27th International Congress of Actuaries

Method for calculating disability pension premiums

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Method for calculating disability pension premium

- Exixting method (based on probability of being disabled, Jx method)
- Alternative method

  -Expected claims when disabled C(x)

    -Disability intensity

    -Selection function

    -Recovery intensity

    -Mortality intensity (active, disabled)

-Discounted claims

-Discountrate

-Degree of disablement

-Discounted C(x) from time of disablement to time of underwriting.

-Probability of being active.

-Probability of being alive.

-Premium reserves for active and disabled.

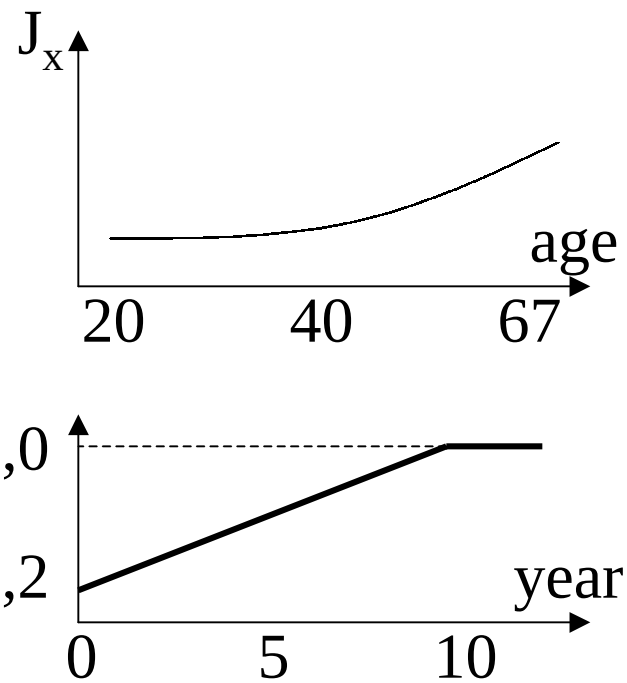
-IBNR

-RBNS

Existing model which has been in use in Norweginans insurance companies since 1964

- This model is based on a possibility of becoming disabled at age x. This function J_x depends on age x.
- The possibility is multiplied with a selection function.
- The selection function depends on the time from the contract was new.
- The contract carries a lower risk in the start because of medical underwriting.

J_x model



- The first graph shows the J_x possibility.
- The second graph shows the selection function.
- $J_{x+t}=J_{x+(t)}\ g(t)$
- Single premium calculated:

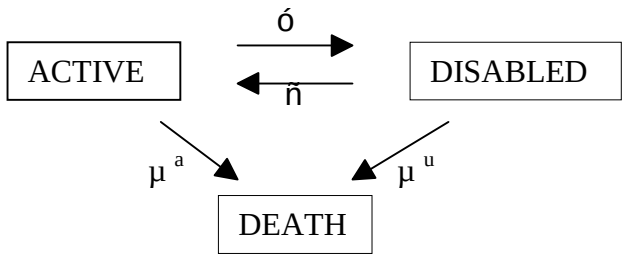
$$\sum_{s=0}^{r-(x+t)} v^s\ S_{t+s}\ \exp\left(-\sum \mu^a_{x+t+s}\right)\ J_{x+t+s}$$

- S is annual pension
- v is discount rate
- μ is mortality intensity

Desirable of an alternative to J_x

- A model build by intensities
- Control all elements seperatly
- Possibilities to calculate premiums and reserves for disability pensions and lump sums
- The flexibility assumes stronger control with the different intensities and the other variables. (Two estimated numbers multiplied give more margines than one single estimat)

