"Life Insurance Options; Pricing and Reserving"

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Summary

A lot of companies selling life business offer some kind of embedded or additional option, which allows the policyholder to alter his life cover at some stage of his policy duration or to prolong it after the maturity date.

The paper examines the influence of antiselection on such options where no underwriting is required to increase or prolong the cover. The following issues are dealt with:

- additional premium for the option,
- necessity of an additional reserve for policyholders having possibility to use option
- necessity of an additional reserve for policies altered or issued by the option without medical underwriting and comparison with reserves of policies with the same parameters, but with medical underwriting.

The proposed method consists in dividing the insured into three categories, which are defined during the medical underwriting process: standard risks, substandard risks and refused risks. A multiple state model is constructed to describe the transitions between this categories, each of them having different mortality rates. Under the assumption that every policyholder in the group of substandard or refused risks takes advantage of the option the behavior of the group of standard risks (healthy people) is the key parameter for pricing and reserving of the option.

Mathematical formulae for premiums and premium reserves are presented together with numerical illustrations.

"Optionen in der Lebensversicherung; Berechnung der Versicherungsbeiträge und Rückstellungen "

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Zusammenfassung

Viele Lebensversicherungsunternehmen bieten eine Art der impliziten oder zusätzlichen Option an, die dem Versicherungsnehmer ermöglicht, den Versicherungsschutz im bestimmten Stadium seiner Versicherungsdauer zu ändern oder nach dem Auslaufen der ursprünglichen Versicherungsdauer zu verlängern.

Die Studie beschäftigt sich mit dem Einfluß der Antiselektion auf die Optionen, bei denen keine Untersuchung des Gesundheitszustands im Falle der Erhöhung oder Verlängerung des Versicherungsschutzes verlangt wird. Folgende Bereiche werden untersucht:

- zusätzliche Versicherungsbeiträge für die Option,
- Notwendigkeit einer zusätzlichen Rückstellung für die Versicherungsnehmer, die von der Option Gebrauch nehmen können,
- Notwendigkeit einer zusätzlichen Rückstellung für die Verträge, die aufgrund der Option verändert wurden oder entstanden sind, ohne daß der Gesundheitszustand untersucht gewesen wäre und Vergleich mit den Rückstellungen der Verträge mit gleichen Parametern, jedoch mit der Untersuchung des Gesundheitszustands.

Die vorgeschlagene Methode beruht auf der Aufteilung der Versicherungsnehmer in drei Kategorien, die während des Prozesses der Untersuchung des Gesundheitszustands bei der Aufnahme neuer Risiken definiert werden (medical underwriting): Standardrisiken, standardüberschreitende Risiken und inakzeptable Risiken. Es wird ein Zustandsmodel (multiple state model) entworfen, der die Übergänge zwischen diesen Kategorien beschreibt, wobei jede Kategorie eine andere Sterblichkeitsrate hat. Unter der Voraussetzung, daß alle Versicherungsnehmer in der Gruppe der standardüberschreitenden und inakzeptablen Risiken von der Option Gebrauch nehmen werden, stellt den Schlüsselparameter für die Berechnung der Versicherungsbeiträge und Rückstellungen das Verhalten der Gruppe „Standardrisiken" (gesunder Menschen) dar.

Es werden mathematische Formeln mit numerischen Bespielen dargestellt.

Contents

1. Introduction

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The paper examines the influence of antiselection on such options where no underwriting is required to increase or prolong the cover. The following issues are dealt with:

- Additional premium for the option,
- Necessity of an additional reserve for policyholders having possibility to use option
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The proposed method consists in dividing the insured into three categories, which are defined during the medical underwriting process: standard risks, substandard risks and refused risks. A multiple state model is constructed to describe the transitions between these categories, each of them having different mortality rates. Under the assumption, that every policyholder in group of substandard or refused risks takes advantage of the option, the behaviour of group of standard risks (healthy people) is the key parameter for pricing and reserving of the option.

Mathematical formulae for premiums and premium reserves are presented together with numerical illustrations.

2. Assumptions

2.1. Dividing of insured portfolio into groups

During the underwriting process are the insured usually classified into three groups, which is used in our model.

2.1.1. Standard risks (Group 1)

The majority of the insured portfolio is accepted into insurance for standard premium rates. This group is called "standard risks".

2.1.2. Substandard risks (Group 2)

It is usual that approximately 6,5 % of applicants for life insurance are insurable for increased premium because of health problems.

2.1.3. Refused risks (Group 3)

Remaining 3,5 % of applicants for life insurance (with significant sum at risk) are refused by insurance companies, because the risk is not acceptable.

2.2. Mortality rates for particular groups

Although we are aware of the fact that the particular groups are not homogenous, we will estimate the average mortality for each group (or better said, relation to some standard mortality). This step allows us further calculations. The mortality is expressed as a percentage of the reference mortality tables.

Mortality for Group 1 (standard risks) is set as *g1* times the mortality of the reference tables. This mortality is better then the population mortality and we estimate it as 70 % of the reference mortality rates.

Mortality for Group 2 (substandard risks) is set as *g2* times the mortality of the reference tables. *g2* equals 100 % plus the average excess-mortality of the portfolio insured for increased premium. We estimate *g2* as 200 %.

Mortality for Group 3 (refused risks) is set as *g3* times the mortality of the reference tables. It is the average mortality of non-acceptable risks considering that the highest still acceptable risk is 500 % of reference tables. We estimate *g3* as 700 %.

The important factor is the ultimate mortality in particular groups when considering the dependence on age. It is possible to use our definition till the age of 70 years. For higher ages, mortality rates in Group 3 would be above 100 %. For the majority of life products (except pensions and whole life) the age at the end of insurance is limited by 70, so we can use the definition for our purposes.

Another important assumption is that mortality of the total portfolio (all three groups together) equals the original mortality expressed by the reference tables.

It means

$$q_x = 0.9 g_1 q_x + 0.065 g_2 q_x + 0.035 g_3 q_x .$$

We derive the formula for g_3

$$g_3 = \frac{q_x - 0.9 g_1 q_x - 0.065 g_2 q_x}{0.035 * q_x} = \frac{1 - 0.9 g_1 - 0.065 g_2}{0.035} .$$

After inserting g_1=0,7 and g_2=2, we get g_3 = 6,85714286. These values is used in the following calculations.

2.3. Mortality after the selection period

When developing the selection mortality tables, we introduce the period of improved mortality at the beginning, which is called selection period. This period is considered to last 5 years in our model.
After this period, the mortality of insured portfolio corresponds to the reference mortality tables. That means, in the insured portfolio are 90 % of insured with g_1 times the reference mortality rates, 6,5 % of insured with g_2 times the reference mortality rates and 3,5 % of insured with g_3 times the reference mortality rates.

2.4. *Realization of the option*

We will assume that all policyholders in Group 2 and Group 3 realize the option, because they are aware of their risk (or the insurance agent will remind them).
Under this assumption, the dependence of the premium loading for covering the increased claim ratio after the realization of the option on the number of options realized by Group 1 policyholders is explored.

We assume that before the realization of the option the total mortality in the insured portfolio corresponds to the reference mortality tables. As above, the insured are divided into groups:

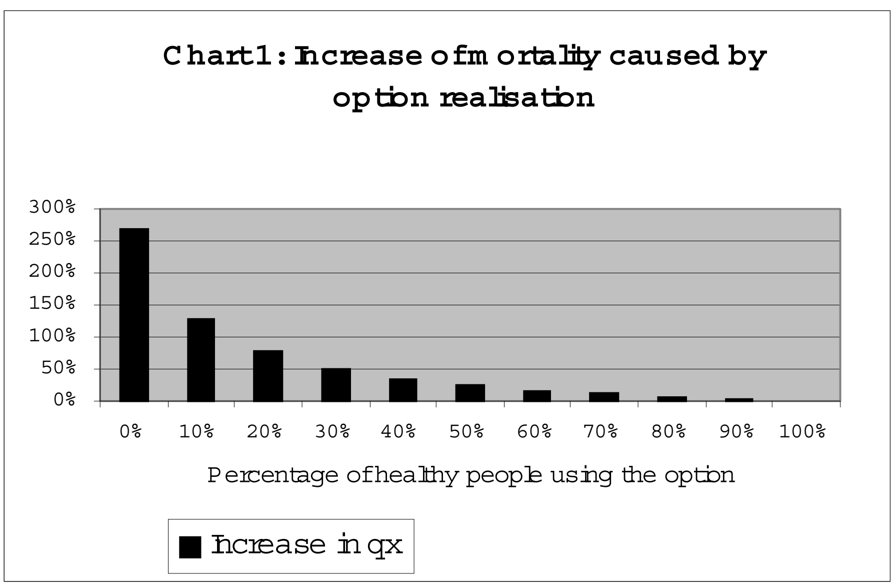
$$q_x = 0{,}9 g_1 * q_x + 0{,}065 g_2 q_x + 0{,}035 g_3 * q_x$$

Let us assume that the mortality rates after the realization of the option increases, because only A % of the insured in Group 1 will realize the option. The mortality of the insured portfolio is than

$$q_x^{option} = 0{,}9 A g_1 q_x + 0{,}065 g_2 q_x + 0{,}035 g_3 q_x /(0{,}9A + 0{,}065 + 0{,}035)$$

Under the assumptions made, the mortality after the realization of the option varies between 100 % of the reference tables when A = 100 % and 370 % of the reference tables when A = 0 %. For A = 50 % the mortality increases by 24,5 %.

