Two-dimensional Hazard Estimation for Longevity Analysis

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Data description

Danish population

- **Time periode: 1974 - 1998**
- **Age: 0 - 98 years**
- **Women and men**
- **Occurrence (integer age, integer year)**
- **Exposure (integer age, integer year)**

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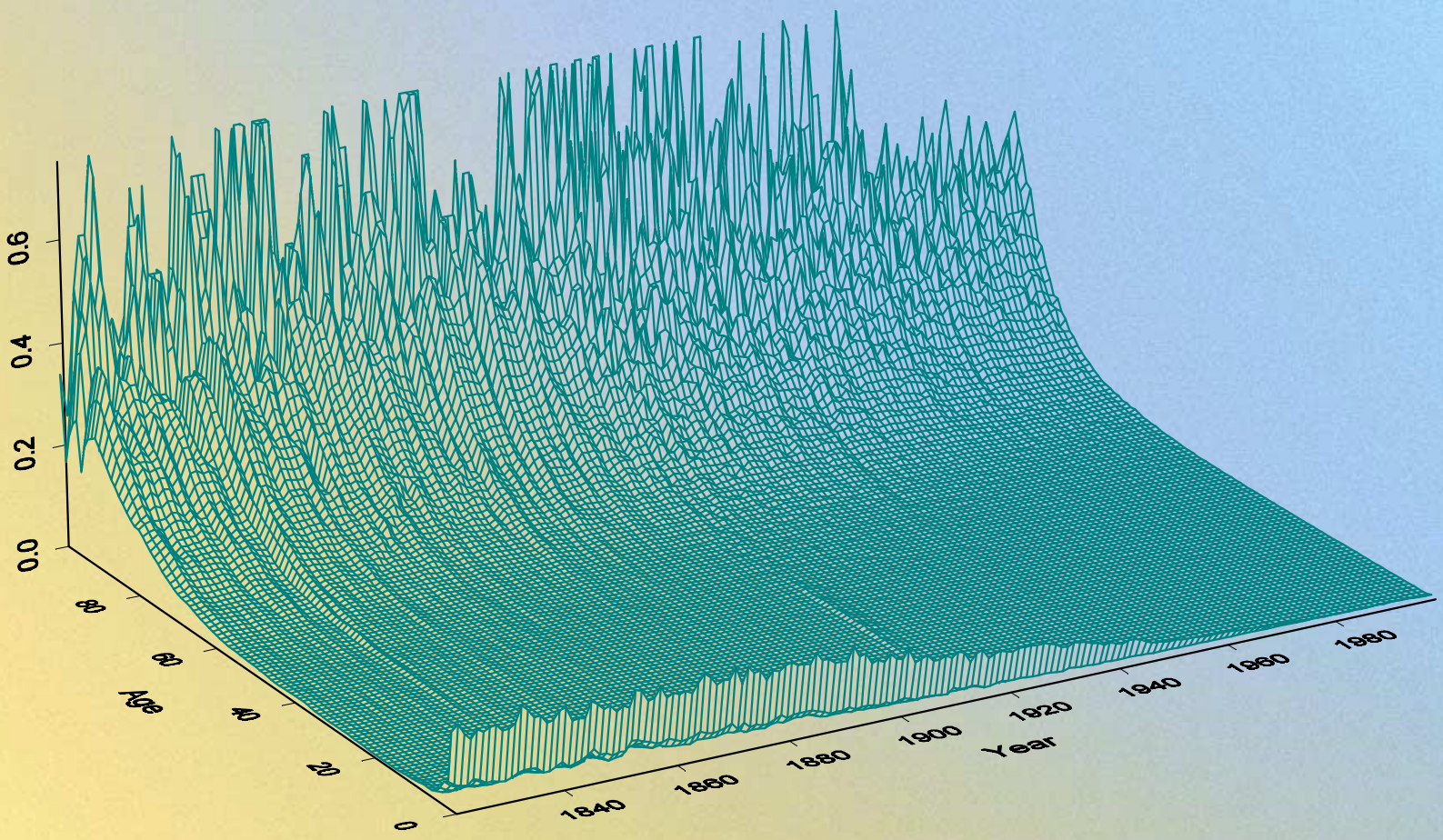
’Raw’ mortality estimator



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Estimation techniques

- Non-parametric statistical model
- Local constant hazard estimation. Two-dimensional kernel smoothing.
  - Nielsen & Linton (Ann. Stat. 1995)
- Bandwidth selection by cross-validation (two-dimensional)
  - Andersen et. al (Springer-Verlag 1993)
  - Nielsen & Linton (Ann. Stat. 1995)
- Pointwise confidence bands by bootstrap
  - Andersen et. al (Springer-Verlag 1993)

