ASSESSING ADMINISTRATION CHARGES FOR THE AFFILIATE IN INDIVIDUAL ACCOUNT SYSTEMS[1]

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ABSTRACT

In any pension system based on capitalization, affiliates have to cover certain explicit costs which in a pay-as-you-go system would be implicit. In this paper we set out a model based on Whitehouse (2000) and Diamond (1999) to enable the explicit costs borne by the affiliate both during his working life and his retirement period to be assessed. It also shows the relationships between the different ways of measuring the costs that make up the total price finally paid by the contributors. Included in the model is the notable effect that some factors - such as gaps in contribution profiles, account transfers and changes in salary profiles - have on projecting the costs borne by the affiliates.

Finally we carry out an international comparison of administration costs from the point of view of the affiliate, focusing special attention on the countries of Latin America and Spain. This has a double objective:

1.-To test the validity of criticisms made by some researchers as to whether the new capitalization systems introduced in Latin America are too expensive to run for the affiliates.

2.-To serve as a reference for the individual pension scheme system in Spain.

(JEL: G23, H55, J26)

Key words: Capitalization, Administration Costs, Pension Funds, Latin America

VALORACIÓN DE LOS COSTES DE ADMINISTRACIÓN PARA EL AFILIADO EN LOS SISTEMAS DE CUENTAS INDIVIDUALES DE CAPITALIZACIÓN

RESUMEN

En un sistema de pensiones basado en la capitalización los afiliados deben hacer frente a unos costes explícitos que en el sistema de reparto son implícitos. En este trabajo se desarrolla un modelo, basado en Whitehouse (2000) y Diamond (1999), que permite evaluar las comisiones explícitas que soporta el afiliado, tanto durante la vida laboral como durante la etapa de jubilación y que, además, muestra la relación entre las diferentes medidas de los costes que integran el precio total que finalmente pagan los cotizantes. En el modelo se introduce el efecto de algunos aspectos que tienen una repercusión muy importante en la proyección de los costes que soportan los afiliados: "vacíos" en los perfiles de aportación, efecto de los traspasos de fondos y cambio en los perfiles de salarios. Por último, se realiza una comparación internacional de los costes de administración desde la óptica de los afiliados con un doble objetivo:

1.-Contrastar la validez de la crítica realizada por algunos investigadores a los nuevos sistemas de capitalización individual implantados en América Latina, en el sentido de que son excesivamente caros de gestionar para los afiliados.

2.-Servir de referencia para el sistema de planes de pensiones individuales en España.

CONTENTS:

LIST OF TABLES

LIST OF GRAPHS

I.-INTRODUCTION.

Over the last few years there has been much debate regarding the nature, viability and scope of public pension systems which has ended up by undermining political support for the more traditional forms of pension provision. It has set in motion an unprecendented process of reforms inspired by the approach of the World Bank, according to Holzmann (2000), which for the second pillar recommends capitalization, defined contributions and private management. Naturally enough, not all researchers agree with the predominant profile or bias of the reform. Included amongst the better-known critics are Orszag and Stiglitz (1999) and Barr (2000), for example, although as Queisser (2000) points out, while discussion of the reforms initially took the form of a heated debate with a noticeable ideological bias, now all the participants are seeking mutual understanding and, more specifically, a greater level of coordination and cooperation.

According to Devesa, Martínez and Vidal (2000), many countries in Latin America have partially or totally transformed their pensions systems into individual capitalization systems in which the ideas of individual responsibility and freedom of choice take on greater importance. Chile pioneered the reforms in 1981 and is perhaps the country which has put them into practice in the most drastic way due to its own particular political conditions. Peru (1993), Colombia (1993), Argentina (1994), Uruguay (1995), Mexico (1995), Costa Rica (1995), Bolivia (1997) and El Salvador (1998) also carried out reforms at a later date, these last two countries being those which most resemble the pioneer model. Other countries in the region - Nicaragua, Venezuela and the Dominican Republic - are currently in the process of carrying out reforms based on individual capitalization accounts.

The pensions systems in all these countries - most of which were pay-as-you-go, although previously they had been collective capitalization - collapsed for various reasons: serious economic problems, evident design flaws, a general lack of trust in politicians, the inability of the State to administer the public systems, the low level of coverage, unfair differences between contributors, regressivity in the distribution of income, bad management of existing funds and high administration costs[2].

Administration costs are currently attracting the interest of various researchers for a number of reasons:

a) Many countries have set up or are considering setting up some sort of system of individual capitalization accounts.

b) Measuring the cost of financial services is much more difficult than measuring the cost of other goods and services.

c) The cost is often not transparent and affects different contributors in different ways according to their level of income and the amount they have accumulated in the funds.

d) Charges which are too high:

[2] Rodríguez and Durán (2000) argue that the fact that in some Latin American countries, pre-reform administration costs on an absolute level reached an amount similar to the benefits paid, leads one to conclude that the excessive costs in the public pay-as-you-go systems were due to the solvency crisis, which in many cases preceded the reform, almost as much as to demographic factors or the design of the benefits. On average, the proportion represented by administration fees in relation to benefits paid was ten times higher than it was in OECD countries.

1.-Discourage affiliates from participating and reduce the real return on capitalization accounts, thereby making it impossible to fulfil one of the basic objectives of the reform.

2.-According to Mitchell (1999), they increase future costs for the State due to the fact that some countries guarantee a minimum retirement pension. Higher administration costs generate a greater number of people who will need to have their pensions supplemented.

These arguments would appear to justify State intervention in the interests of maintaining administration costs at a reasonable level, at least in mandatory capitalization systems. In voluntary systems, the argument for intervention in this sense is not as strong. However, as will be seen later, it does appear to be justified in the case of Spain.

It is not our intention in this paper to try and analyse the costs structure of the pension fund administration industry, as Valdés-Prieto (1999) and Mastrángelo (1999) and others have done in detail. Neither is it our intention to compare costs between different systems, as Mitchell (1998) has done, nor to compare costs with other forms of private industry or to find the most effective way of organizing a capitalization system, as shown in papers by James, Ferrier, Smalhout and Vittas (1999), Thompson (1999) and Rodríguez and Durán (2000).

This paper does have a connection with those mentioned above, and in some aspects is supported by them, but it takes a different approach as it is carried out from the point of view of the individual. Its structure is as follows. After this brief introduction, the next section analyzes the basic charges that affiliates usually have to pay in capitalization systems. A model based on Whitehouse (2000) and Diamond (1999) is then set out so as to enable the explicit costs borne by the affiliate both during his working life and his retirement period to be assessed. The model also shows the relationship between the different ways of measuring those costs which make up the total amount the contributors eventually pay. The fourth section analyzes, mainly, the two measurements most used in the literature:
a'_1 : combines the effect of all the explicit fees as a whole into a single constant percentage which decreases the total amount of all the contributions.
a'_2 : combines the effect of all the explicit fees as a whole into a constant decrease in gross return.
It also looks at some factors which are not usually given much importance in the literature but which undoubtedly have a great effect on projecting the costs borne by affiliates during their entire period as contributors/pensioners, namely: gaps in contribution profiles, the effect of account transfers, and changes in salary profiles. In the fifth section an international comparison is made of administration costs from the point of view of the contributor, focusing special attention on the countries of Latin America. This has a double objective:

1.-To test the validity of criticisms made by some researchers as to whether the new capitalization systems introduced in Latin America are too expensive to run for the affiliates.

2.-To serve as a reference for the individual pension scheme system in Spain.

The paper ends with the main conclusions reached and a full bibliography. Finally there are various appendices where questions relating to the determination of the fund values and salary functions used in the various comparisons are dealt with in detail.

II.-BASIC EXPLICIT CHARGES FOR INDIVIDUAL CAPITALIZATION ACCOUNTS.

In any pensions system based on capitalization, affiliates have to cover explicit costs (which in a pay-as-you-go system are implicit) both throughout their working life and during their retirement period. In the case of workers, the costs take the form of fees paid to the administrators of the system, while for pensioners the cost will depend on the type of pension chosen. It is clear that the way a capitalization system is organized will have a great impact on costs, and that a system of decentralized individual accounts will always have a higher level of costs than other types of capitalization accounts.

The explicit charge structure adopted by any accumulation system is very important due to its long-term character and the fact that the fund accumulation process is exponential. In general terms the charges can be categorized as follows:

a) By amount:

- Fixed. This generates a relatively higher cost for workers on low incomes.
- Variable. This usually covers two types of services: managing the balance in the individual account and collecting contributions.

b) By frequency of payments:

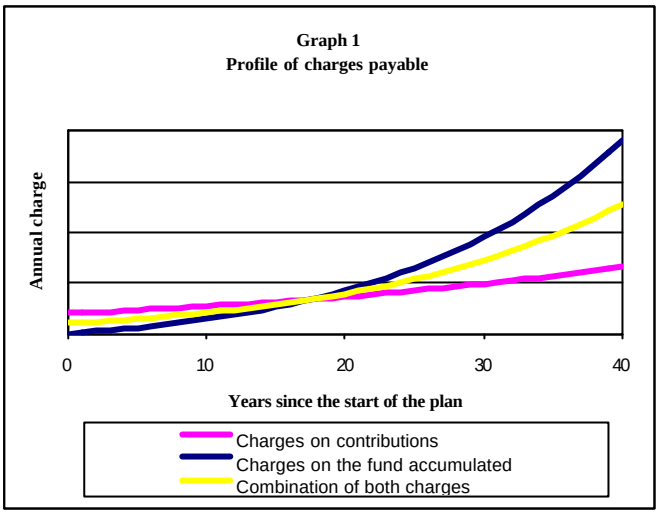
- One-off charges. These usually apply in three cases: to open the individual capitalization account, to transfer the funds accumulated to another scheme, or to convert the assets accumulated into a pension.
- Periodic charges. These cover the costs of management, custody and deposit, and may be paid in advance or in arrears.
- Ongoing charges. These are payable up-front periodically.

c) By the amount on which they are levied:

- On the balance.
- On the contribution.
- On the nominal value of the portfolio.

A suitable combination of these charges or explicit costs would increase coverage and enable some of the costs of the system to be distributed proportionally according to the affiliates' level of income. Whatever charge structure is adopted will have a great impact on the payments the contributor has to make over time, although in the long term they can work out to be equivalent. In support of these claims, Graph 1 shows the time profile for three different types of charges payable by the affiliate. For simplicity's sake it is assumed that each of them is for a single charge type:

1) As a percentage of contributions (without lower limit).
2) As a percentage of the assets in the accumulated fund (without lower limit).
3) A combination of the two above (without lower limit).



The first charge structure proposed means that the affiliate has to pay more at the start. This would not be very positive from the point of view of possibly increasing the system's coverage and would work against affiliates on lower incomes. From the point of view of the administrators, however, it would be very positive since they could recoup the system's initial set-up costs more quickly. The structure would not always work against the affiliate as those who may not be able to make contributions over certain periods would have zero-cost fund management.

The second charge structure, unlike the first, means that the affiliate pays less at the start. However, the amount payable increases considerably as the fund accumulates. Whether or not new contributions are made, the affiliate always has to pay charges, and the amount of these charges gets bigger and bigger over time. This structure could be the most suitable from the point of view of possibly extending coverage.

Both these charge structures could imply cross-subsidies between affiliates. Assuming that managing the individual capitalization account has a fixed cost for the administrator, and given that there is no minimum fee, the first structure would mean that affiliates on higher incomes would subsidize those on lower incomes, while in the second structure it would be the older affiliates with substantially larger funds who would be subsidizing younger affiliates. The third structure, being a combination of the two previous ones, could soften the effects mentioned above.

