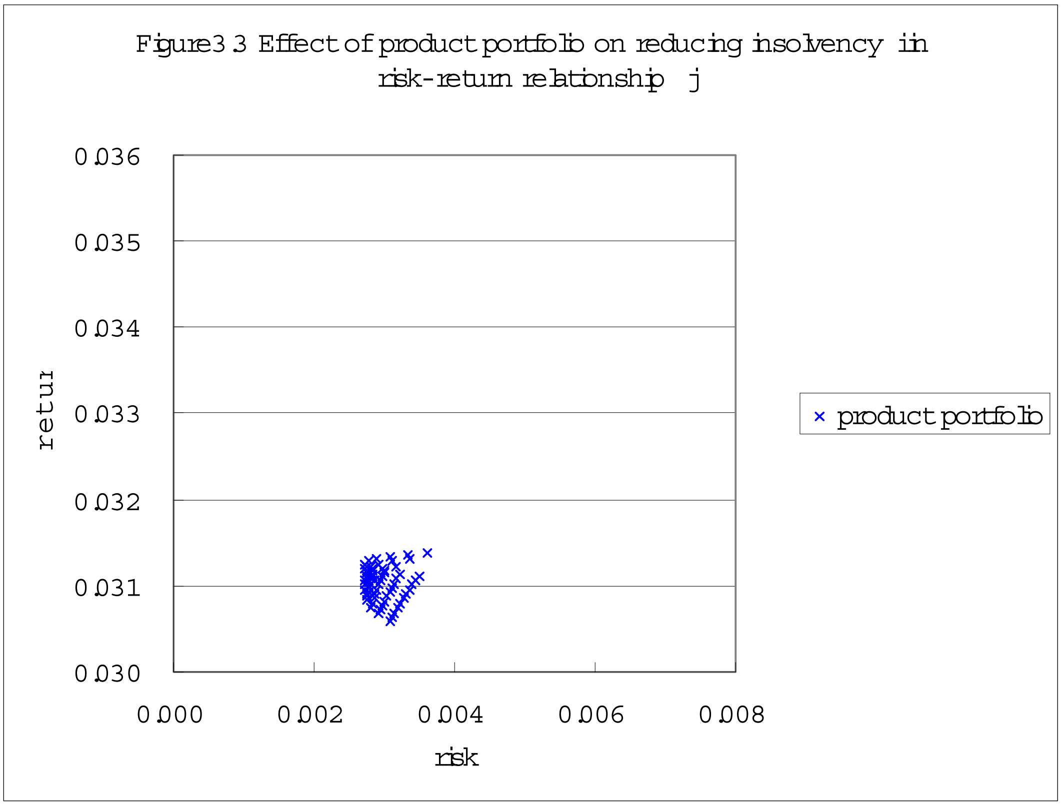


Figure 3.4 Effect of product portfolio on reducing insolvency (in insolvency-return relationship)

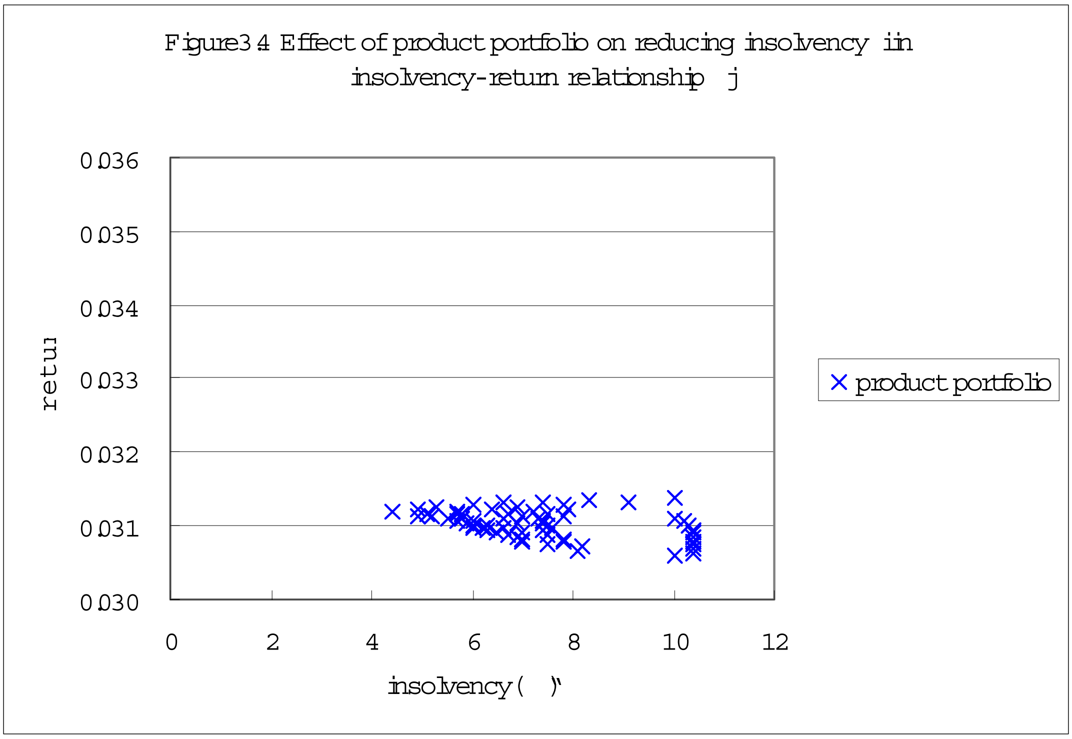


Table3.8 Effects of reducing risk and composition of products

probability of insolvency (%) Case	4.4 (1)	4.9 (2)	4.9 (3)	10.0	10.0	10.0
return	0.03119	0.03122	0.03114	0.03111	0.03059	0.03139
risk	0.00284	0.00275	0.00282	0.00349	0.00308	0.00360
return/risk	10.982	11.353	11.043	8.914	9.932	8.719
investment 1	70	60	60	100		
investment 2			10		100	
investment 3	30	40	30			100

Considering each source of profit, risk does not decrease in terms of the probability of insolvency for a single product and product portfolio for profit derived from mortality, profit resulting from a higher investment return than estimated, and profit resulting from actual expenses less than estimated. This suggests that the effect of a product portfolio on reducing risk results from the profit derived from a higher investment return and the mortality profit canceling out each other's risk.