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Kernel smoothed estimation

$$\hat{\alpha}(t,x) = \frac{\sum_{i=1}^{n} \int_{1974}^{1998} K_{b1}(t-s) K_{b2}\{x - X_i(s)\} dN_i(s)}{\sum_{i=1}^{n} \int_{1974}^{1998} K_{b1}(t-s) K_{b2}\{x - X_i(s)\} Y_i(s) ds} = \frac{\overline{O}_{t,x}}{\overline{E}_{t,x}}$$

$K_{b1}(\bullet) =$ weight function, time direction,

$K_{b2}(\bullet) =$ weight function, age direction,

$X_i(s) =$ age at time s,

$N_i(s) =$ # of deaths at time s $\{0,1\}$,

$Y_i(s) = 1$ if alive, 0 if dead.

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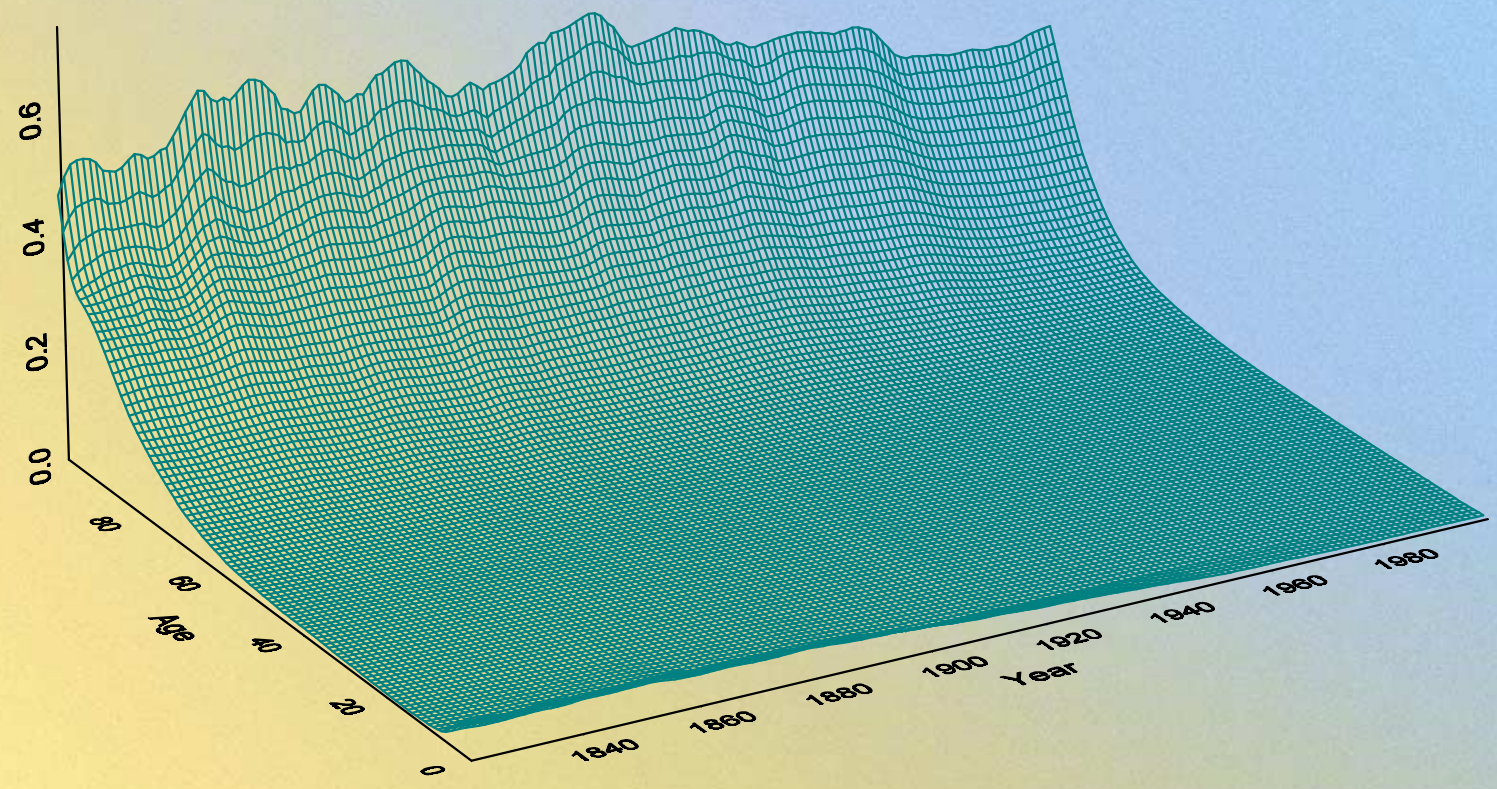
’Smooth’ mortality estimator



