On Mesoamerican Mathematics

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(A *quick and not too scientific* overview for Actuaries)

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Basic Assumptions
(1. Theoretical Postulates)

1. Ontological *Platonism*

   Mathematical entities are culture-independent

2. Epistemological *Relativism*

   At a given time, in a given culture, only some mathematical concepts and practices are known

Basic Assumptions

(1a. Theoretical Postulates Implications)

1. Ontological *Platonism*

   Mathematical entities are culture-independent

   Valid to "reify"

2. Epistemological *Relativism*

   At a given time, in a given culture, only some mathematical concepts and practices are known

   Look for mathematical content into cultural developments:

   Tribute (Economics, Demography)
   Calendrics (Astronomy)
   Architecture (Geometry)

Basic Assumptions
(2. Methodological Restrictions)

3. Extensional Treatment

   Not concerned with intensional aspects (mystical meaning, religious implications, political uses,…)

4. Not-Ethnographic

   Monuments

   Ceramics and other archeological objects

   Original Documents

   Contemporary Accounts

   Cf. "Ethnomathematics"

5. Restrict to *Mesoamerica (Maya & Aztec)*

   Area

   Time-frame

Mesoamerica

(P. Birkoff 1943)

Contents

1. Domain of Discourse

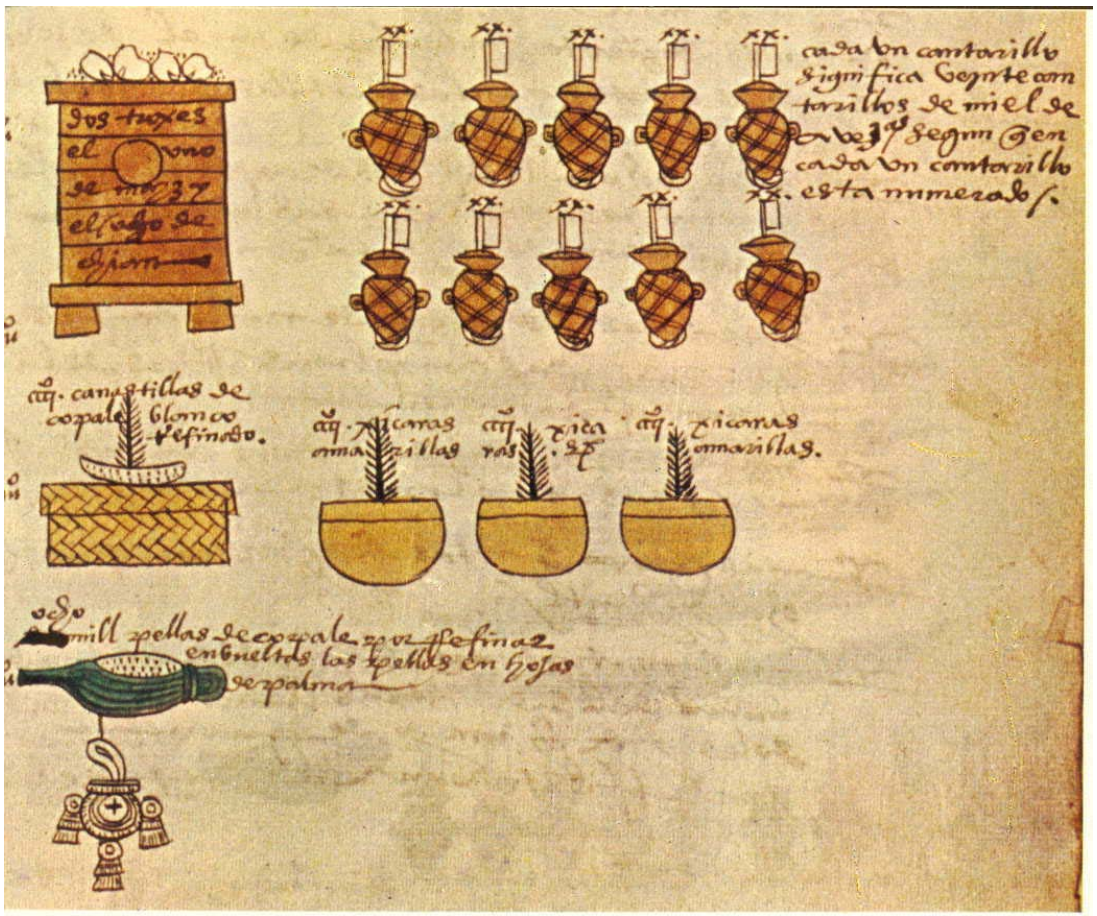
1. Mathematical Applications:
   - Tribute
   - Calendrics

