# "Method for calculating disability pension premiums"

**Kyrre Fossum Pedersen , Magne Nilsen**
**Norway**

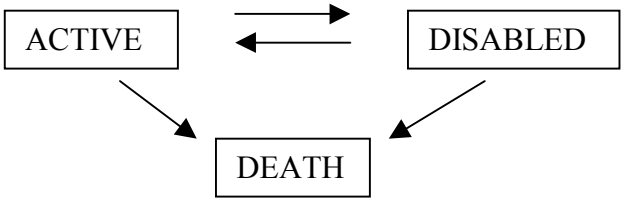
## Summary

This note describes a new model for calculating the premiums and savings for disability pensions. This method is an alternative to the model in use today Jx, the model that has been used in Norway since 1963.

MODEL IN USE TODAY:
Jx - method describes the probability for an "x" year old person disabled. This model is selective, that means that the probabilities are corrected since new members are healthier than the rest of the portfolio. That is due to the demands of health declaration for new members.

THE NEW MODEL:
The new method is based on the intensitiy of beeing disabled, recovered and dead. In addition the model includes selection and correction for the degree of disability.

Schematic view of the different states and the elements included in the new model.




A woman that has been 100% sick for 3 months has 50% chance for full disability the rest of her working life. A woman who has been sick for a period of 12 months has a chance of 70%.
This study is intended for group and individual disablement pension, but has in contrast to Jx also possibility for:

- Calculating disability lump sum.
- Calculating the reserve for disabled persons.
- Different interest rate in premium payment period and the out paying period.
- Probability of beeing dead or alive, active or disabled.
- Model for spouse pension calculations.

In the new/alternative model, there are also possibilities for different mortality tables for disabled and active members.

# " Eines Modell vor berechnungen der Premien und Rücklagen zur Erwerbsunfähligkeitsrente. "

**Kyrre Fossum Pedersen , Magne Nilsen**
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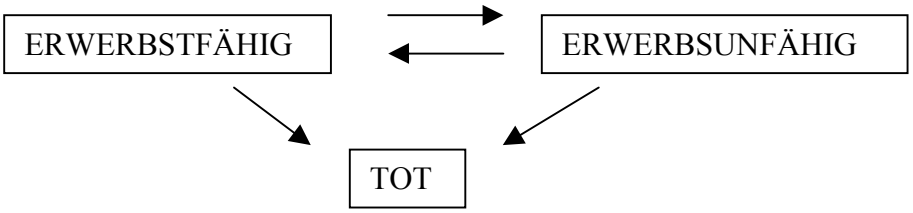
## Zusammenfassung

Das Notat beschreibt ein neues Modell zur Berechnung der Premien und Rücklagen zur Erwerbsunfähligkeitsrente. Diese Methode ist eine Alternative zu den eksistierenden Modellen Jx, die seit 1963 in Norwegen augewendet wurden.

ALTES/EKSISTIRENDES MODELL:
Die Jx - Methode besteht aus einem Modell, das die Wahrscheinlichkeit beschreibt, dass ein x-Jähriger erwerbsunfähig wird. Dieses Modell ist selektiv. Das bedeutet, dass die Wahrscheinlichkeit korrigiert ist, dass in die Ordnung Neueingemeldete gesunder sind als der übrige Bestand. Grund dazu ist die Forderung von Gesundheitsattesten für die Unterzeichnung eines neuen Versichrungvertrages.

NEUES MODELL:
Die neue Methode ist auf die Häufigkeit der Erwerbsunfähigkeit und der Reaktivierung basiert, sowie der Auswahl und Korrektur des Erwerbsunfähigkeitsgrades.

Schematic view of the different states and the elements included in the new model.




Eine Frau, die 3 Monate 100% krankgemeldet war, wird mit 50% Wahrscheinlichkeit erwerbsunfähig bleiben. Eine Frau, die 12 Monate krank gemeldet war, wird mit entsprechender 70% Warhrscheinlichkeit erwerbunfähig bleiben.
Diese neue Methode ist für die kollektive und individuelle Erwerbsunfäighkeitsrente geplant, hat aber auch gegenüber der Jx -Methode Möglichkeiten für:

- Die Berechnung des Erwerbsunfähigkeitskapitals
- Die aus fürliche Berechnung der Reserven für Erwerbsunfäigkeits- rentner.
- Die verschiedene Versinung der Lauf - und Wartezeit.
- Die Wahrscheinlichkeitsberechnung des Todes oder Aktivität zu einem angenommenden Zeitpunkt.

