"SIMULATION OF RISK PROCESSES WITH VARIABLE PREMIUN RATE"

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Summary

The classical Cramer-Lundberg process assumes that the rate of premium income received by the insurance company is a constant. The present paper examines generalization of the classical process in which the rate of premium income varies with time or is stochastic. To construct the generalized process we use the Monte-Carlo simulation. The ruin probability of the insurance company in case of variable premium rate is estimated. The results are illustrated by actual data.

"LA SIMULATION DES PROCESSUS AVES LES RECETTES VARIABLE DES PRIMES D'ASSURANCE "

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Russia

Résumé

On suppose que dans le processus classique de Cramér-Loundberg la vitesse des recettes des primes dans la compagnie d'assurance est une constante. Dans l'ouvrage en question on envisage la généralisation du processus classique dans lequel la vitesse des recettes des primes d'assurance change avec le temps ou se présente stochastique. Pour construire le processus généralisé on utilise la simulation à la base de la méthode Monté-Carlo. On estime la probabilité de la ruine de la campagnie d'assurance en cas de la vitesse variable des recettes des primes d'assurance. Les résultats souf illustrés à la base des données réélles.

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1. INTRODUCTION

The problem of estimating (calculating) the *probability of ruin* has been discussed in many papers on actuarial risk theory. Most of these papers assume that surplus at time t is defined as a sum of initial surplus u, income obtained from premium payments $P(t)$ minus total amount of paid claims $S(t)$:

$$U(t) = u + P(t) - S(t). \qquad \textbf{(1.1)}$$

$S(t)$ is a stochastic process, for example, Poisson process and amount of claims has some distribution function F in these papers. The classical Cramer-Lundberg process assumes that the rate of premium income received by an insurance company is constant and can be written as

$$P(t) = ct,$$

and the risk process in this case can be written by the formula:

$$U(t) = u + ct - \sum_{i=1}^{N_t} Y_i, \qquad \textbf{(1.2)}$$

where $U(t)$ — is surplus at time t, u – initial surplus, c – is a constant premium rate, Y_i – are claim amounts paid out between (i-1)-th and i-th claims, N_t — is the number of claims occurred up to time t. The premium income rate according to the expected value principle is

$$c = (1+q)\, lm,$$

where q — is a strictly positive parameter called *safety loading*, m — is the mean of claim amounts, l — is an intensity of Poisson process.

Let us consider the problem of finding ruin probability of an insurance company. We define the time of ruin as

$$T = \inf\{t \mid U(t) < 0\}$$

and define $T = \infty$ if $U(t) \geq 0$ for all t. We define the ruin probability over a finite time $T_{\max}$

$$y\,(u, T_{\max}) = P[T < T_{\max} \mid U(0) = u] \qquad \textbf{(1.3)}$$

and the probability of ultimate ruin as

$$y\left(u\right)=P[T<\infty\,|U\left(0\right)=u]\,. \qquad \textbf{(1.4)}$$

We can derive explicit formulae **(1.3)** and **(1.4)** for rather a small number of claims distribution.

If we know the so-called *adjustment coefficient* $k>0$, which is non-zero solution of the equation:

$$1+(1+q)km=L_X(-k)\,,$$

and we have a Laplace transform for the distribution of the claim amounts, then we can use a *Cramer's asymptotic ruin formula* [8]:

$$y\,(u)\sim-\frac{qme^{-ku}}{L_X'(-k)+m(1+q)},\quad u\to\infty\,.$$

We are interested in the case when premium income is not proportional with time and moreover the process of premium income is stochastic.

We are going to introduce more general models for this classical Cramer-Lundberg risk process.

In the papers [10-12] the authors discuss perturbed risk processes

$$X\left(t\right)=U\left(t\right)+W\left(t\right),$$

where $U\left(t\right)$ – is the classical Cramer-Lundberg process, $W\left(t\right)$ – is some stochastic perturbation process, for example, $W\left(t\right)=eW_B\left(t\right)$, where $e>0$ is a constant, $W_B\left(t\right)$ — is the standard Brownian motion independent of $U\left(t\right)$. In the papers [1, 4, 5, 7, 15] the authors assume that the rate at which premiums come in is non-constant and is a function of the current surplus. In this case the accumulated premiums at time t can be written by the following formula

$$P\left(t\right)=\int_0^t c\left(U\left(t\right)\right)dt\,. \qquad \textbf{(1.5)}$$

It is possible to consider generalizations of model **(1.5)**. In particular, when premium income rate is a function of time:

$$c=c\left(t\right).$$

Then total premiums up to time t can be written as

$$P\left(t\right)=\int_0^t c\left(t\right)dt\,. \qquad \textbf{(1.6)}$$

The process $P\left(t\right)$ can also be defined as a stochastic process.

In paper (18) the author models premiums with a Poisson process independent of claims process and estimates confidence interval for adjustment coefficient.

In this paper we consider the methods of estimating the probability of ruin for variable and stochastic premium rate.