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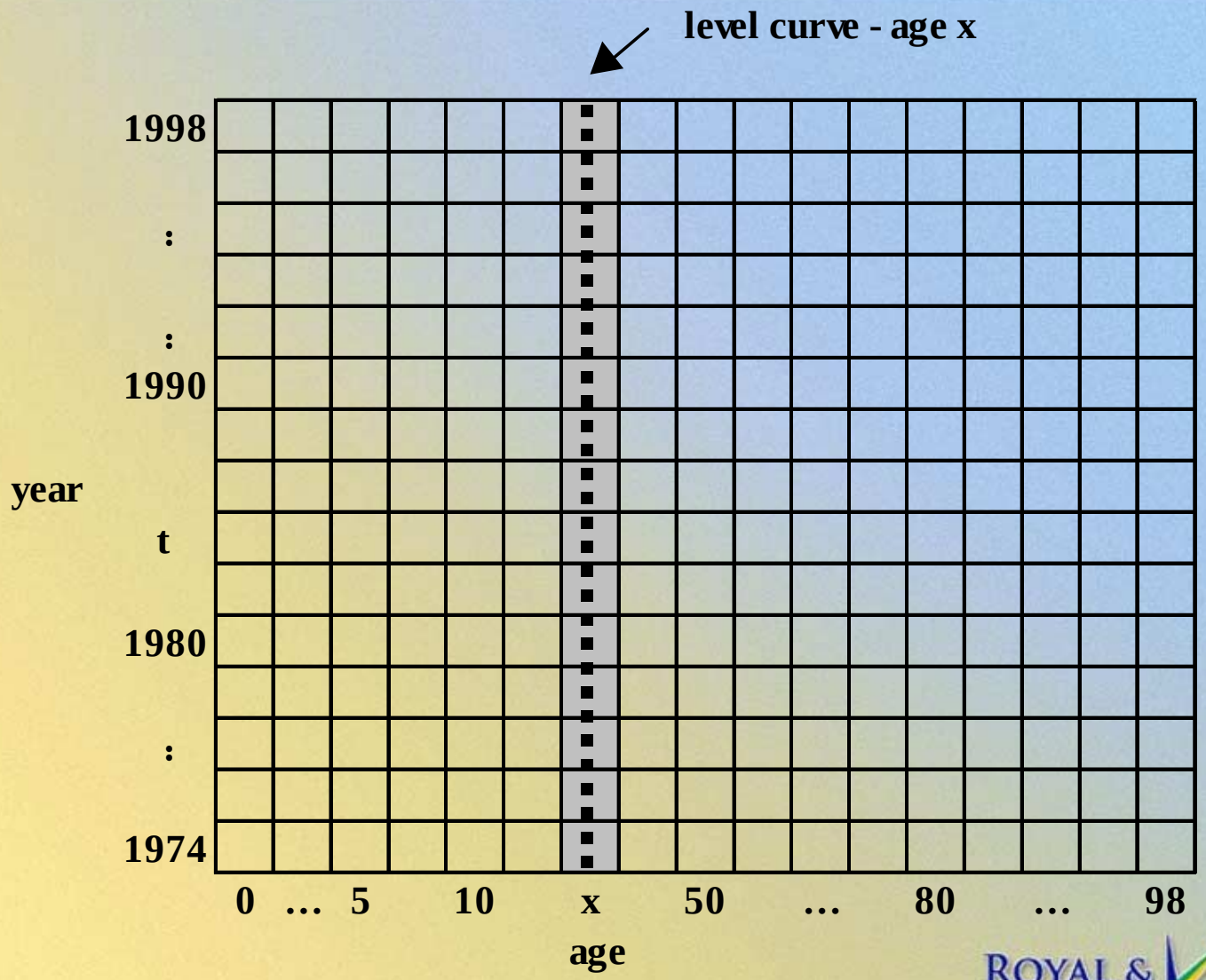
Risk profiles