The concept of “explicit costs” should be clarified here since substantial implicit costs could arise which are not included in the model in Section 3. They are also very difficult to assess in practice. According to Valdés-Prieto (1999), interest groups in some countries, including those forming part of the political system, can influence the pensions system in various different ways. The easiest way to measure this “influence” is by looking at the effect it has on the return of the investments. A decrease in the return is equivalent to an implicit tax charged by the pressure groups on the pension fund. The cost of political influence on the pensions system can be measured as the difference between the return reached in that particular country at that particular time by other big long-term investors and the return actually reached by the pension fund in question.

Iglesias and Palacios (2000) find empirical evidence that capitalized funds managed by the public sector are often used for purposes other than those intended, that they are very difficult to protect from political interference, and that they tend to produce returns far below reference rates. This evidence is consistent and particularly relevant in countries with weak democratic structures. Valdés-Prieto (1999) calls this effect the cost of not privatizing.

Another implicit cost for the affiliate may be found by comparing the implicit return of the pay-as-you-go system with the net return of the capitalization system in those countries where it is possible to choose between systems. In a financially balanced pay-as-you-go system, the implicit internal rate promised by the system, as shown by Devesa, Lejárraga and Vidal (2000), is approximately equal to the rate of growth of the contributing population plus the real growth rate of salaries which, under certain conditions, could be assimilated into the real growth of the gross domestic product. Therefore net returns below what the financially balanced pay-as-you-go system is in theory capable of producing can be considered as another cost for the affiliate.

Finally it should be mentioned that, as Whitehouse (2000) points out, measuring the cost of financial services is a complex task since different systems have very different charges and regulations:

a) Restrictions on charges do not usually exist or, if they do, they are very limited. This tends to be the case in countries where private pension schemes (or individual capitalization accounts) are voluntary or have been built upon already existing voluntary systems. Hence in Spain for example, the administration costs that fund managers and deposit takers can charge are limited to a maximum of 2.6% of the annual assets managed/deposited.

b) "Subsidies" to workers on low incomes. The State, either directly or through special taxes on workers with higher incomes, contributes a fixed percentage of salary to the individual accounts of workers on low incomes. This is the case in Colombia and Mexico. Workers on low incomes are also usually allowed to remain outside the system, as is the case in Uruguay.

c) Limitation of the charges structure and/or setting partial (fixed or variable) ceilings on charges in such a way that pension fund managers can only levy a certain number of charges, and these only under certain conditions. For example:
   1) They may have to choose between a fixed or variable charge.
   2) They may have to choose between a contributions-based charge and an assets-based charge.
   3) They may be allowed to apply only two types of charges, but one of them may have a ceiling while the other may be unrestricted.

d) Multiple and/or competitive portfolios are offered to tender. In other words, the different pension fund managers may have to bid for the right to manage the funds accumulated by members of the scheme. This is the procedure applied in the capitalization part of the new Swedish system, and also in Bolivia.

III.-MODEL FOR MEASURING CHARGES FROM THE POINT OF VIEW OF THE CONTRIBUTOR/PENSIONER.

In this section a model will be set up based on the ideas of Whitehouse (2000) and Diamond (1999) to show the relationship between the different ways of measuring the costs that make up the total price that contributors eventually have to pay. This will enable the explicit costs (implicit costs will not be considered here) borne by the affiliate both during his working life and his retirement period to be assessed. This analysis is therefore an important prerequisite for the comparison of different charge structures.

The elements considered are:
w_t: salaries at time t.
g: real, annual, accumulative growth rate of salaries.
c: percentage of salary contributed to the pension scheme.
r: real, gross, annual, accumulated return obtained by the fund.
a_0: single fixed charge, payable at the start.
a_1: percentage applied on each contribution, giving rise to a periodic charge.
a_2: percentage applied instantaneously on the assets accumulated, giving rise to an ongoing charge.
a_3: percentage applied on the accumulated balance, payable for transferring the account to another scheme.
a_4: percentage applied on the accumulated balance, payable for converting the accumulated fund into a pension.

An individual's salary w_t in a given period t, assuming real exponential growth at rate g, in accordance with salaries for period 0, can be expressed in the following way:

$$w_t = w_0 e^{gt} \qquad \textbf{[1.]}$$

Contribution to the pension fund at time t, net of the periodic charge on contributions (a_1), will be shown as:

$$c(1-a_1)w_0 e^{gt} \qquad \textbf{[2.]}$$

The net amount accumulated in the fund, corresponding to the contribution made at time t and assessed at retirement time T, is given by the formula:

$$c(1-a_1)w_0 e^{gt} e^{r(T-t)} \qquad \textbf{[3.]}$$

If ongoing charge (a_2) is added, the accumulated amount corresponding to the contribution made at time t and assessed at retirement time T will grow according to the difference between the real gross return and the percentage of that charge (r- a_2):

$$c(1-a_1)w_0 e^{gt} e^{(r-a_2)(T-t)} \qquad \textbf{[4.]}$$

By integrating the above expression, see Appendix 1, from moment 0 (the start of the plan) up to time T (when the accumulated funds are withdrawn), the following total amount accumulated is obtained:

$$c(1-a_1)w_0\left(\frac{e^{(r-a_2)T}-e^{gT}}{r-g-a_2}\right) \qquad \textbf{[5.]}$$

Any single charge payable in advance (a_0) will imply a decrease in the net return of the investment, which would translate into a reduction in pension, at time *T*, of:

$$a_0 e^{(r-a_2)T} \qquad \textbf{[6.]}$$

If the charge for converting the accumulated fund (a_4) into an pension(income) is considered, the total net amount accumulated in an individual capitalization system (IA_c) will take on the following expression:

$$IA_c = \left[c(1-a_1)w_0\left(\frac{e^{(r-a_2)T}-e^{gT}}{r-g-a_2}\right)-a_0e^{(r-a_2)T}\right](1-a_4) \qquad \textbf{[7.]}$$

which is the final value that should be reached by the individual capitalization account after deducting all charges and costs, and assuming that the account has not been transferred at any time during the contribution period.

The experience of the countries of Latin America shows that this last assumption is far from realistic. The contributor tends to move his account from one fund to another during his working life. If we assume that a percentage a_3^i is charged on the accumulated fund as a fee for each change of fund, and that in addition "s" changes of fund are made every "T/(s+1)" years, the net total accumulated will be (see Appendix 2):

$$\mathrm{IA_C} = \left\{-a_0e^{(r-a_2)T}\prod_{i=1}^{s}(1-a_3^i)+ \right.$$
$$\left. +c(1-a_1)w_0\left(\frac{e^{(r-a_2)\frac{T}{s+1}}-e^{g\frac{T}{s+1}}}{r-g-a_2}\right)\left(\left\langle\sum_{i=1}^{s}e^{(r-a_2)\left(T-\frac{iT}{s+1}\right)}e^{g\frac{(i-1)T}{s+1}}\prod_{j=i}^{s}(1-a_3^j)\right\rangle+e^{g\frac{sT}{s+1}}\right)\right\}(1-a_4)$$

[8.]

Another effect that can easily be studied is the gap in contributions. It often happens that some people are unable to contribute for a number of years due to unemployment or because they work in the informal sector of the economy or need to care for relatives, etc. During these years without contributions, charges are still made on funds, although charges on contributions are obviously nil. For simplicity's sake assume that the worker makes contributions to the scheme up to time N, which is when contributions cease, although the funds remain invested until time T (which is when he begins to receive the pension).

At the time when the contributions cease, the accumulated fund, net of charges on contributions and assets (a_1 and a_2 respectively), is given by equation (5) simply by substituting N for T:

$$c(1-a_1)w_0\left(\frac{e^{(r-a_2)N}-e^{gN}}{r-g-a_2}\right) \qquad \textbf{[9.]}$$

After N, the time when contributions cease, the fund continues to grow because of the return, net of charges, obtaining an accumulated total of:

$$IA_c=\left[c(1-a_1)w_0\left(\frac{e^{(r-a_2)T}-e^{(r-a_2)(T-N)+gN}}{r-g-a_2}\right)-a_0e^{(r-a_2)T}\right](1-a_4) \qquad \textbf{[10.]}$$

To sum up, the above equations supply the net value of the fund when considering five different specific charges: one fixed and payable in advance (a_0); one on contributions (a_1); one instantaneous on the assets in the fund (a_2); one which penalizes anyone leaving the scheme, which is equivalent to a percentage on the accumulated balance at each time (a_3) and which can be applied "s" number of times; and finally a one-off charge paid by the worker when the accumulated fund is converted into a pension(a_4).

So as later to be able to assess the impact of the charges, we are going to use the amount that would have accumulated in the absence of any charges (IA_{nc}), reducing all the a_s terms to zero:

$$IA_{NC}=cw_0\frac{e^{rT}-e^{gT}}{r-g} \qquad \textbf{[11.]}$$

This simple analytical model enables us to measure the impact of administration costs in various ways:

A) As a reduction in gross return. It tranfers the total effect of all the charges to a constant annual decrease in gross return. It is assumed that the gross return, the time profile of the contributions and the length of the schemeare known. It can be interpreted as the equivalent fee that, applied as a constant instantaneous charge on the accumulated fund, supplies the same final amount that would be obtained if all the charges were taken into account. It is calculated by obtaining a'_2 of the equation resulting from equalling value IA_C of expression (7)[3] and the following equation:

$$c\cdot w_0\frac{e^{(r-a'_2)T}-e^{gT}}{r-g-a'_2} \qquad \textbf{[12.]}$$

B) As a reduction in contributions. It transfers the total effect of all the charges to a single constant percentage, a'_1, which would decrease the amount of all the contributions paid. It is assumed that the gross return, the time profile of the contributions and the length of the schemeare known. It can be interpreted as the equivalent fee that, applied as a constant percentage of contributions, supplies the same final amount that would be obtained if all the charges were taken into account. It is calculated by finding the value of a'_1 of the equation obtained by equalling value I_{AC} of expression (7) and the following equation:

[3] The comparison can also be made by using equations (8) and (10).

$$c(1-a'_1)w_0\frac{e^{rT}-e^{gT}}{r-g} \qquad \textbf{[13.]}$$

it also being possible to arrive at the expression:

$$a'_1 = 1 - \frac{\mathrm{IA_C}}{\mathrm{IA_{NC}}} \qquad \textbf{[14.]}$$

or in other words the complement of the unit of the quotient between the total amount accumulated in the fund, once fees are deducted, and the amount accumulated in the case of there being no fees.

C) As a reduction in the amount of the accumulated fund. This measures the proportion of the final balance in an individual capitalization account which is absorbed by charges. It can be interpreted as the equivalent charges that, applied to the amount in the accumulated fund at the time of retirement, supply the same final amount that would be obtained if all the charges were taken into account. It is calculated by finding the value of a'_4 of the equation obtained by equalling value I_{AC} of expression (7) and the following equation:

$$cw_0\frac{e^{rT}-e^{gT}}{r-g}(1-a'_4) \qquad \textbf{[15.]}$$

reaching the same formula as in case B), whereby:

$$a'_1 = a'_4 = \mathrm{CR} \qquad \textbf{[16.]}$$

where CR stands for charge ratio, as introduced by Diamond (1999). This must be between 0 and 1, as the theoretical value of the fund once the charges have been deducted cannot be more than the value it would reach without taking those charges into consideration. Neither can it be equal to or less than zero. Therefore high costs associated with individual capitalization accounts correspond to high CR values. It can take on the extreme values of the interval in the following cases:

CR = 0 if no charge of any type exists. This is a situation which in practice never comes about and is purely academic.

CR = 1 if the amount of the charges is so high that it completely absorbs the value of the accumulated fund.