2. Model with premium rate dependent on current surplus

Let us consider the case when premium income rate depends on the current value of surplus. If we assume this, the risk process can be written as

$$U(t) = u + \int_0^t c\left(U(t)\right) dt - \sum_{i=1}^{N_t} Y_i \,. \qquad \textbf{(2.1)}$$

We can rewrite **(2.1)** in form of stochastic differential equation:

$$dU(t) = c\left(U(t)\right)dt - dS\,,$$

where $dS = ydp$, $dp = 1$ with probability $l\,dt + o(dt)$ and $dp = 0$ with probability $1 - l\,dt + o(dt)$, and y — is a random variable (claim amount) with distribution function $F(x)$, and $U(0) = u$.

It is shown if $c = c\left(U(t)\right)$ that the probability of ruin can be described by the integral-differential equation

$$c(u)\bar{y}'(u) = l\bar{y}(u) - l\int_{-\infty}^{u} \bar{y}(u-y)\,dF(y), \quad u > 0\,,$$

where $\bar{y}(x) = 1 - y(x)$.

Example 1. Model with interest rate. Let us assume that surplus earns interest at constant force d, so

$$c\left(U(t)\right) = c_0 + dU(t)\,.$$

See for instance [4, 5, 13].

There are exact analytical results for ruin probability in this case, when the claim distribution function is exponential $F(x) = 1 - e^{-ax}$ (See [4, 13]):

$$y(u) = \frac{\Gamma\left(\frac{l}{d}, \frac{c_0}{dm} + \frac{u}{m}\right)}{\Gamma\left(\frac{l}{d}, \frac{c_0}{dm}\right) + \frac{d}{m}\left(\frac{c_0}{dm}\right)^{\frac{l}{d}} e^{-\frac{c_0}{dm}}}\,, \qquad \textbf{(2.2)}$$

where $\Gamma(a,b)=\int_b^\infty x^{a-1}e^{-x}dx$, m — is mean claim amount, l — is the rate of Poisson process [13].

Example 2. Model with premiums by layers. Consider the case when premiums can vary according to the level of current surplus. The larger the surplus, the lesser the risk of ruin. Mathematically, this can be formulated as:

$$c\left(U(t)\right)=\begin{cases} c_0, & 0=U_0\leq U\leq U_1 \\ c_{1,} & U_1<U\leq U_2 \\ \mathrm{M} & \\ c_{k-1} & U_{k-1}<U<U_k=\infty \end{cases} \qquad \textbf{(2.3)}$$

The method of estimation of ruin probability in this case, based on links between the waiting time of the single-server queue (M/G/1) and the risk process, is discussed in [7].

Consider the special case when premium income rate is a function of surplus and is defined by the followingrule: premium income rate is assumed to be $c=c_0$ if surplus is below some barrier b, otherwise it is $c=c_1$:

$$c=\begin{cases} c_0, & u<b \\ c_1, & u\geq b \end{cases}.$$

When claim distribution is exponential we have an explicit solution

$$y(u)=\begin{cases} a\,y_0(u)+b, & u<b \\ y_1(u-b)\left(a e^{-\frac{b(c_0-lm)}{c_0}}+b\right), & u\geq b \end{cases}$$

where

$$y_i(u)=\frac{lm}{c_i}e^{-\left(1-\frac{lm}{c_i}\right)u}, \qquad a=\frac{c_1-lm}{c_1-lm+(c_0-c_1)y_o(b)}, \qquad b=1-a\ .$$

Analytical results for this case were evaluated by D.Dickson [1].

Suppose the claim distribution is other than exponential. Then we cannot find exact analytical results for ruin probability and hence computer simulation of the risk process is a useful method.

See [2] on the links between the surplus process of risk theory and the single-server queue. See [7] for results for Examples 1 and 2.

3. MODEL WITH PREMIUM INCOME RATE DEPENDENT ON TIME

Combining **(1.1)** and **(1.6)** we can write the formula for surplus in this case, when premium income rate is a function of time:

$$U(t) = u + \int_0^t c(t)\,dt - \sum_{j=1}^{N_t} Y_j .$$

Consider some special cases of the premium function.

1. The classical case. In this case $c(t) = c = \text{const}$ .Then

$$P(t) = \int_0^t c dt = ct .$$

2. The harmonic law. Let $c(t) = a + A\cos(wt + b)$, so that the accumulation of premiums at time t is defined as

$$P(t) = \int_0^t (a + A\cos(wt + b))\,dt =$$

$$= at + \frac{A}{w}\left[\sin(wt + b) - \sin b\right].$$

3. Let $c(t) = a + bt$. Then

$$P(t) = \int_0^t (a + bt)\,dt = at + \frac{b}{2}t^2 .$$

4. The generalization of rule 3 is $c(t) = a + bt^k, k \neq -1$. Then the accumulated premiums can be written as

$$P(t) = \int_0^t (a + bt^k)\,dt = at + \frac{b}{n+1}t^{n+1} .$$

5. For $c(t) = a + b/t$, $t > t_0 > 0$ the accumulated premiums can be written as

$$P(t) = \int_{t_0}^t \left(a + \frac{b}{t}\right)dt = at + b(\ln t - \ln t_0) .$$