Table3.9 Profit by source for product portfolio

	(endowment) e = 12.97% P = 1□S = 16.097	(special endowment) e = 12.47% P = 1□S = 133.860	(term) e = 12.95% P = 1□S = 691.936	effect of product portfolio on reducing risk
profit from all sources	return=00.03111 risk=0.00349 Def = 10%	return=0.03059 risk=0.00308 Def = 10%	return= 0.03139 risk= 0.00360 Def = 10%	return= 0.03119 risk= 0.00284 Def = 4.4%
investment profit	return=0.02849 risk=0.00343 Def = 35.5%	return=0.02833 risk=0.00298 Def = 35.1%	return=0.02750 risk=0.00096 Def = 32.0%	return= 0.02819 risk= 0.00260 Def = 34.2%
mortality profit	return=0.02702 risk=0.00005 Def = 35.7%	return=0.02725 risk=0.00067 Def = 36.1%	return=0.02832 risk=0.00362 Def = 36.2%	return=0.02741 risk=0.00111 Def = 36.3%
expense profit	return=0.02964 risk=0.00029 Def = 0.0%	return=0.02904 risk=0.00028 Def = 0.0%	return=0.02962 risk=0.00029 Def = 0.0%	return=0.02964 risk=0.00029 Def = 0.0%

2) Effect of reducing risk with restrictions on ratio for products and assets

The degree of decrease in the probability of insolvency is measured for cases with restrictions on composition ratio. For example, assume that a certain range was identified for the asset composition ratio by management side, and that for the product composition ratio by the sales side. The effect of reducing risk is measured by assuming a constant asset composition ratio and product composition ratio.

The effect of reducing risk is measured by combining the asset portfolio to a product portfolio having a certain product ratio. Assume the restriction imposed on product composition ratio requires 70% in endowment insurance and 30% in term insurance. When insurance policies are sold and investments made with endowment insurance (50% in investment 1, 10% in investment 2, 10% in investment 3) and term insurance (30% in investment 3), the probability of insolvency decreases to 2.0%.

Figure 3.5 Compound effects of asset portfolio and product portfolio (in risk-return relationship)

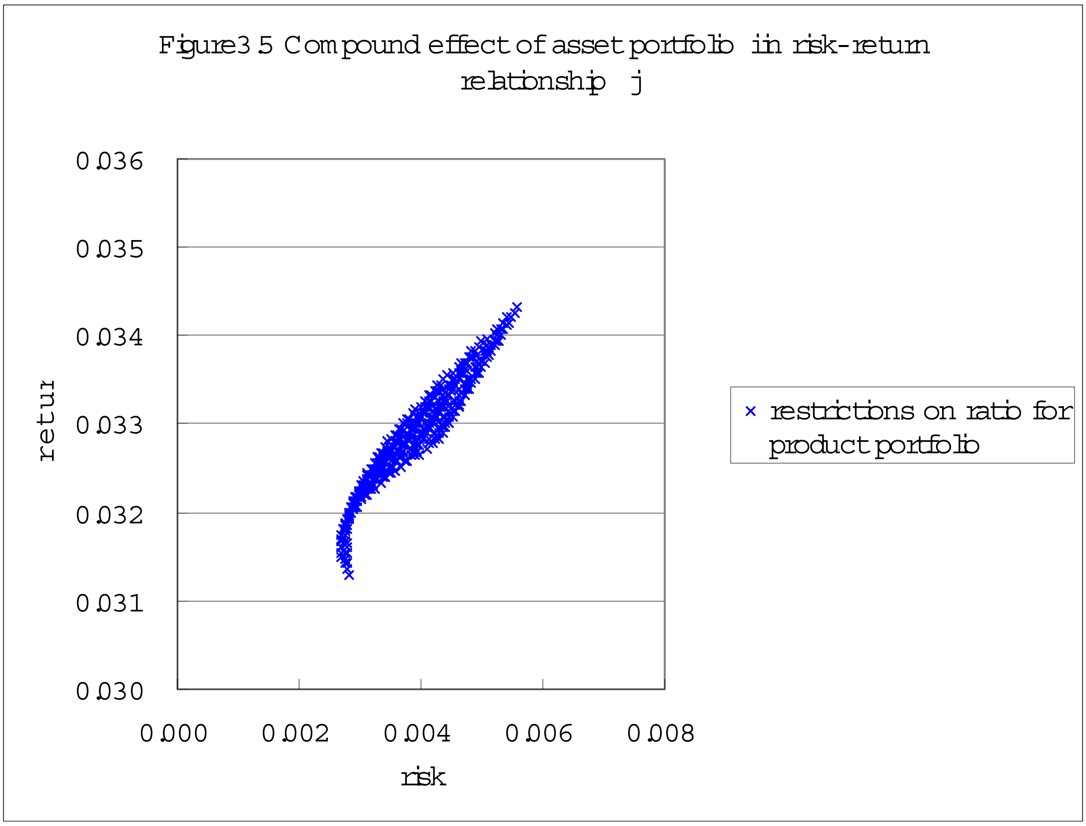


Figure 3.6 Compound effects of asset portfolio and product portfolio (in insolvency-return relationship)