Murthi, Orszag and Orszag (1999) give another view of the CR, which they break up into three components:

$$CR = 1 - \frac{IA_c}{IA_{nc}} = 1 - RAC \times RAL \times RAN \qquad \textbf{[17.]}$$

1. The accumulation ratio (RAC): this includes the decreases brought about by the costs (of administration, management, etc.) incurred by the worker when making contributions to the individual account during working life, assuming that the contributions are made regularly and to a single pension fund.

2. The alteration ratio (RAL): this measures the costs arising from not contributing regularly to a single pension fund, in other words the costs incurred by the participant during his working life arising from:
    - transferring the amount accumulated to another account, and the contributions made at a later date to another alternative fund (transferred account).
    - maintaining the accumulated amount in the original fund, but paying new contributions into alternative funds .
    - maintaining the accumulated amount in the original fund, but without making any more contributions.

3. The annualization ratio (RAN): this shows the costs involved when converting the accumulated fund into an annuity, programmed withdrawal, etc..

IV.-ANALYSIS OF THE SENSITIVITY OF THE CHARGE MEASURES AND VARIATION IN SOME ASSUMPTIONS.

The various charge measures can be analyzed by calculating the value of IA_C by way of equation (7) for different charges. The baseline assumptions are that salaries are 1 monetary unit; the real, annual, accumulative growth rate of individual salaries is 3%; the accumulative annual return on the investments is 5%; and the contributions are made over periods of 40, 30 and 20 years.

Graph 2 shows the relationship between the charges on the assets accumulated by the contributor, a_2, and the charges on contributions, a'_1, assuming the rest to be zero. The horizontal axis represents a_2, with assigned values between 0% and 3%, and the vertical axis shows the equivalent charge, a'_1, that would have to be levied on the contributions in order to obtain the same final pension value. Graph 2 shows that some relatively low charges levied on assets can substantially reduce the value of the pension. For example, a value of a_2 equal to 1% is equivalent to a reduction of approximately 20% of the contributions (or 20% of the final pension value) with forty years of contributions. The relation between both measures is almost linear. Also, given a level of profit growth and a real return, whichever way of measuring the charges is chosen, no great differences will show up when comparing individual schemes or systems in different countries with different charge levels. In other words:

- Doubling the value of a_2, for example, increasing it from 0.5% to 1% anually, would bring about an increase in a'_1 of approximately 90%.
- A similar reduction in a'_1 is brought about if the value of a_2 is decreased in the same proportion.

Graph 3 shows the relation between gross return r and a'_1, assuming that a_2 is equal to 1% and considering the other charges to be zero. It can be seen that an increase of one percentage point in the rate of return brings about an increase of approximately one percentage point in the value of a'_1. This is due to the fact that an increase in the amount of the accumulated fund has to be compensated for by a greater increase in the charges that have to be paid on the contributions (see the relation between a_2 and a'_1).

Graph 4 shows the total amount of the accumulated funds with charges IA_C, and without charges IA_{NC}, with a_2 being equal to 1%, T equal to 40 years, and with various values for the return. It can be seen that due to the accumulative effect, after taking the charges into

consideration the funds have a more gradual upward curve. The area between the two curves would be the arithmetical sum of the charges levied.

The effect of changes in the real accumulative growth of salaries on the value of a'_1, with a_2 being equal to 1%, can be seen in Graph 5. For T= 40, a one point increase in the real growth of salaries implies a decrease in the value of a'_1 of almost one percentage point. Hence for example, for g=3%, a'_1 is approximately 20%, whereas if it is assumed that g=6%, a'_1 is 16%. This can be explained by the fact that, although the amount in the accumulated fund has increased, the effect comes about as a result of an increase in the amounts contributed.

Graphs 6 and 7 show that measuring costs by way of a'_2 is very sensitive to changes in assumptions as to what return will be reached and as to the accumulative growth rate of salaries. A higher return, see Graph 5, will reduce the percentage of charges levied on the fund, although the total charge actually paid would remain constant. If Graph 6 is compared with Graph 3 the opposite case is true. Increases in the growth rate of salaries, Graph 7, bring about an increase not only in the charges levied on the fund but also in the charges actually paid.

So far the only case considered has been that of an individual contributing over a 40-year period. What would happen if the contribution period were less than 40 years, assuming that the individual withdraws what he has accumulated when he stops contributing? This is shown in Graph 8, which relates a'_1 to the time period, with a_2 being equal to 1%. It can be seen that a'_1 increases in an almost straight line with respect to the increase in time, approximately 0.5 percentage points for each additional year. This happens because in the short term most of the assets accumulated come from contributions, whereas over the long term the relative weight of interest increases vis-à-vis that of contributions.

In Graph 9 a'_2 (charges on the amount in the fund) relates to the time period, considering a fixed charge, a_1, of 20% of contributions. There is an inverse relation between both amounts. The relation seen is not linear and, in addition, it gives the opposite effect from what was shown in Graph 8 in such a way that it is the longer-term schemes that appear to be cheaper, although the absolute value of the costs increases. The longer the scheme, the more years there are over which to distribute the charges on contributions, which implies that its impact will be less than when the cost is considered as a percentage of assets.

IV.1-Gaps in contribution profiles.

Graph 10 shows how gaps in making contributions have an effect on average charges as a percentage of contributions or, in other words, on the total accumulated in the pension fund. For a_2 being equal to 1%, and with N=40, a'_1 is approximately 20%. In the middle of the curve, where it is assumed that the worker has contributed for 20 years and stops making contributions to the fund for another twenty years, and with the same a_2, the value of a'_1 is now 26%.

Then again, the opposite effect can be seen in Graph 11. For a_1 being equal to 20%, and with N=40, a'_2 is approximately 1%. However, if contributions are only made for 20 years, then a'_2 is approximately 0.7% and appears to be cheaper.

IV.2.-Effect of account transfers on costs.

Changing funds causes a noticeable increase in the administration costs to be borne by the affiliate throughout his working life. Therefore it only makes sense to do so if the expected increased return of the new fund is likely to compensate for the transfer charge paid, or should it be a case of participants close to retirement age who are seeking funds with safer portfolios. The possibility of measuring this effect via the charges defined as a_3 has already been introduced in the model set out above. With this end in view see Graphs 12 and 13, which are copies of Graphs 8 and 9 but with the effect of the transfers added in, albeit in an exaggerated form to make it easier to see.

In Graph 12, assuming that the return is the same in both funds, that there is a change of fund every 8 years, and with the value of a_2 equal to 1%, the line is superimposed over the line from Graph 8 until the seventh year, then in the eighth year there is a sharp increase in the value of a'_1 due to the inclusion of the account transfer charge. From the ninth year until the fifteenth the two curves get closer to each other, then in the sixteenth there is another sharp increase denoting the new change of fund. The same dynamics follow through successive periods. Maintaining the same assumptions, again in Graph 13 a situation comes about that is symmetrical to that shown in Graph 12, but with respect to Graph 9.

As mentioned earlier, transferring to another fund should only be considered if the net return of the new fund is greater than that of the old one. It would appear, therefore, that the best way of calculating the cost is by determining the equivalent value of a_2, as this would enable the net return expected from the new fund to be deduced easily from historic information and from future expectations of gross return.

IV.3-Changes in salary profiles.

Until now it has been assumed that real salaries have grown at annual accumulative rate g. Although this assumption is valid for the nominal growth of salaries, it does not appear to be the most suitable method when real salaries are being calculated. Nevertheless, it is the method most used in the literature because it is simple to operate and calculate. In this subsection we will try to analyse the impact of the use of different salary functions on measuring administration costs. Two functions will be taken as references: the Gaussian[4] and the Carriere-Shand. The salary profiles compared are shown along with the profile of the exponential function of constant growth in Graph 14. It should be pointed out that both these functions respond much more to the average real behaviour of employed and self-employed workers than the exponential function of constant growth does.

Graph 15 shows the relationship between charges levied on the assets accumulated by the contributor, a_2, and charges on contributions, a'_1, assuming all others to be zero. The horizontal axis represents a_2, which has been assigned values of between 0% and 3.6%. The vertical axis shows the equivalent charge, a'_1, that would have to be levied on contributions in order to obtain the same final pension value. As can be seen, the salary function used has an influence on the equivalent charge, a'_1. For the data used (r =5%, T=40 years, equal actual values for salaries in all three profiles, which means that the value reached by the fund without charges is the same in all three cases), the salary function supplying the lowest charge is the exponential function of constant growth, followed by the Gaussian function, and finally the Carriere-Shand. This is logical since, as the reference charge is levied on average assets

[4] See Appendices 3 and 4.

and it is assumed that the percentage of contribution remains constant throughout working life, the equivalent charge, a'_1, will be greater in whichever function provides less average assets invested in the capitalization account. This, given its functional form, is the exponential function.

Graph 16 shows the relation between charges on contributions, a_1, and charges on the assets accumulated by the contributor, a'_2, assuming the rest to be zero. The horizontal axis represents a_1, which has been assigned values between 0% and 36%, while the vertical axis shows the equivalent charge, a'_2, that would have to be levied on the assets accumulated in the fund by the contributor in order to obtain the same final pension value. In this case the exponential function of constant growth is penalized for precisely the same reason as was mentioned in the paragraph above. According to Graph 16, it appears to be the most expensive function since it provides a higher value of equivalent charges on the assets than the other two functions used. It is therefore clear that most papers that obtain this equivalent measure by working with charge structures based on a_1, as is the case for most countries in Latin America, overvalue it by using the constant exponential function.

V.-INTERNATIONAL COMPARISON OF CHARGES ON INDIVIDUAL CAPITALIZATION ACCOUNTS.

Using the theoretical structure set out above, this section contains an international comparison of charges for affiliates, with special attention focused on the countries of Latin America. In principle it is hoped that this will achieve a double objective:

1.-To test the validity of criticisms made by some researchers as to whether the new capitalization systems introduced in Latin America are too expensive to run for the affiliates.

2.-To serve as a reference for the individual pension schemesystem in Spain.

The international comparison concentrates on what are known as individual schemes. Judging by the experience of countries such as the United Kingdom, Spain, Australia and the United States, these are always much more expensive for the affiliate than company or collective schemes. It is logical that these latter schemes be excluded from the comparison because the area of study is the countries of Latin America, and the systems in those countries are built up around individual capitalization accounts, which are the most similar to individual pension schemes.

V.1.-Charges in the countries of Latin America.

As far as the countries of Latin America are concerned there is a wide range of charge structures, as can be seen from Table 1:

Table 1: Charge structure in the countries of Latin America

Country	Variable charge. Percentage of :		Fixed charge	Discounts
	Balance	Flow		
ARGENTINA	NO	YES	NO	YES Loyalty & Fulfilment
BOLIVIA	YES	YES	NO	NO
CHILE	NO	YES	YES	NO
COLOMBIA	NO	YES	NO	NO
SPAIN	YES	NO	NO	NO
EL SALVADOR	NO	YES	NO	NO
MEXICO	YES	YES	YES	YES Loyalty
PERU	NO	YES	NO	NO
URUGUAY	NO	YES	NO	NO

Source: Devesa and Vidal (2001).

- Charges are levied on contributions in all the countries analyzed.
- Charges are also levied on the balance in the fund in Bolivia and Mexico.
- Fixed charges are levied only in Chile and Mexico.
- In Mexico the possibility of giving discounts on charges when the contributor keeps his account with the same administrator for a certain agreed length of time is being considered. In Argentina, as well as the loyalty discount, there are also fulfilment discounts when certain other requirements are met.

- The authorized charge structure in Spain is quite different from that in Latin American countries. Charges can only be levied on the amount managed/deposited.