4. MODEL WITH PREMIUM INCOME RATE VARYING BETWEEN CLAIMS

Suppose premium income rate can vary but is a constant in each time interval between (i-1)-th and i-th claims. Then **(1.2)** can be written as

$$U(t) = u + \left[\sum_{i=1}^{N_t} c_i s_i + c_{N_t+1}\left(t - T_{N_t}\right)\right] - \sum_{j=1}^{N_t} Y_j , \qquad \textbf{(4.1)}$$

where $T_1, T_2, \ldots$ — are times of claims (we assume that $T_0 = 0$), c_i — is the premium income rate between (i-1)-th and i-th claims (if $c_i = c$ then we get a classical Cramer-Lundberg process), $s_i = T_i - T_{i-1}$.

We can simplify **(4.1)** if we examine surplus at times $t = T_1, T_2, \ldots$ only:

$$U(T_j) = u + \sum_{i=1}^{N_t} c_i s_i - \sum_{j=1}^{N_t} Y_j .$$

The values c_i can be both deterministic and stochastic.

There is another possible way to form the premium income process. Consider the case when mean m of the claim size distribution is unknown. Here the premium rate at time t based upon claims statistics up to time t is

$$c(t) = (1+q)\frac{S(t-)}{t},$$

as the best estimator of the lm is the ratio $S(t-)/t$. The values of the c_i can be written as

$$c_1 = 0,$$
$$c_i = (1+q)\frac{S(T_{i-1})}{T_{i-1}}, \qquad i = 2,3,\ldots,$$

where $S(T_k) = \sum_{j=1}^{k} Y_j$. The total of accumulated premiums at time t is

$$P(t) = \sum_{i=1}^{N_t} c_i s_i + c_{N_t+1}\left(t - T_{N_t}\right),$$

or in the time moments $T_1, T_2, \ldots$

$$P(T_j) = \sum_{i=1}^{N_t} c_i s_i .$$

Note, that in this model $y(0) = 1$, as there is not any surplus growing before the first claim and any small claim leads to ruin.

5. PREMIUM INCOME AS STOCHASTIC PROCESS

In the classical risk model premiums are assumed being received continuously at a constant rate. In reality premium income of an insurance company can be presented as a sum of separate premiums received from clients for insurance policies.

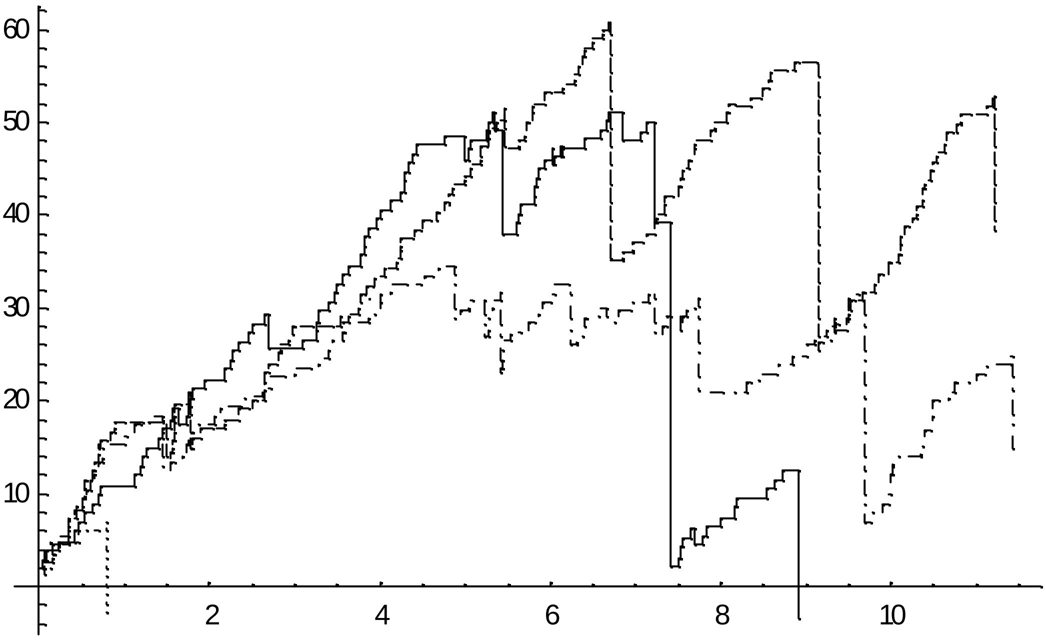

Fig. 1. Sample paths of the risk processes where the premiums and claims occurred in accordance the Poisson process with rates $l^p = 10$ and $l = 1$ respectively.

In this case the classical model of risk does not work and we introduce the following new process.