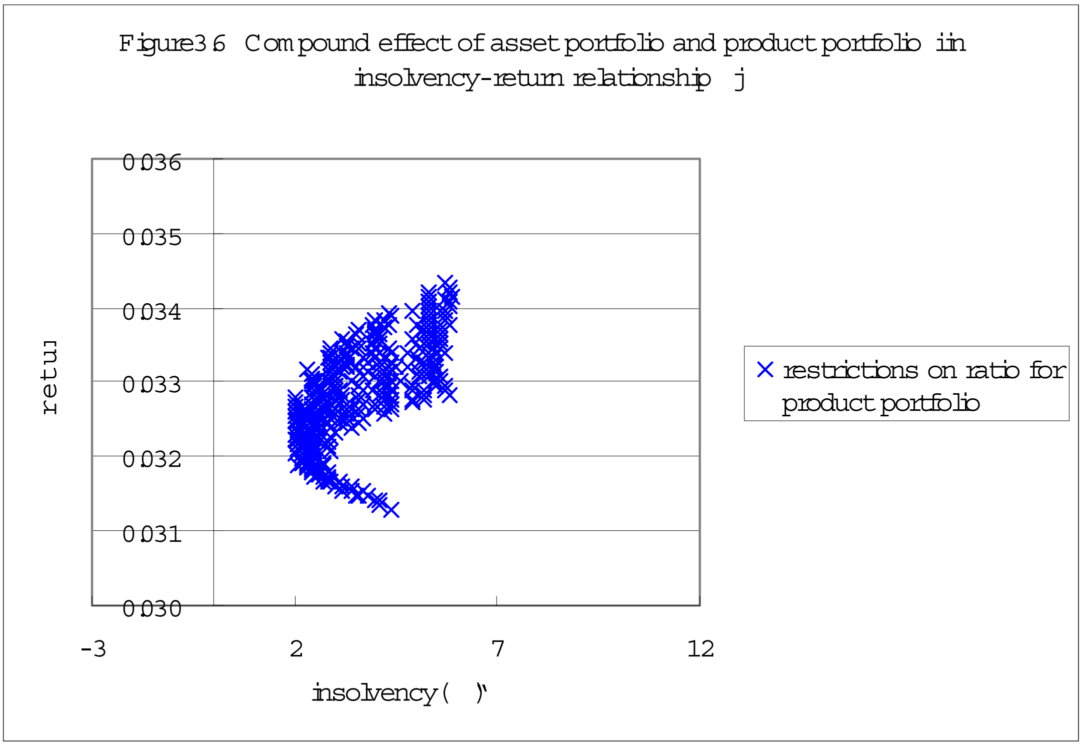


Table 3.10 Effects of reducing risk

Probability of insolvency(%) Case		2.0 (1)	2.0 (2)	2.0 (3)	2.0 (4)	2.0 (5)
return		0.03205	0.03224	0.03230	0.03249	0.03255
risk		0.00286	0.00297	0.00301	0.00320	0.00327
return/risk		11.206	10.855	10.731	10.153	9.954
Product 1		70	70	70	70	70
	Investment 1	50	50	50	40	40
	Investment 2	10			10	10
	Investment 3	10	20	20	20	20
Product 2						
	Investment 1					
	Investment 2					
	Investment 3					
Product 3		30	30	30	30	30
	Investment 1		10	10	10	
	Investment 2	30	10			10
	Investment 3		10	20	20	20
Total						
	Investment 1	50	60	60	50	40
	Investment 2	40	10		10	20
	Investment 3	10	30	40	40	40

The effect of reducing risk is measured by combining the product portfolio to an asset portfolio having a certain asset ratio. Assume the restriction imposed on investment composition ratio requires 70% of investment to be made in investment 1 and 10% of investment in investment 2, 20% of investment in investment 3. If investments were made for endowment insurance (50% in investment 1, 20% in investment 3), and for term insurance (20% by investment 1, 10% in investment 2), the probability of bankruptcy decreases to 2.2%.

Figure 3.7 Compound effects of asset portfolio and product portfolio (in risk-return relationship)

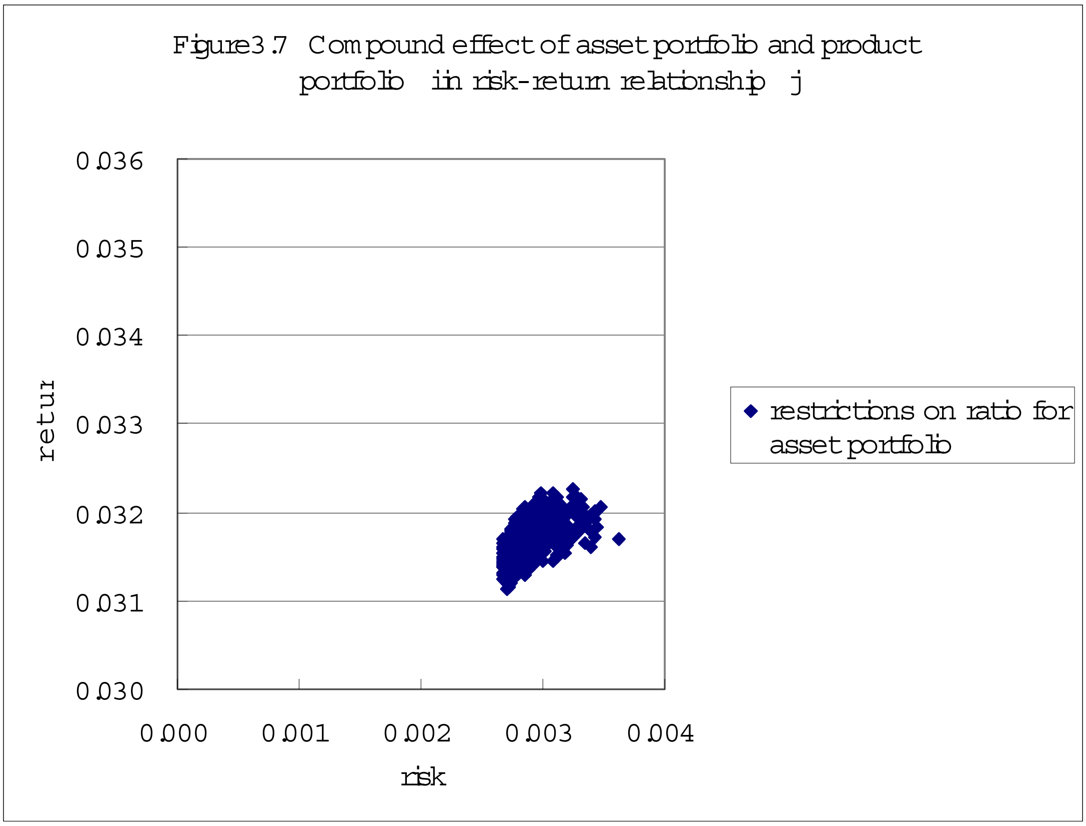


Figure 3.8 Compound effects of asset portfolio and product portfolio (in insolvency-return relationship)