The dependence of increased mortality on A is presented in Chart 1.



3. *Mathematical model*

3.1. *Transition Probabilities*

We have defined the division of the insured portfolio into groups of proportions p=(p1, p2, p3) and the multiplication of the standard mortality in particular groups g_1, g_2 a g_3.

We get

(1) $p_1 + p_2 + p_3 = 1$,

(2) $p_1 g_1 + p_2 g_2 + p_3 g_3 = 1$.

We look for a transition matrix *M* within the groups of insured, which preserves the proportions of the groups for all q_x. This holds if

$$pM = p\begin{pmatrix} 1-g_1q-aq-bq & aq & bq \\ 0 & 1-g_2q-cq & cq \\ 0 & 0 & 1-g_3q \end{pmatrix} = (1-q)p,$$

where q stands for q_x.

Consequently, the constants *a, b, c* have to fulfil

$$p_1(1-g_1q-aq-bq) = (1-q)p_1,$$

$$p_1aq + p_2(1-g_2q-cq) = (1-q)p_2,$$

$$p_1bq + p_2cq + p_3(1-g_3q) = (1-q)p_3.$$

From the second equation we obtain

$$a = \frac{p_2(g_2+c-1)}{p_1},$$

and from the third equation

$$b = \frac{p_3(g_3-1)-p_2c}{p_1}.$$

Because of (1), (2) the equations are linearly dependent, by inserting into the third equation we can only confirm the calculation. It is possible to choose *c* and calculate *a* and *b*.

3.2. First years after the writing the business

We decided to use the selection period of 5 years. After this period, the mortality of the insured portfolio should be close to reference mortality tables. That corresponds to the fact that group proportions of the insured portfolio converge to *p*.

The above derived transition matrix assures the convergence, but the portfolio is close enough to “stable status” after much longer time, than the expected 5 year selection period. We look for a matrix, which gets the portfolio close to the stable status already after 5 years.

The wanted transition matrix is denoted M_{start}.

We choose M_{start} in the form

$$M_{start} = \begin{pmatrix} 1-x-y-qg_1 & x & y \\ 0 & 1-z-qg_2 & z \\ 0 & 0 & 1-qg_3 \end{pmatrix}$$, which is consistent with the matrix M.

We look for the values *x, y* and *z* such that

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1-x-y-qg_1 & x & y \\ 0 & 1-z-qg_2 & z \\ 0 & 0 & 1-qg_3 \end{pmatrix}^5 \cong \begin{pmatrix} p_1 & p_2 & p_3 \end{pmatrix}$$

This means that the insured that were originally in group of “healthy” G1, are after five years represented in groups G1, G2 and G3 in the same ratios as in the population.

Under the assumption $qg_1 \cong 0$ it comes from the first equation

$$(1-x-y)^5 = p_1 = 0{,}9 \cong 0{,}979^5$$

and therefore

$$x + y = 1 - 0{,}979$$

x is the transition probability from Group 1 to Group 2 and *y* is the transition probability from Group 2 to Group 3. We choose $x=9\,y$, because the probability that a healthy insured passes into group of substandard risks is much higher then that he passes in one year into group of refused risks. For z the value 0,16 is used.

From this it results

$$M_{start} = \begin{pmatrix} 0{,}979-qg_1 & 0{,}0189 & 0{,}0021 \\ 0 & 0{,}84-qg_2 & 0{,}16 \\ 0 & 0 & 1-qg_3 \end{pmatrix}$$

This model corresponds to the matrix *M* used during the following policy years. The difference is, that the morbidity does not depend on q_x (or age).

4. *Renewal option*

The renewal option allows the policyholder to prolong the coverage at the end of original policy duration. New policy has the same sum assured as the original policy. The main advantage for policyholder is the possibility of avoiding the underwriting.

The premium of the original insurance is increased by the loading to cover the price of the option. During the original policy duration the reserve is built to cover the increased risk for those insured, which use the option.

The procedure to calculate the increased premium of the original insurance and the reserve of the option, that is built during the policy duration of the original contract and released during the renewed insurance, is as follow:

a) The number of insured in groups G1, G2 and G3 is modelled for the original insurance using matrices M_{start} and M.
b) The number of insured after the realization of the option is modelled using the matrix M_{option}

$$M_{option} = \begin{pmatrix} A & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

c) The insured portfolio of the renewed insurance is modelled using the matrix M.
d) The corresponding insurance written as new business without any option is modelled using matrices M_{start} and M.
e) Provisions of the insurances (written as new business) are modelled by recursive methods, which are used to calculate the net premium: net premium provision at the beginning of policy duration equals zero.
f) By the same recursive method is modelled the provision of the insurance with option. For the net premium of the renewed insurance is inserted the net premium calculated in previous step. The net premium of the original insurance consists of two parts:
   - Net premium calculated in step e)
   - Premium for the option (increase of the premium so that the net provision of both connected policies is at the beginning of the first policy zero)
g) The provision of the option is the difference between the provision calculated in step f) and the provision calculated in step e). By this provision must be increased the standard provision if the insurer includes renewal options to his life business.

The calculation is demonstrated on an example of the endowment product. Age at entry is 50, insurance period of both original and renewed contract is 10 years and sum insured equals 10 000.

Ad a)
Matrix M_{start} is used to model first 5 years of the policy duration. We can observe convergence to the stable status (number of policyholders decreases and the percentages in groups G1, G2 a G3 converge). Using matrix M in the following years assures further convergence of percentages to wanted values.

We define the number of policyholders in group G1 in age x+r as n_{x+r}^{G1}, the number of policyholders in group G2 in age x+r as n_{x+r}^{G2} and the number of policyholders in group G3 in age x+r as n_{x+r}^{G3}.

Because of the underwriting, $n_x^{G1} = 10000$, $n_x^{G2} = n_x^{G3} = 0$.

For r = 1 to 5

$$n_{x+r}^{G1} = (0{,}979 - g_1\, q_{x+r})\; n_{x+r-1}^{G1},$$
$$n_{x+r}^{G2} = 0{,}0189\; n_{x+r-1}^{G1} + 0{,}84\, g_2\, q_{x+r}\, n_{x+r-1}^{G2},$$
$$n_{x+r}^{G3} = 0{,}0021\; n_{x+r-1}^{G1} + 0{,}16\; n_{x+r-1}^{G2} + n_{x+r-1}^{G3}\,(1 - g_3\, q_{x+r})$$

and

for r > 5

$$n_{x+r}^{G1} = (1 - a\, q_{x+r} - b\, q_{x+r} - g_1\, q_{x+r})\; n_{x+r-1}^{G1},$$
$$n_{x+r}^{G2} = a\, q_{x+r}\; n_{x+r-1}^{G1} + (1 - g_2\, q_{x+r} - c\, q_{x+r})\; n_{x+r-1}^{G2},$$
$$n_{x+r}^{G3} = b\, q_{x+r}\; n_{x+r-1}^{G1} + c\, q_{x+r}\; n_{x+r-1}^{G2} + (1 - g_3\, q_{x+r})\; n_{x+r-1}^{G3}.$$

Table 1: Development of percentages per group

Age	G1 Number	Percentage	G2 Number	Percentage	G3 Number	Percentage
50	10000	100,00%	0	0,00%	0	0,00%
51	9715	97,88%	189	1,90%	21	0,21%
52	9431	95,85%	338	3,43%	70	0,71%
53	9147	93,93%	453	4,66%	138	1,41%
54	8863	92,14%	541	5,62%	216	2,24%
55	8579	90,48%	605	6,38%	298	3,14%
56	8430	90,45%	595	6,38%	296	3,17%
57	8268	90,41%	584	6,39%	293	3,20%
58	8094	90,37%	573	6,39%	290	3,24%
59	7906	90,33%	560	6,40%	286	3,27%
60	7703	90,29%	547	6,41%	281	3,30%

Ad b)
In accordance to the assumptions, A % of insured in group G1 and all insured in groups G2 and G3 use the option. Any change of the health status during the realisation of the option is not possible, so the transition matrix is

$$M_{opce} = \begin{pmatrix} A & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Using formulae the transition is described as (n is the moment before the realisation of the option, n + ε is the moment after the realisation)

$$n^{G1}_{x+n+\varepsilon} = \mathrm{A}\ n^{G1}_{x+n},$$
$$n^{G2}_{x+n+\varepsilon} = n^{G2}_{x+n},$$
$$n^{G3}_{x+n+\varepsilon} = n^{G3}_{x+n}.$$

For A = 50 % the result of **b)** and **c)** is shown in Table 2.

Table 2: Development of percentages per group (renewal option)

	G1		G2		G3	
Age	Number	Percentage	Number	Percentage	Number	Percentage
50	10000	100,00%	0	0,00%	0	0,00%
51	9715	97,88%	189	1,90%	21	0,21%
52	9431	95,85%	338	3,43%	70	0,71%
53	9147	93,93%	453	4,66%	138	1,41%
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58	8094	90,37%	573	6,39%	290	3,24%
59	7906	90,33%	560	6,40%	286	3,27%
60	7703	90,29%	547	6,41%	281	3,30%
60	3852	82,31%	547	11,68%	281	6,01%
61	3743	82,84%	509	11,26%	267	5,90%
62	3627	83,40%	471	10,82%	251	5,78%
63	3505	83,97%	434	10,39%	235	5,63%
64	3376	84,56%	398	9,97%	219	5,47%
65	3243	85,14%	364	9,56%	202	5,30%
66	3105	85,72%	332	9,16%	185	5,12%
67	2963	86,27%	302	8,79%	169	4,93%
68	2817	86,80%	274	8,45%	154	4,75%
69	2668	87,30%	248	8,13%	140	4,57%
70	2515	87,77%	225	7,84%	126	4,40%

Ad d)
Separate policies without option are modelled using matrices M_{start} and M. In both examples, at the beginning of policy duration all policyholders are in group G1.
While the result for first insurance is already calculated, the second insurance is modelled in Table 3.