- Die Ausarbeitung der Rentenberechnung von Eheleuten.

Das Modell enthält auch die Möglichkeiten der Einbeziehung verschiedener Todesfälle. Einerseits für die Erwerbsunfähigen und anderseits für die Aktiven.

# ” Method for calculating disability pension premiums.”

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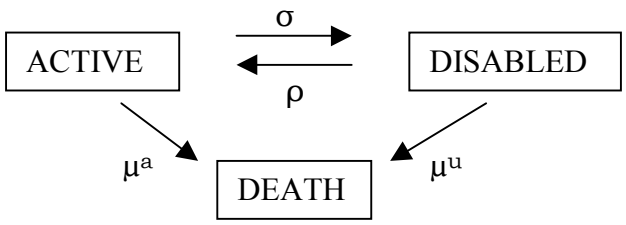
The following is a description of a method of estimating premiums and premiums reserves for group disability pension. The disability pension gives income to persons who develop illness during the insurance period. The payments starts after a period of illness exceeding the waiting period. The intensities and parts of the model are also applicable for calculating lump sum disability products.

The new model is based on intensities. It is selective, and adjusts for vorying payments according to the degree of disability.

Schematic view of the different states and the elements included in the new model.




σ – disability intensity
$\mu^a$ – death intensity for active
$\mu^u$ – death intensity for disable
ρ – recovery intensity

Each year represents the potential beginning of a sequence of payments that may last to pention age . The sequence of payments is represented by s, which is the number of periods following a possible case of disability which occured during the time r.

If the insured becomes disabled during the year r, the expected payment sequence of the period equals $\varphi_{rs}$ .

- r is the number of years from the point of calculation.
- s is the number of periods since the insured became sick.
- x is calculating age
- p is pension age

| age | time from first illness | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| x+1 | $\varphi_{1.1}$ | $\varphi_{1.2}$ | $\varphi_{1.3}$ | $\varphi_{1.4}$ | $\varphi_{1.5}$ | $\varphi_{r1.6}$ | $\varphi_{1.7}$ | $\varphi_{1.8}$ |
| x+2 | $\varphi_{2.1}$ | $\varphi_{2.2}$ | $\varphi_{2.3}$ | $\varphi_{2.4}$ | $\varphi_{2.5}$ | $\varphi_{r2.6}$ | $\varphi_{2.7}$ | $\varphi_{2.8}$ |
| x+3 | $\varphi_{3.1}$ | $\varphi_{3.2}$ | $\varphi_{3.3}$ | $\varphi_{3.4}$ | $\varphi_{3.5}$ | $\varphi_{r3.6}$ | $\varphi_{3.7}$ | $\varphi_{3.8}$ |
| : | | | | | | | | |
| : | | | | | | | | |
| p-3 | $\varphi_{p-x-3.1}$ | $\varphi_{p-x-3.2}$ | $\varphi_{p-x-3.3}$ | | | | | |
| p-2 | $\varphi_{p-x-2.1}$ | $\varphi_{p-x-2.2}$ | | | | | | |
| p-1 | $\varphi_{p-x-1.1}$ | | | | | | | |

The number of periods after illness could be represented as the number of years as in the preceeding table. From now on, the number of periods will be devided into months, in order to more easily describe the problems concerning waiting period and recovery.

The costs when becoming disabled within any given year equals the risk of several sequences of payments.

The disability intensity and the selection function gives the propability of beginning one sequence this year. The intensities of recovery and death, as well as the expected degree of disability, leads to the expected payments for this sequence. However, the payments are limited by the waiting period. Waiting period is a period of no payment at the beginning of a illness. Usually, the waiting period is a period of 12 months, and the payments cease at pension age. At other times, the waiting period equals 3 months. You can also use two waiting periods at the same policy, 3 months as waiting period for starting the payment, and then payments for 9 months until the payments cease at the second waiting time (12 months).

As an example, the following parameters are used when the expected future contributions are to be estimated:

- The insured is x = 60 years
- The pension age is p = 67 years
- The waiting period is k = 3 months

After one year, the insured has the probability $\sigma_{61}$ of being disabled, with expected degree of disability $Upg_{61}$. These variables are depending on age.