Alternative method for calculating disability pension premiums



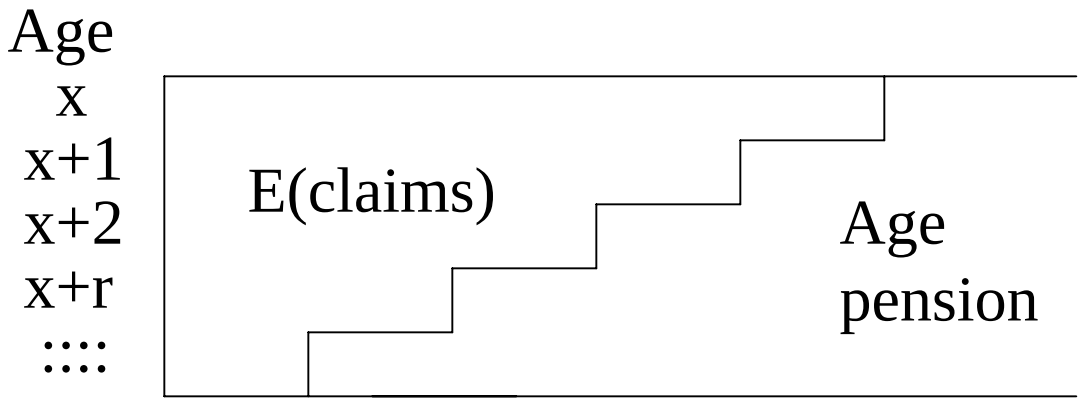
- ó is the disability intensity
- μ^a is the death intensity for active
- μ^u is the death intensity for disable
- ñ is the recovery intensity

Variables in the new model

- Disablement intensity.
- Degree of disablement.
- Recovery intensity.
- Selection, which means that new incurance contracts have less risk in the begining

Sequenses of expected payments

(months from possible disablement to age pension)



- Each year r has a possibility to start a sequence of payment towards age pension
- Sum of the values in each row in the matrix multiplied with the disable intensity, is the expected values of this sequence of claims.

Some parameters

- r is year from calculating point
- s is the number of months since possible sickness
- x is age on calculating point
- b is pension age
- z(x,r) number of months from possible sickness to pension age.

Expected values by monthly claims

Age	time from possible sickness (months)						
	1	2	3	4	5	6	7
x+1	$\varphi_{1.1}$	$\varphi_{1.2}$	$\varphi_{1.3}$	$\varphi_{1.4}$	$\varphi_{1.5}$	$\varphi_{1.6}$	$\varphi_{1.7}$
x+2	$\varphi_{2.1}$	$\varphi_{2.2}$	$\varphi_{2.3}$	$\varphi_{2.4}$	$\varphi_{2.5}$	$\varphi_{2.6}$	$\varphi_{2.7}$
x+r	$\varphi_{r.1}$	$\varphi_{r.2}$	$\varphi_{r.3}$	$\varphi_{r.4}$	$\varphi_{r.5}$	$\varphi_{r.6}$	$\varphi_{r.7}$

- s is the time from possible sickness in age x+r.
- The expected value of one sequence discounted to time r:

$$C(x+r) = \acute{o}_{x+r} \sum_{s=1}^{z\,(x,r)} v^{s/12}\, \varphi_{rs}$$

(when v = 1/(1+i) and i is the annual discount rate.)

The single risk premium - E(x)

$$E(x) = \sum_{r=1}^{b-x} v^r \, {}_rp_x^{aa} \; C(x+r)$$

$${}_rp_x^{aa} = \exp\left(-\int_0^{b-x} (\mu^a_{x+\tau} + \sigma_{x+\tau})\,d\tau\right)$$

C(x+r) will be defined after we have looked at the different variables which define φ.

E(x) and C(x+r) is solved by:

- σ_x is intensities for a defined waiting period for exaple 3 or 12 months. This intensity will then be the one year possibility that the insured will be sick for a period longer than 3 or 12 months.
- σ_x^{uk} is the intensity we use in lumpsum insurance. This intensity is the one year risk for life long disability.
- μ_x^a is the death intensity for active insured.