Table 3 : Policies written in age x+n as new business

G1			G2		G3	
Age	Number	Percentage	Number	Percentage	Number	Percentage
60	10000	100,00%	0	0,00%	0	0,00%
61	9593	97,86%	189	1,93%	21	0,21%
62	9184	95,87%	328	3,43%	67	0,70%
63	8774	94,10%	427	4,58%	123	1,32%
64	8364	92,56%	493	5,46%	179	1,98%
65	7957	91,28%	533	6,12%	227	2,60%
66	7618	91,03%	516	6,17%	234	2,80%
67	7270	90,83%	497	6,21%	237	2,96%
68	6913	90,65%	477	6,25%	236	3,09%
69	6546	90,51%	455	6,29%	231	3,20%
70	6171	90,39%	432	6,33%	224	3,28%

Ad e)

While the portfolio has been modelled from lower ages to the older ones, during the calculation of the reserves first the reserve at the end is calculated and then the reserves for preceding ages.

The reserve for one policy in each group is to be calculated fist. The total reserve is calculated by the multiplication of the unit reserve by number of policies. Reserve for all groups is the sum of reserves for each group.

For all groups G1, G2 and G3 the reserve at the end of policy duration equals the sum insured

$$V^{G1}_{x+n,n} = V^{G2}_{x+n,n} = V^{G3}_{x+n,n} = K \,.$$

Reserves in preceding years are calculated by the following formulae:

$$V^{G1}_{x+n,r} =$$
$$v\left(g_1 q_{x+n+r} K + \left(1 - g_1 q_{x+n+r} - a q_{x+n+r} - b q_{x+n+r}\right) V^{G1}_{x+n,r+1} + a q_{x+n+r} V^{G2}_{x+n,r+1} + b q_{x+n+r} V^{G3}_{x+n,r+1}\right) - P_{x+n,n},$$

where v denotes the discount factor.

In the first step is inserted $P_{x+n,n} = \dfrac{A_{x+n,n}}{a_{x+n,n}}$ for $P_{x+n,n}$, real net premium is determined later.

$$V^{G2}_{x+n,r} = v\left(g_2 q_{x+n+r} K + \left(1 - g_2 q_{x+n+r} - c q_{x+n+r}\right) V^{G2}_{x+n,r+1} + c q_{x+n+r} V^{G3}_{x+n,r+1}\right) - P_{x+n,n}$$

$$V^{G3}_{x+n,r} = v\left(g_3 q_{x+n+r} K + \left(1 - g_3 q_{x+n+r}\right) V^{G3}_{x+n,r+1}\right) - P_{x+n,n}\,.$$

Different formulae is used for first 5 policy years, when using for transitions the matrix M_{start}.

$$V^{G1}_{x,r} = v\left(g_1 q_{x+r} K + \left(0{,}979 - g_1 q_{x+r}\right) V^{G1}_{x,r+1} + 0{,}0189 V^{G2}_{x,r+1} + 0{,}0021 V^{G3}_{x,r+1}\right) - P_{x,n},$$

$$V_{x,r}^{G2} = v\left(g_2 q_{x+r} K + \left(0{,}84 - g_2 q_{x+r}\right) V_{x,r+1}^{G2} + 0{,}16 V_{x+n,r+1}^{G3}\right) - P_{x,n},$$

$$V_{x,r}^{G3} = v\left(g_3 q_{x+r} K + \left(1 - g_3 q_{x+r}\right) V_{x,r+1}^{G3}\right) - P_{x,n}.$$

The reserves in each group is multiplied by the number of policyholders in each group and added together to get the total reserve for the whole portfolio.

We get a negative reserve, because of too high net premium used in the calculation (real net premium is lower because of the selection period). We look for such premium, that the total reserve at the beginning equals zero.
Linear interpolation is used to find this value. Let's choose the initial values $P_{x+n,n}^1 = 0{,}8 \frac{A_{x+n,n}}{a_{x+n,n}}$ and $P_{x+n,n}^2 = \frac{A_{x+n,n}}{a_{x+n,n}}$.

The premium for which the total reserve equals zero is calculated using the linear relation between reserve and premium

$$P_{x+n,n} = P_{x+n,n}^1 - \frac{V_{x,n}^1}{V_{x,n}^1 - V_{x,n}^2}\left(P_{x+n,n}^2 - P_{x+n,n}^1\right),$$

where
$V_{x,n}^1$ is the initial reserve using net premium $P_{x+n,n}^1$ and
$V_{x,n}^2$ is the initial reserve using net premium $P_{x+n,n}^2$.

The net reserves for separate policies written in ages *x* and *x+n*, both with the policy duration *n,* are obtained by inserting the real net premium into the calculation (Tables 4 and 5).

The premium for the option is calculated by analogous method. The net premium for entry age *x+n* is used for the insurance written during the realisation of the option, which corresponds to the design of the option. The net premium for entry age *x* increased by the premium for option is to be used for the original insurance.

When calculating the reserve before the end of the first policy duration, the reserve is calculated by the formulae

$$V_{EndOfFirstPolicy}^{G1} = K + V_{BeginningOfNewPolicy}^{G1} \cdot A,$$

$$V_{EndOfFirstPolicy}^{G2} = K + V_{BeginningOfNewPolicy}^{G2},$$

$$V_{EndOfFirstPolicy}^{G3} = K + V_{BeginningOfNewPolicy}^{G3}.$$

Net premium for the option is again calculated by the linear interpolation (initial values are zero and 10 % of original net premium).

The result for A = 0,5 is presented in Table 6.

Table 4 : Reserves without option — Net premium without option: 833,79358

First policy

Age	G1 No.	Percentage	Reserve per policy	Total reserve	G2 No.	Percentage	Reserve per policy	Total reserve	G3 No.	Percentage	Reserve per policy	Total reserve	**Total reserve**	**Reseve per policy**
50	10000	100,00%	0,00	-2	0	0,00%	1296,81	0	0	0,00%	2528,60		-2	0
51	9715	97,88%	783,39	7610865	189	1,90%	1882,14	355724	21	0,21%	3053,06	64114	8030703	809
52	9431	95,85%	1606,07	15147010	338	3,43%	2503,84	846111	70	0,71%	3592,52	251281	16244402	1651
53	9147	93,93%	2470,83	22601445	453	4,66%	3169,51	1436820	138	1,41%	4153,21	571434	24609699	2527
54	8863	92,14%	3380,70	29964227	541	5,62%	3887,80	2102004	216	2,24%	4743,17	1023535	33089765	3440
55	8579	90,48%	4338,88	37224508	605	6,38%	4668,37	2822560	298	3,14%	5372,59	1598674	41645742	4392
56	8430	90,45%	5349,31	45095164	595	6,38%	5522,59	3285287	296	3,17%	6059,26	1790750	50171201	5383
57	8268	90,41%	6416,78	53057069	584	6,39%	6510,10	3803727	293	3,20%	6824,58	1999366	58860162	6436
58	8094	90,37%	7543,50	61056609	573	6,39%	7576,70	4339654	290	3,24%	7700,77	2231461	67627724	7551
59	7906	90,33%	8735,58	69063515	560	6,40%	8735,58	4894420	286	3,27%	8735,58	2497451	76455386	8736
60	7703	90,29%	10000,00	77033675	547	6,41%	10000,00	5467482	281	3,30%	10000,00	2812547	85313704	10000

Table 5 : Reserves without option — Net premium without option: 919,60779

Second policy

Age	G1 No.	Percentage	Reserve per policy	Total reserve	G2 No.	Percentage	Reserve per policy	Total reserve	G3 No.	Percentage	Reserve per policy	Total reserve	**Total reserve**	**Reseve per policy**
60	10000	100,00%	0,00	-2	0	0,00%	2662,10	0	0	0,00%	4812,53	0	-2	0
61	9593	97,86%	741,97	7117493	189	1,93%	3053,82	577173	21	0,21%	5135,08	107837	7802503	796
62	9184	95,87%	1524,14	13996955	328	3,43%	3467,12	1138499	67	0,70%	5447,05	364575	15500029	1618
63	8774	94,10%	2351,45	20630818	427	4,58%	3913,40	1671915	123	1,32%	5752,25	709028	23011761	2468
64	8364	92,56%	3230,05	27016954	493	5,46%	4408,78	2175158	179	1,98%	6059,99	1085080	30277192	3350
65	7957	91,28%	4166,79	33155054	533	6,12%	4973,89	2653468	227	2,60%	6384,91	1449298	37257820	4274
66	7618	91,03%	5169,40	39382216	516	6,17%	5634,47	2907577	234	2,80%	6752,19	1582540	43872334	5242
67	7270	90,83%	6245,52	45405576	497	6,21%	6489,78	3226285	237	2,96%	7199,82	1707132	50338993	6289
68	6913	90,65%	7402,13	51169801	477	6,25%	7484,67	3568849	236	3,09%	7793,07	1837924	56576575	7419
69	6546	90,51%	8649,77	56624844	455	6,29%	8649,77	3936628	231	3,20%	8649,77	1999858	62561329	8650
70	6171	90,39%	10000,00	61705699	432	6,33%	10000,00	4320308	224	3,28%	10000,00	2237612	68263619	10000