Suppose a premium p is not constant but a variable and depends on many parameters. Hence the process $U(t)$ will be

$$U(t) = u + \sum_{i=1}^{M_t} p_i - \sum_{j=1}^{N_t} Y_j \,, \qquad \textbf{(5.1)}$$

where M_t denotes the number of premiums up to time t, p_i is amount of i-th premium. For simplicity the number of accumulated premiums M_t in time interval $(0,t]$ – is a Poisson process with rate l^p and time intervals between two consequent premiums are distributed exponentially with parameter $1/l^p$.

Let us introduce the following definitions. Let the claim c.d.f. be $F(x)$, premiums c.d.f. is $G(x)$. Then time intervals between the claims — s_i — have a c.d.f. $H(x)$ and time interval between premiums — x_i — have a c.d.f. $K(x)$. In case of Poisson processes distribution functions $H(x), K(x)$ are exponential with parameters $1/l$ and $1/l_p$ respectively. The distribution functions $F(x)$, $H(x), G(x)$ and $K(x)$ can be both continuous and discrete.

In the special case when the premiums p_i are constant and equal p the process **(5.1)** can be written as

$$U(t) = u + \sum_{i=1}^{M_t} p - \sum_{j=1}^{N_t} Y_j = u + M_t\, p - \sum_{j=1}^{N_t} Y_j \ .$$

If the number of premiums is large then instead of p_i we can take the mean of premiums distribution function. Hence

$$p = E[p_i]. \qquad (5.2)$$

Let us show that with this assumption we can reduce **(5.1)** to the classical model. We have

$$ct = E[pM_t] = pE[M_t] = p\,l^{p} t \ . \qquad (5.3)$$

Then $c = p\,l^{p}$. The **(5.1)** with **(5.3)** and **(5.2)** can be written as

$$ct = E[p_i M_t] = E[p_i]E[M_t] = p\,l^{p} t$$

Example 3. Assume that premium income process is a Poisson process. The value of premiums is constant and equal p. The following table gives results for this simple model.

l	l_p	p	a	u_0	$T_{\max}$	N	t	y	y_{exact}
10	1	0,25	0,5	25	500	20000	43,84	0,0755	0,065668
10	1	0,25	0,45	25	500	20000	77,33	0,2674	0,25497
10	1	0,25	0,5	10	500	20000	20,668	0,31215	0,2943
10	1	0,25	0,45	10	500	20000	37,14	0,555	0,539138

Here l – is the intensity of the claim process;

l_p – is the intensity of the premium income process;

p – is the premium size;

a – is the parameter of the exponential distribution;

u_0 – is the initial reserve;

$T_{\max}$ – is the up time limit;

N – is the number of simulations;

t – is the calculated mean of the time to ruin;

y – is the estimation of the probability of ruin;

y_{exact} – is the probability of ruin in the classical case with $c = 2{,}5$.

6. ESTIMATING OF RUIN PROBABILITY BY SIMULATION

Monte-Carlo can be one of the possible methods of estimating ruin probability. The main idea of this method is multiple simulation of the risk processes and calculation of the relative frequency of processes leading to ruin. But we cannot get the information about the risk processes when the time goes to the infinity by straightforward simulation. Therefore, this method allows us to estimate the ruin probability in finite time $T_{\max}$ not only for the classical Cramer-Lundberg processes but also for ordinary and delayed renewal processes and others.

Consider the following problem. We would like to evaluate ruin probability of an insurance company given the information about all premiums and claims occurring in some time period. If we know a constant premium income rate, initial surplus and claims distribution then we can try to use the results for classical Cramer-Lundberg process. If we can find an adjustment coefficient and Laplace transform for our claims distribution then we can also use the famous asymptotic Cramer-Lundberg formula.

But in real insurance practice the process $P(t)$ cannot be expressed by ct exactly and therefore we can use Monte-Carlo simulation.

We suggest the following method of estimating ruin probability. With the actual data on premiums and claims we shall construct a suitable risk model and then try to estimate ruin probability by simulations.

We denote $Z_i = I(t_i < T_{\max})$ as indicator of ruin of i-th process for the time $T_{\max}$. For N realizations of stochastic process we can write a number of ruined processes

$$N_r = \sum_{i=1}^{N} Z_i \,, \qquad \textbf{(6.1)}$$

and we can also calculate approximate values of ruin probability and complimentary 'survival' (non-ruin) probability as

$$y_N = \frac{1}{N}(Z_1 + Z_2 + ... + Z_N) = \frac{N_r}{N}, \qquad \textbf{(6.2)}$$

$$d_N = 1 - y_N$$

respectively.

We can write sample variance as

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{N}\left(Z_i - \psi_N\right)^2 =$$
$$= \frac{1}{n-1}\left(N_r\left(1-\psi_N\right)^2 + \left(N - N_r\right)\psi_N^2\right) =$$
$$= \frac{1}{n-1}\left(N\psi_N^2 - 2N_r\psi_N + N_r\right).$$

We will discuss a concrete example.