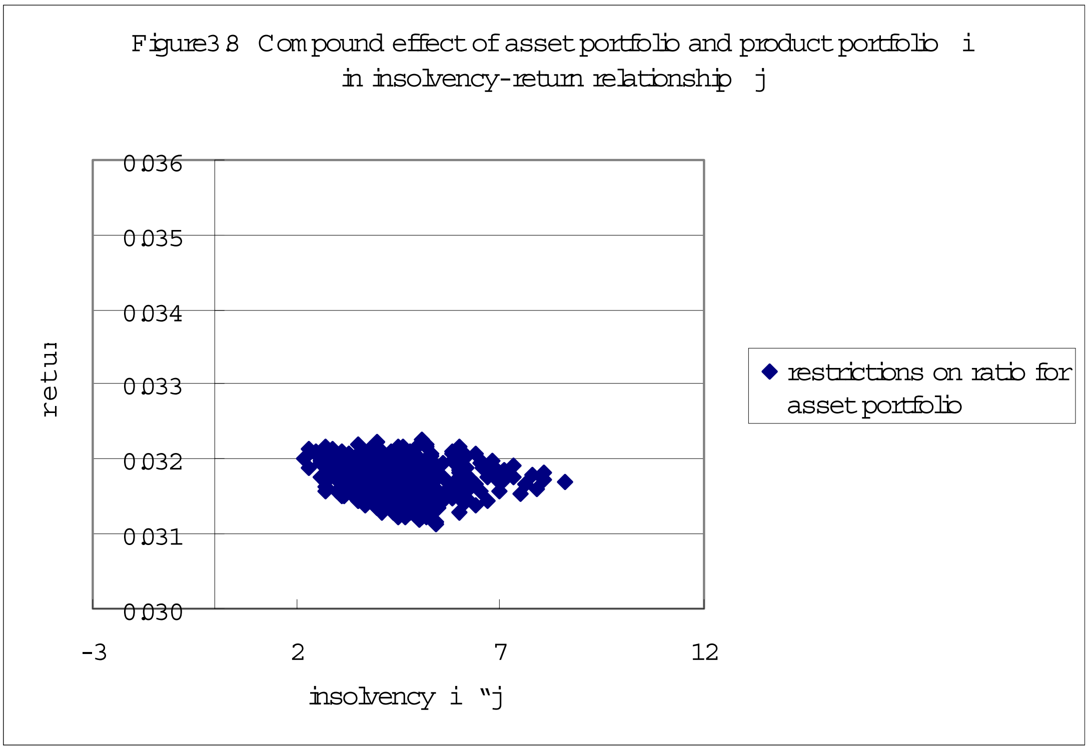


Table 3.11 Effects of reducing risk

Probability of insolvency (%) Case		2.2% (1)	2.3% (2)	2.3% (3)	2.5% (4)	2.5% (5)
return		0.03202	0.03189	0.03215	0.03206	0.03208
risk		0.00289	0.00282	0.00299	0.00295	0.00296
return/risk		11.080	11.309	10.753	10.868	10.838
Product 1		70	70	70	60	60
	Investment 1	50	50	40	40	40
	Investment 2		10	10	10	
	Investment 3	20	10	20	10	20
Product 2					10	10
	Investment 1					
	Investment 2					10
	Investment 3				10	
Product 3		30	30	30	30	30
	Investment 1	20	20	30	30	30
	Investment 2	10				
	Investment 3		10			
Total						
	Investment 1	70	70	70	70	70
	Investment 2	10	10	10	10	10
	Investment 3	20	20	20	20	20

In actual business practices, a decision must be made for selecting mixed assets from cases(1) to (5) in Table 3.10 (Effects of reducing risk). What must be considered in selecting a case is whether the probability of insolvency is stable regarding the diversity in asset investment and variable correlation coefficient ρ. The stability in the probability of insolvency regarding variable correlation factor ρ is verified as described below.

5 Effects of reducing and optimal composition ratio for each correlation factor in investment model

Up to this point, we assumed the correlation coefficient in the investment model as ρ = 0.5. In this section, however, variable correlation coefficient ρ = -0.5 to 1.0 is used to calculate the effects of asset portfolios and product portfolios on reducing compound risks and the optimum composition ratio.

1) Effects of reducing risk and optimum composition

The degree of decrease resulting from the compound effect of asset portfolios and product portfolios on the probability of insolvency (Def %) is examined by applying variable correlation factor among investment models ρ. As shown in the table below, if the correlation coefficient among investment models changes between -0.5 to 1.0, the probability of insolvency changes from 0.2% to 3.9%.