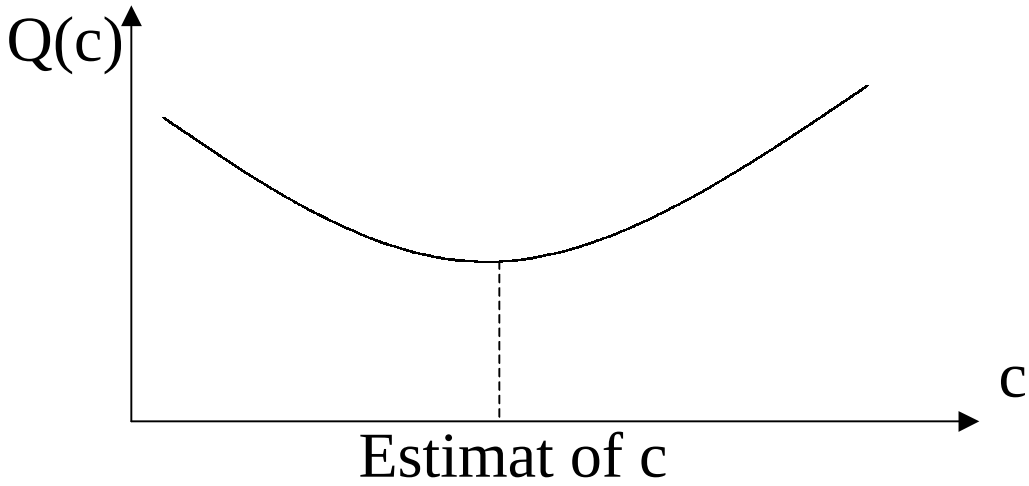
Disability intensities

- Solved by Gomperts Makeham $\sigma_x = \alpha + \beta c^x$

α , β & c are estimated by Bisection. Method of Least Squared is solved by a likelihood function:

$$Q(c) = \Sigma (s_t - \alpha - \beta c^x)$$

This function can be solved as linear regresion, where α and β will be estimated, c^x will then be the variable part with c as a constant. α and β can be estimated for many different values of c.