2. Final Remarks

Contents

1. Domain of Discourse
2. Mathematical Applications:
   - Tribute:
     - Aztec Numerical Notation
     - Cadastral Registers
   - Calendrics
3. Final Remarks

Aztec Numerical Notation



Mendoza 36r (frg)

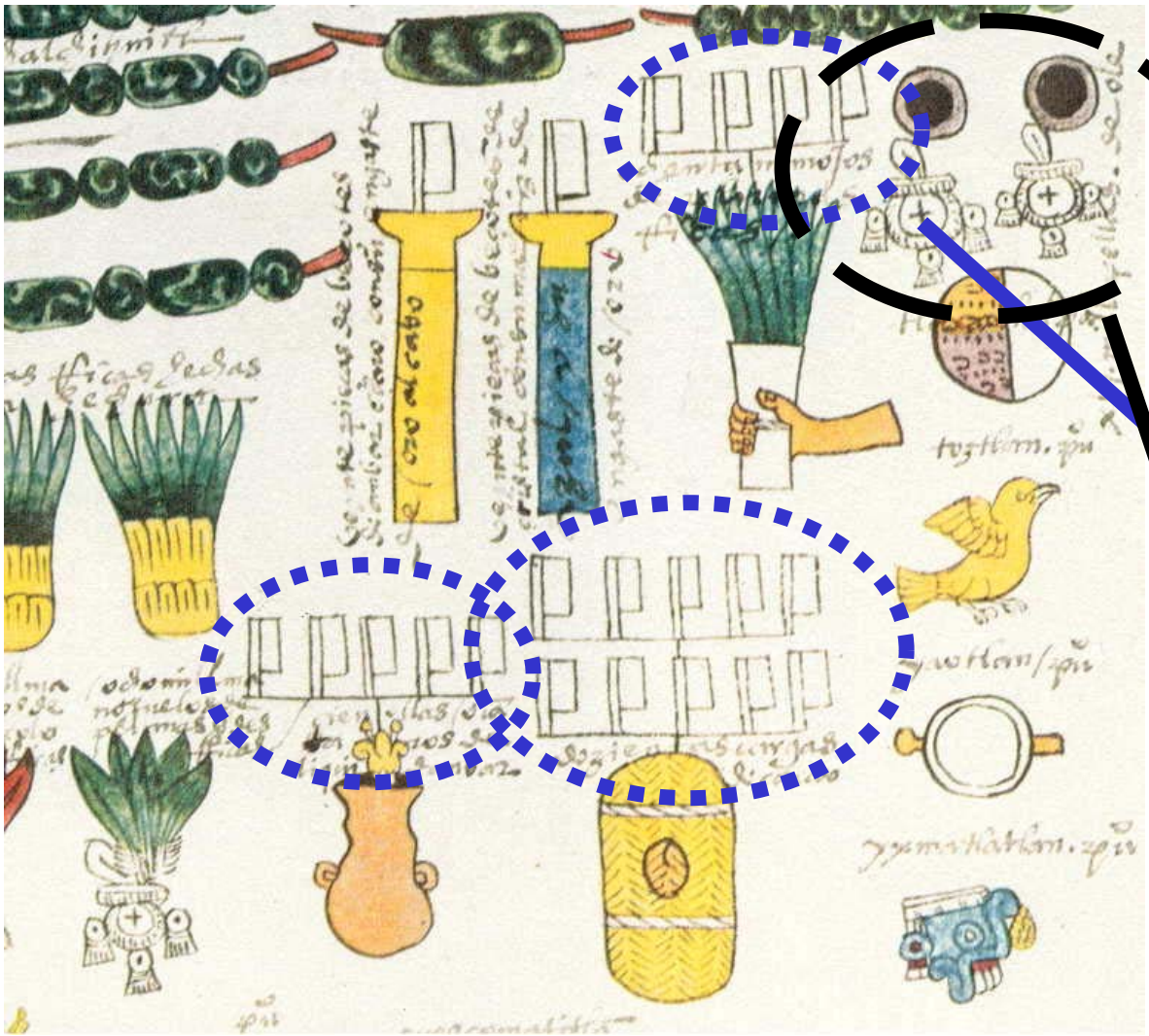
20

400