The preceeding example is based on 61 years as the starting age of the period of disability. In this case, a sequence of monthly payments starts one year after the point of estimation. This disability pension pays monthly for 72 months, since 6 years remain until the pension age. (The three first amounts equals 0, since the waiting period is 3 months.)

$( 0, 0, 0 , \varphi_{(1,4)} , \varphi_{(1,5)}, \varphi_{(1,6)}, \varphi_{(1,7)}, \varphi_{(1,8)}, \ldots, \varphi_{(1,72)} )$

These are the monthly payments, adjusted for the propability that the insured has recovered or died. Intensity of recovery depends on the age of the insured at the time when disability occured and the time from this point, given in months. The probability for death is determined by the death intensity of active and disabled, also the probability of at any time becoming disabled.

The probability of becoming disabled at age 61 is $\sigma_{61}$ . Hence, the costs if the insured becoming disabled at 61 can explicitly be derived. As this is one year after the point of estimation, r equals 1.

After correcting for the degree of disability, the discounted expected value of the costs can be displayed as follows, when the disability starts in period r:

$$\sigma_{x+r} \sum_{s=1}^{12(p-x-r)} v^{s+r} \varphi_{(r,s)} = \sigma_{61} \sum_{s=1}^{72} v^{s+1} \varphi_{(1,s)}$$ , where v = 1/(1+i) and i is the discount rate.

The next part of the costs are those where the insured becomes disable at the age of 62. This appears as above, but with r =2:

$$\sigma_{x+r} \sum_{s=1}^{12(p-x-r)} v^{s+r} \varphi_{(r,s)} = \sigma_{62} \sum_{s=1}^{60} v^{s+2} \varphi_{(2,s)}$$

These payments run over a period of 60 months, since a person of 62 years reaches pension age in 5 years.

As an example, the single premium for a 60-year-old worker equals the discounted sum of the risk of the sequense of payment for a 61-year-old, 62-year-old, 63-year-old, and so on until the age of retirement.

On a general form, this may be expressed as:

$$\sum_{r=0}^{p-r} \sigma_{x+r} \, Upg_{x+r} \sum_{s=1}^{12(p-x-r)} v^{s+r} \varphi_{(r,s)}$$

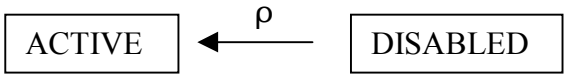
**The data material**

The background datas are collected from a database of the Norwegian social security system, including 90000 cases of illness records. Each of theese records describes, by dates, the entire illness period with all the illness intervals since the patient have been recovered for shorter, temporary periods. The data also includes disability percentage. In case of death, this has also been included in the material.

**Recovery**

Recovery means that a disabledperson is back to work, and therefore no longer gets disability pension. Experience shows that persons who have been declared disabled seldom recover. To be declared disabled, the person must have been ill for a period of two years. Within thees two years, a relatively large presentage of the persons recovers. Generally, recovery depends on the amount of time since the start of the illness and the fact there is very few recovering after 3 years. In Norway, disability pensions usually start its payments after 12 months of illness. These 12 months are called waiting period, and this usually states the limitation of the payments. In some cases the payments may start as soon as 3 months after illness. Hence, a certain part of the policies start their payments before the insured is declared disabled. It is therefore appropriate to bring recovery into the estimate of this type of insurance.

Recovery intensities are based on the data material. It relates to age in years and time of illness in months. The recovery intensity then constitutes the frequency of recovery of each single month after the person has become ill. The recovery intensity is referred to as $\rho$.



Definition:

When: $C_t$ is the status at the time t
U is the status disabled
A is the status active

$P(C_{t+h} = A \mid C_t = U) = \rho h$
$\pi_t = P(C_t = A \mid C_0 = U)$
$\pi_{t+h} = \pi_t + (1 - \pi_t)\rho h$
$(\pi_{t+h} - \pi_t)/h = (1 - \pi_t)\rho$
$\lim_{h \to 0} (\pi_{t+h} - \pi_t)/h = d\pi_t/dt$

$$\begin{cases} \pi_o = 0 \\ d\pi_t/dt = \rho - \rho\pi_t \end{cases}$$

Integrating the expression gives:
I.) $\pi_t = 1 - \exp(-\rho t)$
II.) $-\ln(1 - \pi_t) = \rho t$
III.) $\rho = -\ln(1 - \pi_t)/t$

,when $\rho$ is not constant, the expressions I.), II.) and III.) are approximations which are true when t is small.