Table 6: Reserving for policies with renewal option

Net premium for the option **19,958458**
Net premium for the option/Net premium **2,39%**

Age	G1 No.	Percentage	Reserve per policy	Total reserve	G2 No.	Percentage	Reserve per policy	Total reserve	G3 No.	Percentage	Reserve per policy	Total reserve	**Total reserve**
50	10000	100,00%	0,00	0	0	0,00%	2310,62	0	0	0,00%	3403,75	0	0
51	9715	97,88%	780,89	7586535	189	1,90%	3003,18	567601	21	0,21%	4054,88	85153	8239288
52	9431	95,85%	1598,40	15074672	338	3,43%	3735,16	1262203	70	0,71%	4744,65	331868	16668743
53	9147	93,93%	2455,08	22457357	453	4,66%	4509,95	2044475	138	1,41%	5485,68	754767	25256599
54	8863	92,14%	3353,78	29725663	541	5,62%	5329,26	2881353	216	2,24%	6294,65	1358332	33965348
55	8579	90,48%	4297,72	36871373	605	6,38%	6191,55	3743498	298	3,14%	7193,97	2140647	42755518
56	8430	90,45%	5291,13	44604689	595	6,38%	7090,24	4217859	296	3,17%	8216,93	2428429	51250976
57	8268	90,41%	6365,85	52635976	584	6,39%	8196,61	4789123	293	3,20%	9408,31	2756308	60181407
58	8094	90,37%	7497,77	60686513	573	6,39%	9377,98	5371359	290	3,24%	10833,40	3139210	69197082
59	7906	90,33%	8692,27	68721090	560	6,40%	10636,67	5959569	286	3,27%	12587,65	3598734	78279393
60	7703	90,29%	9955,35	76689737	547	6,41%	11966,66	6542749	281	3,30%	14812,53	4166094	87398580
60	3852	82,31%	-89,30	-343938	547	11,68%	1966,66	1075267	281	6,01%	4812,53	1353547	2084876
61	3743	82,84%	680,89	2548634	509	11,26%	2496,24	1269643	267	5,90%	5135,08	1369841	5188118
62	3627	83,40%	1485,70	5389110	471	10,82%	3041,63	1431938	251	5,78%	5447,05	1368816	8189863
63	3505	83,97%	2330,07	8166595	434	10,39%	3611,93	1566632	235	5,63%	5752,25	1352427	11085653
64	3376	84,56%	3220,39	10873265	398	9,97%	4221,10	1679889	219	5,47%	6059,99	1324220	13877374
65	3243	85,14%	4164,00	13503082	364	9,56%	4887,38	1778709	202	5,30%	6384,91	1288856	16570647
66	3105	85,72%	5169,40	16049947	332	9,16%	5634,47	1869997	185	5,12%	6752,19	1252050	19171993
67	2963	86,27%	6245,52	18504725	302	8,79%	6489,78	1959608	169	4,93%	7199,82	1220194	21684527
68	2817	86,80%	7402,13	20853895	274	8,45%	7484,67	2051897	154	4,75%	7793,07	1201519	24107310
69	2668	87,30%	8649,77	23077059	248	8,13%	8649,77	2148382	140	4,57%	8649,77	1207898	26433339
70	2515	87,77%	10000,00	25147726	225	7,84%	10000,00	2245238	126	4,40%	10000,00	1259534	28652498

Before the benefit payment and option realisation

To build up the sufficient reserve, all policyholders must pay extra premium.

Table 7: Increase of net premium for some entry ages

Entry age of the original insurance	Premium loading for n = 10, if A = 0 %	Premium loading for n = 10, if A = 50 %	Premium loading for n = 10, if A = 100 %
20	0,30 %	0,25 %	0,19 %
30	0,64 %	0,52 %	0,39 %
40	1,51 %	1,24 %	0,97 %
50	2,73 %	2,39 %	2,06 %

Changes in the mortality

Even in the situation, when all policyholders use the option, the extra premium is positive. The reason is, that the mortality during the renewed policy is not influenced by the underwriting selection. If the policy is written as new business, all insured belong to group G1. If the policies are written as a result of the option, policyholders are divided into groups G1, G2 and G3.

Building and the release of the option reserve

Reserve is accumulated during the policy duration of the original contract and then released during the renewed contract.

Table 8: Option reserves

	Policies with option										
	G1		G2		G3		Agregated figures G1+G2+G3				
Age	Reserve per policy	Total reserve	Reserve per policy	Total reserve	Reserve per policy	Total reserve	Grand total	Reserve per policy with option	Reserve per policy without option	Option reserve per policy	Total option reserve
50	0	0	2311	0	3404	0	0	0	0	0	1
51	781	7586535	3003	567601	4055	85153	8239288	830	809	21	208585
52	1598	15074672	3735	1262203	4745	331868	16668743	1694	1651	43	424341
53	2455	22457357	4510	2044475	5486	754767	25256599	2594	2527	66	646900
54	3354	29725663	5329	2881353	6295	1358332	33965348	3531	3440	91	875583
55	4298	36871373	6192	3743498	7194	2140647	42755518	4509	4392	117	1109776
56	5291	44604689	7090	4217859	8217	2428429	51250976	5499	5383	116	1079775
57	6366	52635976	8197	4789123	9408	2756308	60181407	6580	6436	144	1321245
58	7498	60686513	9378	5371359	10833	3139210	69197082	7726	7551	175	1569357
59	8692	68721090	10637	5959569	12588	3598734	78279393	8944	8736	208	1824007
60	9955	76689737	11967	6542749	14813	4166094	87398580	10244	10000	244	2084876
60	-89	-343938	1967	1075267	4813	1353547	2084876	446	0	446	2084877
61	681	2548634	2496	1269643	5135	1369841	5188118	1148	796	352	1591596
62	1486	5389110	3042	1431938	5447	1368816	8189863	1883	1618	265	1151855
63	2330	8166595	3612	1566632	5752	1352427	11085653	2656	2468	188	785005
64	3220	10873265	4221	1679889	6060	1324220	13877374	3476	3350	125	499338
65	4164	13503082	4887	1778709	6385	1288856	16570647	4351	4274	77	292888
66	5169	16049947	5634	1869997	6752	1252050	19171993	5293	5242	51	183414
67	6246	18504725	6490	1959608	7200	1220194	21684527	6314	6289	25	86303
68	7402	20853895	7485	2051897	7793	1201519	24107310	7428	7419	8	26909
69	8650	23077059	8650	2148382	8650	1207898	26433339	8650	8650	0	0
70	10000	25147726	10000	2245238	10000	1259534	28652498	10000	10000	0	0

Option reserve per policy is calculated as total reserve of policy with option minus reserve of policy without option. Reserve for one policy is calculated as portfolio reserve divided by the number of policies as shown in Table 8.

5. *Increasing Option*

The basic difference between the renewal and increasing option is the time of the realisation of the option – for increasing option already during the policy duration of the original policy. Increase could be regular (for example annual) or irregular (related to some event or in the predefined time). Increase can be derived from some index (inflation) or predefined.

The regular increase after 5, 10 and 15 policy years is analysed in the following example. The calculation is demonstrated on an example of the endowment product. Age at entry is 40, insurance period is 20 years and sum insured equals 10 000. Policyholder can refuse every increase without losing right for future increases. The insured sum is increased by 30 %; the premium is increased by standard premium corresponding to the increase of sum assured and actual age.

The percentages of the insured in each risk group are modelled like in the case of renewal option. Increasing option model is more complicated, because policyholders refusing one increase are further insured.

The option design defines a division of the portfolio into groups according to two criteria:

1) Health status (that means corresponding to groups G1, G2 and G3),
2) Regular premium.

Sum assured is not unique criterion, because policyholders with the same sum assured could have used for example only the first or only the second option, and are therefore paying different premium.

Number of insured in group *Gi* with total premium *P* after *r* policy years is notated $n_r^{Gi,P}$. The subgroup of insured belonging to group *Gi* and paying premium *P* is called *[Gi ; P]*.

Further notation:

- Insured sum at the beginning of the policy K^0,
- Premium for unitised insured sum at the beginning of the policy P^0,
- Increase in the premium for unitised insured sum during the first option P^1,
- Increase in the premium for unitised insured sum during the second option P^2,
- Increase in the premium for unitised insured sum during the third option P^3.

Analogous to the renewal option assumptions, let's assume that all policyholders in groups G2 and G3 always use the option to increase their cover without medical underwriting.

The insured in group G1 always ad hoc decide about the increase of their cover. The probability that the insured in group G1 increase his cover during the *j-th* option is called B_j.

Individual reserve for the insured in group *Gi*, who pays premium *P*, is called $V_{x+n,n}^{Gi,P}$.