There are actual data of the premium income and claim amounts for some period. Consequently, we can analyze processes $P(t)$ and $S(t)$ for this period. There are dates of premium income (dates of contract agreements), validity period of policies and cost of these insurance policies (a value of premium). There are claim dates and claim amounts for claims.

Premium income

The period is from May 10, 1994 to November 27, 1997. The number of the records is 32791. In calculating we used the trial-version of the Palisade Corp. BestFit 4.0.4. The fitting to the lognormal distribution with density function

$$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\left(\ln x - \mu\right)^2}{2\sigma^2}\right) \qquad \textbf{(6.3)}$$

gives the following results:

$\mu = 3.04128363077844$

$\sigma = 0.882439444918749$

Shift 0.220034539604291

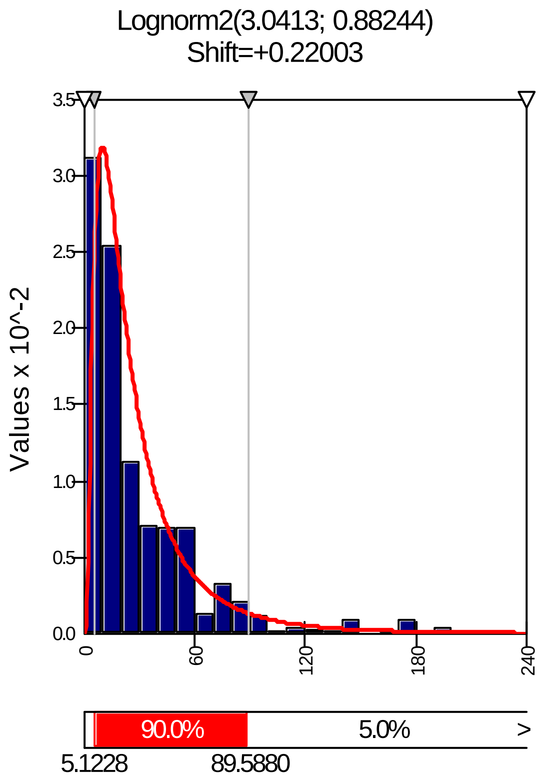

Fig. 2. The fit of Lognorm2 distribution to data of the premium income.

Time intervals between premiums

Intervals between the premiums were from 0 to 4 days. Table 1 contains frequencies of these intervals.

Table 1. Frequencies of time intervals between premiums.

Time interval	Frequency	%
0	38603	97.0
1	1121	2.8
2	49	.1
3	14	.0
4	4	.0
Total	39791	100.0

Claim amounts

The period is from May 10, 1994 to November 27, 1997. The number of the records is 3207. The fitting to the lognormal distribution **(6.3)** gives these parameters

$m = 5.04436473762969$

$s = 1.10229132652852$

Shift 2.83850139509413

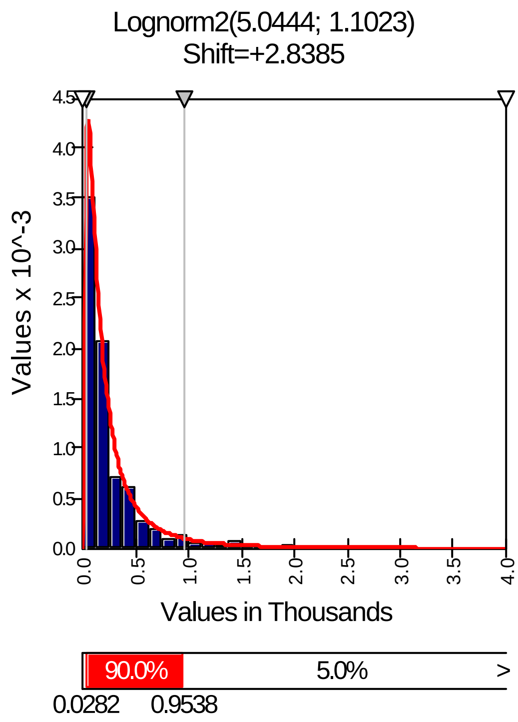

Fig. 3. The fit of Lognorm2 distribution to data of the claim amounts.

Time intervals between claims

In Table 2 there are frequencies of the time intervals between the claims.

Table 2. Frequencies of time intervals between claims.

Time interval	Frequency	%
0	2509	78.3
1	491	15.3
2	33	1.0
3	104	3.2
4	32	1.0
5	14	0.4
6	8	0.2
7	5	0.2
8	2	0.1
10	1	0.0
11	1	0.0
13	2	0.1
14	1	0.0
15	1	0.0

17	1	0.0	Total	3206	100.0
18	1	0.0			

Random number generation

Before we start simulations we need to choose a method of generating random numbers. Below we discuss different possible methods.

