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INFERENCE ABOUT MORTALITY IMPROVEMENTS IN LIFE ANNUITY PORTFOLIOS

Annamaria Olivieri
University of Parma, Italy

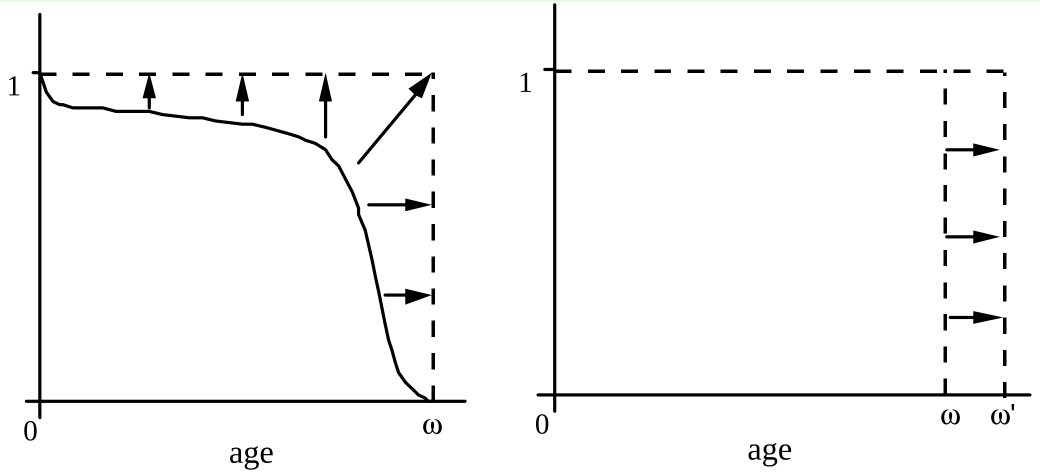
Ermanno Pitacco
University of Trieste, Italy

Outline

- Mortality trends $\rightarrow$ projected tables
- Uncertainty in mortality trends $\rightarrow$ longevity risk
- Monitoring mortality and adjustments in projected tables $\rightarrow$ (bayesian) inferential model
- Reserving and solvency requirements under the inferential model

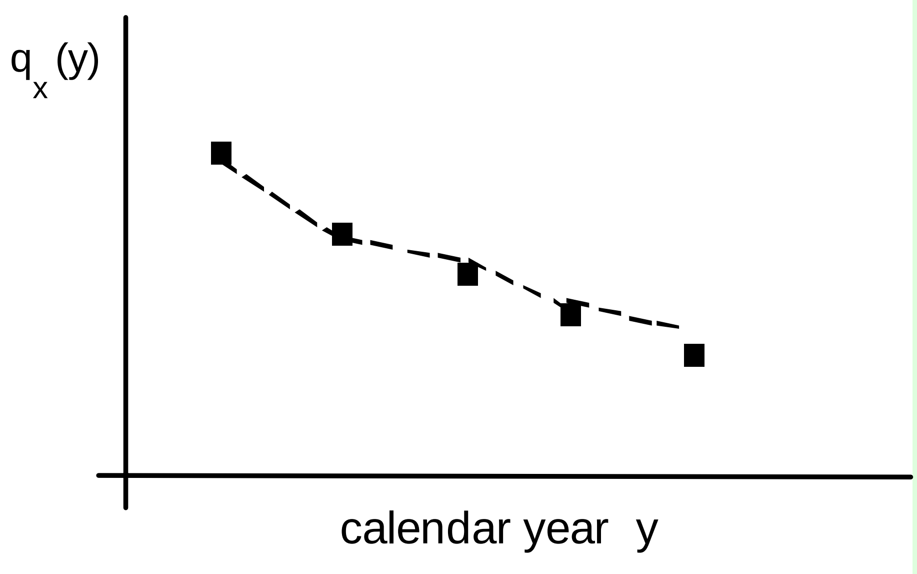
Mortality trends and projected tables

Survival function



rectangularization expansion

Mortality profile at age x



- Mortality projections
  - Mortality is expressed as a function of the future calendar year y
  - Types of projection models:
    - based on the mortality profiles: extrapolation of $q_x(y) \Rightarrow$ possible inconsistencies
    - based on mortality laws (Gompertz, Makeham, …) $\Rightarrow$ scenario building through choice of the mortality law parameters

The longevity risk

- The future mortality trend is random $\Rightarrow$ possible systematic deviations from the forecasted mortality
- A “model” (or a “parameter”) risk is inherent in any mortality projection $\Rightarrow$ LONGEVITY RISK