The procedure of the calculation of premium loading and additional reserve is following:

a) The numbers of policyholders in groups G1, G2 and G3 during the original policy (without possibility to increase premium and insured sum) are modelled using matrices M_{start} and M
b) The same calculation is done for fictive policies written after 5, 10 and 15 years with the policy duration 15, 10 and 5 years, respectively.
c) Provisions of the insurances (written as new business) are modelled by recursive methods, which are used to calculate the net premium: net premium reserve at the beginning of policy duration equals zero.
d) The portfolio of policies with option is to be modelled next. The policies are divided into groups with different health status and different premium. Percentages are modelled by transition matrices, which allow only the change of the health status during every 5years period and only the change in premium during the realization of the option.
e) The provision of the insurance with option is modelled by the same recursive method. The net premium calculated in the previous step is inserted for the net premium of the renewed insurance, taking into account real age at entry and policy duration of the additional cover. The net premium of the original insurance consists of two parts:
    - Net premium for policies with no option
    - Premium for the option (increase of the premium so that the net reserve at the beginning of the first policy is zero)

f) The reserve of the option is the difference between the provision calculated in step e) and the reserve calculated in step c). By this reserve must be increased the standard reserve if the insurer includes increasing options to his life policy.

Formulae used in paragraphs **a)** and **b)** have been derived in the previous text. The calculation and formulae of the stand alone policies - paragraph **c) -** is described in the example of renewal option.

The example of the numerical calculation is demonstrated in Table 9.

Follows the modelling of the portfolio with options – paragraph **d)**.

The transition matrix M^{start} is used to model the selection period. Insured sum keeps the original level.

Now we have three groups of insured divided by health status and insured sum. Numbers of insured in these groups are $n_r^{G1,P^0K^0}, n_r^{G2,P^0K^0}, n_r^{G3,P^0K^0}$, while at the beginning was $n_0^{G2,P^0K^0} = n_0^{G3,P^0K^0} = 0$ (only standard risks can write policy with option).

The same transition matrix is used for initial five years of policy duration like in the case of renewal option:

$$M_{start} = \begin{pmatrix} 0{,}979 - qg_1 & 0{,}0189 & 0{,}0021 \\ 0 & 0{,}84 - qg_2 & 0{,}16 \\ 0 & 0 & 1 - qg_3 \end{pmatrix}$$

Formulae for numbers of policyholders in each group:

$n_{x+r}^{G1,P^0K^0} = (0{,}979 - g_1\, q_{x+r})\; n_{x+r-1}^{G1,P^0K^0}$,

$n_{x+r}^{G2,P^0K^0} = 0{,}0189\; n_{x+r-1}^{G1,P^0K^0} + 0{,}84\, g_2\, q_{x+r}\; n_{x+r-1}^{G2,P^0K^0}$,

$n_{x+r}^{G3,P^0K^0} = 0{,}0021\; n_{x+r-1}^{G1,P^0K^0} + 0{,}16\; n_{x+r-1}^{G2,P^0K^0} + n_{x+r-1}^{G3,K^0}\; (1 - g_3\, q_{x+r})$.

Because the selection period is in our case the same as the period, after which can be used the first option, the transition matrix for the option is to be used now.

Table 9: Reserves of policies without option

Reserves without option Net premium without option **344,64606**

age	G1 number	Percentage	Reserve per policy	Total reserve	G2 number	Percentage	Reserve per policy	Total reserve	G3 number	Percentage	Reserve per policy	Total reserve	Portfolio reserve	Reserve per policy
40	10000	100,00%	0,00	2	0	0,00%	1543,16	0	0	0,00%	2399,88	0	2	0
41	9763	97,89%	303,73	2965188	189	1,90%	1767,03	333969	21	0,21%	2692,66	56546	3355703	336
42	9528	95,85%	621,23	5919061	342	3,44%	1983,75	677717	71	0,72%	2986,42	212384	6809162	685
43	9296	93,87%	953,58	8864588	464	4,68%	2190,99	1016166	143	1,45%	3281,01	470696	10351450	1045
44	9067	91,96%	1302,34	11807690	560	5,68%	2386,31	1337252	232	2,35%	3578,78	830261	13975203	1418
45	8839	90,14%	1668,72	14750164	636	6,48%	2565,24	1630359	331	3,38%	3878,34	1285431	17665954	1802
46	8782	90,13%	2054,50	18043207	632	6,48%	2722,85	1719538	330	3,38%	4180,78	1378537	21141282	2170
47	8720	90,13%	2471,21	21548359	627	6,48%	3098,79	1943197	328	3,39%	4485,56	1470531	24962087	2580
48	8651	90,12%	2905,19	25132442	622	6,48%	3487,04	2169611	326	3,39%	4793,36	1561322	28863375	3007
49	8575	90,12%	3357,29	28789332	617	6,48%	3888,34	2398393	323	3,40%	5103,99	1650522	32838248	3451
50	8492	90,11%	3828,66	32513952	611	6,48%	4304,29	2629596	321	3,40%	5418,96	1738270	36881819	3914
51	8402	90,11%	4320,65	36300142	604	6,48%	4736,94	2863372	318	3,41%	5740,53	1824864	40988377	4396
52	8303	90,10%	4834,69	40140459	597	6,48%	5188,64	3099885	315	3,42%	6071,29	1910647	45150991	4900
53	8195	90,09%	5372,49	44026119	590	6,48%	5662,35	3339440	311	3,42%	6415,08	1996237	49361796	5427
54	8077	90,09%	5935,91	47945140	581	6,48%	6161,61	3582288	307	3,43%	6776,81	2082409	53609837	5979
55	7950	90,08%	6527,02	51887753	572	6,49%	6690,50	3829020	303	3,43%	7162,73	2170341	57887115	6559
56	7811	90,07%	7148,46	55839744	562	6,49%	7254,70	4080410	298	3,44%	7583,76	2262220	62182373	7170
57	7662	90,07%	7802,97	59783977	552	6,49%	7860,19	4336989	293	3,45%	8053,01	2360575	66481541	7815
58	7500	90,06%	8493,81	63703196	540	6,49%	8514,17	4599513	287	3,45%	8590,24	2469448	70772157	8498
59	7326	90,05%	9224,73	67578589	528	6,49%	9224,73	4868586	281	3,46%	9224,73	2594913	75042088	9225
60	7138	90,05%	10000,00	71380448	514	6,49%	10000,00	5143492	275	3,46%	10000,00	2745620	79269560	10000

No change in the health status is possible during the increase of the cover. Only premium and insured sum increases, insured sum from K^0 to $1,3\ K^0$ and premium from P^0K^0 to $\left(P^0K^0+0{,}3P^1K^0\right)$.

The assumption for groups G2 and G3 is, that all insured increase their cover. The transition probability from group [G2 ; P^0K^0] to group [G2 ; $P^0K^0+0{,}3P^1K^0$] equals 1.

The transition probability from group [G3 ; P^0K^0] to group [G3 ; $P^0K^0+0{,}3P^1K^0$] is 1.

The transition probability for the transition between groups [G1 ; P^0K^0] and [G1 ; $P^0K^0+0{,}3P^1K^0$] equals B_1. The insured stays in group [G1 ; P^0K^0] with the probability $1-B_1$.

First option can be expressed by the formulae

$$n_{x+5+\varepsilon}^{G1,P^0K^0+0,3P^1K^0} = \mathrm{B}_1\ n_{x+5}^{G1,P^0K^0},$$

$$n_{x+5+\varepsilon}^{G1,P^0K^0} = (1-\mathrm{B}_1)\, n_{x+5}^{G1,P^0K^0},$$

$$n_{x+5+\varepsilon}^{G2,P^0K^0+0,3P^1K^0} = n_{x+5}^{G2,P^0K^0},$$

$$n_{x+5+\varepsilon}^{G3,P^0K^0+0,3P^1K^0} = n_{x+5}^{G3,P^0K^0},$$

$$n_{x+5+\varepsilon}^{G2,P^0K^0} = n_{x+5+\varepsilon}^{G3,P^0K^0} = 0.$$

6 groups of insured with different health status or different annual premium exist after the realization of the option. The description of transitions between these groups follows.

Till the next option, only health status can vary.

Transitions between groups with the same premium and different health status [G1 ; P], [G2 ; P] and [G3 ; P] are modelled using transition matrix

$$\begin{pmatrix} 1-g_1q-aq-bq & aq & bq \\ 0 & 1-g_2q-cq & cq \\ 0 & 0 & 1-g_3q \end{pmatrix},$$

which corresponds to formulae

$$n_{x+r}^{G1,P} = (1-a\,q_{x+r}-b\,q_{x+r}-g_1\,q_{x+r})\ n_{x+r-1}^{G1,P},$$
$$n_{x+r}^{G2,P} = a\,q_{x+r}\ n_{x+r-1}^{G1,P} + (1-g_2\,q_{x+r}-c\,q_{x+r})\ n_{x+r-1}^{G2,P},$$
$$n_{x+r}^{G3,P} = b\,q_{x+r}\ n_{x+r-1}^{G1,P} + c\,q_{x+r}\ n_{x+r-1}^{G2,P} + (1-g_3\,q_{x+r})\ n_{x+r-1}^{G3,P}.$$

The variables *a=0,21667, b=0,08333, c=2,00000* are used in the numerical examples.

Values $P = P^0K^0$ or $P = \left(P^0K^0 + 0{,}3P^1K^0\right)$ are inserted for P in the period from sixth to tenth policy year.

The premium and insured sum are changed during the second increase.

a) For policyholders, which did not use the first option, the insured sum can increase from K^0 to $1{,}3\ K^0$ and premium can increase from P^0K^0 to $\left(P^0K^0 + 0{,}3P^2K^0\right)$.
b) For policyholders, which used the first option, the insured sum can increase from $1{,}3\ K^0$ to $1{,}3^2\ K^0$ and premium can increase from $\left(P^0K^0 + 0{,}3P^1K^0\right)$ to $\left(P^0K^0 + 0{,}3P^1K^0 + 0{,}39P^2K^0\right)$, where $0{,}39 = 1{,}3 * 0{,}3$ (already increased insured sum increases by another 30 %).