Care should be taken when comparing charges in these countries because they are not applied in the same way:

- In some countries charges are levied on the maximum compulsory contribution, while in others they are levied on the gross contribution. For this reason Table 2 includes a column to show the capitalizable contribution, in other words the amount that actually goes into the fund once all the relevant charges have been paid, since this is considered to be more suitable for carrying out the comparisons.

- In some countries the premium earmarked for invalidity and death benefits is collected at the same time as the charge. However, in other countries the contributor has to pay it separately. In addition to this, the contingencies covered vary from country to country. Table 2 includes a column showing the amount earmarked for this insurance, as well as the net charge once the insurance premium has been deducted from the amount.

Table 2 shows the net charge as a percentage of the contribution, both gross (charge ratio, a'_1) and capitalizable.

In Bolivia a charge is also made for administering the portfolio, the ceiling for which is 0.2285%. The joint effect of this charge along with the original charge on contributions is shown in the table in brackets. In those countries marked with an asterisk the charges include an amount for financing watchdog committees. According to Demarco and Rofman (1999), if this type of charge were deducted, the differences between countries would be narrowed, especially in Argentina and Peru. Then again, the collection of contributions in Argentina is carried out by the same public service that collects taxes and which does not charge the administrators for their services.

As far as insurance is concerned, additional information and recent data on the cost of these contingencies can be found in AFP-ag (2000b) and FIAP (2000). It should be pointed out that Mexico has the highest figure for this contingency, which is managed by a public organisation (the IMSS).

A case which stands out from the rest is Bolivia, which has the lowest administration costs of all the reformed systems in Latin America. According to Von Gersdoff (1997), due to the small size of the country and in order to avoid the high commercial expenses borne by other countries in the area, the authorities decided to authorize only two administrators chosen by international public tender (Demsetz type competition) to provide a service with lower operating costs. Transfers between administrators were prohibited until 1 January 2000, and the system is expected to be opened up to new administrators in May 2002. Transfers are currently suspended until the merger between the two administrators already in the market has been completed and another one enters.

Table 2: Charges (31/12/2000)						
Country	**Total fees as % of wages**	**Cost of insurance as % of wages**	**Net fees as % of wages**	**Net contribution as % of wages**	**Net charge on contribution (charge ratio) gross (a'_1)**	**net**
	a	b	c=a-b	d	c/(d+c)	c/d
ARGENTINA*	3.28	1.19	2.09	7.72	21.30	27.07
BOLIVIA*	2.50	2.00	0.5	10.00	4.76 (9.5)	5.0 (9.59)
CHILE	2.31	0.70	1.61	10.00	13.87	16.10
COLOMBIA*	3.49	1.86	1.63	10.00	14.02	16.30
EL SALVADOR	3.15	1.32	1.83	8.53	17.66	21.45
MEXICO 1[5]	4.48	2.50	1.98	7.07	21.88	28.01
MEXICO 2[6]	4.48	2.50	1.98	12.07	14.09	16.40
PERU*	3.73	1.34	2.39	8.00	23.00	29.88
URUGUAY	2.68	0.64	2.04	12.32	14.21	16.56
Source: AIOS (2001), FIAP (2000) and authors.						

In addition to the income they receive from individual capitalization accounts, administrators are also paid for managing the collective capitalization fund associated with the Bolivida[7] subprogram, see De la Serna (2001). This implies that there is a cross-subsidy from the collective program towards the individual one, which would explain the low level of charges for contributors in comparison with other countries.

Although the Bolivian model appears to be a valid one, Valdés-Prieto (1999) points out that the bidding system can also create great weaknesses due to the fact that, after their tender has been accepted, the winning administrators take on the role of private monopolies bound by tariff regulations where the tariff is the tender originally offered. The main problems that could arise are:

a) The administrator has an incentive to reduce the quality of the administration service in order to reduce costs and increase profits. The regulating authority can only avoid this happening by applying a full watchdog system to check the quality of the service and by being able impose sanctions without appeal to a legal process that would be slow and technically unable to assess the weaknesses.

b) The administrator has the incentive to renegotiate tariffs with the authorities. It is not difficult to see that one of the most tempting offers the administrator could make to the authority in order for it to accept an increase in tariffs would be to reallocate pension fund investments to those areas favoured by the authority, financed by low yield government bonds. The affiliate cannot escape by changing administrator because he has been deprived of the right to choose. The basic problem is that a concessionary administrator has the State as its only client and is, in practice, an extension of the state machinery.

[5] In Mexico, the State contributes 5.5% of the minimum wage, which is estimated as 2.2% of the average wage.

[6] This also includes the 5% contribution to the housing account included in the individual account, which is managed by the Instituto del Fondo Nacional para la Vivienda de los Trabajadores (**INFONAVIT**).

[7] In Bolivia there is a social welfare program coordinated through two accounts: "Bolivida" and the "Cuenta de Acciones Populares" (CAP), managed through a collective capitalization fund and the resources for which come from the privatization of public companies.

Administration costs are much higher for the affiliate in Argentina and Peru, since in these countries the costs of setting up the system and the lower capitalizable contribution are still reflected. These costs should decrease substantially over the medium and long term because, as the companies attract more affiliates and funds, they can benefit from economies of scale. It is to be expected that these benefits would be passed on to the contributors.

Until very recently Argentina had the highest administration costs for the affiliate of all the new pensions systems studied, when measured as the net charge on contributions (charge ratio). This in spite of the fact that they have fallen sharply since the system was set up. The biggest savings have been made in the sales force and advertising costs. However, the high level of profits made by the administrators would appear to indicate that the market is not very competitive. Another fact that seems to support this hypothesis, see Rofman (2000), is that if every affiliate were to choose the cheapest administrator according to his level of income, the average charge for the system, including insurance costs, would fall from 3.41% (at the start of 2000) to 2.95%. According to Rofman (2001), various legislative measures have recently been approved aimed at reducing these high costs, namely: putting a ceiling on charges, reducing the number of transfers per year, and assigning undecided new affiliates to the cheapest administrators according to their expected level of income. These measures would probably bring about an increase in price competition.

Administration costs, including insurance cover, have clearly been falling. According to data supplied by AFP-ag (2000a), costs in Chile fell from 4.87% of salary in 1983 to 2.31% at the end of 2000. This decrease may be due to a combination of factors including experience, greater efficiency, the incorporation of new technology and rationalization of the transfer process. Other measures could bring about an even greater decrease: allowing administrators to give discounts on charges to those affiliates who do not transfer their individual accounts for a certain period, as has recently been allowed in Argentina, or letting them share infrastructures and subcontract sales staff.

Table 3 shows the calculation of charges equivalent to an annual percentage of assets (a'_2) for different salary functions.

Table 3: Charges as annual % of the accumulated Fund (a'_2)

Country	EXPONENTIAL		GAUSSIAN		CARRIERE	
YEARS	T=30	T=40	T=30	T=40	T=30	T=40
ARGENTINA	1.52	1.11	1.35	1.02	1.33	0.98
BOLIVIA	0.53	0.45	0.50	0.43	0.49	0.42
CHILE	0.94	0.68	0.84	0.63	0.82	0.60
COLOMBIA	0.94	0.69	0.85	0.64	0.83	0.61
EL SALVADOR	1.22	0.89	1.09	0.82	1.07	0.79
MEXICO 1	1.57	1.14	1.40	1.05	1.37	1.01
MEXICO 2	0.95	0.69	0.85	0.64	0.83	0.62
PERU	1.66	1.21	1.48	1.11	1.45	1.07
URUGUAY	0.96	0.70	0.86	0.65	0.84	0.62
SPAIN	2.35					
USA	0.30-0.65					

Source: authors' calculation.

The assumptions are: charges levied in each country as per Table 2; contributions for 30 and 40 years; real return of 5%; and real growth in salaries of 3.13% in the case of the exponential function. The data for the United States is obtained from the hypothesis put

forward by Genetski (1999), assuming that the American pensions system were to be transformed into a system of individual accounts as in Chile, for the fifth year of functioning. The data for Spain is the simple average of the data obtained by the authors themselves from a sample made up of 23 individual pension schemes from nine different entities. Six of the 23 schemes charge the participant the maximum authorised by law (Ley de Planes y Fondos de Pensiones en España), which is 2.6% on the assets managed/deposited. The equivalent charges on contributions (a'_1), assuming contributions between 30 and 40 years, real return of 5%, and real growth in salaries of 3.13% for the exponential function, will fluctuate between 29% and 37.5%.

It is immediately noticeable that the salary function of constant exponential growth overvalues the equivalent charges projected as an annual percentage of assets when compared to the other two salary functions. This effect has not been detected before in the specialized literature and, as can be seen from the data in Table 3, it overestimates the projected charges by up to 15%.

Table 4: Charges as an annual % of the accumulated fund (a'_2) with projection of the historic real annual gross return of the funds (31/12/00)

COUNTRY	% historic real annual gross return of the funds [8]	Exponential T=30	Exponential T=40
ARGENTINA	11.11	1.19	0.83
BOLIVIA	11.10	0.47	0.40
CHILE	10.90	0.74	0.52
COLOMBIA	7.84	0.83	0.59
EL SALVADOR	12.88	0.92	0.64
MEXICO1	9.47	1.30	0.91
MEXICO2	9.47	0.79	0.55
PERU	5.30	1.63	1.18
URUGUAY	9.13	0.81	0.57
Source: AIOS (2001), ASOFONDOS (2000), FIAP (2000) and authors			

Table 4 shows the charges recalculated as an annual percentage of the accumulated fund for each of the countries of Latin America, assuming the historic gross annual real return of the funds achieved from the time the system was set up is maintained. With this assumption, and given that the real return reached in all the countries was higher than that used in Table 4 (and in most countries almost double), the estimated charges decrease. It can also be seen that the *most expensive* and the *cheapest* countries are still the same ones, with the reduction in estimated charges for El Salvador standing out due to the extraordinary real return reached up to now. Therefore it can be appreciated that measure (a'_2) is quite sensitive to the passing of time and to the projected real return. But which are the true estimated charges, those in Table 3 or those in Table 4? It would be equally arbitrary to assume that the same real return would be obtained in every country, or that the return obtained up to now will be maintained in the future, although the latter assumption would perhaps be the most appropriate. It is symptomatic that when most authors make international comparisons they

[8] The historic real annual gross return of the funds is since the inception of the various capitalization systems, except for Peru and Colombia, which is for the last 84 and 36 months respectively.

project the data on charges that are applicable now in each country, yet they adopt assumptions of real return that make them all equal.

V.2.-Charges in other countries with individual capitalization accounts.

Table 5 shows the estimated charges in a number of countries outside Latin America. The current pensions system in Australia was set up in 1992. Although there are no regulations governing the structure or level of charges, according to Whitehouse (2000) there is usually a combination of a fee as a percentage of assets, plus a fixed charge per account and/or a charge as a percentage of contributions. There are two types of pension schemes: industry funds (collective pension schemes) and the master trust (individual pension schemes). As mentioned earlier, the references used are those for individual pension schemes, which are the most expensive in all countries, and this is the data used in Table 5.

Table 5: Estimated charges in other countries with individual accounts.

EXPONENTIAL, T=40		
COUNTRIES	**Without account transfer**	
	a'_1	a'_2
AUSTRALIA	35.5	2.09
UNITED KINGDOM	25	1.33
POLAND	20.5	1.06
KAZAKHSTAN	11.45	0.55

Source: authors' calculation based on Whitehouse (2000), Murthi, Orszag and Orszag (2000) , Chlon, Gora and Rutkowski (1999) and Andrews (2001)

The way the current pensions system is organized in the United Kingdom can be considered very complex for the affiliate, according to Murthi, Orszag and Orszag (1999), due to the great number of options available. This means that some affiliates, see Murthi, Orszag and Orszag (2000), may make very inappropriate decisions. The system is decentralized, privately run and voluntary. There are very strict regulations governing the sales process, although these do not apply to administration costs. This lack of regulation means there is a great variety of charges, many of which the affiliates may not understand or even perceive.