If the intensity is seen as constant throughout this space of time of one month, it may be written as:

IV.) $\rho = -\ln(1 - \pi)$

,when $\pi$ is the probability of being recovered during this period.

Generating the recovery intensity:
x = the age of a person who has become ill.
$A_x$ = the total number of persons of age x who become disabled.
$D_{x,i}$ = Persons of age x at t = 0, who have died at the time i.
$R_{x,i}$ = Persons of age x at t = 0, who recover at the time i.
$G_{x,i}$ = Persons of age x at t = 0, whose records are no longer valid. Therefore, the information concerning further development is unattainable.
$\pi_1$ = Probability of recovery within one month.
$\pi_{x,s}$ = Probability of recovery, given age and times since illness occurred.
$\rho_{x,s}$ = Intensity of recovery, given age and times since illness occurred

An estimate of the recovery probability $\pi$ is used to find the intensity, $\sigma$.
The estimate of $\pi$ is expressed as ${}^{est}\pi$.

${}^{est}\pi_{x,s}$ = (Number of recoveries during the period s) / (Number of disabled during the period s)

$$^{est}\pi_{x,s} = R_{x,s} / \left( A_x - \sum_{j=o}^{s-1} D_{x,j} - \sum_{j=o}^{s-1} R_{x,j} - \sum_{j=o}^{s-1} G_{x,j} \right)$$

For a x-year-old ill person (rounded off to the closest month), the recovery intensity (from equality IV), for a period s:

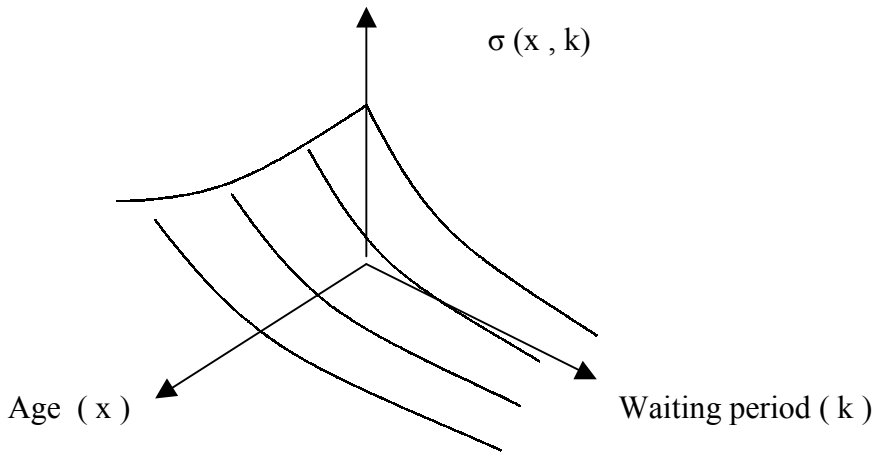
$$^{est}\rho_{x,s} = -\ln(1 - (^{est}\pi_{x,s})) = -\ln\{1 - (R_{x,s} / \left( A_x - \sum_{j=o}^{s-1} D_{x,j} - \sum_{j=o}^{s-1} R_{x,j} - \sum_{j=o}^{s-1} G_{x,j} \right) )\}$$

This recovery intensity therefore depends on age and the time since the first point of illness.

**Disability frequency and degree of disability**

For group lump sum disability in Norway we need the probability of any x-year-old becoming and staying disabled for at least 2 years. The group lump sum disability insurance therefore has a waiting period of 2 years. This probability then derives the frequency of any x-year-old being ill for a period longer than 2 years. From here on, this disability probability is referred to as $\sigma^{uk}$

This model demands disability intensities which imply the probability of an x-year-old becoming disabled under different circumstances. The criterions for considering someone disabled depends on different waiting period.
Therefore, this is a disability intensity which describes the probability of getting ill longer than a waiting period of 3 or 12 months. The disability intensity for a waiting period of 3 months is $\sigma^3$, while the disability intensity for a karens-time of 12 months is $\sigma^{12}$.