Table3.12 Effects of reducing risk based on each correlation coefficient and composition

Investment model	Correlation coefficient	ρ = -0.5	ρ = 0.0	ρ = 0.5	ρ = 0.8	ρ = 1.0
Compound effect	Effect	ret=0.03163 risk=0.00234 Def = 0.2%	ret=0.03192 risk=0.00266 Def = 1.2%	ret=0.03195 risk=0.00286 Def = 2.0%	ret=0.03203 risk=0.00314 Def = 3.3%	ret=0.03342 risk=0.00434 Def = 3.9%
	Composition (Product 1)	Invest1 = 60% Invest2 = 30% Invest3 =	Invest1 = 40% Invest2 = 30% Invest3 =	Invest1 = 50% Invest2 = 10% Invest3 = 10%	Invest1 = 40% Invest2 = 10% Invest3 = 10%	Invest1 = Invest2 = Invest3 = 50%
	(Product 2)	Invest 1 = Invest 2 = Invest 3 =	Invest 1 = Invest 2 = Invest 3 =	Invest 1 = Invest 2 = Invest 3 =	Invest 1 = Invest 2 = Invest 3 =	Invest 1 = Invest 2 = Invest 3 =
	(Product 3)	Invest1 = 10% Invest2 = Invest3 =	Invest1 = Invest2 = 30% Invest3 =	Invest1 = Invest2 = 30% Invest3 =	Invest1 = Invest2 = 30% Invest3 = 10%	Invest 1 = Invest 2 = Invest3 = 50%

2) Consideration of optimum composition

The first row in the table below shows an index for risk with the optimum composition when the correlation coefficient among investment models is -0.5 to 1.0. The rows beneath show how the risk index changes when the correlation coefficient changes between -0.5 and 1.0, while maintaining the optimum composition for each correlation coefficient. For example, if the correlation changes from ρ = 0.5 to ρ = 0 at an optimum composition of ρ = 0.5, the probability of bankruptcy decreases from 1.8% to 1.3%, thus indicating a reduction in risk. Conversely, the probability of bankruptcy increases to 3.7% when the correlation is increased to ρ = 0.8.

Figure 3.9 Change of probability of insolvency by optimum composition

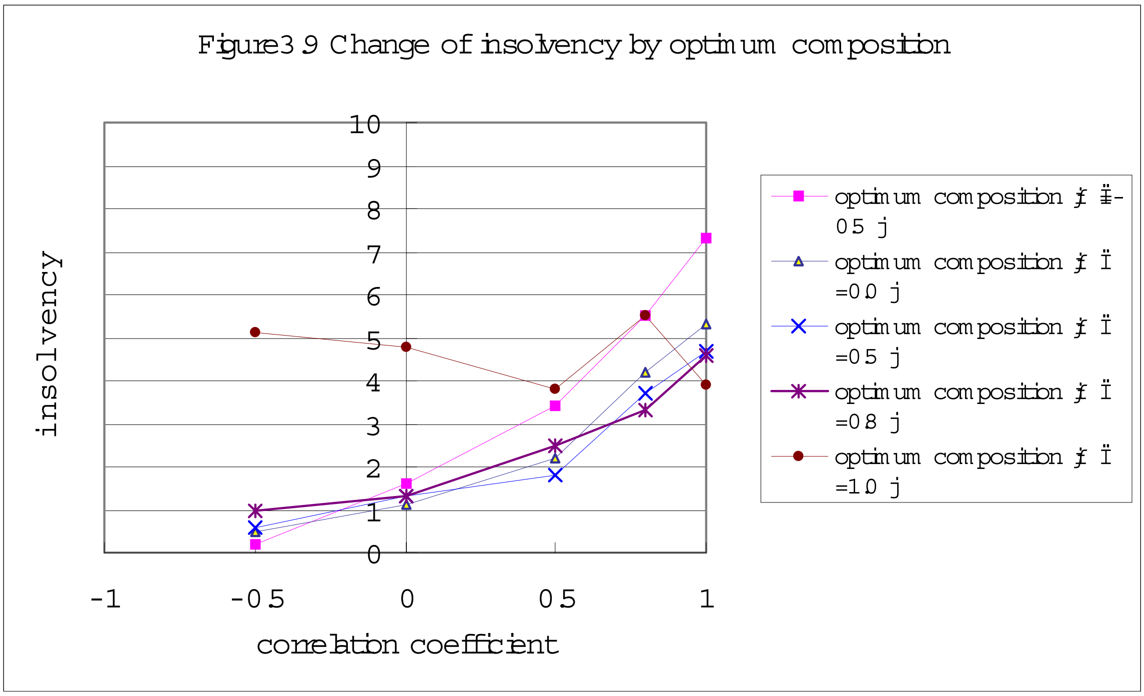


Table3.13 Change of risk reduction effect by optimum composition

Investment model	Correlation coefficient	ρ = -0.5	ρ = 0.0	ρ = 0.5	ρ = 0.8	ρ = 1.0
Compound effect	Optimum composition (each ρ)	ret=0.03236 risk=0.00273 Def = 0.2%	ret=0.03219 risk=0.00280 Def = 1.1%	ret=0.03280 risk=0.00345 Def = 1.8%	ret=0.03206 risk=0.00310 Def = 3.3%	ret=0.03342 risk=0.00434 Def = 3.9%
	Optimum Composition (ρ = -0.5)	ret=0.03236 risk=0.00273 Def = 0.2%	ret=0.03246 risk=0.00313 Def = 1.6%	ret=0.03251 risk=0.00346 Def = 3.4%	ret=0.03239 risk=0.00384 Def = 5.5%	ret=0.03238 risk=0.00421 Def = 7.3%
	Optimum Composition (ρ = 0.0)	ret=0.03209 risk=0.00243 Def = 0.5%	ret=0.03219 risk=0.00280 Def = 1.1%	ret=0.03222 risk=0.00311 Def = 2.2%	ret=0.03213 risk=0.00342 Def = 4.2%	ret=0.03211 risk=0.00372 Def = 5.3%
	Optimum Composition (ρ = 0.5)	ret=0.03262 risk=0.00287 Def = 0.6%	ret=0.03274 risk=0.00320 Def = 1.3%	ret=0.03280 risk=0.00345 Def = 1.8%	ret=0.03267 risk=0.00371 Def = 3.7%	ret=0.03264 risk=0.00397 Def = 4.7%
	Optimum Composition (ρ = 0.8)	ret=0.03203 risk=0.00237 Def = 1.0%	ret=0.03211 risk=0.00262 Def = 1.3%	ret=0.03214 risk=0.00285 Def = 2.5%	ret=0.03206 risk=0.00310 Def = 3.3%	ret=0.03205 risk=0.00330 Def = 4.6%
	Optimum Composition (ρ = 1.0)	ret=0.03338 risk=0.00450 Def = 5.1%	ret=0.03357 risk=0.00468 Def = 4.8%	ret=0.03366 risk=0.00467 Def = 3.8%	ret=0.03346 risk=0.00444 Def = 5.5%	ret=0.03342 risk=0.00434 Def = 3.9%