First of all, we examine the *uniform variates*, which are random numbers that lie within a range from 0 to 1. There is the so-called *linear congruential generator* which generates a sequence of integrals $W_1, W_2, \ldots$. This algorithm starts with explicit setting of the first integer W_1 with the following recursive calculation of the remaining integers

$$W_{i+1} = \left(kW_i + C\right) \bmod p\,. \qquad \textbf{(6.4)}$$

Here k, C, p — are constants. k is called the *multiplier*, and C and p are integers called the *increment* and *modulus* respectively. A uniform random number from [0;1] is obtained by dividing W_i by modulus

$$U_i = \frac{W_i}{p}. \qquad \textbf{(6.5)}$$

The period of such generator equals $p-1$. The constants k, C, p ought to be chosen very carefully. For example, the following values would be suitable: $p = 2^{32} - 1 = 2147483647$, $C = 0$, $k = 16807$, $W_1 = 12345$.

There are many other uniform random number generators with other periods and other CPU timing, for example, nonlinear congruential generators, generators with shuffle, generators based on data encryption and others. For details and references see[9].

For modeling the *standard Poisson process* with the rate l, note that the interarrival times $d_i = T_i - T_{i-1}$ have exponential distribution with c.d.f. $F(x) = 1 - e^{-ax}$ with parameter $a = 1/l$.

Exponential deviates X with parameter a can be obtained from uniform deviates in interval $[0,1]$ based on the so-called *inversion method*. Let $F(x)$ — be some c.d.f., then $F^{-1}(x)$ — is an inverse of the $F(x)$. Then

$$X \sim F^{-1}(U)$$

where $U \sim uniform(0,1)$. For the random deviate with *exponential* distribution we can write

$$Exp(a) \sim -\frac{1}{a}\ln(1-U) \sim -\frac{1}{a}\ln(U). \quad \textbf{(6.6)}$$

The same method can be easily used to produce *Pareto* random deviates ($F(x)=1-x^{-c}$):

$$Pa(c) \sim U^{-1/c}.$$

The random number with *Weibul* c.d.f. ($F(x)=1-e^{-\left(\frac{x}{b}\right)^c}, b>0, c>0$) can be calculated as

$$Weib(c) \sim b(-\ln U)^{1/c}.$$

For the generation of a pair of *standard normal* random deviates, we can use the *Box-Muller method*:

$$\begin{aligned} N_1(0,1) &\sim \sqrt{-2\ln U_1}\sin(2pU_2), \\ N_2(0,1) &\sim \sqrt{-2\ln U_2}\cos(2pU_1), \end{aligned} \quad \textbf{(6.7)}$$

where U_1, U_2 — is a pair of random deviates with a uniform distribution in the interval (0,1). To construct a deviate $N(m, s^2)$ we can use the following formula

$$N(m, s^2) \sim s^2 N(0,1) + m. \quad \textbf{(6.8)}$$

The random deviate with lognormal distribution can be obtained from the standard normal random deviates

$$Lognorm(m, s) \sim m\exp(s\,N(0,1)). \quad \textbf{(6.9)}$$

Example 4. Estimation of the probability of ruin by simulation. We construct N processes with parameters:

$T_{\max} = 2500$,

$N = 1000$,

$p \sim Lognorm2(20.932, 0.8824) + 0,22$,

$Y \sim Lognorm2(155.1457, 1.1023) + 2,8385$.

From **(6.1)** and **(6.2)** we can easily estimate the probability of ruin. For initial surplus varying from 0 to 40000 with step 1000 we have calculated following values of $y(u)$:

Table 3. The probability of ruin for Example 4.

u	$y(u)$	u	$y(u)$
0	0.947	21000	0.047
1000	0.796	22000	0.051
2000	0.694	23000	0.036
3000	0.591	24000	0.041
4000	0.511	25000	0.039
5000	0.423	26000	0.021
6000	0.413	27000	0.022
7000	0.353	28000	0.022
8000	0.265	29000	0.014
9000	0.257	30000	0.022
10000	0.223	31000	0.009
11000	0.199	32000	0.012
12000	0.178	33000	0.015
13000	0.165	34000	0.008
14000	0.131	35000	0.01
15000	0.126	36000	0.009
16000	0.106	37000	0.004
17000	0.107	38000	0.006
18000	0.079	39000	0.007
19000	0.074	40000	0.004
20000	0.069		

