Mathematical Programming and Pension Funds

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Mathematical Programming and Pension Funds

Let

r be the yield rate of the fund;

r_u be the upper value of the fund's yield rate;

r_l be the lower value of the fund's yield rate;

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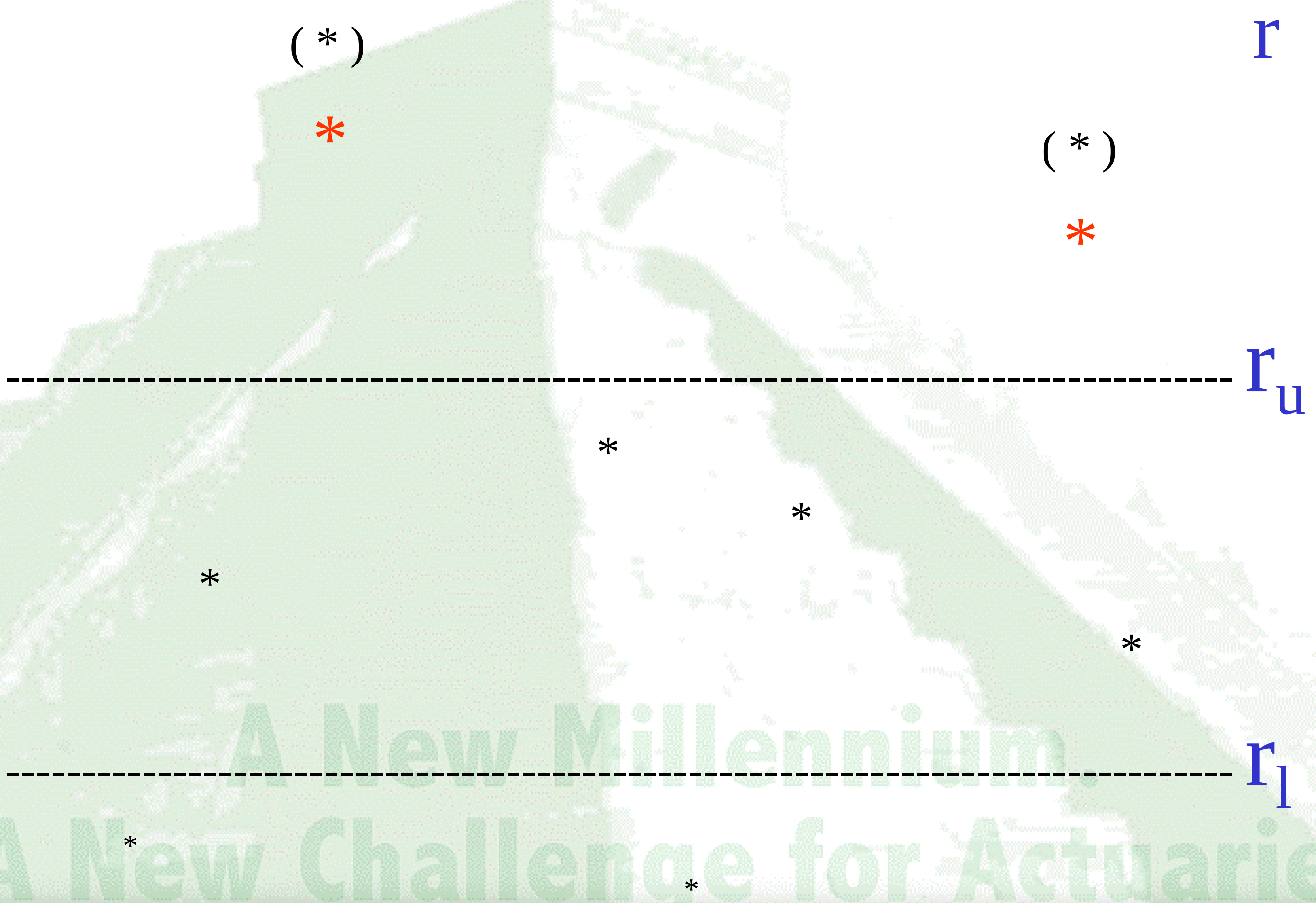