As can be seen, administration costs in the United Kingdom are very much higher than in the countries of Latin America, higher even than those in Peru and Argentina. This high level of costs does not appear to be due to a lack of competition nor to excessive profits for the administrators, but rather because of the complexity of the system and the special regulations governing the sales process. Looking at the data it is not surprising that the authorities are very worried and are setting up a new type of scheme called stakeholder pensions, which aims to be a cheaper option for workers on low incomes than the individual accounts. This new option will come into effect in October 2001 and the given aim is that annual administration costs will not exceed 1% of the total assets administered.

In Poland, according to Chlon, Gora and Rutkowski (1999), which has also recently set up a new pensions system, charges are levied on contributions and assets, but there is no

fixed charge. The result of this is that charges are lower than in the United Kingdom and Australia, and also lower than those of the most expensive countries in Latin America. In Kazakhstan, according to Andrews (2001), where the pensions system has recently been reformed and a system of individual accounts introduced, regulations limit administration charges to a maximum of 1% on contributions, plus 10% on the return. This means that charges are relatively low for the affiliates. However, the data for both Poland and Kazakhstan should be considered with care since the systems are still in their infancy.

V.3.-Charges for transferring accounts and converting accumulated savings into a pension.

Table 6 shows how some factors which are not usually considered very important in the literature undoubtedly have a great effect on estimating the costs borne by the affiliates during their whole period as contributors/pensioners: transferring accounts and converting the savings accumulated into a pension. Due to the lack of data on charges for transfers and conversion for each of the countries, the same charges have been applied to all of them. Neither of these two aspects is less worthy of consideration for that.

Table 7 includes the percentage of accounts transferred in 2000 in each of the countries with individual capitalization accounts. In Bolivia, according to AIOS (2001), transfers have been suspended until the merger between the two administrators existing in the market is completed and another one enters. There is also a high level of transferred accounts in the United Kingdom according to Murthi, Orszag and Orszag (2000).

Table 6: Estimated charges for transferring accounts and converting savings into a pension.

Country	EXPONENTIAL, T=40					
	Without change of fund		With a change of fund every 8 years (4 changes) a_3=0,02		With a change of fund every 8 years (4 changes) and conversion charge a_3=0,02 a_4=0,05	
	a'_1	a'_2	a'_1	a'_2	a'_1	a'_2
ARGENTINA	21.30	1.11	24.82	1.33	28.58	1.58
BOLIVIA	9.50	0.45	13.50	0.66	17.82	0.90
CHILE	13.87	0.68	17.27	0.89	21.84	1.14
COLOMBIA	14.02	0.69	17.87	0.90	21.97	1.15
EL SALVADOR	17.66	0.89	21.35	1.11	25.28	1.36
MEXICO 1	21.88	1.14	25.37	1.36	29.11	1.61
MEXICO 2	14.09	0.69	17.94	0.91	22.04	1.15
PERU	23.00	1.21	26.45	1.43	30.13	1.68
URUGUAY	14.21	0.70	18.05	0.91	22.15	1.16

Source: authors' calculations.

The very high number of transfers between administrators in Chile, see Devesa and Vidal (2001), was historically one of the biggest problems as it meant an unnecessary increase in administration costs. In 2000, according to AIOS (2001), advertising accounted for almost 28% of administration costs in Chile and over 50% in Argentina, Peru and Uruguay. In order to solve the problem to a certain extent, from October 1997 the Chilean authorities demanded additional procedures to effect transfers between administrators and limited the taking on of

new sales staff. These measures brought about a sharp drop in the number of transfers. The percentage of transferred accounts reached a maximum of 76% in 1983, but was still an extremely high 57% in 1997. In 2000 it was 11%. Other countries which reformed their pensions systems at a later date learned from the experience of Chile and introduced measures to limit the number of transfers and the number of sales staff. Outside Latin America, few authors have tried to quantify the effect of changing funds. Murthi, Orszag and Orszag (2000) estimate that charges in the United Kingdom are 52% higher for those who change accounts every seven years than they are for those who remain in the same fund the whole time, since the charge ratio can rise from 21.7% to 33%.

Table 7: Account transfers (30/06/2000)

Country	Transfers (2000) %
ARGENTINA	9.86
BOLIVIA	0.0
CHILE	10.97
COLOMBIA	8.43
EL SALVADOR	21.51
MEXICO	0.50
PERU	0.89
URUGUAY	12.05
Source: AIOS (2000)	

Table 8: % Increase in estimated charges for transferring accounts and converting savings into a pension

COUNTRIES	With a change of fund		With a change of fund and conversion charge	
	$\%-a'_1$	$\%-a'_2$	$\%-a'_1$	$\%-a'_2$
ARGENTINA	16.53	19.82	34.18	42.34
BOLIVIA	42.10	46.66	87.57	100
CHILE	24.51	30.88	57.46	67.65
COLOMBIA	27.46	30.43	56.70	66.67
EL SALVADOR	20.89	24.72	43.15	52.81
MEXICO 1	15.95	19.30	33.04	41.23
MEXICO 2	27.32	31.88	56.42	66.67
PERU	15.00	18.18	31.00	38.84
URUGUAY	27.02	30.00	55.88	65.71
Source: authors' calculations.				

Annuities suffer from marketing problems in various countries of Latin America. In Chile, see AFP-ag (2000c), it is well known that annuity brokers charge a high rate of commission, around 7% of the savings accumulated by the affiliate[9]. Current legislation gives some affiliates the incentive to opt directly for an annuity in order to access free disposal funds, which are obtained through a high rate of commission for the insurance company salesperson which is generally shared with the affiliate. In Argentina, according to Palacios

[9] In the case of Poland, Chlon, Gora and Rutkowski (1999) estimate the conversion cost at 5%.

and Rofman (2001), over 80% of retirees who acquire an annuity contract it from an insurance company with links to the administrator which previously managed the funds. In Colombia and Peru, markets for this type of income are still in their infancy and are less competitive with much more concentration.

The results shown in Table 6 are most revealing. Each effect has a great impact, and this is proportionally even more noticeable the lower the original level of commission charged to the contributors. To make this clearer, Table 8 shows the data from Table 6 transformed into increases in charges over the original figure for each country.

V.4.-Charges in Spain.

Spain itself has been left till the end deliberately, since the level of charges the participants of individual schemes pay appears to be extraordinarily high when compared to the data used for the other countries under analysis. The main characteristics of the "industry" in Spain are as follows. There were 4,412,312 participants in individual schemes at December 2000. There are currently 558 schemes of this type. The average number of participants per scheme is just 7,907. Other than individual schemes, there are 536,120 participants in other types of scheme. There are 78 administrators which manage individual schemes. The largest administrator has 78 individual schemes. The assets managed is much more concentrated, with the ten largest administrators absorbing 75%. The Herfindahl[10] index is 0.101, whereby it can be considered a moderately concentrated market as far as managed assets are concerned.

It is not easy to carry out an analysis of the administrators in Spain because insurance companies and mutual funds can also be pension fund managers. Excluding these, we have obtained data on 47 administrators for 1999 from the statistical and accounting documents the Dirección General de Seguros publishes on its website (http//:www.dgseguros.mineco.es). The average annual return on equity (ROE) for 1999 was 36.47%. Seven of the administrators registered losses. The ten most profitable administrators - mainly linked to groups which manage greater assets - accounted for 94% of the profits of all of those analyzed, their average ROE being 49.50%. It may be assumed that the profits from pension fund management for those insurers which play a dual role could be at least as large as they are for pure administrators since in many cases they share infrastructures with other branches of insurance. The above data appear to indicate an alarming lack of competition in the market. The tariffs charged to participants are in no way justified and are way above the international average according to Tables 3, 4 and 6. It seems that the generous tax treatment of pension schemes is “shared” by the administrators in the form of higher charges.

The current level of a similar type "product", such as investment accounts, shows an average charge of 1.38% as a percentage of assets. The upper limit is occupied by common stock funds in Euros at 1.85% and the lower limit by what are known as FIAMM (money market investments) at 0.95%. In the year 2000, the economic authorities in Spain decided to

[10] The Herfindahl-Hirschman Index (HHI) is used to measure the degree of concentration of an industry. It tends to grow if the number of companies decreases or if just a few capture a large share of the market. It is used as an indicator by the authorities in North America. The formula for calculating it is:

$$\text{HHI} = \sum_{i=1}^{n} \left(\frac{Pi}{TP}\right)^2$$

where P_i: assets of administrator i, and TP: total assets managed by the industry as a whole.

lower the maximum charges levied by the administrators. Even then voices were raised in the specialized Spanish press in favour of reducing them further, since according to the study carried out by Lipper Analítical[11], the administration charges levied by Spanish administrators are the highest of all the countries compared, whether for fixed or variable income funds. However, underwriting charges and back-end fees are generally lower in Spain.

V.5.-Some problems with the international comparison of charges.

Making an international comparison of the administration costs of the different pension systems and/or the level of commission they charge their affiliates is complex and the results questionable, at least in the following aspects:

a) Differences in design and regulations, see Mastrángelo (1999), with regard to minimum return, collection functions and administration of additional resources.

b) For Valdés-Prieto (1999) the amounts of the start-up costs of the different systems vary a great deal and are to a large extent defined by what regulations govern the system from the start. Hence in Peru and Uruguay, for example, new workers who start working for an employer are free to choose between remaining in the old system or entering the new one. In Colombia, those workers who decide to enter the new system are free to return to the old one once they have been there the minimum three years. In Argentina and Chile, however, new workers who start work for an employer have to enter the new system. This ensures that in these countries the costs of attracting clients from the old system are not recurring but part of the set-up costs. Finally, in Bolivia, Mexico and El Salvador the authority almost eliminated this source of set-up costs by making it compulsory to transfer to the new system.

c) In principle it would appear that pension fund administrators could take advantage of economies of scale, and so the size of the market in each country should establish a limit to the number of administrators, to the average level of costs they can reach, and consequently to the level of charges[12].

In Uruguay for example, as can be seen in Table 9, the number of affiliates for the whole of the welfare system is around 552,000 people, which is less than the number for a single administrator in many other countries. In fact the administration industry registered losses until 1999 in the smaller countries: Bolivia, El Salvador and Uruguay. In 2000 they had started making a proft in all three countries.

There can also be large differences between countries that have adopted a mandatory system of individual accounts and those where it is voluntary.

[11] Published in the business supplement of El PAÍS, pages 14 and 15, 13 May 2001.

[12] On this aspect there are many authors who are not in complete agreement. Whitehouse (2000) finds no significant relationship between fees and the size of the administrator either in Latin America or the United Kingdom, although this does not mean that there may not be one between costs and the size of the funds administered, which other researchers have in fact found.

Table 9: Affiliates, number of administrators and assets managed. (31/12/2000)							
COUNTRY	YEAR STARTED	%Funds / GNP	AFP	Affiliates			
				TOTAL	AVERAGE	BIGGEST	SMALLEST
ARGENTINA	1994	7.1	13	8,103,974	623,384	1,539,581	109,978
BOLIVIA	1997	10.8	2	569,790	284,896	293,173	276,617
COLOMBIA	1994	4.0	7	3,713,349	530,479	959,543	35,097
CHILE	1981	59.8	9	6,154,023	683,781	2,461,695	27,194
EL SALVADOR	1998	3.6	5	784,276	156,856	292,647	28,731
MEXICO	1997	3.0	13	16,574,262	1,274,944	2,671,234	206,903
PERU	1993	5.4	4	2,355,144	588,787	614,114	563,520
URUGUAY	1996	3.9	6	551,983	91,998	208,972	31,924
Source: AIOS (2001), FIAP (2000) and authors.							

d) There can be noticeable differences in the quality and quantity of the service provided by companies in different countries.

e) The charge structure does not have the same effect on all workers, the salary level being very important.

f) In the case of measuring costs as a percentage of the annual assets of the funds, it has been assumed that no changes are made to the charge structure during the whole time horizon under consideration. Going by the experience of Latin America, this is a very restrictive assumption.