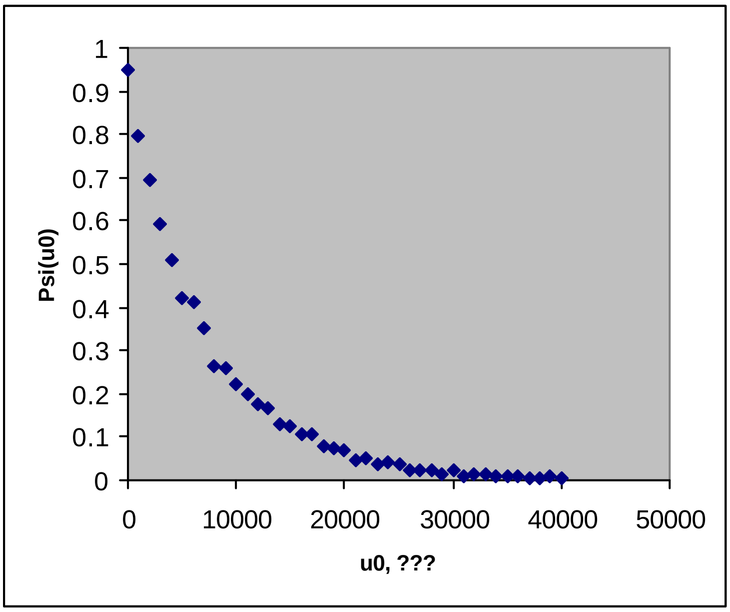

Fig. 4. The visualization of Table 2.

The obtained results can be fit by the exponential curve $y\left(u\right)=0.91392e^{-0.000138152u}$.

Example 5. Severity and time to ruin. Let us show the results of simulation of 20 000 processes with parameters

$u=5000$,

$T_{\max}=3000$,

$p \sim Lognorm2\left(20.932, 0.8824\right)+0.22$,

$Y \sim Lognorm2\left(155.1457, 1.1023\right)+2.8385$.

The fitting of the Lomax (Pearson-2) distribution with c.d.f.

$$F\left(x\right)=1-\frac{b^{q}}{\left(x+b\right)^{q}}$$

to the data of severity gives the following parameters: $b=581.5212$, $q=1.8068$, shift = 0.0056 (see Fig. 5).

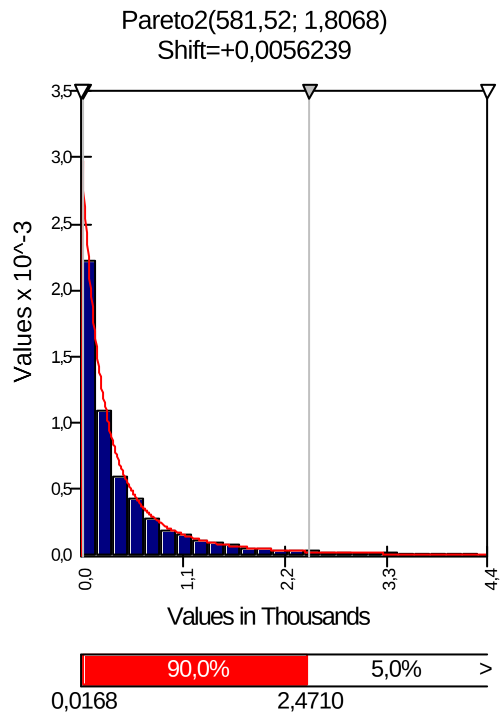

Fig. 5.

The fitting of the Inverse Gaussian distribution with density function

$$f(x) = \sqrt{\frac{\lambda}{2\pi x^3}} \exp\left(-\frac{\lambda (x-\mu)^2}{2\mu^2 x}\right)$$

to the data of the time to ruin (if the process is ruined) gives the following parameters $\mu = 24.1549$, $\lambda = 9{,}0753$, shift = -1,336.

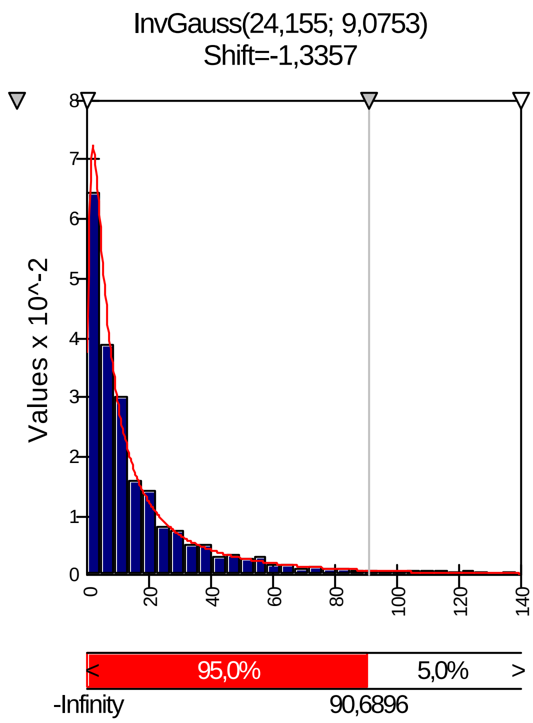

Fig. 6.

7. ON THE PRECISION OF THE RESULTS OBTAINED

Suppose the simulation gives us n values of ruin probability. Each of these values is obtained as a result of N runs. Then the estimate of y can be obtained as empirical mean:

$$\hat{y} = \frac{1}{n}(y_1 + y_2 + ..y_n), \qquad \textbf{(7.1)}$$

and variance $s_y^2 = \mathrm{Var}\, y$ as empirical variance:

$$\hat{s}_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \hat{y}\right)^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - n\hat{y}^2\right). \qquad \textbf{(7.2)}$$

The question is how large n and N we should choose in order to obtain a given level of precision. For Monte-Carlo method we can only get some confidence interval for ruin probability. We will show what n we need to choose to obtain a result close enough to true answer with a given probability.