$\sigma^{uk}$ is the disability probability/intensity for 24 months waiting period
$\sigma^{12}$ is the disability probability/intensity for 12 months waiting period
$\sigma^3$ is the disability probability/intensity for 3 months waiting period

The above is a three-dimentional depiction of a hypothetical function describing how the disability function relates to age and waiting period.

In all cases, the frequency of disability describes the probability of becoming disabled regardless of the degree of disability. Expected degree of disability is therefore made a part of

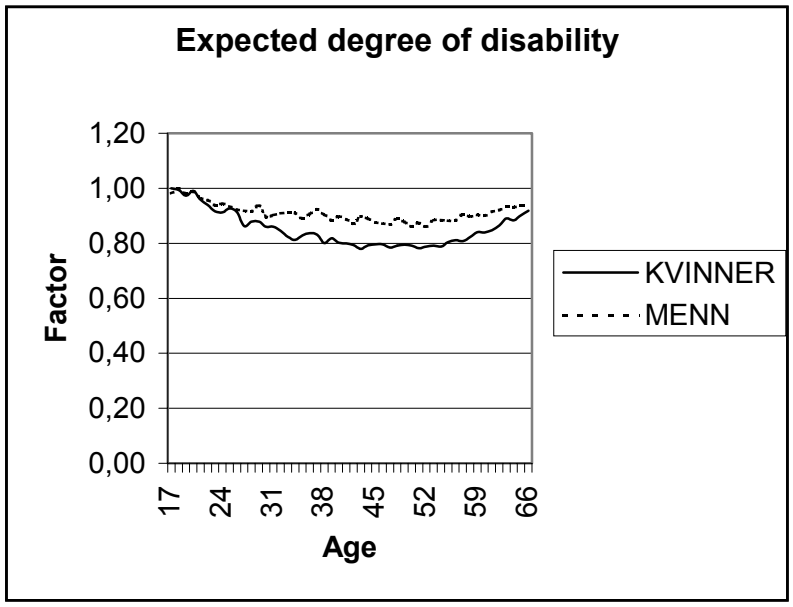
the calculations in order to include the reduced payments in relation to the degree of disability.

To estimate $Upg_x$, we start to find a discreet probability distribution that represents the degree of disability for each age, each gender separately. Expected degree of disability is referred to as $Upg_x$ depending on age. Expected degree of disability is calculated based on average degree of disability among the 90.000 disabled in the data base, described as $Upg_x$, and depending on age. It turns out that age has significance in the estimation of the degree og disability.

P(u|x)= (Number of new disabled x-year-old persons with u % degree of disability) / (Total number of new disabled x-year-old persons)

$$Upg_{(x)} = E(u|x) = \sum_{i=0}^{20} u_i * P(u|x) \quad \text{, når } u_o = 0\%, u_1 = 5\% \; u_2 = 10\%, ....,u_{20} = 100\%$$

E(u | x) is referred to as $Upg_x$ and are estimated for each sex separately as well as together. The results are given in chapter 5. Graphically, the results look as follows:




(Kvinner=women)
(Menn=men)

The fact that the young disabled persons have a high average degree of disability implies that young people who become disabled are either seriously injured, therefor they become 100 % disable. The average degree of disability is lowest for disabled in the early fifties. The factor increases later on (as the graph above shows).

$S_j^+$ is the expected payment corrected for disability degree. $S_j$ is the payment if you get 100% disability pension

$$S_j^+ = \sum_{i=0}^{20} P(u \mid x)\, S_j^u$$

, when $u_o = 0\%$, $u_1 = 5\%$ $u_2 = 10\%$, ....,$u_{20} = 100\%$
, and $S_j^u$ is the payments with disable degree u%.

(since $S_j^u = S_j^+ u_i$ the expression above becomes: )

$$S_j^+ = S_j \sum_{i=0}^{20} P(u_i \mid x)\, u_i$$

These factores can be used directly as table values or used via functions solved by regression.