- Let f(t,y) denote the pdf of the random lifetime, T, referring to people born in year y
- Description of the future mortality scenario, considering uncertainty in mortality evolution $\Rightarrow$ family of pdf's
- Let H(y) denote an hypothesis about mortality trend for people born in year y

$$\{f(t,y \mid H(y));\ H(y) \in \mathcal{H}(y)\}$$

- In particular, in a parametric context

$$\{f(t,y \mid \theta(y));\ \theta(y) \in \Theta(y)\}$$

- Addressing one generation only

$$\{f(t \mid H);\ H \in \mathcal{H}\}$$

$$\{f(t \mid \theta);\ \theta \in \Theta\}$$

- Unconditional pdf (parametric discrete case)

$$f(t) = \sum_{\theta \in \Theta} f(t \mid \theta)\, g(\theta)$$

with $g(\theta) = \Pr\{\tilde{\theta} = \theta\}$

- Conditional and unconditional expected values and variances can be calculated

- In particular, the following result holds

$$\mathrm{Var}(T) = E[\mathrm{Var}(T \mid \tilde{\theta})] + \mathrm{Var}[E(T \mid \tilde{\theta})]$$

total variance

random fluctuations

systematic deviations

Bayesian inference

- Homogeneous set of n individuals (same generation) considered at time (=age) τ
- Residual duration of life of the h-th individual: $T_h-\tau$
- $T_1, T_2, \ldots, T_n$ iid under any given scenario
- Sampling pdf

$$f_\tau(t \mid \theta) = \begin{cases} 0 & t \le \tau \\ \dfrac{f(t \mid \theta)}{\int_\tau^{+\infty} f(u \mid \theta)\,du} & t > \tau \end{cases}$$

- Multivariate sampling pdf

$$f_\tau(t_1, t_2, \ldots, t_n \mid \theta) = \prod_{h=1}^{n} f_\tau(t_h \mid \theta)$$

- (Prior) predictive pdf (restricted to $[\tau, +\infty)$)

$$f_\tau(t) = \sum_{\theta \in \Theta} f_\tau(t \mid \theta)\, g(\theta)$$

- Given m deaths observed in the age interval $[\tau, \tau']$ at the ages $\underline{x} = (x_1, x_2, \ldots, x_m)$, the (posterior) predictive pdf is $f_\tau(t \mid m, \underline{x})$

- Steps of the inferential procedure
  - Update the opinion about the possible evolution of mortality (i.e. about the probability distribution over Θ) $\Rightarrow$ posterior pdf

$$g(\theta \mid m,\underline{x}) \propto g(\theta)\, L(\theta \mid m,\underline{x})$$

    with $L(\theta \mid m,\underline{x})$ likelihood function
  - Calculate the (posterior) predictive pdf

$$f_\tau(t \mid m,\underline{x}) = \sum_{\theta \in \Theta} f_\tau(t \mid \theta)\, g(\theta \mid m,\underline{x})$$

Implementation of the inferential model

- Weibull law

$$f(t \mid \alpha,\beta) = \frac{\alpha}{\beta}\left(\frac{t}{\beta}\right)^{\alpha-1} e^{-\left(\frac{t}{\beta}\right)^{\alpha}}$$

- Discrete parameter space

$$\Theta = \{(\alpha_i,\beta_j)\}$$

$$g(\alpha_i,\beta_j) = \Pr\{\tilde{\alpha} = \alpha_i \wedge \tilde{\beta} = \beta_j\}$$

In the examples: i=1,2,…,5; j=1,2,…,5

- Analyses described in the paper
  - Case I: prior pdf $g(\alpha,\beta)$ with high concentration on the "central" scenario (α_3,β_3); five samples generated from different mortality scenarios (no. of deaths forced to be equal to the no. expected under the assumed scenario)
  - Case II: uniform prior pdf $g(\alpha,\beta)$; five samples generated as in case I
  - Case III: prior pdf $g(\alpha,\beta)$ as in case I; samples generated from scenario (α_3,β_3) (no. of deaths forced to be equal to the no. expected under such scenario)

- Some examples: case I

α , β	82	83.5	85.2	87	89
7	0.0025	0.0075	0.03	0.0075	0.0025
8	0.0075	0.0225	0.09	0.0225	0.0075
9.15	0.03	0.09	0.36	0.09	0.03
10.45	0.0075	0.0225	0.09	0.0225	0.0075
12	0.0025	0.0075	0.03	0.0075	0.0025

Prior pdf $g(\alpha,\beta)$

α , β	**82**	83.5	85.2	87	89
7	0.19441	0.30156	0.15176	0.00121	0.00000
8	0.22923	0.07390	0.00552	0.00001	0.00000
9.15	0.04085	0.00153	0.00001	0.00000	0.00000
10.45	0.00002	0.00000	0.00000	0.00000	0.00000
12	0.00000	0.00000	0.00000	0.00000	0.00000

Case Ia: posterior distrib. $g(\alpha,\beta \mid m,\underline{x})$; actual scenario (α_1,β_1)

Actuarial applications

- Reserving and solvency aspects are investigated (mortality in one generation is considered)
- The reserve can be calculated according to a prudential basis or a fair value principle; in both cases, a realistic demographical hypothesis is needed
- Required solvency reserve (SR)
  = reserve + required solvency margin
  - assessed in particular according to the changing mortality scenario