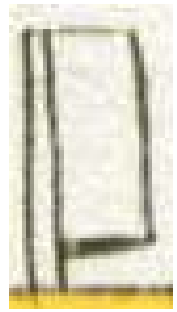
8000

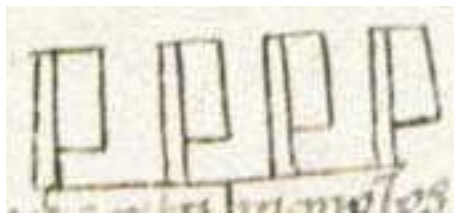
Vigesimal

Aztec Numerical Notation

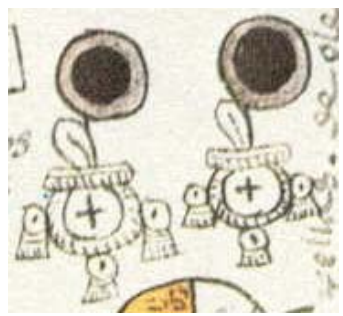


20



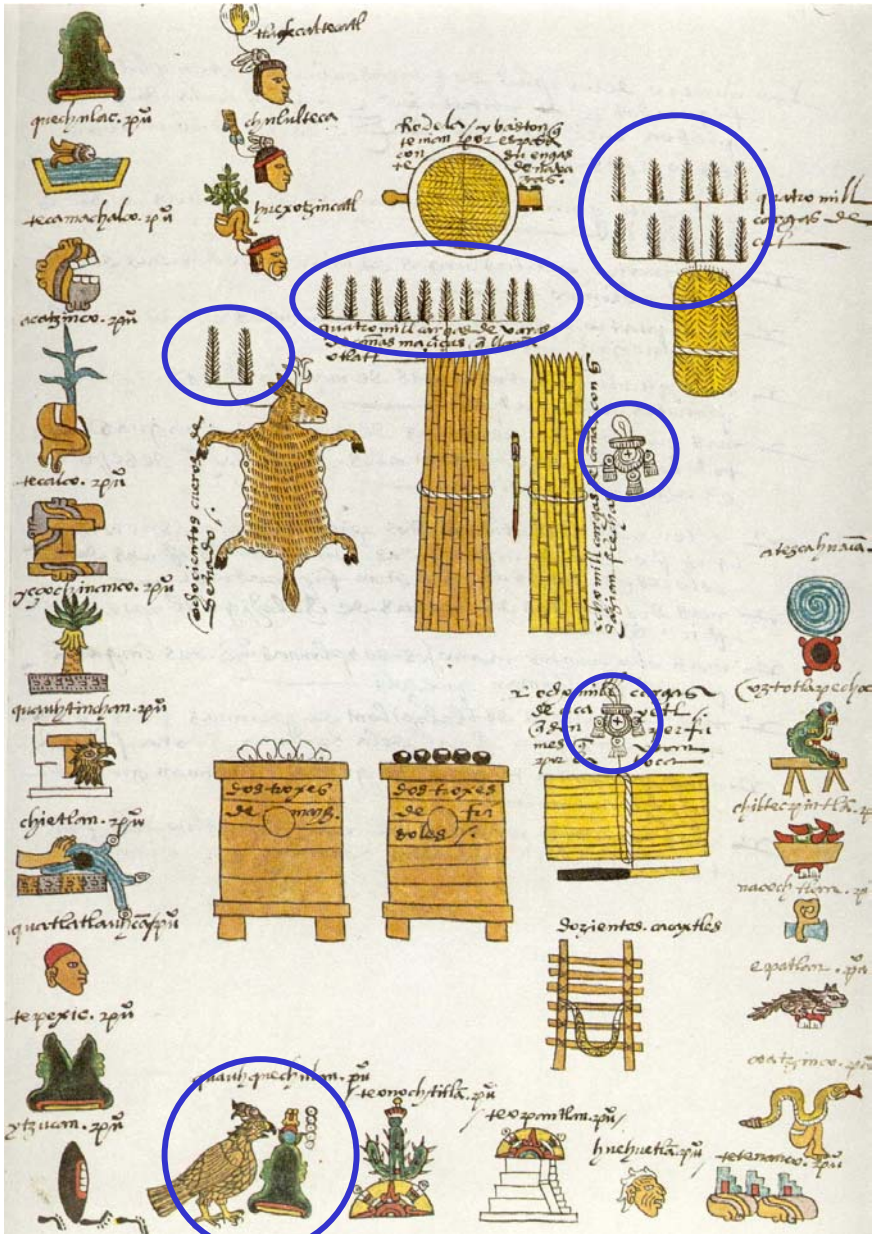


20+20+20+20



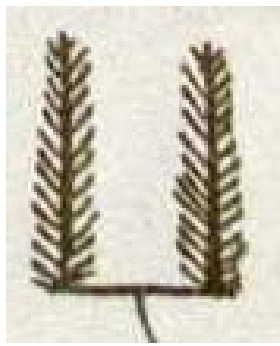
Mendoza 37r (frg.)