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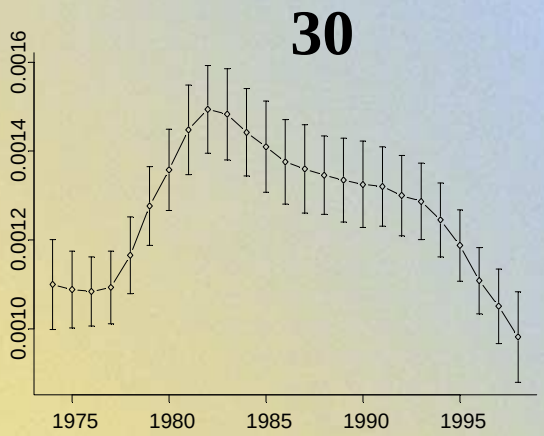
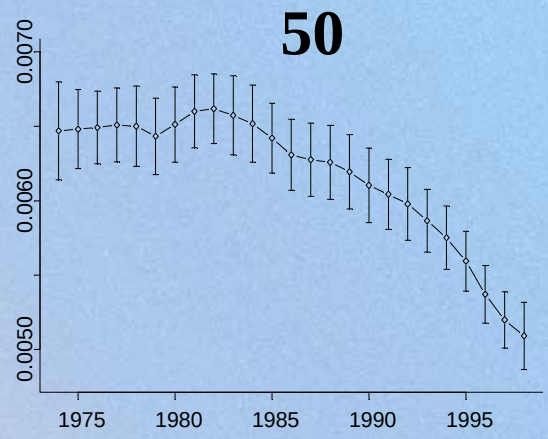
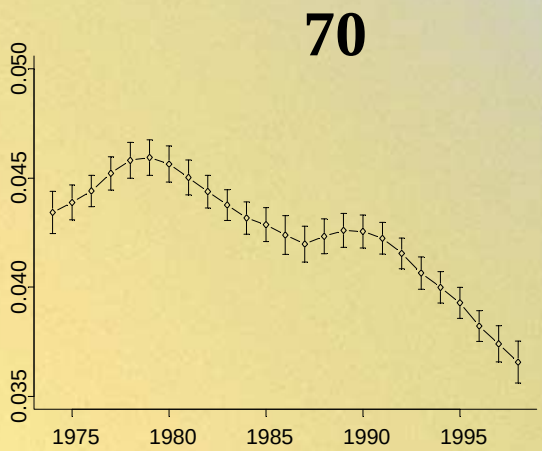
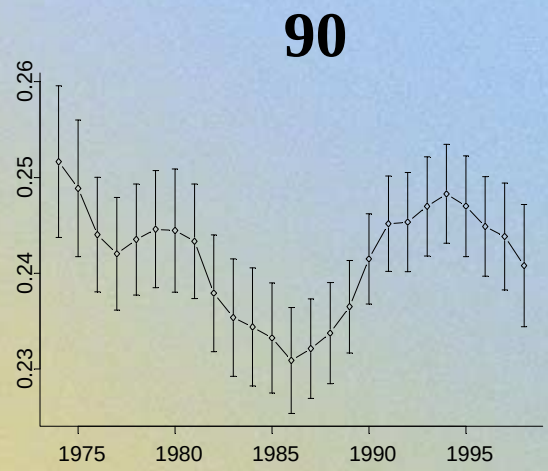
Risk Profiles Men
30, 50, 70, 90 years









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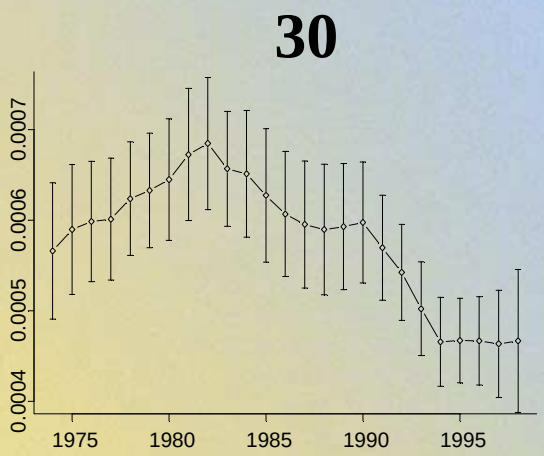
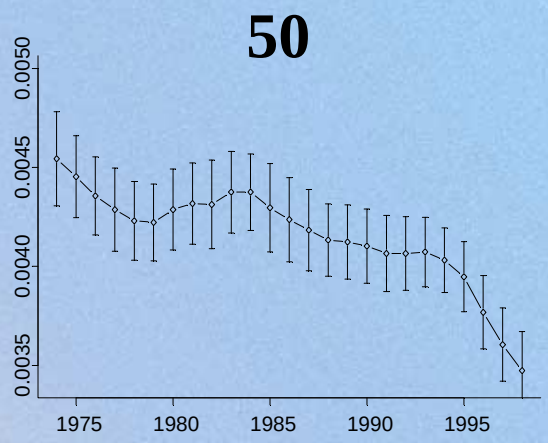
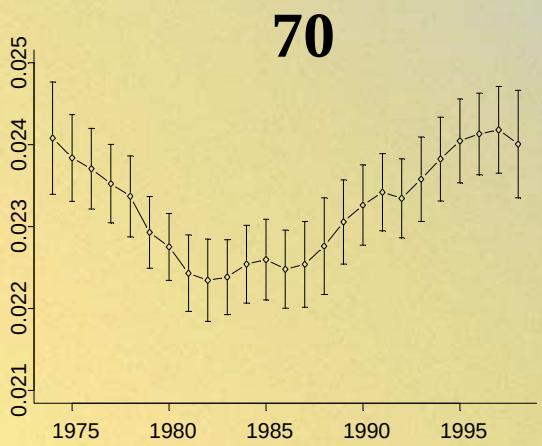
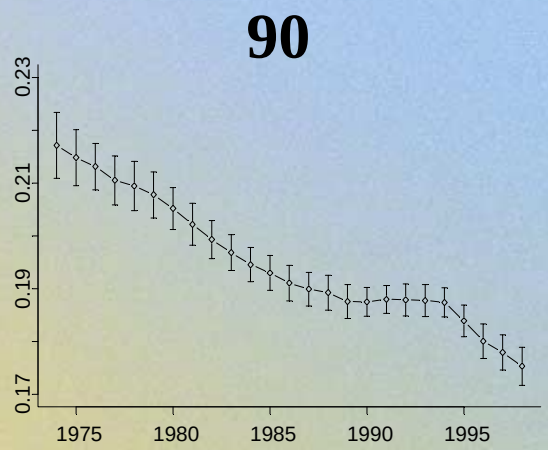
Risk Profiles Women
30, 50, 70, 90 years