From central limit theorem it follows that

$$\sqrt{n}\left(\hat{y} - y\right) \xrightarrow{D} N\left(0, s_y^2\right). \qquad \textbf{(7.3)}$$

Then 95% confidence interval is:

$$\hat{y} \pm \frac{1,96\hat{s}_y}{\sqrt{n}} = \left[\hat{y} - \frac{1,96\hat{s}_y}{\sqrt{n}}, \hat{y} + \frac{1,96\hat{s}_y}{\sqrt{n}}\right]. \qquad \textbf{(7.4)}$$

For example, if we need to evaluate a number n of realizations so that the error is smaller than e with probability 95% we can write the following formula

$$n = \frac{1,96^2 s_y^2}{e^2}. \qquad \textbf{(7.5)}$$

In practice the value s_y^2 is unknown. But we can take some initial number n' of the values $y_1, y_2, \ldots, y_{n'}$, calculate the variance $s_y'^2$ and then calculate $n = 1,96^2 s_y'^2 / e^2$.

Example 6. There is a histogram of the y_i with normal curve with parameters:

$n = 1615$,

$T_{\max} = 2500$,

$u = 5000$,

$p \sim Lognorm2(20.932, 0.8824) + 0,22$,

$Y \sim Lognorm2(155.1457, 1.1023) + 2,8385$

in the Fig. 7.

Estimations of $\hat{y}(u)$ and $\hat{s}_y^2$ are equal $\hat{y} = 0.45507$ and $\hat{s}_y^2 = 0.000179$. Therefore we can give the result:

$$y = 0.45509 \pm 0.00065. \qquad \textbf{(7.6)}$$

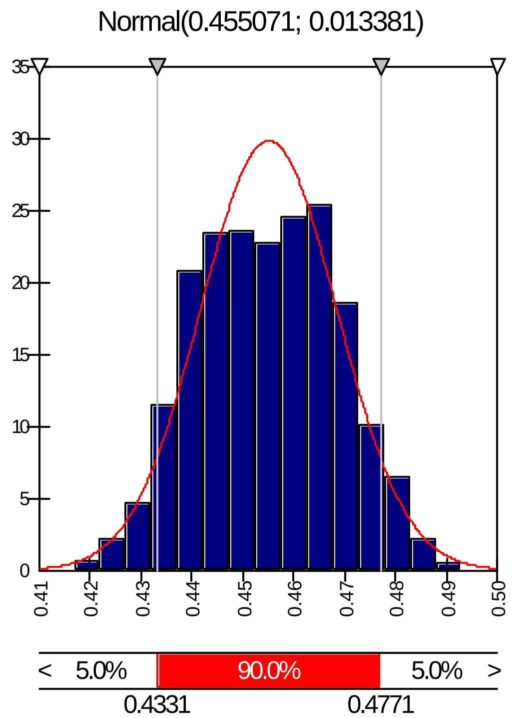

Fig. 7. The histogram of the estimations of $y\ (u)$**.**

8. Conclusion

The risk models described in this paper do not aim at an exhaustive coverage of real processes occurring in insurance practice. However, the impossibility of obtaining analytical results compels the researcher to use the methods proposed in this paper.

Regrettably, the Monte-Carlo method requires considerable time expenditure in order to reach an acceptable precision, and in direct simulation a slight gain in the precision of result is achieved at a considerable increase of calculation time. Besides, in this case we cannot obtain the ruin probability at infinity. But these disadvantages are offset by the simplicity of constructing models for a number of complex risk processes.

For example, the classical risk model assumes that the number of claims N_t by time t obeys the Poisson distribution with rate l. More realistic is the method in which claims are put in in accordance with the inhomogeneous Poisson process whose rate changes with time. A change of the rate $l\,(t)$ can describe various phenomena, for example, a seasonal increase in the frequency of fires in dry and hot weather, a rise in traumatism, a rise in road accidents on an ice-slick, and so on. The application the inhomogeneous Poisson process to premium income can help describe the various cycles and trends in concluding contracts with insurance clients. It is rather difficult to obtain specific results within the framework of classical theory, but simulation of the inhomogeneous Poisson process both for putting in claims and premium income can yield ruin probability values for similar processes.

It is assumed in the classical risk model that the number of claims by the moment t - N_t has a Poisson distribution with parameter l. A more realistic model would be the one describing claims arising by time-inhomogeneous Poisson process with intensity varying over time. Changes in intensity $l(t)$ can reflect different events, for example, seasonal factors: more fires in summer, more car accidents during snowy winter, etc.

The application of time-inhomogeneous Poisson process describing claims arising might be useful for understanding all sorts of cycles and seasonal trends in the work of an insurance company.

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