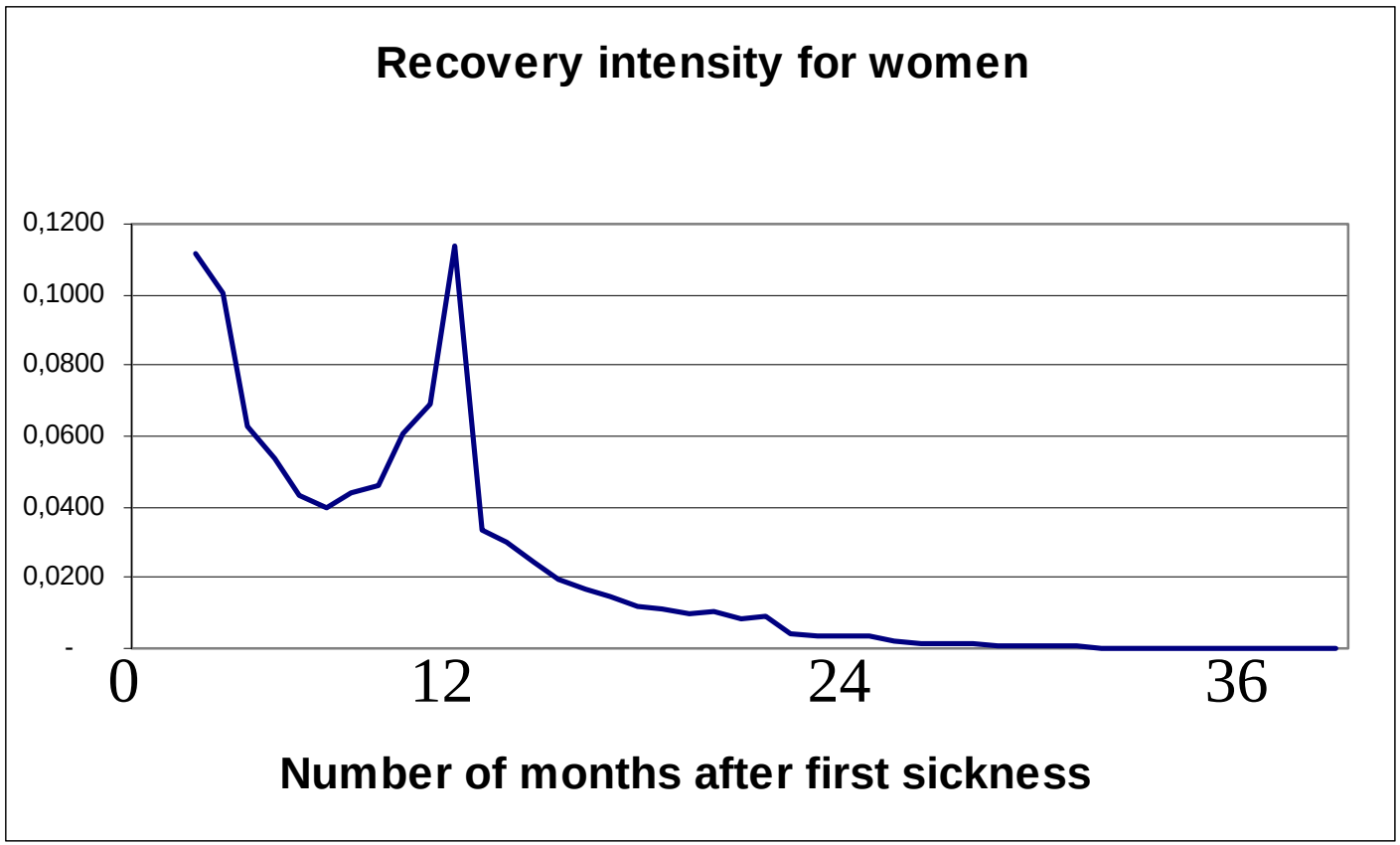


- Each fixed c gives different optimal solutions of α and β via linear regresson.
- Each c gives then different values of Q(c).
- Min(Q) gives the estimates for the Gomperts Makham function.
- This is a common numerical method and is called Bisection.

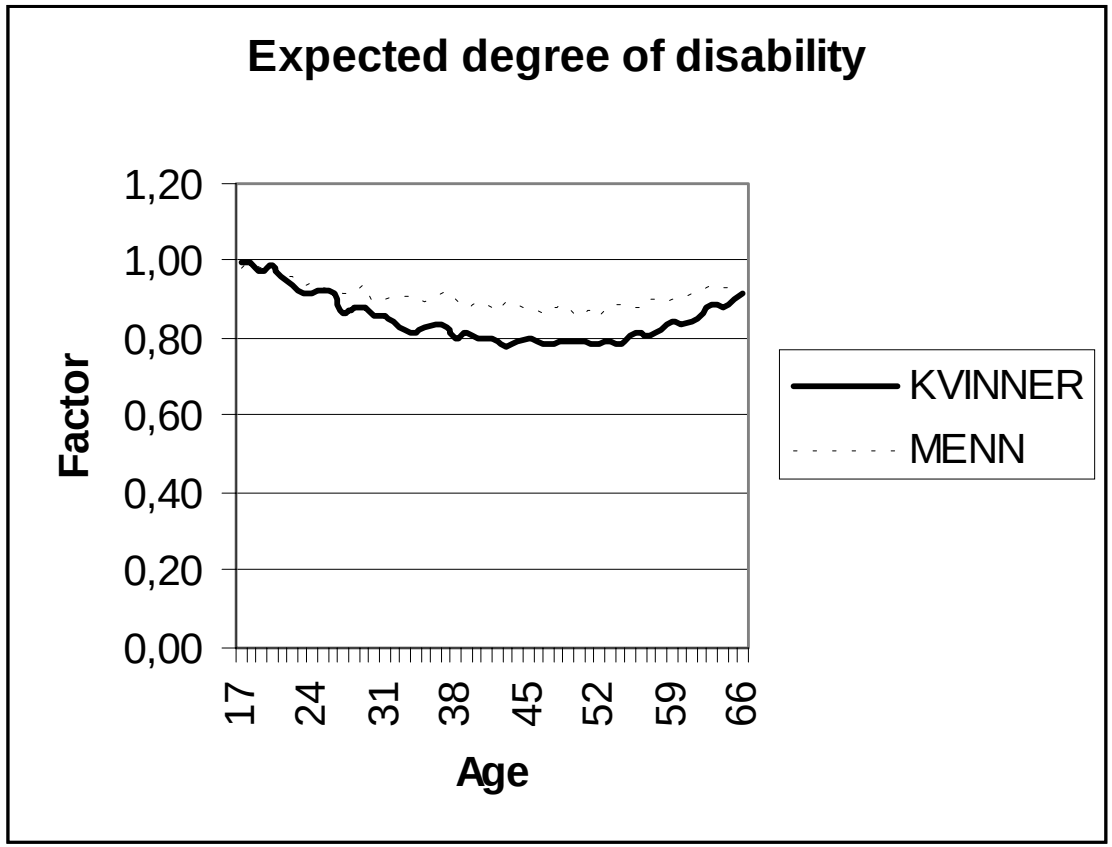
Variables which solve φ_{rs} and then $C(x+r)$ is:

1) Recovery intensity.
2) Degree of disability.
3) Death intensity for disabled.