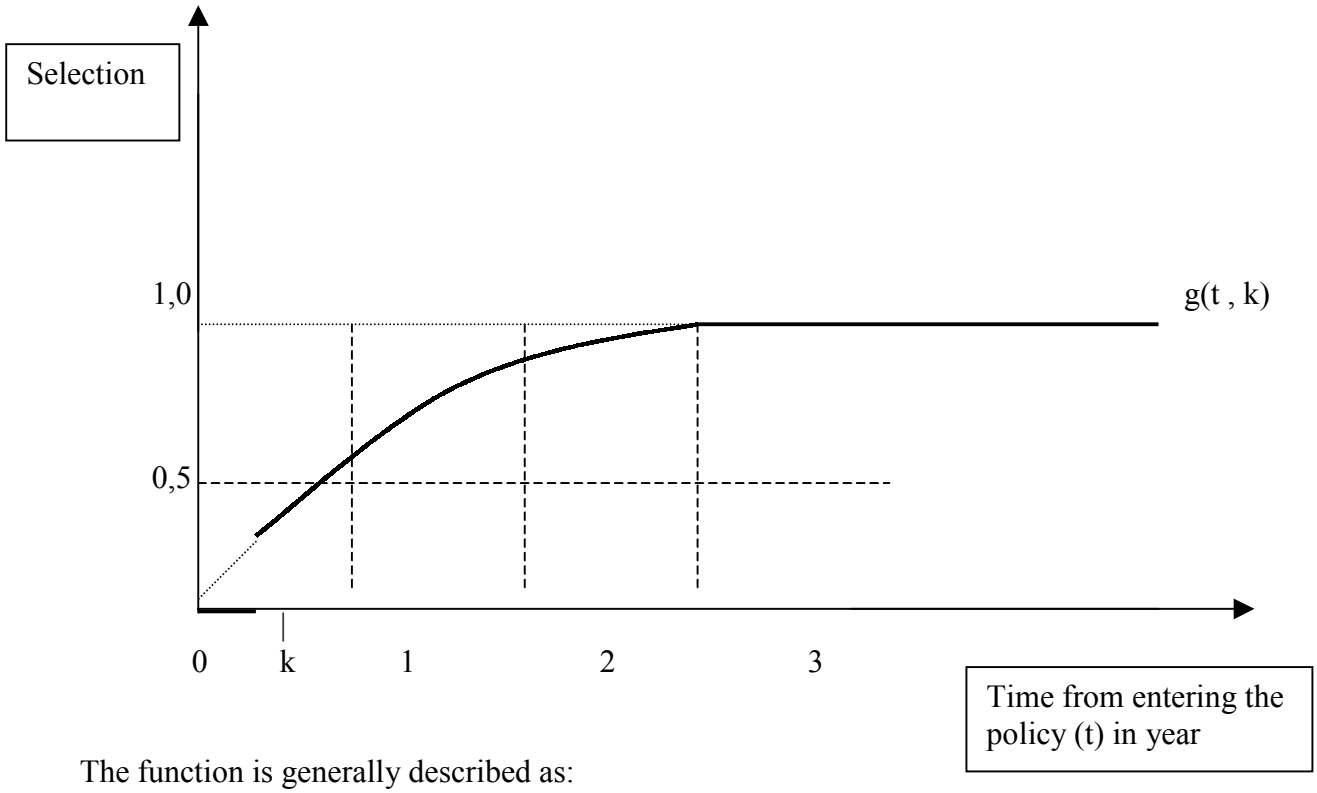
**Mortality**
At this point, some analyses have been done, which show that there is a genuine higher mortality for disabled persons in relation to the mortality of working people. The statistical material from 90.000 disabled-records was not sufficient in drawing any significant conclusion.

**Selection**
Selection is a function included in the calculations which are to consider a new insurance policy and the fact that it carries a lower disability risk in the beginning because of the medical underwriting.

The selection function:
The selection function mostly depends on the time since a person enters the policy, where t is used as a variable name. The function also depends on the waiting period k. The insured is healty at the time the person entires the policy. Hence, no payment can take place before the end of the karens-time, and the selection factor is 0 when $t < k$. t is given in years in part of year. 3months is for example 0.25 years.

The time from the entrance of the policy is measured in years. Since the waiting period is usually given in months, it needs to be recalculated into years, i.e. 3 months equals 0,25 years, 12 months equals one year.

Figure:




The function is generally described as:

$$g(t , k) = f(t)$$

, when the arrangement opens for the possibility of payments in relation to karens-time, but also lower probability of payments because of a declaration of health at the time of establishing a policy.