All this makes it difficult to carry out an international comparison of administration costs due to the different baseline assumptions.

VI.-SUMMARY AND CONCLUSIONS.

This paper has set out a model based on the ideas of Whitehouse (2000) and Diamond (1999) to enable the explicit costs borne by the affiliate during both his working life and his retirement period to be assessed. It also shows the relationship between the different ways of measuring the costs that make up the total price that contributors eventually have to pay. In addition to this, the different charge structures usually applied to the participants of the pension funds can be compared. This is made possible by transforming the original charge structure into an equivalent single measure:
a'_1 : combines the effect of all the explicit fees as a whole into a single constant percentage which decreases the total amount of all the contributions made.
a'_2 : combines the effect of all the explicit fees as a whole into a constant decrease in gross return.

Assuming that it was necessary to show a single measure for assessing the management costs for the affiliate, the most appropriate would be determined according to the charge structure applied and by its relative importance. Hence for example in the case of most Latin American countries - those where charges payable by affiliates are levied mainly on contributions - the use of a'_1 would be the most appropriate. In the case of Spain, however, the use of a'_2 would be preferable due to the charge structure used.

The basic contribution of the paper vis-à-vis the reference models used has been the analytical introduction of some aspects which are not usually given much importance in the literature but which, as has been shown, have a great effect on projecting the costs borne by the affiliates during the whole of their period as contributors/pensioners: the effect of account transfers and the changes in assumptions regarding salary profiles. Changing assumptions regarding salary profiles from the standard usually used (exponential function of constant growth) shows that converting charges levied on contributions into other equivalent charges levied on the fund or the accumulated assets brings about an important change in the projected costs for the affiliate, which can reach 15%.

The specific application of the model to the new capitalization systems of Latin America leads us to reject the criticisms made by some researchers in the sense that they are too expensive to administer for the affiliate. These criticisms appear to be clearly disproportionate when a comparison is made with other countries considered more developed, such as Australia, the United Kingdom or Spain.

Nevertheless, it must be acknowledged that there is still room for improvement - many countries have costs well above 1% of annual assets - and that these systems will always have higher costs for the affiliate than other types of capitalization account. A reduction to half the current level of charges levied during the accumulation phase could increase the accumulated value individual capitalization accounts in Latin America by between approximately 6% and 11%.

Another important aspect is that the projected costs for the affiliate grow substantially if the effects of charges for transferring accounts and converting savings into pensions are taken into account. The lower the level of the original costs, the higher the proportional growth of costs when these are included. The effect of these charges can increase projected costs by up to 100%, in the extreme case of Bolivia.

As far as the second aim in carrying out the international comparison is concerned, it must be said that the level of charges paid by the participants of individual pension schemes in Spain appears to be extraordinarily high. Also the great concentration of managed assets in the hands of very few financial groups and the high concentration of profits within the administration industry seem to point to a clear size effect and indicate symptoms of a lack of real competition in the market. All this leads one to think that the ceiling on charges authorized for the Spanish market is very high. It may possibly have been valid when the legislation governing pension schemes and funds was introduced in 1987, but today, with greater experience, greater efficiency and, more than anything, the incorporation of new technology designed for the management and administration of funds, these ceilings should be modified downwards and the possibility considered of encouraging loyalty or fulfilment discounts along the lines of those being introduced in some countries of Latin America and, in a very limited way, in Spain.

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APPENDIX 1. DETERMINING THE VALUE OF THE ACCUMULATED FUND.

Starting from:

$$\int_{t=0}^{T} c(1-a_1)w_0 e^{gt} e^{(r-a_2)(T-t)} dt \qquad \textbf{[18.]}$$

which can be expressed as:

$$\int_{t=0}^{T} c(1-a_1)w_0 e^{t(g-r+a_2)+T(r-a_2)} dt = c(1-a_1)w_0 e^{T(r-a_2)} \int_{t=0}^{T} e^{t(g-r+a_2)} dt \qquad \textbf{[19.]}$$

from which is immediately obtained:

$$\int_{t=0}^{T} e^{t(g-r+a_2)} dt = \frac{1}{g+a_2-r}\left[e^{t(g-r+a_2)}\right]_0^T = \frac{1}{g+a_2-r}\left[e^{T(g-r+a_2)}-1\right] = \frac{1-e^{-T(r-g-a_2)}}{r-g-a_2} \qquad \textbf{[20.]}$$

whereby the total amount accumulated is:

$$c(1-a_1)w_0\left(\frac{e^{(r-a_2)T}-e^{gT}}{r-g-a_2}\right) \qquad \textbf{[21.]}$$

APPENDIX 2. DETERMINING THE VALUE OF THE ACCUMULATED FUND WITH ACCOUNT TRANSFERS.

Assuming that a percentage, a_3^i, is charged on the accumulated fund as a fee each time the account is transferred, and that in addition "s" changes of fund are made every "T/(s+1)" years, the total net amount accumulated (apart from the charge for converting it into income) will need to be deduced section by section. The contributions made up to the first change, therefore, will generate an accumulated value at time "T" equal to:

$$e^{(r-a_2)(T-\frac{T}{s+1})}\left[c(1-a_1)w_0\left(\frac{e^{(r-a_2)\frac{T}{s+1}}-e^{g\frac{T}{s+1}}}{r-g-a_2}\right)-a_0e^{(r-a_2)\frac{T}{s+1}}\right]\prod_{i=1}^{s}(1-a_3^i) \qquad \textbf{[22.]}$$

In the same way, for the contributions made between the first and second change we get:

$$e^{(r-a_2)(T-\frac{2T}{s+1})}\left[c(1-a_1)w_0e^{g\frac{T}{s+1}}\left(\frac{e^{(r-a_2)\frac{T}{s+1}}-e^{g\frac{T}{s+1}}}{r-g-a_2}\right)\right]\prod_{i=2}^{s}(1-a_3^i) \qquad \textbf{[23.]}$$

The expressions are deduced until the contributions made between the penultimate and final changes are reached:

$$e^{(r-a_2)(T-\frac{sT}{s+1})}\left[c(1-a_1)w_0e^{g\frac{(s-1)T}{s+1}}\left(\frac{e^{(r-a_2)\frac{T}{s+1}}-e^{g\frac{T}{s+1}}}{r-g-a_2}\right)\right]\prod_{i=s}^{s}(1-a_3^i) \qquad \textbf{[24.]}$$

Finally, for the contributions made after the final change up to the end:

$$\left[c(1-a_1)w_0e^{g\frac{sT}{s+1}}\left(\frac{e^{(r-a_2)\frac{T}{s+1}}-e^{g\frac{T}{s+1}}}{r-g-a_2}\right)\right] \qquad \textbf{[25.]}$$

with which the sum of all the final values obtained earlier, along with the incorporation of the charge for conversion into a pension, can be expressed as:

$$\begin{aligned}\mathrm{IA_C} = &\left\{-a_0e^{(r-a_2)T}\prod_{i=1}^{s}(1-a_3^i)+\right.\\ &\left.+c(1-a_1)w_0\left(\frac{e^{(r-a_2)\frac{T}{s+1}}-e^{g\frac{T}{s+1}}}{r-g-a_2}\right)\left(\sum_{i=1}^{s}e^{(r-a_2)\left(T-\frac{iT}{s+1}\right)}e^{g\frac{(i-1)T}{s+1}}\prod_{j=i}^{s}(1-a_3^j)+e^{g\frac{sT}{s+1}}\right)\right\}(1-a_4)\end{aligned} \qquad \textbf{[26.]}$$

APPENDIX 3. THE GAUSSIAN AND CARRIERE-SHAND FUNCTIONS.

A) The Gaussian Function.

In this case, following Devesa and Vidal (1997), it is assumed that real salaries will increase with age until reaching a maximum, W_M, which will coincide with the high point of the career, age M. From this point onwards real salaries will gradually decrease. Therefore the income level at age x will depend not on the initial level, but on the maximum level reached at age M. That is:

$$W_x^{CP} = W_M e^{-\frac{(x-M)^2}{d}} \quad \textbf{[27.]}$$

where:

W_x^{CP}**:** real salary at age *x* for self-employed workers.
W_M**:** maximum level of real salary.
M**:** age at which maximum salary level is reached.
d**:** constant parameter, which will always take on a positive value.

Assuming that t ="j-x" and that, in addition, "M-x"= h, with j: retirement age, then:

$$W_0 = W_M \; e^{-\frac{(-h)^2}{d}} \quad \textbf{[28.]}$$

whereby

$$W_t = W_M \; e^{-\frac{(-h+t)^2}{d}} \quad \textbf{[29.]}$$

If the corresponding value of W_t is substituted in equation (4)

$$c(1-a_1)w_M \, e^{-\frac{(-h+t)^2}{d}} e^{(r-a_2)(T-t)} \quad \textbf{[30.]}$$

and then integrating between 0 and *T*:

$$IA_c = c(1-a_1)w_M \, e^{T(r-a_2)} \int_{t=0}^{T} e^{-\frac{1}{d}\left[(t-h)^2 + td(r-a_2)\right]} dt \quad \textbf{[31.]}$$

An expression is arrived at where there is an integral which cannot be solved exactly, and that makes it impossible to determine the amount of administration costs analytically. One way of

getting round this difficulty would be to use approximate methods for solving integrals, although the use of them here has been considered inappropriate because of their complexity.

B) The Carriere-Shand function.

The second alternative chosen is the income function put forward by Carriere and Shand (1998). In this it is considered that salaries increase with age, but at ever decreasing rates due to the merit factor being less as time goes by:

$$W_x = e^{\left[x\,d+\frac{b}{l}(1-e^{-l\,x})\right]} \qquad \textbf{[32.]}$$

where:
δ: rate of real salary growth for each age.
β y λ: constant parameters to be estimated, which will determine that part of salary growth linked to merit and which is assumed to decrease with age.

The disadvantage of this function is that two parameters have to be estimated. Carriere and Shand (1998) estimate them using data for the United States.

Assume, as in the case above, that t ="j-x", the salary function over time is:

$$W_t = W_0\, e^{\left[t\,d+\frac{b}{l}(1-e^{-l\,t})\right]} \qquad \textbf{[33.]}$$

If it is substituted in equation (4), we get:

$$c\ (1-a_1)\,W_O e^{\left[t\,d+\frac{b}{l}(1-e^{-l\,t})\right]} e^{(r-a_2)(T-t)} \qquad \textbf{[34.]}$$

and integrating between 0 and T:

$$IA_c = c(1-a_1)W_O\ e^{T(r-a_2)} \int_{t=0}^{T} e^{\left[t(d-r+a_2)+\frac{b}{l}(1-e^{-l\,t})\right]} dt \qquad \textbf{[35.]}$$

the same problem arises as in the Gaussian function: the integral cannot be solved analytically.

C) Approximation by means of a polynomial function.