1 - RECOVERY

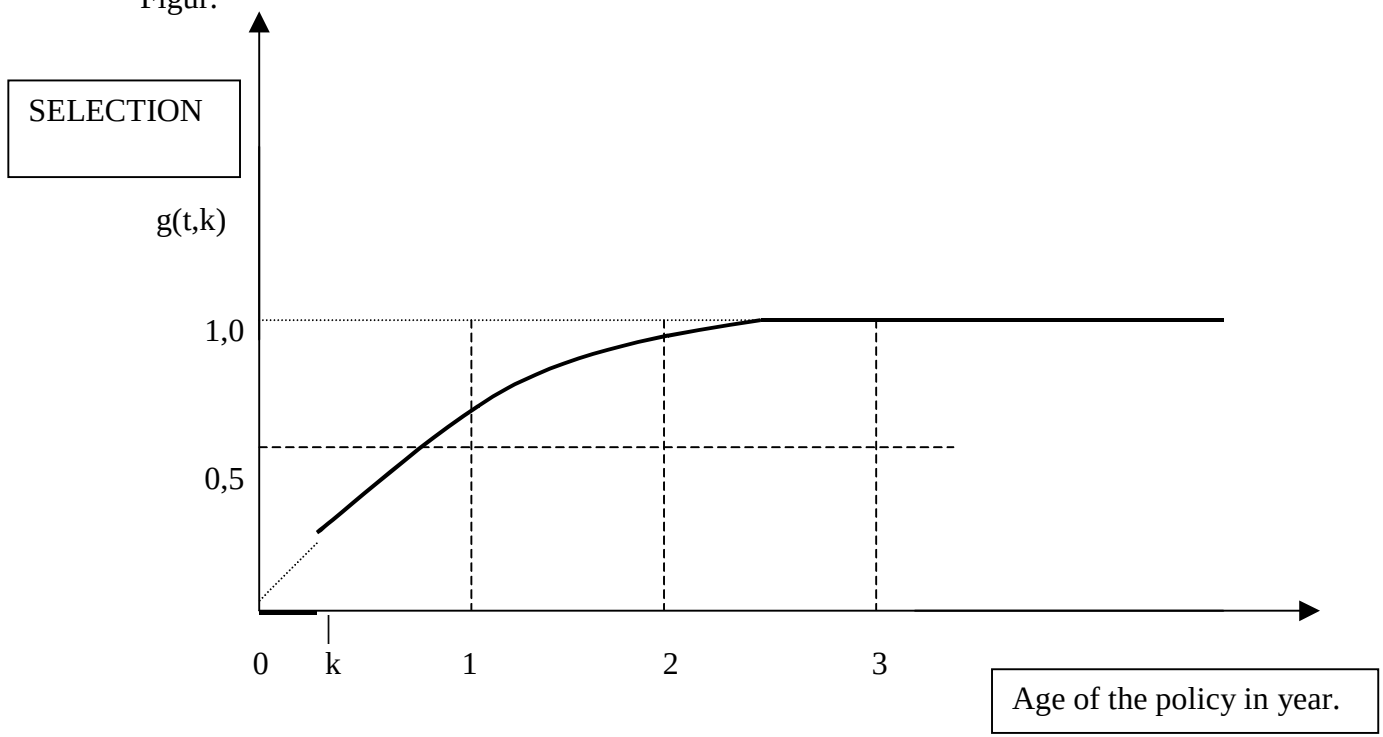

Recovery intensity for women
0,1200
0,1000
0,0800
0,0600
0,0400
0,0200
-
0
12
24
36
Number of months after first sickness

2 - DEGREE OF DISABILITY



3 - SELECTION

Figur:



- Upg_x is the degree of disability on age x
- g is the selection function
- k is the karens time
- s is the annual pension payments