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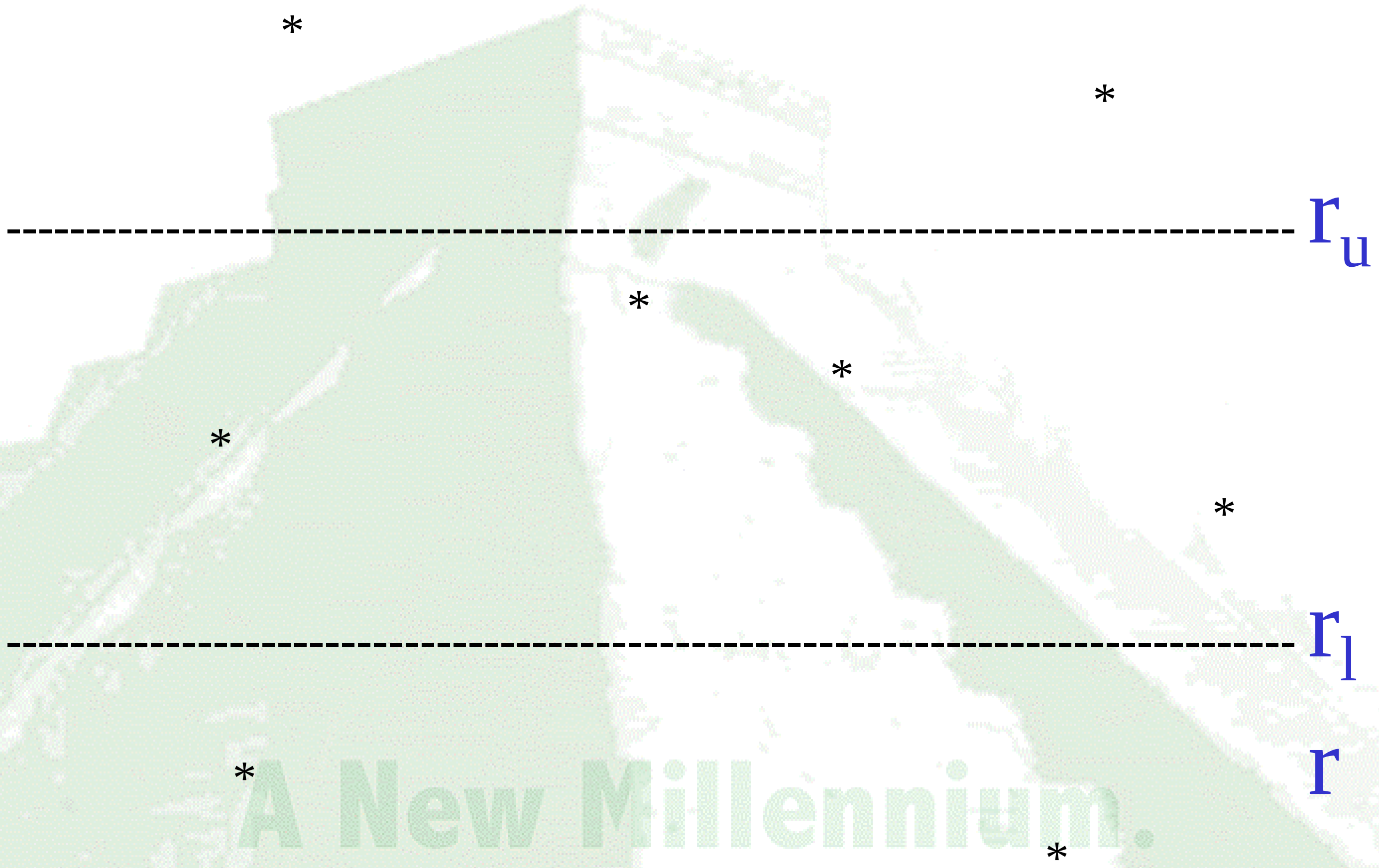