To solve the problem arising in the two cases above, it was decided to use a salary function that could be integrated. The polynomial function was chosen for this due to its versatility in adjusting to the different types of functions and the ease with which it can be integrated. The procedure to be followed is to first choose the type of function (Gaussian, Carriere, etc.) that best reflects the real or theoretical baseline data so that, later on, the coefficients of the polynomial that works best with the function chosen can be determined.

$$W_x = b_n\, x^n + b_{n-1}\, x^{n-1} + ... + b_1\, x + b_0 \qquad \textbf{[36.]}$$

where:
b_s: are the coefficients to be estimated.

n: degree of the polynomial chosen.

Assuming, as in the case above, that t = "j-x", the salary function over time is:

$$W_t = W_0 (b_n t^n + b_{n-1} t^{n-1} + ... + b_1 t + b_0) = W_0 \phi_n \quad \textbf{[37.]}$$

where ϕ_n is the polynomial of degree "n".

If it is substituted in equation (4), we get:

$$c (1-a_1) W_0 \phi_n e^{(r-a_2)(T-t)} \quad \textbf{[38.]}$$

and if we integrate between 0 and *T*:

$$IA_c = c(1-a_1)W_0 e^{T(r-a_2)} \int_{t=0}^{T} f_n \; e^{-t(r-a_2)} \, dt \quad \textbf{[39.]}$$

This integral is solved (see Appendix 4) by applying the section by section integration method "n-1" times; producing the following expression:

$$IA_c = c(1-a_1)W_0 e^{T(r-a_2)} \left| e^{-t(r-a_2)} \left(\frac{f_n}{(a_2-r)} - \frac{d f_n}{(a_2-r)^2} + \frac{d^2 f_n}{(a_2-r)^3} - ... + (-1)^n \frac{d^n f_n}{(a_2-r)^{n+1}} \right) \right|_{t=0}^{T} =$$

$$= c(1-a_1)W_0 \left[\sum_{s=0}^{n} (-1)^s \frac{d^s f_n(t=T)}{(a_2-r)^{s+1}} - e^{T(r-a_2)} \left(\sum_{s=0}^{n} (-1)^s \frac{s! \, b_s}{(a_2-r)^{s+1}} \right) \right]$$

[40.]

where:

$d^s \phi_n$: derived from order "s" of the polynomial of degree "n".
$d^s \phi_n(t=T)$: the value for t =T of the derived order "s" of the polynomial of degree "n".

To analyze the actual effect that the change in salary profile has on the variation in charges, the functions used were:

a) Carriere-Shand. Based on a salary equal to 1 and the values of the parameters supplied by the authors for data on the United States, the calculation was made with the following fourth degree polynomial function:

$$W_t^{CS} = -1.98722E-06\, t^4 + 0.000406231\, t^3 - 0.03094774\, t^2 + 1.04283129\, t - 11.2898888$$

[41.]

with an R^2 coefficient of 0.9992.

b) Gaussian. First of all a value for coefficient "d" was found in order to enable the same actual value for salaries, based on an initial unitary salary, to be obtained as was obtained with the previous function. Secondly, a sixth degree polynomial function was assigned to

the function thus obtained in order to reach an analytical result. The salary function obtained is:

$$w_t^G = -2.2437\text{E-}10\,t^6 + 7.33585\text{E-}8\,t^5 - 8.7928\text{E-}6\,t^4 + 0.00047559\,t^3 - 0.0123648\,t^2 + 0.203273842\,t - 1.01204121 \quad \textbf{[42.]}$$

with an R^2 coefficient of 0.9999.

c) Exponential. This presents no difficulties with regard to the analytical determination of the value of the fund, as was mentioned in Section II. The value of exponential growth was calculated in order to compare it with the other functions. Based on an initial unitary salary, this would enable the same actual value of salaries as found in the previous functions to be obtained. The salary function used is:

$$w_t^E = e^{0.0313418\,t} \quad \textbf{[43.]}$$

APPENDIX 4. DETERMINING THE VALUE OF THE ACCUMULATED FUND IN THE CASE OF A POLYNOMIAL SALARY FUNCTION.

By using equation (39) as the basis and substituting the polynomial expression ϕ_n by its value according to variable "t", we have:

$$IA_c = c(1-a_1)W_0\, e^{T(r-a_2)} \int_{t=0}^{T} f_n\ e^{-t(r-a_2)}\, dt =$$

$$= c(1-a_1)W_0\, e^{T(r-a_2)} \int_{t=0}^{T} (b_n t^n + b_{n-1} t^{n-1} + ... + b_1 t + b_0)\ e^{t(a_2-r)}\, dt \qquad \textbf{[44.]}$$

From this equation will be calculated, firstly, the indefinite integral. This can be solved section by section, making:

$u = b_n t^n + b_{n-1} t^{n-1} + ... + b_1 t + b_0 \Rightarrow du = [n\, b_n t^{n-1} + (n-1)\, b_{n-1} t^{n-2} + ... + b_1]\, dt = d\,\phi_n$

$$dv = e^{t(a_2-r)}\, dt \Rightarrow v = \frac{e^{t(a_2-r)}}{a_2 - r}$$

$$\int (b_n t^n + b_{n-1} t^{n-1} + ... + b_1 t + b_0)\ e^{t(a_2-r)}\, dt =$$

$$= (b_n t^n + b_{n-1} t^{n-1} + ... + b_1 t + b_0)\frac{e^{t(a_2-r)}}{a_2 - r} - \int \frac{e^{t(a_2-r)}}{a_2 - r}(n\, b_n t^{n-1} + (n-1)\, b_{n-1} t^{n-2} + ... + b_1)\ dt =$$

$$= (b_n t^n + b_{n-1} t^{n-1} + ... + b_1 t + b_0)\frac{e^{t(a_2-r)}}{a_2 - r} - \qquad \textbf{[45.]}$$

$$- \frac{1}{a_2 - r} \int e^{t(a_2-r)} (n\, b_n t^{n-1} + (n-1)\, b_{n-1} t^{n-2} + ... + 2b_2 t + b_1)\, dt =$$

The new integral that appears is formed by the same exponential function as before (divided by a constant) and a polynomial of one degree less than the one before (the first one being derived from the original polynomial). The new integral will have to be solved once again by integrating section by section, just as was done in the first step:

$u = n\, b_n t^{n-1} + (n-1)\, b_{n-1} t^{n-2} + ... + 2b_2 t + b_1 \Rightarrow du = [n\,(n-1)\, b_n t^{n-2} + (n-1)\,(n-2)\, b_{n-1} t^{n-3} + ... + 2b_2]\, dt = d^2\,\phi_n$

$$dv = e^{t(a_2-r)}\, dt \Rightarrow v = \frac{e^{t(a_2-r)}}{a_2 - r}$$

whereby equation (44) can be expressed as:

$$= (b_n t^n + b_{n-1} t^{n-1} + ... + b_1 t + b_0) \frac{e^{t(a_2 - r)}}{a_2 - r} -$$

$$- \frac{1}{a_2 - r} \left[(n\, b_n t^{n-1} + (n-1)\, b_{n-1} t^{n-2} + ... + 2 b_2 t + b_1) \right] \frac{e^{t(a_2 - r)}}{a_2 - r} +$$

$$+ \frac{1}{a_2 - r} \int \frac{e^{t(a_2 - r)}}{a_2 - r} \left[n\,(n-1) b_n t^{n-2} + (n-1)(n-2)\, b_{n-1} t^{n-3} + ... + 2 b_2 \right] dt =$$

$$= \frac{e^{t(a_2 - r)}}{a_2 - r}\, f_n - \frac{e^{t(a_2 - r)}}{(a_2 - r)^2}\, df_n + \frac{1}{(a_2 - r)^2} \int e^{t(a_2 - r)}\, d^2 f_n\, dt$$

The new integral is formed by the same exponential function as before and by a polynomial of one degree less than the one above (the second one derived from the original polynomial). It will therefore be necessary to repeat the same process "n" times until the following result is reached:

$$\int (b_n t^n + b_{n-1} t^{n-1} + ... + b_1 t + b_0)\, e^{t(a_2 - r)}\, dt =$$

$$= \frac{e^{t(a_2 - r)}}{a_2 - r}\, f_n - \frac{e^{t(a_2 - r)}}{(a_2 - r)^2}\, df_n + \frac{e^{t(a_2 - r)}}{(a_2 - r)^3}\, d^2 f_n + ... + (-1)^n \frac{e^{t(a_2 - r)}}{(a_2 - r)^{n+1}}\, d^n f_n = \qquad \textbf{[46.]}$$

$$= e^{t(a_2 - r)} \sum_{s=0}^{n} (-1)^s \frac{d^s f_n}{(a_2 - r)^{s+1}}$$

If the result of equation (46) is substituted in equation (44) we get:

$$IA_c = c(1 - a_1) W_0\, e^{T(r - a_2)} \left| e^{t(a_2 - r)} \sum_{s=0}^{n} (-1)^s \frac{d^s f_n}{(a_2 - r)^{s+1}} \right|_{t=0}^{T} =$$

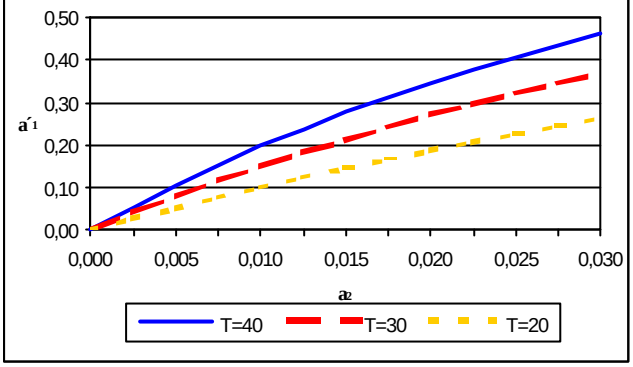
$$= c(1 - a_1) W_0\, e^{T(r - a_2)} \left[e^{T(a_2 - r)} \sum_{s=0}^{n} (-1)^s \frac{d^s f_n(t = T)}{(a_2 - r)^{s+1}} - \sum_{s=0}^{n} (-1)^s \frac{d^s f_n(t = 0)}{(a_2 - r)^{s+1}} \right] = \qquad \textbf{[47.]}$$

$$= c(1 - a_1) W_0 \left[\sum_{s=0}^{n} (-1)^s \frac{d^s f_n(t = T)}{(a_2 - r)^{s+1}} - e^{T(r - a_2)} \sum_{s=0}^{n} (-1)^s \frac{d^s f_n(t = 0)}{(a_2 - r)^{s+1}} \right] =$$

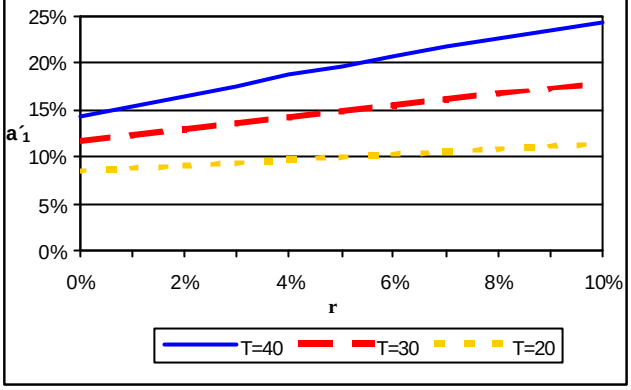
$$= c(1 - a_1) W_0 \left[\sum_{s=0}^{n} (-1)^s \frac{d^s f_n(t = T)}{(a_2 - r)^{s+1}} - e^{T(r - a_2)} \sum_{s=0}^{n} (-1)^s \frac{s!\, b_s}{(a_2 - r)^{s+1}} \right]$$

where:
$d^s \phi_n(t = T)$: represents the value for t = T derived from order "s" of the original polynomial of degree "n".