Continuing calculation of single premiums

- The expected value of one sequence discounted to time r:

$$C(x+r) = \acute{o}_{x+r} \sum_{s=1}^{z\,(x,r)} v^{s/12}\, \varphi_{rs}$$

$$\varphi_{rs} = Upg * \; g * \; S * \; {}_sp^{ii}{}_{x+r}$$

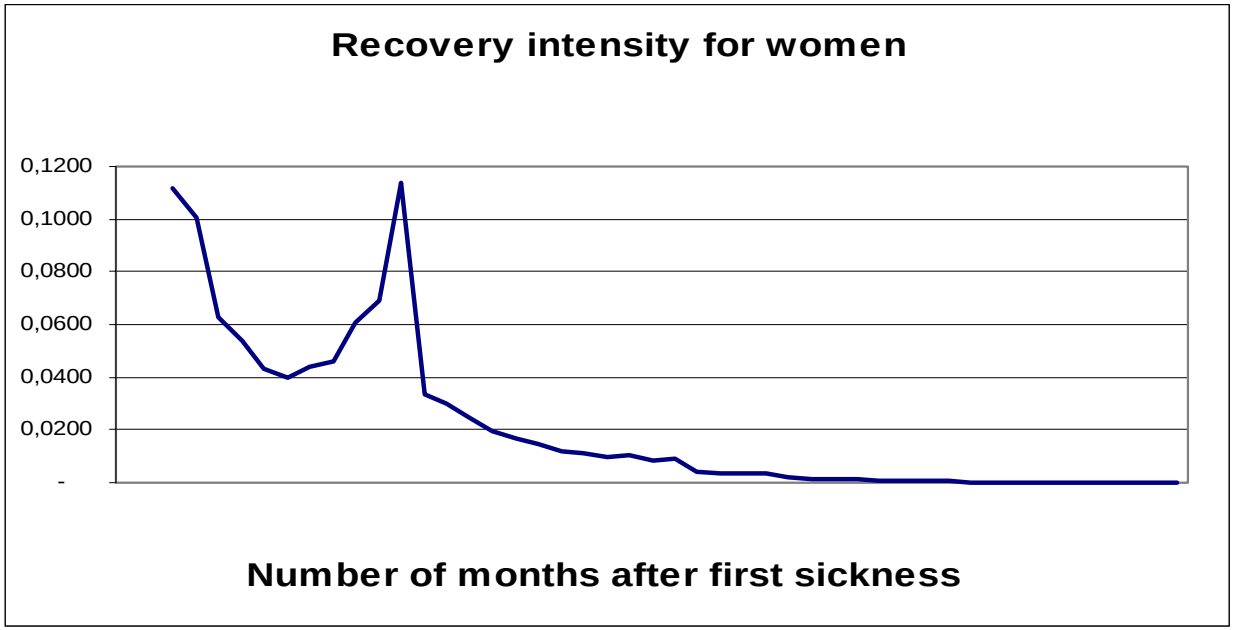
$${}_sp_x^{ii} = \exp\left(-\int_0^s (\mu^u_{x+r+(\tau/12)} + \rho_\tau)\, d\tau\right)$$

Single premium:

$$E(x) = \sum_{r=1}^{b-x} v^r \; {}_rp_x^{aa} \; C(x+r)$$

$${}_rp_x^{aa} = \exp\left(-\int_0^r (\mu^a_{x+\tau} + \sigma^{uk}_{x+\tau})d\tau\right)$$

Since the σ in the calculation of p^{aa} is the lump sum intensity, will the calculation describe the possibility that the person not have died or has been life long disabled on age x+r.



- The recovery intensity is estimated so that temporarly recovery does not count. The recovery intensity in this model is calculated by lasting recovery alone.
- (In same way as the disability intensity for lump sum payments only calculate lasting disabled.)

Use of the method

The single premium, and derivates can solve:

- Premium reserves for active V^a and disable V^u seperatly.
- Annulal premiums : $E(x+1) - E(x)$

IBNR & RBNS

- Some insurance companies want to reserve rbns and ibnr for disability pension. The payments usually start 12 months after the first day of sickness. Ibnr and rbns is then reserves for persons who has been sick for a period- s shorter than the waiting period- k.
- Rbns: $A(x,s)=V^a+(V^u-V^a)\exp(-\sum_{w=s}^{12}\rho_w)$
- Ibnr: $\sum_{all\_members}(k/12)\,\sigma(V^u-V^a)-\sum_{all\_members}(A(x,s)-V^a)$