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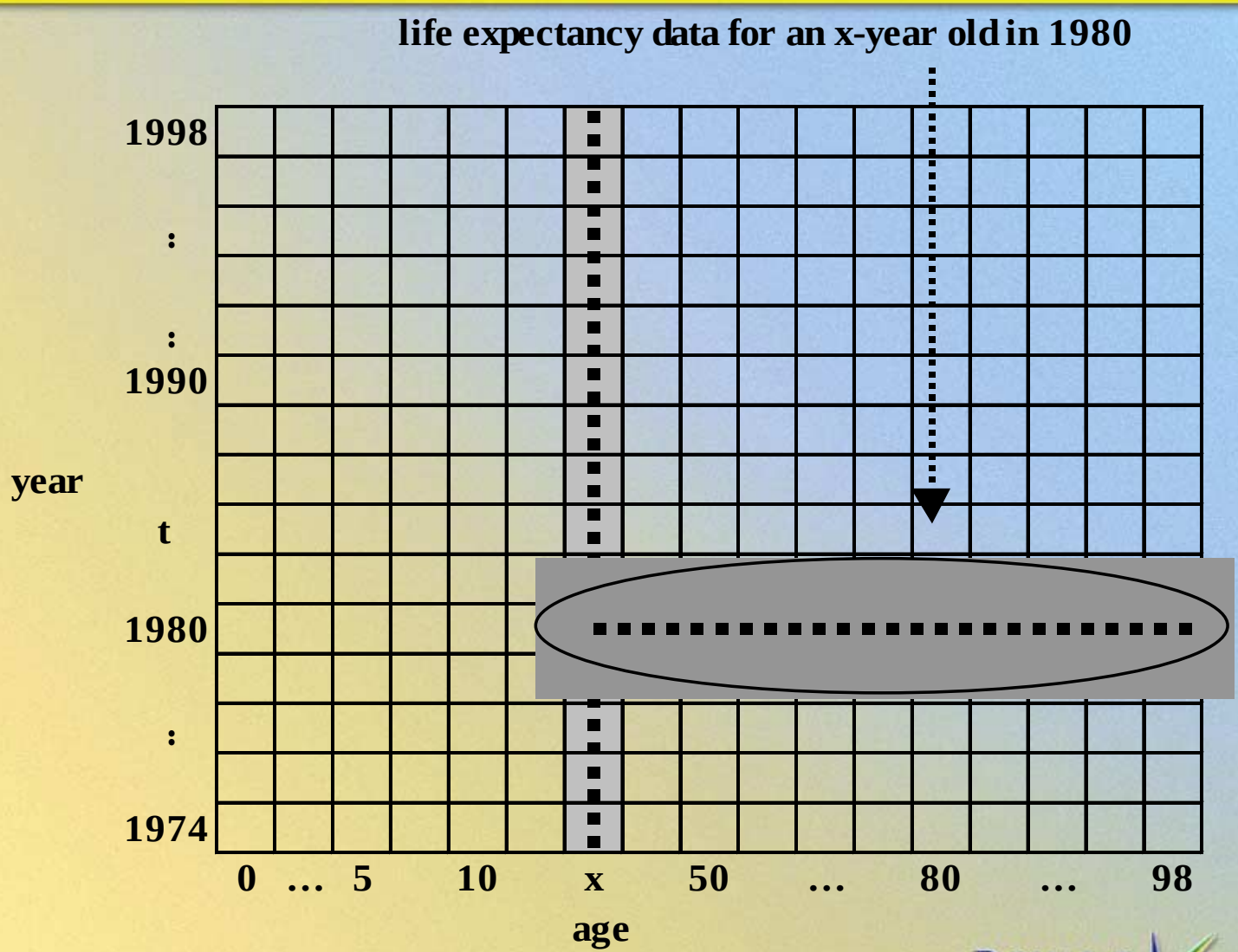
Life expectancy