The function is based on the following assumptions:

- f(t)is concave, when t>k and t<3 .
- f(t) is increasing for all t.
- f(t) runs through the x-axis in the origo.
- f{t = 1(years)} = 0,5
- g{t > 3(years) , k} = 1,0
- g(t , k) = 0 , when t<k

g(t , k) depends on time since entering the insurance and the length of the waiting period, as earlier stated. The dependance on waiting period means that the function equals 0 when time since entering is less than the waiting period. This means no payments occur when t < k.

Result:

$$g(t , k) = \begin{cases} 0 & \text{, when } t > k \\ (7t-t^2)/12 & \text{, when } k \le t < 0.25 \\ 1 & \text{, when } t \le 0.25 \end{cases}$$

, both t and k are given in part of years.

**The single premium**

Earlier on, $\varphi_{rs}$ was used as a symbol of expected payment. From now on the single premium will be more precisely described, using the functions and symbols above.

The premium reserve is set equal to the single premium for the disabilitypension.The yearly premium is calculated based on the difference between this year`s and the next year`s premium. In this way, the single premium directly relates to both premium and premium reserves within group pension insurance. Therefore, the single premium is the only calculation element further treated.

If two waiting periods are used, the shortest time activates the payments while the longest stops the payments, k1 < k2. If only one karens-time is in use, k1 is set equal to the actual karens-time, while k2 is set to ∞.

The single premium for a member of thegroup disability pension, where the insured has the age x+t:

$$E_{x+t} =$$

Disability intensity
Discount relating to the yearly discount rate
Expected degree of disability
Selection function
Payment in case of disability, considering reactivation

$$\sum_{j=0}^{r-(x+t)} \left[ \sigma_{(x+t+j)} \left\{ \sum_{i=0}^{\text{Min}\{k2-k1,\ 12(r-x-t-j)\}} v^{(j+(i/12))}\, \text{Upg}_{(x+t+j)}\, g(t+j+(i/12),\, k)\, (S_{(x+t+i+12j)}/12)(\exp(-\sum_{v=0}^{i} \rho^{(s)}_{(x+t+j,v+k1)})) \right\} \right.$$

$$* \left\{ \left[ \exp(-\sum_{s=0}^{j} \mu^{a}_{(x+t+s)})(\exp(-\sum_{s=0}^{j} \sigma_{(x+t+s)})) + \exp(-\sum_{s=0}^{j} \mu^{u}_{(x+t+s)})(1-\exp(-\sum_{s=0}^{j} \sigma_{(x+t+s)})) \right.\right.$$

$$\left.\left. * \exp(-\sum_{s=0}^{j-1} \sigma^{uk}_{(x+t+s)}) \right\} \right]$$

Probability that the insured is not dead at point j
Probability that the insured has not become continuously disabled (Disabled
Lump sum intensity, with karens-time 2 years or 24months)

Compared with the notation in the introduction to this article is:

$$\varphi_{ji} = Upg_{(x+t+j)}\, g(t+j+(i/12), k)\ (S_{(x+t+i+12j)}/12)(\exp(-\sum_{v=0}^{i}\rho^{(s)}_{(x+t+j,v+k1)}))\ \}$$

$$* \{ [\ \exp(-\sum_{s=0}^{j}\mu^{a}_{(x+t+s)})(\exp(-\sum_{s=0}^{j}\sigma_{(x+t+s)})) + \exp(-\sum_{s=0}^{j}\mu^{u}_{(x+t+s)})(1-\exp(-\sum_{s=0}^{j}\sigma_{(x+t+s)}))$$

$$* \exp(-\sum_{s=0}^{j-1}\sigma^{uk}_{(x+t+s)})$$

This $\varphi_{ji}$ is now a part of the single premium.

**The current method**

In Norway, there the same method for calculation of disability pension premiums has been used since 1964, called the $J_x$ –method. It deals with the probability of getting disability pension at age x. The method is selective, the selection time being 10 years. This means that the probability of being disabled at age x+t is lower the first 10 years after entering the policy.

The $J_x$ probability is based on waiting time 3 months. For a waiting period of 12 monts the premiums for 3 monts is multiplied by a factor 0,85. This factor is a rough estimate.