All policyholders in groups G2 and G3 increase their cover. Expressed by formulae,

$$n_{x+10+\varepsilon}^{G2,P^0K^0+0,3P^2K^0} = n_{x+10}^{G2,P^0K^0},$$

$$n_{x+10+\varepsilon}^{G3,P^0K^0+0,3P^2K^0} = n_{x+10}^{G3,P^0K^0},$$

$$n_{x+10+\varepsilon}^{G2,P^0K^0+0,3P^1K^0+0,39P^2K^0} = n_{x+10}^{G2,P^0K^0+0,3P^1K^0},$$

$$n_{x+10+\varepsilon}^{G3,P^0K^0+0,3P^1K^0+0,39P^2K^0} = n_{x+10}^{G3,P^0K^0+0,3P^1K^0},$$

$$n_{x+10+\varepsilon}^{G2,P^0K^0} = n_{x+10+\varepsilon}^{G3,P^0K^0} = n_{x+10+\varepsilon}^{G2,P^0K^0+0,3P^1K^0} = n_{x+10+\varepsilon}^{G3,P^0K^0+0,3P^1K^0} = 0.$$

The probability, that healthy insured in group G1 increase his cover, is B_2. Expressed by formulae,

$$n_{x+10+\varepsilon}^{G1,P^0K^0+0,3P^2K^0} = B_2\ n_{x+10}^{G1,P^0K^0},$$

$$n_{x+10+\varepsilon}^{G1,P^0K^0} = (1-B_2)\, n_{x+10}^{G1,P^0K^0},$$

$$n_{x+10+\varepsilon}^{G1,P^0K^0+0,3P^1K^0+0,39P^2K^0} = B_2\ n_{x+10}^{G1,P^0K^0+0,3P^1K^0},$$

$$n_{x+10+\varepsilon}^{G1,P^0K^0+0,3P^1K^0} = (1-B_2)\ n_{x+10}^{G1,P^0K^0+0,3P^1K^0}.$$

Similar method is applied for the following policy duration.

- Only the health status is varying another 5 years.
- Realization of the third option.
- Last 5 years of policy duration, again only the health status can vary (premium equals values $\left(P^0K^0\right)$, $\left(P^0K^0 + 0{,}3P^1K^0\right)$, $\left(P^0K^0 + 0{,}3P^2K^0\right)$, $\left(P^0K^0 + 0{,}3P^3K^0\right)$, $\left(P^0K^0 + 0{,}3P^1K^0 + 0{,}39P^2K^0\right)$, $\left(P^0K^0 + 0{,}3P^1K^0 + 0{,}39P^3K^0\right)$,

$\left(P^0K^0 + 0{,}3P^2K^0 + 0{,}39P^3K^0\right)$, $\left(P^0K^0 + 0{,}3P^1K^0 + 0{,}39P^2K^0 + 0{,}507P^3K^0\right)$, which correspond to all possibilities of using the options).

Table 10: Increasing option, number of policyholders per group

age	G 1 No.	No. of insured with insured sum								G 2 No.	No. of insured with insured sum							
		10000	13000	13000	13000	16900	16900	16900	21970		10000	13000	13000	13000	16900	16900	16900	21970
		Using/Not-using the option									Using/Not-using the option							
		0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1		0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1
		Premium paid by insured in the subgroup									Premium paid by insured in the subgroup							
		349,2	501,7	599,3	890,4	826,9	1205,3	1302,9	1741,6		349,2	501,7	599,3	890,4	826,9	1205,3	1302,9	1741,6
40	10000	10000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	9763	9763	0	0	0	0	0	0	0	189	189	0	0	0	0	0	0	0
42	9528	9528	0	0	0	0	0	0	0	342	342	0	0	0	0	0	0	0
43	9296	9296	0	0	0	0	0	0	0	464	464	0	0	0	0	0	0	0
44	9067	9067	0	0	0	0	0	0	0	560	560	0	0	0	0	0	0	0
45	8839	8839	0	0	0	0	0	0	0	636	636	0	0	0	0	0	0	0
45	8839	7955	884	0	0	0	0	0	0	636	0	636	0	0	0	0	0	0
46	8782	7904	878	0	0	0	0	0	0	632	11	620	0	0	0	0	0	0
47	8720	7848	872	0	0	0	0	0	0	627	23	604	0	0	0	0	0	0
48	8651	7786	865	0	0	0	0	0	0	622	36	587	0	0	0	0	0	0
49	8575	7718	858	0	0	0	0	0	0	617	49	568	0	0	0	0	0	0
50	8492	7643	849	0	0	0	0	0	0	611	63	547	0	0	0	0	0	0
50	8492	6879	764	764	0	85	0	0	0	611	0	0	63	0	547	0	0	0
51	8402	6805	756	756	0	84	0	0	0	604	16	2	63	0	524	0	0	0
52	8303	6725	747	747	0	83	0	0	0	597	33	4	62	0	500	0	0	0
53	8195	6638	738	738	0	82	0	0	0	590	50	6	60	0	474	0	0	0
54	8077	6542	727	727	0	81	0	0	0	581	68	8	59	0	447	0	0	0
55	7950	6439	715	715	0	79	0	0	0	572	86	10	58	0	419	0	0	0
55	7950	5795	644	644	644	72	72	72	8	572	0	0	0	86	0	10	58	419
56	7811	5695	633	633	633	70	70	70	8	562	22	2	2	82	0	9	54	390
57	7662	5585	621	621	621	69	69	69	8	552	44	5	5	78	1	9	50	360
58	7500	5467	607	607	607	67	67	67	7	540	66	7	7	75	1	8	46	330
59	7326	5341	593	593	593	66	66	66	7	528	87	10	10	71	1	8	42	299
60	7138	5204	578	578	578	64	64	64	7	514	108	12	12	67	1	7	38	269

before the increase of the cover

Table 10 (contd)

Age	G 3 No.	No. of insured with insured sum 10000	13000	13000	13000	16900	16900	16900	21970	Total number of policyholders in groups G1, G2, G3
		Using/Not-using the option								
		0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1	
		Premium paid by insured in the subgroup								
		349,2	501,7	599,3	890,4	826,9	1205,3	1302,9	1741,6	
40	0	0	0	0	0	0	0	0	0	10000
41	21	21	0	0	0	0	0	0	0	9973
42	71	71	0	0	0	0	0	0	0	9941
43	143	143	0	0	0	0	0	0	0	9903
44	232	232	0	0	0	0	0	0	0	9859
45	331	331	0	0	0	0	0	0	0	9806
45	331	0	331	0	0	0	0	0	0	9806
46	330	4	325	0	0	0	0	0	0	9744
47	328	9	319	0	0	0	0	0	0	9675
48	326	14	312	0	0	0	0	0	0	9599
49	323	19	304	0	0	0	0	0	0	9515
50	321	25	295	0	0	0	0	0	0	9424
50	321	0	0	25	0	295	0	0	0	9424
51	318	6	1	25	0	286	0	0	0	9324
52	315	13	1	26	0	275	0	0	0	9215
53	311	20	2	26	0	264	0	0	0	9096
54	307	27	3	26	0	251	0	0	0	8966
55	303	35	4	26	0	238	0	0	0	8825
55	303	0	0	0	35	0	4	26	238	8825
56	298	8	1	1	35	0	4	25	225	8672
57	293	17	2	2	34	0	4	24	210	8507
58	287	26	3	3	34	0	4	23	195	8328
59	281	36	4	4	33	0	4	21	179	8135
60	275	45	5	5	32	1	4	20	163	7927

Comment to the table 10:
Row „Using/Not-using the option" means:
1 – option is used; premium and insured sum is increased
0 - option is used; premium and insured is unchanged
? – in the moment is not known further development of the cover

Reserves for the policies with option **(paragraph e)** are calculated from the end of policy duration.

$V_{x+n,n}^{Gi,P} = K$ for i=1,2,3 and all P.

Reserves back to the beginning of policy duration are calculated by the recursive method. The premium on the level of sum of premium is deducted for the original cover plus all increases plus premium for the option. The premium for the option is calculated so that the portfolio reserve at the beginning equals zero.

As we defined before, during every five-years period can vary only health status, but not the premium and insured sum.

$$V_{x+n,r}^{G1,P} =$$
$$v\left(g_1 q_{x+n+r} K + \left(1 - g_1 q_{x+n+r} - a q_{x+n+r} - b q_{x+n+r}\right) V_{x+n,r+1}^{G1,P} + a q_{x+n+r} V_{x+n,r+1}^{G2,P} + b q_{x+n+r} V_{x+n,r+1}^{G3,P}\right) - P,$$

where for P can be inserted any value of premium depending on the development of the policy (previous using of the option) and K is the insured sum corresponding to P.