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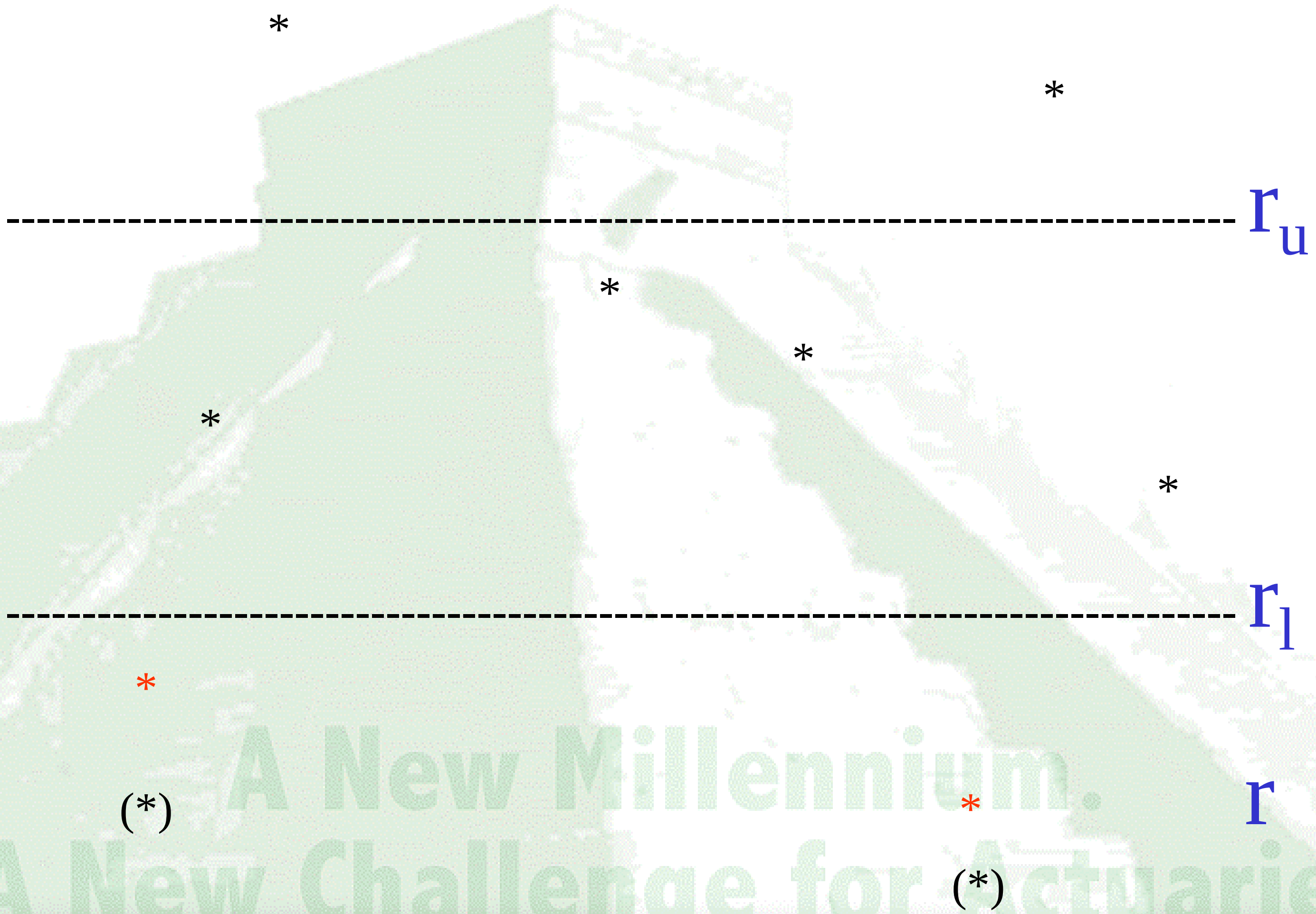

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Let

r_i be the yield rate of the i^{th} personal account of those accounts where the yield rate is less than r_l and E_i be its recent balance;

r_j be the yield rate of the j^{th} personal account of those accounts where the yield rates is greater than r_u and G_j be its recent balance;

$\underline{r}_i$ be the new r_i and $\underline{r}_j$ be the new r_j

R be the size of the yield adjustment reserve

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$\underline{r}_i \leq \underline{r}_l$ (i=…)

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$$\underline{r}_i \leq r_l \ (i=\ldots)$$

$$\underline{r}_j \geq r_u \ (j=\ldots)$$

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$$\underline{r}_i \leq r_l \ (i=\ldots)$$

$$\underline{r}_j \geq r_u \ (j=\ldots)$$

$$R-\Sigma_i x_í + \Sigma_j y_j \geq 0$$

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$$\underline{r}_i \le r_l \text{ (i=…)}$$

$$\underline{r}_j \ge r_u \text{ (j=…)}$$

$$R-\Sigma_i x_í + \Sigma_j y_j \ge 0$$

$$R-\Sigma_i x_í + \Sigma_j y_j \le 0.05*(\Sigma E_i + \Sigma_i x_í + \Sigma G_j - \Sigma_j y_j$$

+the balance of those personal accounts which do not change in the yield redistribution)

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$\underline{r}_i \leq r_l$ (i=…)

$\underline{r}_j \geq r_u$ (j=…)

$R-\Sigma_i x_í + \Sigma_j y_j \geq 0$

$R-\Sigma_i x_í + \Sigma_j y_j \leq 0.05*(\Sigma E_i + \Sigma_i x_í + \Sigma G_j - \Sigma_j y_j$ +the balance of those personal accounts which do not change in the yield redistribution)

min $d(\ldots \underline{r}_i \ldots \underline{r}_j \ldots)$

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$d(\ldots r_i \ldots r_j \ldots)$ is a measure expressing how much the r_i and r_j are outside of the interval $[r_l, r_u]$

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$$d(\ldots\underline{r}_i \ldots\underline{r}_j\ldots) = \max_{i,j} (\ldots\underline{r}_j - r_u \ldots r_l - \underline{r}_i\ldots)$$

can be considered as a basis for a fair redistribution.