$J_{x+t}$ = (number of disabled (x + t)-year-olds) / (number of (x + t)-year-olds)

$J_{(x+t)} = J_{x+t}$ * Select function

**Conclusion**

The alternative method described in comparison to the method of the $J_x$:

| Possibilities: | New method | $J_x$ -method |
|---|---|---|
| Calculation of premiums for group disability pension | Yes | Yes |
| Calculation of premiums for individual disability pension | Yes | Yes |
| May be used for modelling disability lump sum | Yes | No* |
| Simplest in use | No | Yes |
| Calculation of reserve for disabled pensions explicity | Yes | No |
| Possibility of different rate of interest in the active and disabled period. | Yes | No |
| Different mortality for active and disabled | Yes | No |
| May be used for spuse pension because different mortality for active and disabled | Yes | No |

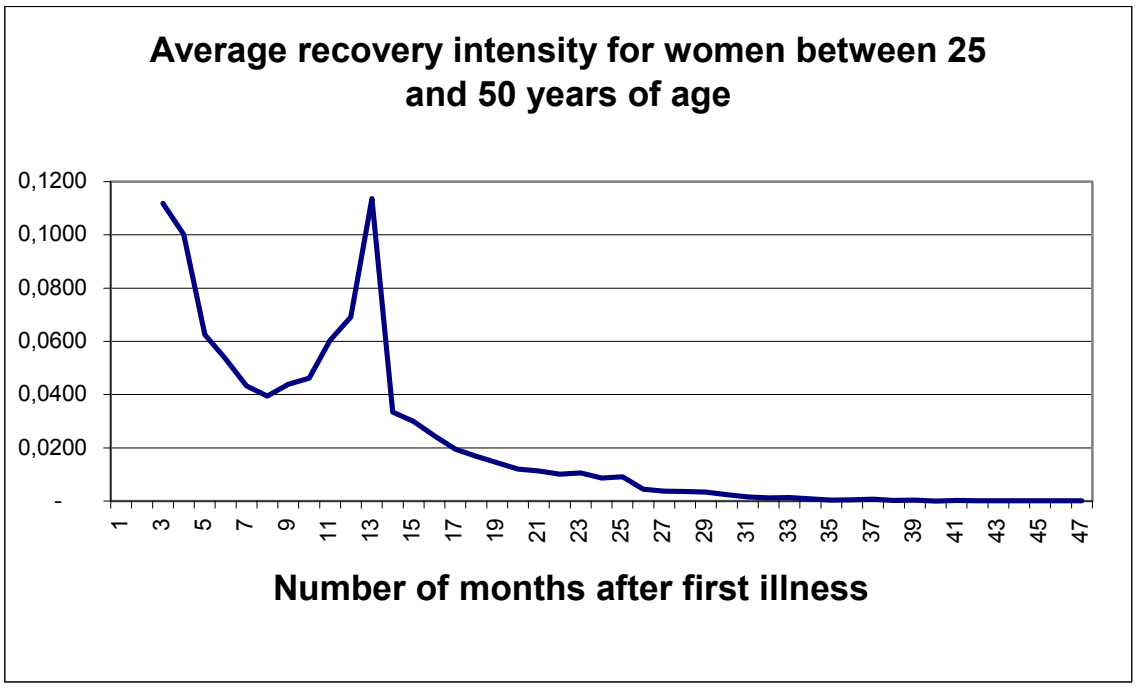
*May be solved through a complicated process, giving certain incorrectnesses.

The advantages of the new method as compared to the $J_x$ –method are mainly due to the use of intensities. Intensities are more flexible and can better approximate the probability of the insured party being active, disabled or dead at any given time.

**Results**

A woman who has been ill for longer than three months will have a 40.5% possibility to get recovered before 9 months. The possibility to get recovered the second, third and fourth year are respectively 26.3%, 3.2%, and 0.2%. After the fourth year the recover is nearly equal 0.

The graph below shows the monthly recovery intensity or reactivating intensity. This example is an average for women between 25 to 50 years.




The recovery intensity is estimated for those who have been ill for longer than two months.

After a illness over a period for 12 months will the payments been redusend from the social security system. In this period will the social security system give a compensation equal to the loss of income. Those who have private insurance policies will get less income after this period. The recovery intensity have for that reason a peak about 12 months of illness. Apart from this it would be a smooth decreasing curve.