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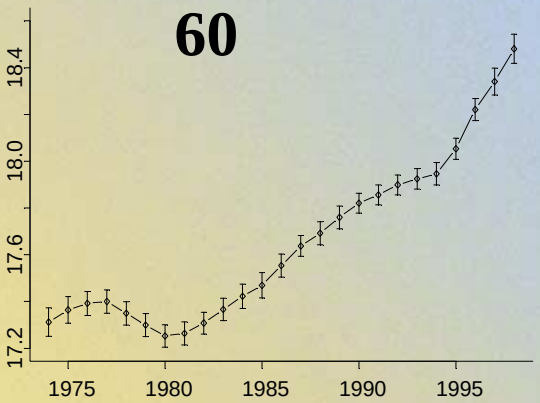
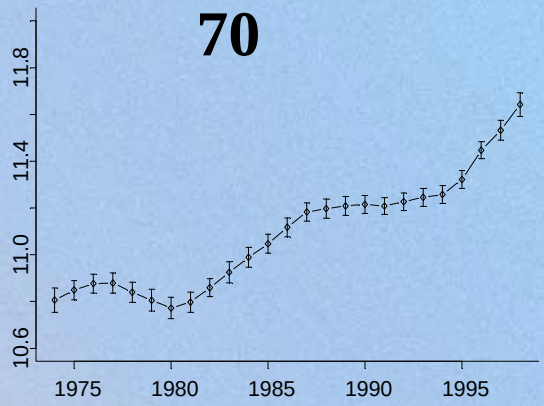
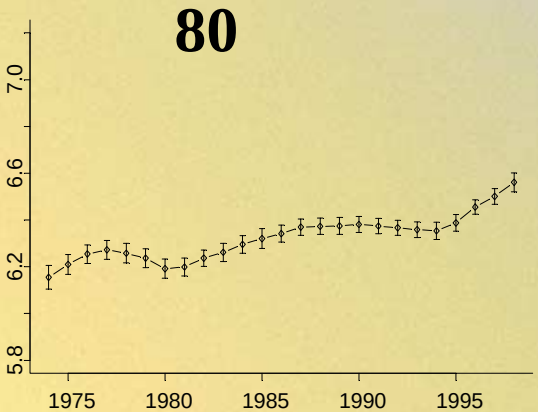
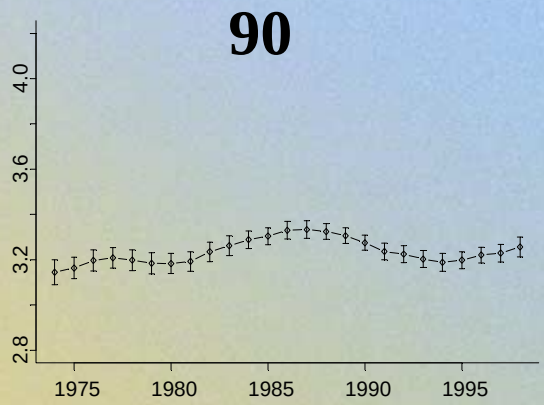
Life expectancy, Men
60, 70, 80, 90 years