6 Conclusion

This paper has described the measurement of the effects of asset portfolios and product portfolios on reducing risk under certain conditions using the OMNI model first. In the discussion of product portfolios, reduction in risk was examined as the mutual cancellation of risk between mortality profit and profit derived from investment. In the discussion of asset portfolios, the reduced risk resulting from correlation among investment models was examined. Under the conditions specified in this paper, the effect of product portfolios on reducing risk was found to be equivalent to the effect of correlation among investment models having a ρ = 0.5 correlation coefficient.

In actual business practices, it is important to consider the restrictions imposed on product composition and asset composition. Consequently, the effects of asset portfolios and product portfolios on reducing risk were measured for cases subject to restrictions. As a result, the approach in which asset composition is determined after determining product composition was found to further reduce risk.

Moreover, the effect of compound portfolios on reducing risk was measured with variable correlation among investment models, and each optimum composition ratio was determined. By focusing on optimum composition, changes in risk were examined by changing the correlation in investment models.

As a result, it was proved that risk increases when the correlation is high in an investment model, and that risk increases considerably when the correlation of the investment model increases while maintaining the optimum composition.

For these reasons, consideration must be given to each factor used in determining gross premiums. I suggest that we need to at least know the expected level of risk corresponding to the correlation coefficient of investment models, and discuss risk measures.

I am sure that this paper may help those involved in the management of life insurance.

Appendix 1 Overview of the OMNI model

The OMNI model is a multi-period model that focuses on the correlation between the scenarios based on the various stochastic models. Moreover, the flexibility of the OMNI model allows the scenarios to be suitably modified.

The OMNI model consists of an external environment model (established by stochastic model's scenarios) and a life insurance company model (dedicated to insurance company management).

The external environment model uses the probability differential equations for the market interest rate scenario and inflation scenario, and imports the correlation between scenarios.

The life insurance company model refers the scenario derived from the external environment model, especially the market rate of interest level and diversification. The investment yield process of life insurance products is derived from its correlation with the market rate of interest process.

In this way, it becomes possible to apply modern portfolio theories to product mix of life insurance products.

In other words, the probability of insolvency can be calculated in advance and applied to the pricing of life insurance products. One possible modification to the OMNI model is the approach to actual business practices by improving the probability theoretical scenario.

A path length used in simulating the market rate of interest and inflation rate using the external environment model was 15 years (terms). There were a total of 1000 paths.

The life insurance company model calculates the asset share for each period and judges whether it satisfies the payments guaranteed by the contract at insurance maturity. If asset share falls below guaranteed payment, the condition is defined as insolvency. The percentage of simulations that show unsatisfied guaranteed payments in 1000 simulations represents the probability of insolvency.

The OMNI model is as outlined as follows:

1) External environment model

The external environment model consists of following three processes:

Mortality process: based on a life table

Market rate of interest process: CIR model

$$d\delta_t = a(b-\delta_t)dt + \sqrt{\delta_t}\sigma_\delta dB_\delta$$

$$(a = 0.5, \qquad b = 3.0\%, \sigma_\delta = 1.0)$$

Inflation rate process:

$$d\xi_t = c(d-\xi_t)dt + \sigma_\xi dB_\xi$$

$$(c = 0.5, d = 1.2\%, \sigma_\xi = 1.0)$$

This inflation rate process is the probability differential equation modified from the inflation model of the Wilkie model.

Correlation between market rate of interest and inflation rate

$$dB_\delta dB_\xi = \rho_{\delta\xi} dt + o(dt)$$

$$(\rho_{\delta\xi} = 1)$$

2. Life insurance company model

The life insurance company model consists of the part generating actual mortality, rate of yield, expense rate corresponding to the external environment model, and the part generating proceeds at maturity, its diversification, and the probability of insolvency within the company.

A. Actual basis rates of calculation

A) Mortality process: follows binomial distribution.

a) Basis mortality

mortality: q(x) at age x based on a life table

b) Mortality scenario

scenario for living at age x: $\tilde{L}(x)$ scenario for dead at age x: $\tilde{D}(x)$

mortality scenario at age x: $\tilde{q}(x) = \tilde{D}(x) / \tilde{L}(x)$

c) Logic for generating mortality process

step 1) Living at age 0 $\tilde{L}(0)$ =100000 (per 100,000 population)

step 2) Determine the number of people living and dead at age x (x = 0 - 105).

step 2-1) Generate a uniform random number (R) [0 - 1].

step 2-2) Determine the distinguishing value (for dead) *m* at which accumulation value of binomial distribution based on mortality basis exceeds R.

$$R<(\Sigma_{k=1,m}\ {}_{L(x)}C_k\ (1-q(x))^{L(x)-k}\ q(x)^k)$$

step 2-3) Number of dead at age x is $\tilde{D}(x) = m$

mortality scenario is $\tilde{q}(x) = \tilde{D}(x) / \tilde{L}(x)$

step 2-4) Number of living at age (x+1) is

$$\tilde{L}(x+1) = \tilde{L}(x) - \tilde{D}(x)$$

Return to step 2-1) and repeat for x = 0 to 105.