- A portfolio of time-continuous straight life annuities is considered; annual benefit (instantaneous rate): b=1
  - n insurance covers, iid under any mortality scenario
- Financial risk is disregarded
- Random present value of future benefits
  - at the individual level: Y_t
  - at the portfolio level: $\hat{Y}_t$

- Conditional and unconditional expected value and variance of Y_t and $\hat{Y}_t$ are calculated; in particular, the following holds

$$\mathrm{Var}(\hat{Y}_t \mid \theta) = n\,\mathrm{Var}(Y_t \mid \theta)$$

$$\mathrm{Var}(\hat{Y}_t) = n\,E(\mathrm{Var}(Y_t \mid \tilde{\theta})) + n^2\,\mathrm{Var}(E(Y_t \mid \tilde{\theta}))$$

- Required solvency reserve
  - under a given mortality scenario

$$SR_t^{(\theta)} = \inf\{y : \Pr\{\hat{Y} > y \mid \theta\} \le 1 - \varepsilon\}$$

  - under the set of possible mortality scenarios

$$SR_t = \inf\{y : \Pr\{\hat{Y} > y\} \le 1 - \varepsilon\}$$

- # Numerical implementation

	prior	posterior case Ia	posterior case Ib	posterior case Ic	posterior case Id	posterior case Ie
$E(Y_{65})$	13.190	12.416	12.713	13.132	13.616	14.220
$V_{65} = E(\hat{Y}_{65})$	13190.11	12416.068	12713.379	13131.95	13616.32	14220.036
$E(Var(Y_{65} \mid \tilde{\alpha},\tilde{\beta}))$	26.701	31.240	29.135	26.585	23.986	20.990
$Var(E(Y_{65} \mid \tilde{\alpha},\tilde{\beta}))$	0.454	0.299	0.453	0.238	0.494	0.497
$Var(Y_{65})$	27.155	31.538	29.588	26.822	24.481	21.488
$Var(\hat{Y}_{65})$	480304.577	329871.128	481900.405	264239.086	518299.286	518420.382

Case I: prior and posterior valuations

(scenarios a, b, c, d, e:
characterized by increasing effects of
rectangularization and expansion)

ε	α_1,β_1	α_2,β_2	α_3,β_3	α_4,β_4	α_5,β_5
0.5	99.911%	99.958%	99.990%	99.996%	100.010%
0.8	101.128%	101.124%	101.054%	101.006%	100.845%
0.9	101.782%	101.672%	101.580%	101.470%	101.263%
0.95	102.380%	102.118%	102.095%	101.764%	101.613%
0.99	103.399%	102.954%	102.931%	102.548%	102.129%
$V_{65} = E(\hat{Y}_{65} \mid \alpha,\beta)$	12060.105	12481.497	13149.624	14008.583	15078.668

(Conditional) solvency reserve $\dfrac{SR_{65}^{(\alpha,\beta)}}{V_{65}}$

ε	prior	posterior case Ia	posterior case Ib	posterior case Ic	posterior case Id	posterior case Ie
0.5	99.839%	99.760%	100.019%	100.096%	97.993%	99.558%
0.8	103.500%	103.742%	104.924%	101.712%	104.040%	105.509%
0.9	106.627%	107.278%	106.088%	105.196%	108.801%	106.378%
0.95	109.654%	108.546%	107.233%	106.921%	109.797%	106.909%
0.99	113.954%	110.220%	111.616%	111.865%	111.067%	107.774%
$V_{65} = E(\hat{Y}_{65})$	13190.110	12416.068	12713.379	13131.950	13616.320	14220.036

Case I: solvency reserve $\dfrac{SR_{65}}{V_{65}}$

	age τ' = 65		age τ' = 75		age τ' = 85		age τ' = 95	
	prior	posterior	prior	posterior	prior	posterior	prior	posterior
$g(\alpha_3,\beta_3)$	0.36	0.63961	0.79552	0.89912	0.95334	0.99137	0.99999	1
$E(Y_{\tau'})$	13.19	13.174	8.833	8.812	5.221	5.203	2.819	2.819
$E(Var(Y_{\tau'} \mid \tilde{\alpha},\tilde{\beta}))$	26.701	26.948	21.597	21.512	12.027	11.947	4.92	4.92
$Var(E(Y_{\tau'} \mid \tilde{\alpha},\tilde{\beta}))$	0.454	0.236	0.213	0.071	0.02	0.004	0	0
$SR_{\tau'} / V_{\tau'}$	106.63%	105.48%	105.57%	102.68%	102.84%	102.84%	103.44%	103.44%

Monitoring on a five year basis; actual scenario (α_3,β_3)

(solvency requirement $\varepsilon = 0.9$)

Final remarks

- The longevity risk has a strong impact on the global riskiness of a life annuity portfolio
- Monitoring mortality and implementing inferential procedures may help in reducing the magnitude of longevity risk
- The Bayes model presented in this paper seems a valid tool to this regard; in particular, a test concerning the monitoring of mortality shows the soundness of the model itself