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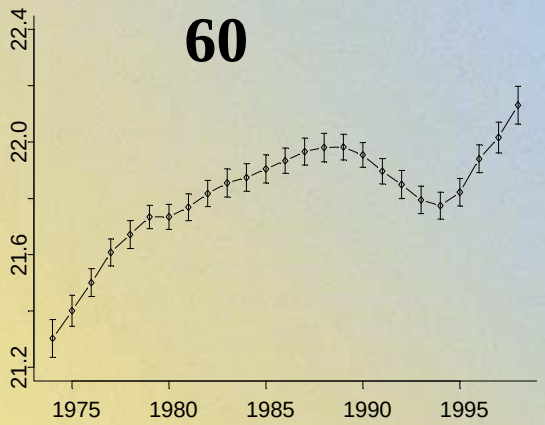
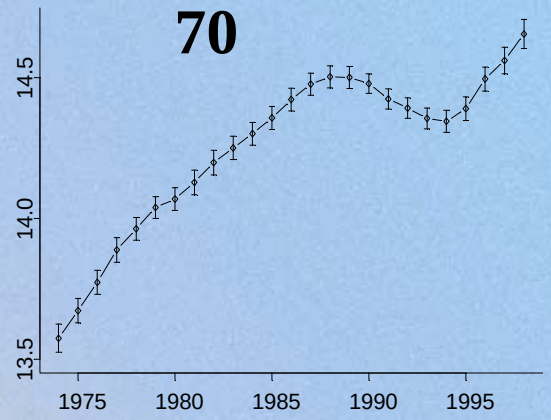
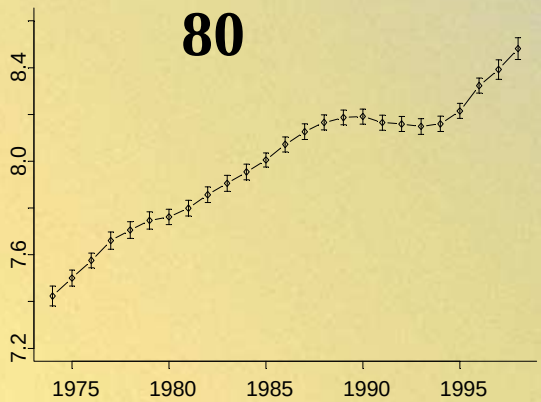
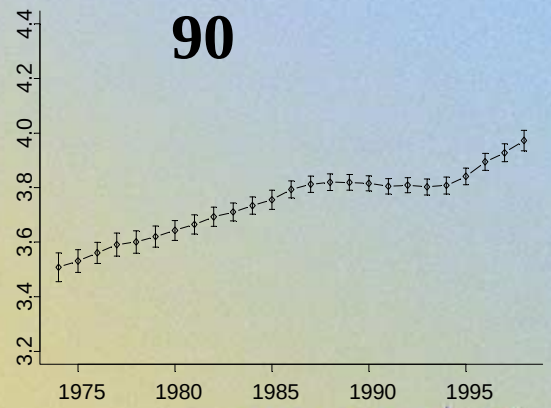
Life expectancy, Women
60, 70, 80, 90 years









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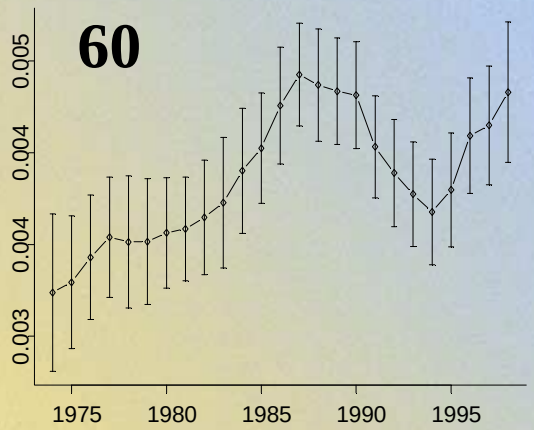
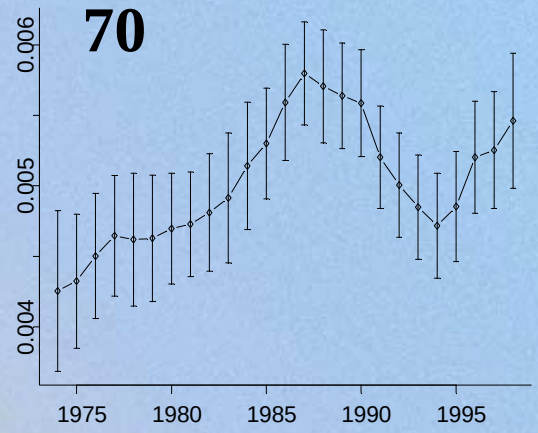
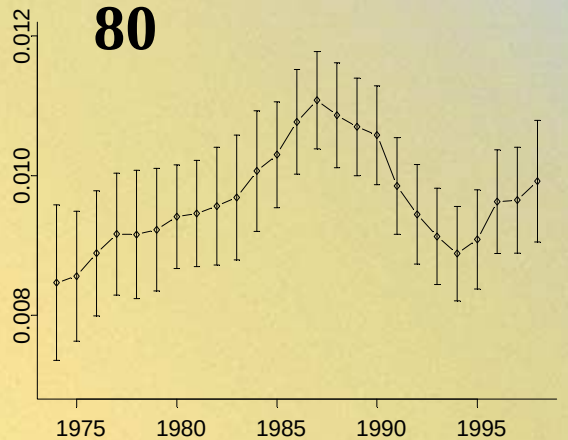
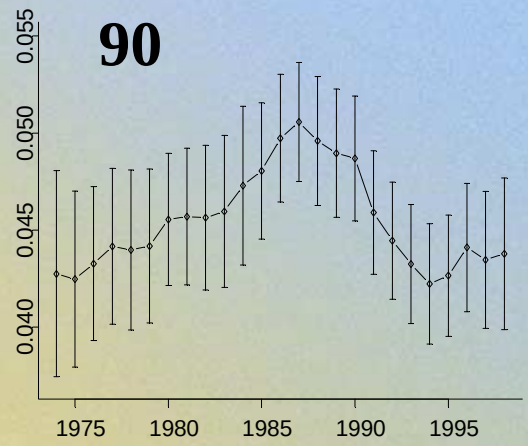
Survival probability until 99 years
Men - 60, 70, 80, 90 years