B. Investment yield process for insurance products

$$d\delta_t = a(b \times \alpha - \delta_t)dt + \sqrt{\delta_t} \cdot \beta \cdot \sigma_\delta dB_\delta$$
$$(a = 0.5, b = 3.0\%, \sigma_\delta = 1.0)$$

a) Endowment insurance 1 $\alpha = 1, \beta = 1$

Endowment insurance 2 $\alpha = 1.1, \beta = 1.21$

Endowment insurance 3 $\alpha = 1.2, \beta = 1.44$

b) Endowment insurance : $\alpha = 1, \beta = 1$

Ten times term endowment insurance: $\alpha = 1, \beta = 1$

Term insurance: $\alpha = 1, \beta = 1$

C) Operating expense rate process

Lemma 1 increase rate of new contracts ≈ increase rate of received premiums = r_n

Lemma 2 increase rate of operating expenses E: $z_n \geq$ base up rate ≈ inflation rate in one period before ξ_{n-1}

Lemma 3 operating expenses E_t changes together with inflation rate

(initial value E_0 = P×10%)

$E_t = E_{t-1}(1+\xi_t)$

The inflation rate in the period $t=-1$: $\xi_{-1}=0$. Assume that increases in operating expenses up to one period before are canceled out by the increase in new contracts resulting from operating efforts in this period. Therefore, operating expenses increase only with the amount derived from this period's inflation rate.

$z_n = \xi_n$, $r_n = \xi_{n-1}$

From Lemma 1 to 3, the following equation is generated:

$$\tilde{e} = \frac{\text{operating expenses } E_n}{\text{operating income } P_n} = \frac{E_{n-1}(1+z_{n-1})}{P_{n-1}(1+r_{n-1})} = \frac{E_0(1+z_{n-1})\cdots(1+z_0)}{P_0(1+r_{n-1})\cdots(1+r_0)} = e_0\frac{(1+\xi_{n-1})\cdots(1+\xi_0)}{(1+\xi_{n-2})\cdots(1+\xi_{-1})}$$

$$= e_0\frac{1+\xi_{n-1}}{1+\xi_{-1}} = e_0(1+\xi_{n-1})$$

E_n: operating expenses

P_n: operating income

(initial value e_0 = 10%)

D. Correlation between market rate of interest and yield on investment

$$dB_i dB_j = \rho_{ij}dt + o(dt)$$

The following shows correlation ρ_{ij} between the market rate of interest and return on investment in the following four cases (-1,-0.5,0,0.5).

	Market rate	Product 1	Product 2	Product 3
Market rate	1			
Retrun on investment Product 1	1	1		
Return on investmentProduct 2	ρ	ρ	1	
Return on investment Product 3	ρ	ρ	1	1

B. Expected calculation base rate and calculation of reserve

A) Definition of expected calculation base rate

The expected calculation base rate is found based on the external environment model.

$i = b$ * 0.9 (b: average regression level of CIR model)

let $e = e_0$ *(1+γ) (Note that e is found where probability of insolvency is 10%.)

q: mortality

B) Calculation of reserve and premiums

Calculate the expected reserve and gross premiums corresponding to the sum payable.

The types of insurance are endowment insurance, special endowment insurance with tenfold proceeds, and term insurance.

$$\text{at beginning of period}: V_t = P(1-e) - Sq_{x+t} + V'_{t-1}$$
$$\text{at end of period}: V'_t = \frac{1+i}{1-q_{x+t}} V_t$$
$$\text{sum payable at maturity}: S_1 = \frac{1+i}{1-q_{x+n}} V_n = S \times \frac{1}{\alpha}$$

sum payable at death: S

endowment insurance : $\alpha = 1$, ten times term endowment insurance : $\alpha = 10$,
term insurance : $S_1 = 0$

C) Generating asset share
Asset share is generated based on actual calculation base rate $\tilde{i}_t$, $\tilde{q}_{x+t}$, $\tilde{e}_t$,in 2. A. ,and gross premiums calculated in 2.B B).

$$\text{at beginning of period}: As_t = P - E_{t-1} - S\tilde{q}_{x+t} - CV_{t-1} + As'_{t-1}$$
$$\text{at end of period}: As'_t = \frac{1+\tilde{i}_t}{1-\tilde{q}_{x+t}} As_t$$

C V_{t-1}: refund for cancellation

D) Fixing probability of bankruptcy for single product (10%)
Find the average distribution of difference(R_k)between asset share (As) at maturity and insurance benefits at maturity(S_1) when generating N paths.
Endowment insurance R_1 is shown as an example.

$$R_1^k \equiv As^k - S_2$$
$$\textit{Mean}: \overline{R}_1^k = \sum_{k=1}^{N} R_1^k / N$$
$$\textit{Variance}: \sigma_1^2 = \sum_{k=1}^{N} (R_1^k - \overline{R}_1^k)^2 / N$$

Generate the gross premium where the probability of insolvency, Pro(R_1^k<0;k = 1-N), is 10%.

E) Optimization of compound product for minimizing probability of insolvency
Generate the optimum product share where sales share (or sales numbers) [1]* of endowment insurance, ten times term endowment insurance, and term insurance are equal to ω.
Find the value of ω_i which satisfies equation 1 (minimum probability of insolvency), then find the value of ω_i which also satisfies equation 2 among the combination of three products ωi(i = 1to3) in each path of k = 1 to N.

$$\textit{Min}\ \text{Pro}(R_\omega^k < 0; k = 1 \sim N) \qquad (1)$$
$$\textit{Max}\ \left(\sum_{k=1}^{N} R_\omega^k / N\right) \qquad (2)$$
$$\textit{Subject to}$$
$$R_\omega^k = \sum_{i=1}^{3} \omega_i R_i^k ; \sum_{i=1}^{3} \omega_i = 1 \qquad (3)$$

Find the optimum value $\omega_0 = (\omega_1, \omega_2, \omega_3)$ which satisfies 1) to 3) above.
Mean and variance are as follows:

$$Mean : \overline{R}_{\omega 0} = \sum_{k=1}^{N} R_{\omega 0}^{k} / N$$

$$Variance : \sigma_{\omega 0}^{2} = \sum_{k=1}^{N} (R_{\omega 0}^{k} - \overline{R}_{\omega 0})^{2} / N$$

1* When the premium rate is based on S, the ratio based on the number of cases equals the ratio based on the sum payable. When based on P, the ratio based on the number of cases equals the ratio based on the premium.