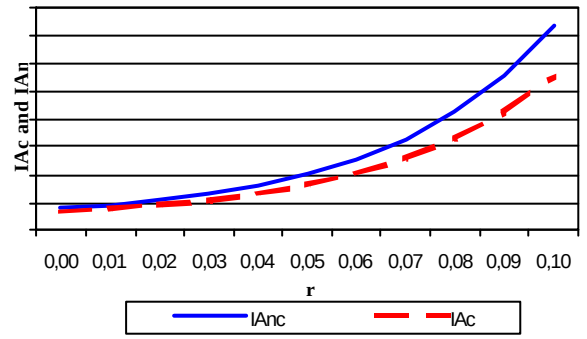
Graph 2
Relation between æ and a´1



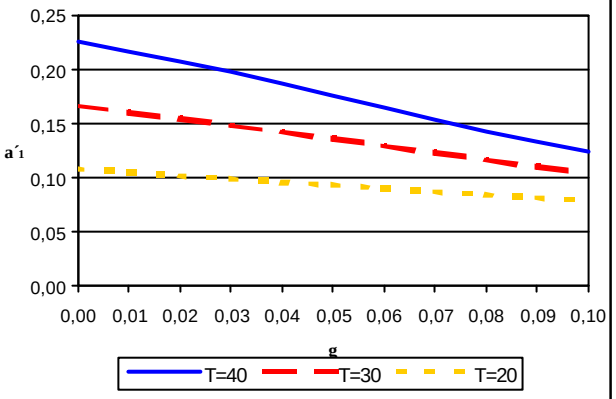
Graph 3
Relation between r and a´1, with æ=0.01



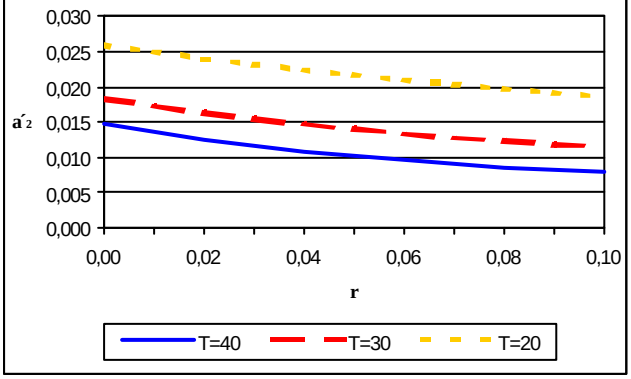
Graph 4
Relation between r and IAc, IAnc, with T=40, æ=0.01



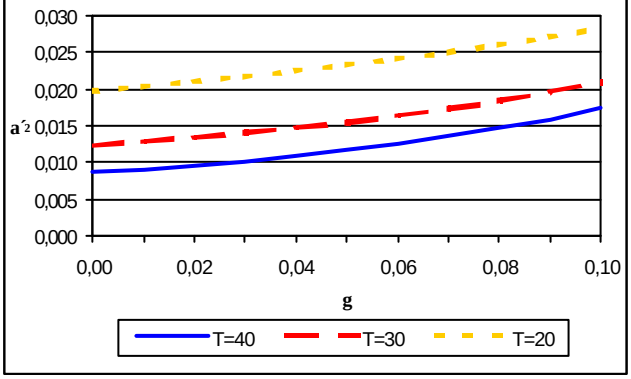
Graph 5
Relation between g and a´1, with æ=0.01



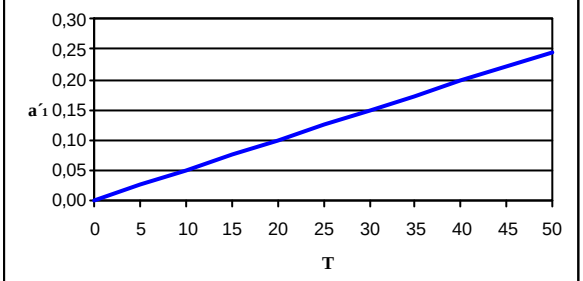
Graph 6
Relation between r and a´2, with a1=0.20



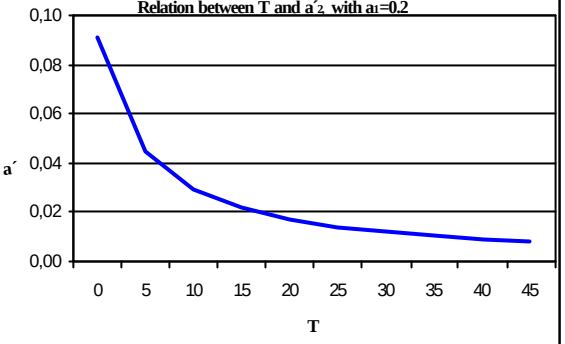
Graph 7
Relation between g and a´2, with a1=0.2



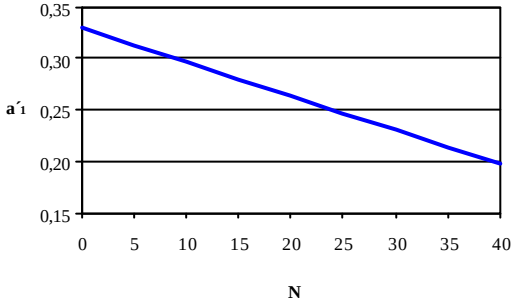
Graph 8
Relation between T and a´1, with a2=0.01



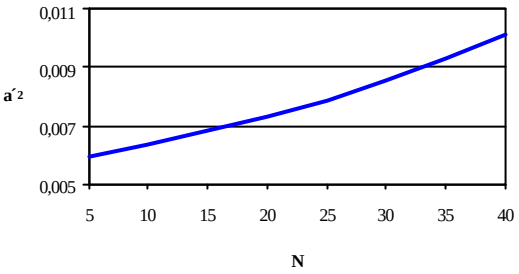
Graph 9
Relation between T and a´2, with a1=0.2



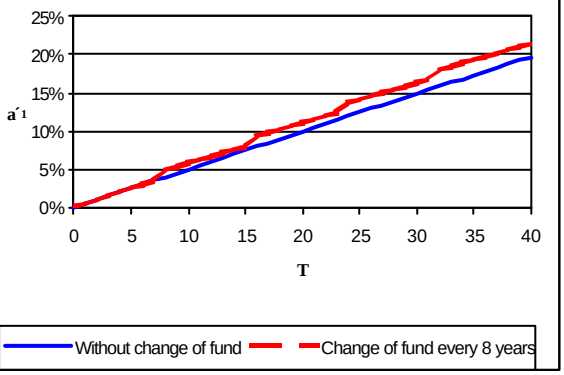
Graph 10
Relation between N and a´1, with a2=0.01 and T=40



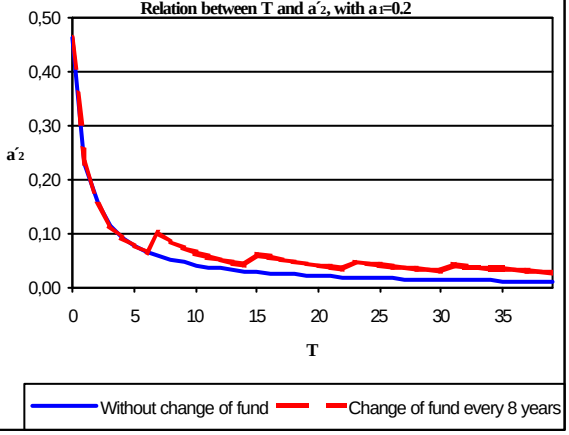
Graph 11
Relation between N and a´2, with a1=0.2 and T=40

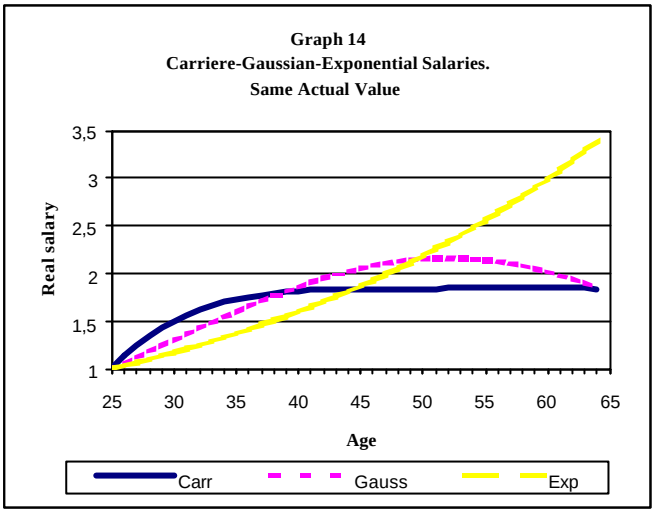


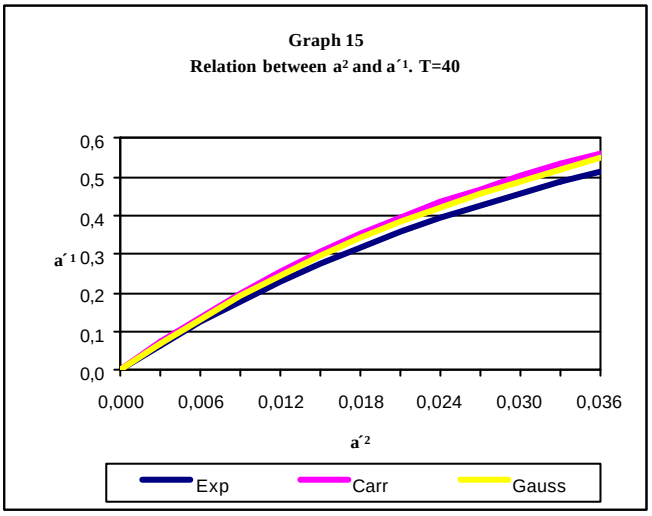
Graph 12
Relation between T and a´1, with a2=0.01

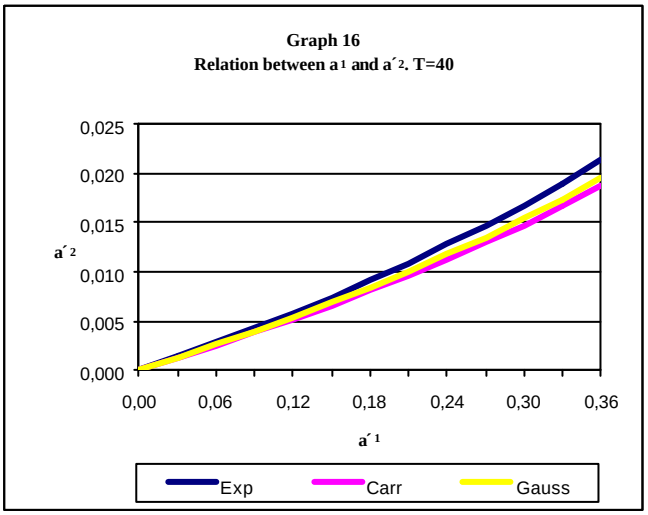


Graph 13
Relation between T and a´2, with a1=0.2




Graph 14
Carriere-Gaussian-Exponential Salaries.
Same Actual Value
Real salary
3,5
3
2,5
2
1,5
1
25
30
35
40
45
50
55
60
65
Age
Carr
Gauss
Exp


Graph 15
Relation between a² and a´¹. T=40
a´¹
0,6
0,5
0,4
0,3
0,2
0,1
0,0
0,000
0,006
0,012
0,018
0,024
0,030
0,036
a´²
Exp
Carr
Gauss


Graph 16
Relation between a¹ and a´². T=40
a´²
0,025
0,020
0,015
0,010
0,005
0,000
0,00
0,06
0,12
0,18
0,24
0,30
0,36
a´¹
Exp
Carr
Gauss