In the recent regulation

$$d(\ldots\underline{r}_i \ldots\underline{r}_j\ldots) = \Sigma_k\, c_k s_k,$$

where $\underline{r}_j - r_u$ and $r_l - \underline{r}_i$ are denoted by s and for the weights c $c_l < c_k$ if $s_l > s_k$

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- Unisex annuities
- P_x denotes the annuity factor, i.e. the capital needed to be present *now* to pay 1 HUF to a member of x age.

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- $C_{M,x,j}$ is the sum of the expected present value of those x age males' personal account balances who will be pensioner j years later;
- $C_{F,x,j}$ is the same for female members;
- $P_{M,x}$ is the annuity factor calculated only on the male members' mortality;
- $P_{F,x}$ is the annuity factor calculated only on the female members' mortality

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$$\Sigma_j \, \Sigma_x \, C_{M,x.j} + \Sigma_j \, \Sigma_x \, C_{F,x,j} =$$

$$\Sigma_j \, \Sigma_{\,x} \, (C_{M,x,j} \,/\, P_x)*P_{M,x} + \Sigma_j \, \Sigma_{\,x} \, (C_{F,x,j} \,/\, P_x)*P_{F,x}$$

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$$\Sigma_j \, \Sigma_x \, C_{M,x.j} + \Sigma_j \, \Sigma_x \, C_{F,x,j} =$$

$$\Sigma_j \, \Sigma_x \, (C_{M,x,j} / P_x)*P_{M,x} + \Sigma_j \, \Sigma_x \, (C_{F,x,j} / P_x)*P_{F,x}$$

$$P_x = (\Sigma_j \, C_{M,x,j}*P_{M,x} + \Sigma_j \, C_{F,x,j}*P_{F,x}) \, /$$

$$(\Sigma_j \, C_{M,x,j} + \Sigma_j \, C_{F,x,j})$$

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$$\Sigma_j \, \Sigma_x \, C_{M,x.j} + \Sigma_j \, \Sigma_x \, C_{F,x,j} =$$

$$\Sigma_j \, \Sigma_{\,x} \, (C_{M,x,j} \,/\, P_x)*P_{M,x} + \Sigma_j \, \Sigma_{\,x} \, (C_{F,x,j} \,/\, P_x)*P_{F,x}$$

$$1/P_x = \lambda*1/P_{M,x} + (1-\lambda)*1/P_{F,x}$$

$$(\, 0 \leq \lambda \leq 1 \,)$$

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minimise an l_p (e.g. l_∞ or l_2) distance of the

$$P_{x+j} \text{ and } \underline{P}_{x+j}$$

subject to

$$A + \Sigma_x C_{M,x} + \Sigma_x C_{F,x} \leq$$

$$L(\underline{P}_x) + \Sigma_x (C_{M,x} / P_x) * P_{M,x} + \Sigma_x (C_{F,x} / P_x) * P_{F,x}$$

$$1/P_{F,x+k} \leq 1/P_{x+k} \leq 1/P_{M,x+k} \quad (k = 0,1,2\ldots)$$

Mathematical Programming and Pension Funds

minimise an l_p (e.g. l_∞ or l_2) distance of the

$$P_{x+j} \text{ and } \underline{P}_{x+j}$$

subject to

$$A + \Sigma_x C_{M,x} + \Sigma_x C_{F,x} \leq$$
$$L(P_x) + \Sigma_x (C_{M,x} / P_x)^* P_{M,x} + \Sigma_x (C_{F,x} / P_x)^* P_{F,x}$$

$$1/P_{F,x+k} \leq 1/P_{x+k} \leq 1/P_{M,x+k} \quad (k = 0,1,2\ldots)$$

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$$A + \Sigma_x C_{M,x.j} + \Sigma_x C_{F,x,j}$$

$$- L(P_x) - \Sigma_x (C_{M,x,j} / P_x) * P_{M,x} - \Sigma_x (C_{F,x,j} / P_x) * P_{F,x}$$

$$= U - V$$

$$1/P_{F,x+k} \leq 1/P_{x+k} \leq 1/P_{M,x+k} \quad (k = 0,1,2\ldots)$$

an l_p (e.g. l_∞ or l_2) distance of the P_{x+j} and $\underline{P}_{x+j}$ is not greater than a small value

$$U, V \geq 0$$

$$\min (U + V)$$

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A starting "safe" $\underline{P}_x$ can be chosen by solving the MP

$$\Sigma_j \Sigma_x C_{M,x.j} + \Sigma_j \Sigma_x C_{F,x,j} =$$

$$\Sigma_j \Sigma_x (C_{M,x,j} / \underline{P}_x)*P_{M,x} + \Sigma_j \Sigma_x (C_{F,x,j} / \underline{P}_x)*P_{F,x}$$

$$1/P_{F,x+k} \le 1/P_{x+k} \le 1/P_{M,x+k} \quad (k = 0,1,2\ldots)$$

maximise the l_p (e.g. l_∞ or l_2) norm of $\underline{P}_{x+j}$

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