For groups G2 and G3 holds

$$V_{x+n,r}^{G2,P} = v\left(g_2 q_{x+n+r} K + \left(1 - g_2 q_{x+n+r} - c q_{x+n+r}\right) V_{x+n,r+1}^{G2,P} + c q_{x+n+r} V_{x+n,r+1}^{G3,P}\right) - P$$

and

$$V_{x+n,r}^{G3,P} = v\left(g_3 q_{x+n+r} K + \left(1 - g_3 q_{x+n+r}\right) V_{x+n,r+1}^{G3,P}\right) - P.$$

Because transitions in the first 5 policy years are defined differently (by transition matrix M^{start}), the reserve calculation must be derived separately.

$$V_{x,r}^{G1,P} = v\left(g_1 q_{x+r} K + \left(0{,}979 - g_1 q_{x+r}\right) V_{x,r+1}^{G1,P} + 0{,}0189 V_{x,r+1}^{G2} + 0{,}0021 V_{x,r+1}^{G3,P}\right) - P,$$

$$V_{x,r}^{G2,P} = v\left(g_2 q_{x+r} K + \left(0{,}84 - g_2 q_{x+r}\right) V_{x,r+1}^{G2,P} + 0{,}16 V_{x+n,r+1}^{G3,P}\right) - P,$$

$$V_{x,r}^{G3,P} = v\left(g_3 q_{x+r} K + \left(1 - g_3 q_{x+r}\right) V_{x,r+1}^{G3,P}\right) - P.$$

In the moment before the realization of *j-th* option, the reserve is calculated by the formulae

$$V_{x,r}^{G1,P} = B_j \cdot V_{x,r+\varepsilon}^{G1,P} + (1 - B_j) \cdot V_{x,r+\varepsilon}^{G1,P+\Delta P},$$

$$V_{x,r}^{G2,P} = V_{x,r+\varepsilon}^{G2,P+\Delta P},$$

$$V_{x,r}^{G3,P} = V_{x,r+\varepsilon}^{G3,P+\Delta P},$$

where ΔP is the premium increase during the realization of the *j-th* option by the insured, which paid premium P before.
The calculation is shown on the numerical example in Table 11.

The total reserve for subgroups of policies shown in Table 12 is calculated after the multiplication of the policy reserves by the number of policies.

Reserve of the option **(paragraph f)** is calculated as the difference between reserve of the policy with option and corresponding policy without option (that means the policy with the same premium and insured sum, which was subject to medical underwriting during every increase). Results are presented in Table 13.

Some results of the numerical example are pointed out now.

1) Individual reserve in a particular group can decrease. Groups of insured are defined by premium. The insured stays within group till he increases his cover. Because of the assumption, that every "ill" insured uses the option (thus leave their groups), group becomes "healthier" after the option. Individual reserve for so "positively" influenced group decreases.

2) Total reserve of the option before the end of policy duration is negative. Premium for the option is paid during the whole policy duration. Even the policyholders, who cannot increase the premium any more, are paying the premium for the option (in our case, during last 5 policy years. Total reserve is decreased by the deduction of this premium loading.

As well as in the case of renewal option, it is important to reserve for the antiselection effect of the option. It is important for the calculation of the option reserve, how many policyholders use their right to increase their cover.

Table 11: Reserves for increasing option

Age	G1 No.	Policy reserve (G1)								G2 No.	Policy reserve (G2)							
		for insured with insured sum									for insured with insured sum							
Age	No.	10000	13000	13000	13000	16900	16900	16900	21970	No.	10000	13000	13000	13000	16900	16900	16900	21970
		Using/Not-using the option									Using/Not-using the option							
		0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1		0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1
		Premium paid in the subgroup									Premium paid in the subgroup							
		349,19	501,73	599,33	890,42	826,91	1205,33	1302,93	1741,59		349,19	501,73	599,33	890,42	826,91	1205,33	1302,93	1741,59
40	10000	0								0	2350							
41	9763	291								189	2553							
42	9528	595								342	2726							
43	9296	914								464	2860							
44	9067	1251								560	2940							
45	8839	1608								636	2950							
45	8839	1611	1580							636	2631	2950						
46	8782	2013	2127							632	3011	3466						
47	8720	2432	2697							627	3402	3998						
48	8651	2867	3290							622	3805	4546						
49	8575	3321	3908							617	4221	5110						
50	8492	3794	4551							611	4649	5693						
50	8492	3796	4554	3777		4530				611	4398	5366	4649		5693			
51	8402	4290	5227	4512		5516				604	4838	5964	5302		6568			
52	8303	4807	5930	5280		6546				597	5296	6588	5982		7481			
53	8195	5347	6666	6083		7623				590	5775	7241	6694		8435			
54	8077	5913	7436	6925		8751				581	6279	7926	7443		9439			
55	7950	6506	8244	7807		9935				572	6809	8648	8231		10497			
55	7950	6507	8245	7807	6503	9936	8240	7803	9930	572	6671	8468	8052	6809	10263	8648	8231	10497
56	7811	7132	9096	8737	7666	11182	9790	9431	12084	562	7239	9241	8895	7865	11395	10055	9709	12453
57	7662	7790	9992	9715	8890	12494	11422	11145	14353	552	7848	10070	9801	8997	12609	11565	11295	14552
58	7500	8485	10938	10748	10183	13880	13144	12955	16748	540	8505	10966	10778	10221	13920	13195	13008	16819
59	7326	9220	11938	11841	11550	15345	14967	14869	19282	528	9220	11938	11841	11550	15345	14967	14869	19282
60	7138	10000	13000	13000	13000	16900	16900	16900	21970	514	10000	13000	13000	13000	16900	16900	16900	21970

Table 11 (cont'd)

	G3	Policy reserve							
		for insured with insured sum							
vìk	poèet	10000	13000	13000	13000	16900	16900	16900	21970
		Using/Not-using the option							
		0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1
		Premium paid in the subgroup							
		349,19	501,73	599,33	890,42	826,91	1205,33	1302,93	1741,59
40	0	3521							
41	21	3898							
42	71	4286							
43	143	4685							
44	232	5101							
45	331	5534							
45	331	4547	5534						
46	330	4913	6029						
47	328	5291	6539						
48	326	5684	7068						
49	323	6093	7619						
50	321	6523	8198						
50	321	5679	7100	6523		8198			
51	318	6036	7585	7048		8900			
52	315	6410	8091	7596		9633			
53	311	6805	8626	8174		10406			
54	307	7229	9200	8794		11233			
55	303	7693	9825	9468		12134			
55	303	7146	9115	8758	7693	11210	9825	9468	12134
56	298	7570	9692	9387	8480	12055	10875	10571	13593
57	293	8042	10334	10089	9358	12996	12045	11800	15220
58	287	8582	11070	10892	10363	14073	13385	13207	17083
59	281	9220	11938	11841	11550	15345	14967	14869	19282
60	275	10000	13000	13000	13000	16900	16900	16900	21970

Table 12: Total reserve for subgroups of policies

Age	G1 No.	Total of reserve for groups of poliyhoder – Insured sum in the subgroup: 10000	13000	13000	13000	16900	16900	16900	21970
	Using/Not-using the option	0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1
	Premium in the subgroup	349,19	501,73	599,33	890,42	826,91	1205,33	1302,93	1741,59
40	10000	-1	0	0	0	0	0	0	0
41	9763	2838874	0	0	0	0	0	0	0
42	9528	5669837	0	0	0	0	0	0	0
43	9296	8500948	0	0	0	0	0	0	0
44	9067	11344916	0	0	0	0	0	0	0
45	8839	14212401	0	0	0	0	0	0	0
45	8839	12815870	1396531	0	0	0	0	0	0
46	8782	15912300	1868390	0	0	0	0	0	0
47	8720	19083215	2351867	0	0	0	0	0	0
48	8651	22324418	2846375	0	0	0	0	0	0
49	8575	25630065	3351060	0	0	0	0	0	0
50	8492	28995241	3865207	0	0	0	0	0	0
50	8492	26108518	3480535	2886723	0	384672	0	0	0
51	8402	29194864	3952372	3411736	0	463399	0	0	0
52	8303	32324932	4431284	3945374	0	543457	0	0	0
53	8195	35491443	4916204	4486540	0	624685	0	0	0
54	8077	38684498	5405683	5033747	0	706866	0	0	0
55	7950	41895913	5898492	5585669	0	789803	0	0	0
55	7950	37708220	5308917	5027377	4187693	710863	589575	558293	78941
56	7811	40612011	5754983	5527840	4850395	786111	688257	663019	94397
57	7662	43510605	6200834	6029182	5517236	861545	787597	768525	109972
58	7500	46391369	6644584	6529391	6185831	936868	887243	874444	125612
59	7326	49240527	7084169	7026253	6853521	1011753	986803	980368	141259
60	7138	52036347	7516361	7516361	7516361	1085697	1085697	1085697	156823

Age	G2 No.	Total of reserve for groups of poliyhoder – Insured sum in the subgroup: 10000	13000	13000	13000	16900	16900	16900	21970
	Using/Not-using the option	0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1
	Premium in the subgroup	349,19	501,73	599,33	890,42	826,91	1205,33	1302,93	1741,59
40	0	0	0	0	0	0	0	0	0
41	189	482477	0	0	0	0	0	0	0
42	342	931408	0	0	0	0	0	0	0
43	464	1326389	0	0	0	0	0	0	0
44	560	1647792	0	0	0	0	0	0	0
45	636	1874798	0	0	0	0	0	0	0
45	636	0	1874798	0	0	0	0	0	0
46	632	33425	2150429	0	0	0	0	0	0
47	627	78177	2415021	0	0	0	0	0	0
48	622	135791	2666016	0	0	0	0	0	0
49	617	207636	2900616	0	0	0	0	0	0
50	611	295038	3116522	0	0	0	0	0	0
50	611	0	0	295038	0	3116522	0	0	0
51	604	77001	10549	331472	0	3443429	0	0	0
52	597	172309	23817	367940	0	3738638	0	0	0
53	590	287457	40043	404421	0	3998792	0	0	0
54	581	424154	59491	440871	0	4219952	0	0	0
55	572	583271	82309	477281	0	4399694	0	0	0
55	572	0	0	0	583271	0	82309	477281	4399694
56	562	158061	22421	21582	645897	3072	91752	526427	4856825
57	552	343835	49023	47712	705902	6820	100814	568756	5240782
58	540	558474	80000	78636	763183	11284	109478	603805	5546412
59	528	802792	115497	114552	817631	16495	117726	631141	5768876
60	514	1077993	155710	155710	868785	22491	125491	649981	5900002