Appendix 2 Characteristics of Stochastic Model for Interest Rate

The general stochastic model for interest rate is described in the equation below.

$$dr = (\alpha + r\beta)dt + r^{\gamma}\sigma dZ \cdots\cdots (0)$$

The stochastic model for interest rate handles randomly changing interest r as a stochastic variable. The values α and β in the above equation are constants that show how the average of interest rate drifts, and γ, σ are constants that show random fluctuations in the diversification of interest rate around the average interest rate. For this reason, the first term on the right side is called as the drift term and the second term is called the diffusion term. The value t and Z represent the time and the standard Brownian movement, respectively.

Appendix Table 2.1 shows the main stochastic models for interest rate. The average interest rate and its variance were generated analytically. There are two techniques using Ito integral calculus and Stratonovich integral calculus. Ito integral calculus was used because it only depends on past information and is most widely used in the financial field.

Appendix Table 2.2 classifies stochastic models for interest rate by the degree of interest rate r included in the drift term and diffusion term, by focusing on the structure of stochastic models for interest rate.

The drift term illustrates the average nature of interest rate changes, and the average interest rate is calculated in the same equation for models categorized as having the same drift terms (models on the same row in Appendix Table 2.2). If there is no drift term, the average does not change over time. When the drift term is constant, the average linearly increases or decreases over time. When the drift term is the first-degree expression with negative β, the average converges to a certain value. When the drift term is the first-degree expression with positive β, the average diverges infinitely.

The diffusion term illustrates the degree of random change in the interest rate, and as the degree of diffusion term increases, the variance of interest rate change also tends to increase.

Appendix Table 2.1 Stochastic models of interest rate

No.	Name of model	Equation in model, mean and variance of interest rate	Flexibility, Flexible parameter	Fixed parameter
0	Unrestricted model (general model)	$dr = (\alpha + r\beta)dt + r^{\gamma}\sigma\, dZ$ Mean : $(r_0 + \alpha/\beta)\exp(\beta\, t) - \alpha/\beta$ Variance : analytical solution is unknown, however found through a simulation.	4 α, β γ, σ	none
1	Merton model	$dr = \alpha\, dt + \sigma\, dZ$ Mean : $\alpha t + r_0$ Variance : $\sigma^2 t$	2 α, σ	$\beta = 0.0$ γ = 0.0
2	Dothan model	$dr = r\sigma\, dZ$ Mean : r_0 Variance : $r_0^2(\exp(\sigma^2 t) - 1)$	1 σ	α = 0.0 $\beta = 0.0$ γ = 1.0
3	GBM model	$dr = r\beta\, dt + r\sigma\, dZ$ Mean : $r_0 \exp(\beta\, t)$ Variance : $r_0^2 \exp(2\beta\, t)(\exp(\sigma^2 t) - 1)$	2 β, σ	α = 0.0 γ = 1.0
4	Vasicek model	$dr = (\alpha + r\beta)dt + \sigma\, dZ$ Mean : $(r_0 + \alpha/\beta)\exp(\beta\, t) - \alpha/\beta$ Variance : $\sigma^2/(-2\beta) \cdot (1 - \exp(2\beta\, t))$	3 α, β, σ	γ = 0.0 $\beta < 0$
5	CIR(SR)model	$dr = (\alpha + r\beta)dt + \sqrt{r}\sigma\, dZ$ Mean : $(r_0 + \alpha/\beta)\exp(\beta\, t) - \alpha/\beta$ Variance : $\sigma^2/(2\beta)\{\alpha/\beta - 2(r_0 + \alpha/\beta)\exp(\beta\, t) + (2r_0 + \alpha/\beta)\exp(2\beta\, t)\}$	3 α, β, σ	γ = 0.5 $\beta < 0$
6	Brennan& Schwarz model	$dr = (\alpha + r\beta)dt + r\sigma\, dZ$ Mean : $(r_0 + \alpha/\beta)\exp(\beta\, t) - \alpha/\beta$ Variance : when $r_\infty = -\alpha/\beta, \eta = -\sigma^2/(2\beta)$ $(r_\infty^2\eta)/(1-\eta) + \{4r_\infty\eta(r_0 - r_\infty)\}/(1-2\eta) \cdot \exp(\beta\, t)$ $-(r_0 - r_\infty)^2 \exp(2\beta\, t) + \{(1-\eta)(1-2\eta)r_0^2$ $-2r_\infty r_0(1-\eta) + r_\infty^2\}/\{(1-\eta)(1-2\eta)\}\exp(2\beta + \sigma^2 t)$	3 α, β, σ	γ = 1.0 $\beta < 0$
7	CIR VR model	$dr = r^{3/2}\sigma\, dZ$ Mean : r_0 Variance : $\approx r_0^2(\exp(\sigma^2 r_0 t) - 1)$	1 σ	α = 0.0 $\beta = 0.0$ γ = 1.5
8	CEV model	$dr = r\beta\, dt + r^{\gamma}\sigma\, dZ$ Mean : $r_0 \exp(\beta\, t)$ Variance : analytical solution is unknown, however found through a simulation	3 β, γ, σ	α = 0.0

Appendix Table 2.2 Classification of stochastic models of interest rate

order of interest rate r			diffusion term 0	0.5	1	1.5	mean of interest rate r(t)
drift term	0	0			**Dothan**	**CIR(VR)**	r_0
		α	**Merton**				$\alpha t + r_0$
	1	$r\beta$ (**CEV**)			**GBM**		$r_0 \exp(\beta t)$
		$\alpha + r\beta$ (unrestricted)	**Vasicek**	**CIR(SR)**	**Brennan & Schwarz**		$(r_0 + \alpha/\beta)\exp(\beta t) - \alpha/\beta$