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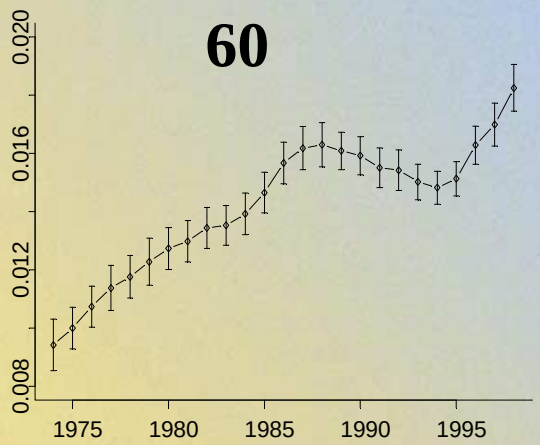
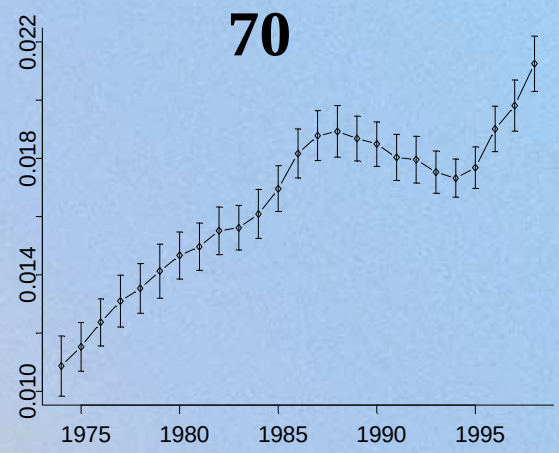
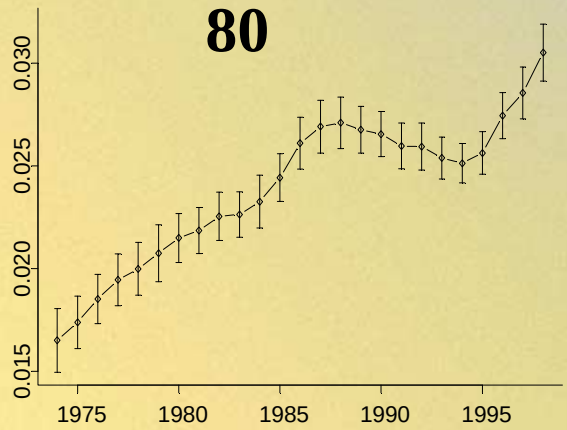
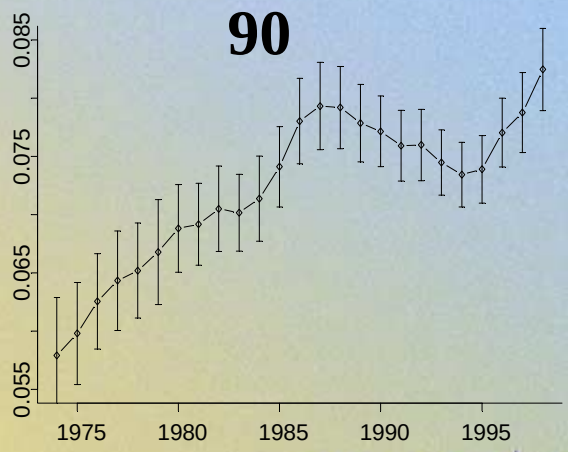
Survival probability until 99 years
Women - 60, 70, 80, 90 years









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Main longevity results

- Risk profiles
  - No certain conclusion
  - Multiplicative or additive longevity models do not fit this dataset
- Life expectancy
  - Life expectancy increased from 1974 to 1998
- Survival probability until 99 years.
  - Men - uncertain conclusion
  - Women - survival probability increased

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Conclusion

- The non-parametric two-dimensional hazard estimator can be a valuable first step when analyzing mortality changes
- Summary statistics are more easy to interpret than hazards
- Bootstrapped confidence available
- Life expectancy and survival probability show tendency towards decreasing mortality from 1974 - 1998

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