Table 12 (cont'd)

Age	G 3 No.	Total of reserve for groups of policyhoder								Total reserve for groups G 1, G 2, G 3
		Insured sum in the subgroup								
		10000	13000	13000	13000	16900	16900	16900	21970	
		Using/Not-using the option								
		0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1	
		Premium in the subgroup								
		349,19	501,73	599,33	890,42	826,91	1205,33	1302,93	1741,59	
40	0	0	0	0	0	0	0	0	0	-1
41	21	81863	0	0	0	0	0	0	0	3 403 214
42	71	304787	0	0	0	0	0	0	0	6 906 033
43	143	672161	0	0	0	0	0	0	0	10 499 497
44	232	1183368	0	0	0	0	0	0	0	14 176 076
45	331	1834030	0	0	0	0	0	0	0	17 921 229
45	331	0	1834030	0	0	0	0	0	0	17 921 229
46	330	20977	1962097	0	0	0	0	0	0	21 947 619
47	328	47139	2085445	0	0	0	0	0	0	26 060 864
48	326	79319	2203545	0	0	0	0	0	0	30 255 464
49	323	118318	2315768	0	0	0	0	0	0	34 523 464
50	321	165052	2422195	0	0	0	0	0	0	38 859 254
50	321	0	0	165052	0	2422195	0	0	0	38 859 254
51	318	36954	5159	179629	0	2541942	0	0	0	43 648 506
52	315	81296	11401	194777	0	2648668	0	0	0	48 483 894
53	311	133913	18860	210610	0	2742401	0	0	0	53 355 369
54	307	195976	27709	227278	0	2823133	0	0	0	58 249 358
55	303	268551	38112	244984	0	2892007	0	0	0	63 156 087
55	303	0	0	0	268551	0	38112	244984	2892007	63 156 087
56	298	63572	9044	8760	293921	1250	41882	263316	3051868	69 036 663
57	293	138546	19782	19313	320634	2764	45857	281150	3195700	74 872 886
58	287	226651	32484	31963	349310	4589	50130	298807	3327524	80 648 072
59	281	330395	47533	47145	380957	6789	54852	316838	3453529	86 347 400
60	275	454057	65586	65586	417082	9474	60245	335970	3580619	91 944 126

where the nullification of the total reserve at the beginning of policy duration results in the **premium for the option 4,54**, which is **1,32 % of the premium for the original cover**.

Table 13: Reserve of the option

	Policies with option/Policy reserve for subgroup								Policies without option/Policy reserve for subgroup							
	Insured sum								Insured sum							
Age	10000	13000	13000	13000	16900	16900	16900	21970	10000	13000	13000	13000	16900	16900	16900	21970
	Using/Not-using option								Using/Not-using option							
	0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1	0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1
	Annual premium								Annual premium							
	349,19	501,73	599,33	890,42	826,91	1205,33	1302,93	1741,59	349,19	501,73	599,33	890,42	826,91	1205,33	1302,93	1741,59
40	0								0							
41	341								336							
42	695								685							
43	1060								1045							
44	1438								1418							
45	1828								1802							
45	1611	2758							1802	1802						
46	2016	3279							2170	2318						
47	2438	3817							2580	2881						
48	2877	4376							3007	3467						
49	3334	4955							3451	4075						
50	3810	5557							3914	4707						
50	3796	4554	3923		6384				3914	4707	3914		4707			
51	4293	5231	4647		7214				4396	5359	4639		5675			
52	4812	5937	5403		8080				4900	6046	5395		6690			
53	5354	6676	6193		8988				5427	6765	6185		7750			
54	5922	7448	7022		9946				5979	7518	7011		8860			
55	6517	8258	7892		10966				6559	8309	7877		10022			
55	6507	8245	7807	6592	9936	8358	8240	11076	6559	8309	7877	6559	10022	8309	7877	10022
56	7133	9097	8738	7725	11184	9869	9722	12860	7170	9142	8785	7709	11241	9842	9485	12151
57	7791	9993	9717	8924	12497	11466	11307	14792	7815	10021	9746	8919	12531	11456	11181	14397
58	8486	10939	10749	10195	13881	13161	13015	16915	8498	10953	10764	10198	13898	13162	12973	16770
59	9220	11938	11841	11550	15345	14967	14869	19282	9225	11943	11845	11554	15350	14971	14874	19287
60	10000	13000	13000	13000	16900	16900	16900	21970	10000	13000	13000	13000	16900	16900	16900	21970

Table 13 (cont'd)

Age	Reserve of the option for 1 policy in the subgroup								Reserve of the option for all policies in the subgroup								Reserve of the option
	Insured sum								Insured sum								
Age	10000	13000	13000	13000	16900	16900	16900	21970	10000	13000	13000	13000	16900	16900	16900	21970	
	Using/Not-using option								Using/Not-using option								
	0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1	0-0-0	1-?-?	0-1-?	0-0-1	1-1-?	1-0-1	0-1-1	1-1-1	
	Annual premium								Annual premium								
	349,19	501,73	599,33	890,42	826,91	1205,33	1302,93	1741,59	349,19	501,73	599,33	890,42	826,91	1205,33	1302,93	1741,59	
40	0	0	0	0	0	0	0	0	-3	0	0	0	0	0	0	0	-
41	5	0	0	0	0	0	0	0	47511	0	0	0	0	0	0	0	4751
42	10	0	0	0	0	0	0	0	96870	0	0	0	0	0	0	0	9687
43	15	0	0	0	0	0	0	0	148048	0	0	0	0	0	0	0	14804
44	20	0	0	0	0	0	0	0	200873	0	0	0	0	0	0	0	20087
45	26	0	0	0	0	0	0	0	255275	0	0	0	0	0	0	0	25527
45	-191	957	0	0	0	0	0	0	-1515642	1770917	0	0	0	0	0	0	25527
46	-154	961	0	0	0	0	0	0	-1216670	1753430	0	0	0	0	0	0	53676
47	-142	936	0	0	0	0	0	0	-1122169	1680636	0	0	0	0	0	0	55846
48	-130	909	0	0	0	0	0	0	-1021440	1602979	0	0	0	0	0	0	58153
49	-118	880	0	0	0	0	0	0	-914987	1521671	0	0	0	0	0	0	60668
50	-104	850	0	0	0	0	0	0	-804009	1438780	0	0	0	0	0	0	63477
50	-118	-153	10	0	1677	0	0	0	-812234	-117105	8225	0	1555885	0	0	0	63477
51	-103	-128	8	0	1540	0	0	0	-704290	-97193	7032	0	1376350	0	0	0	58189
52	-88	-109	8	0	1390	0	0	0	-595150	-81664	6465	0	1192316	0	0	0	52196
53	-73	-89	8	0	1237	0	0	0	-486771	-66275	6897	0	1014153	0	0	0	46800
54	-57	-70	11	0	1087	0	0	0	-381269	-51347	8554	0	846789	0	0	0	42272
55	-43	-51	14	0	944	0	0	0	-281033	-37238	11500	0	695452	0	0	0	38868
55	-53	-64	-70	33	-86	49	363	1054	-305887	-41438	-44912	24854	-6182	4200	56412	701634	38868
56	-38	-45	-47	17	-57	27	237	709	-214821	-28596	-29950	12422	-4056	2271	35388	440933	21359
57	-24	-28	-29	4	-35	10	126	395	-135452	-17542	-18260	3181	-2413	806	18026	228368	7671
58	-13	-14	-15	-3	-17	-1	42	145	-71016	-8763	-9016	-2129	-1136	-86	5706	77074	-936
59	-5	-5	-5	-5	-5	-5	-5	-5	-24822	-2758	-2758	-3167	-306	-352	-589	-2206	-3696
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

6. Conclusion

The paper analyses two types of life insurance options – renewal option and increasing option. Both options could have various designs, some of them more standard (annual indexation of premium), some of them offered less often (increase of some insured every *n* years) and some of them more theoretical.

The subject of actuarial work is the analysis of the risk related to the option.

When the presented model is applied to a concrete design of the option, the model must be appropriately redesigned, especially the transition matrix related to the option realization. There are usually more possibilities how to model a particular option. When designing the model, both the relevance of the model and the practical point of view should be taken into account. The result of including all input parameters to the model could be a model, which is too complicated for calculation purposes. Available data should be kept in mind as well.

We saw that one of the most important parameters is the percentage of policyholders, which use the option. Actuary should regularly analyse this parameter to asses the influence of the option on the future cash flow.

The model presented in this paper could as well be used in the cash flow modelling and embedded value calculation, especially to estimate the mortality rates after any realization of the option.

7. References

Swiss Re: Selection of Risks in Life Assurance, Zürich.
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