Vigesimal Additive?



Mendoza 42r

More Compound Numbers

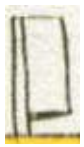


Standard Aztec Numerical Notation

Basic Numbers

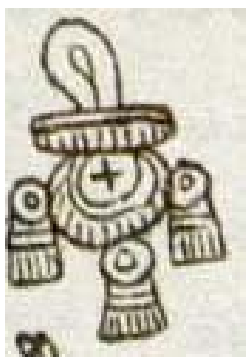
1

20

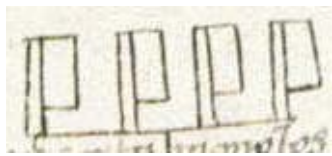


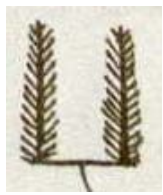
400

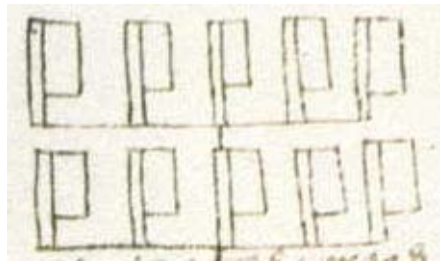
8000

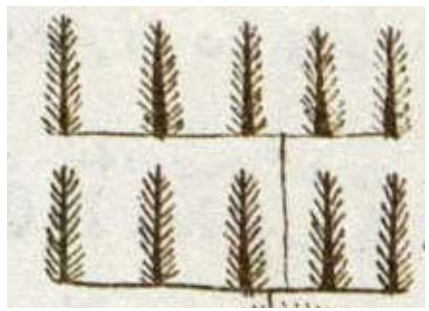


Compound Numbers









Vigesimal
Additive

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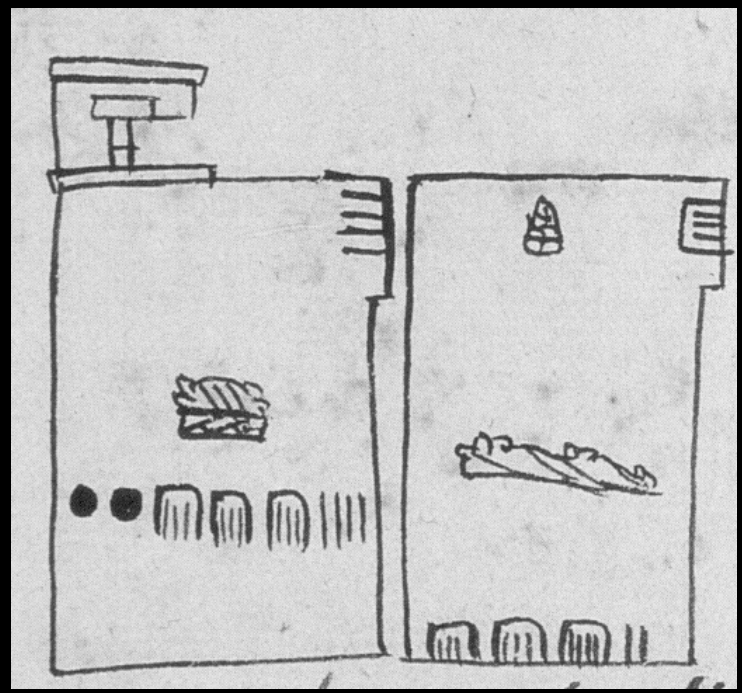
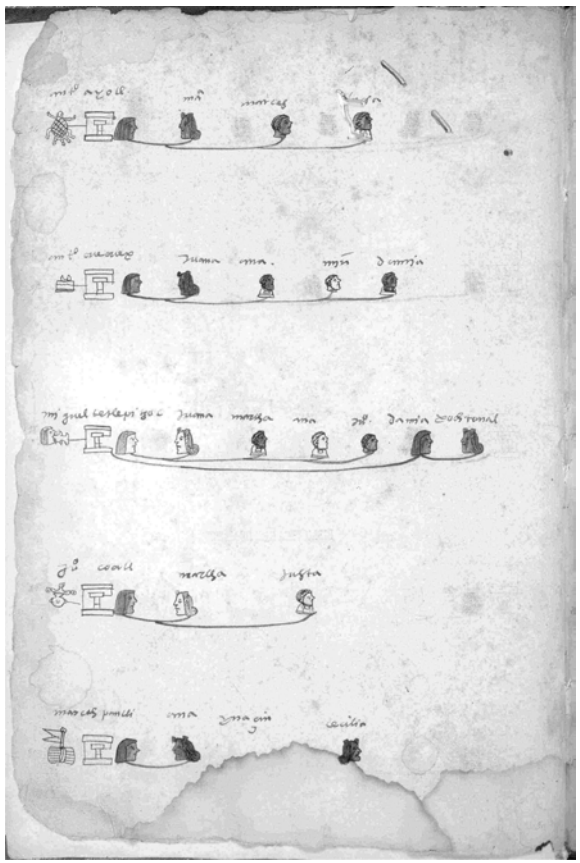
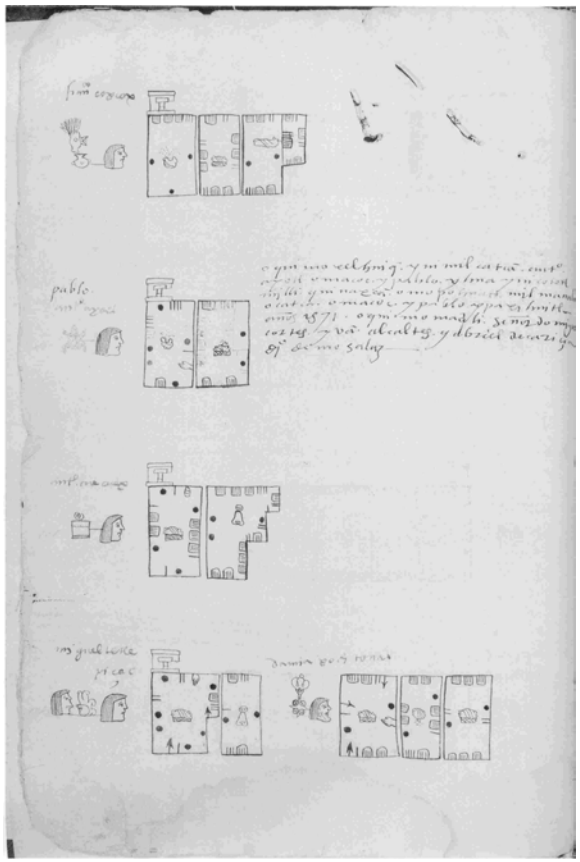


Aztec Notational Variants (Texcoco)

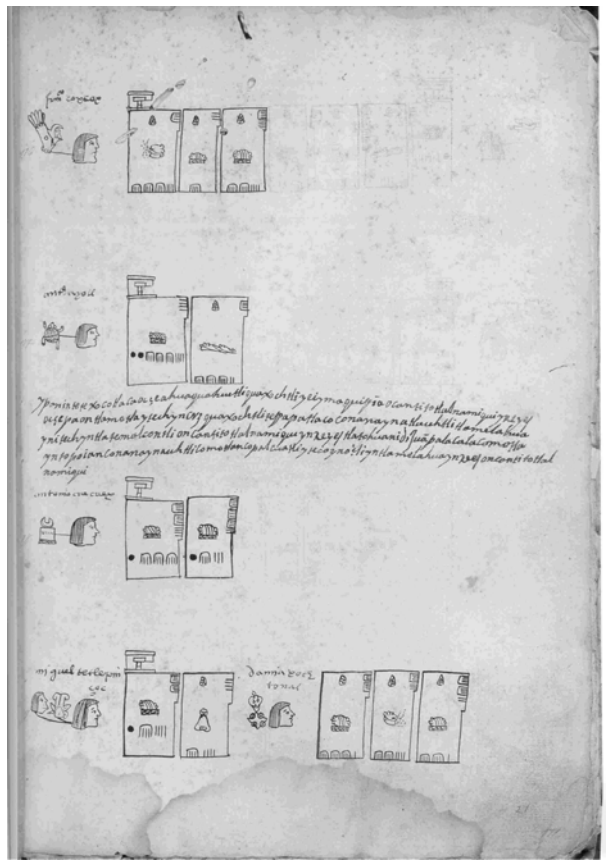
P1: Census



P2: Land Plots

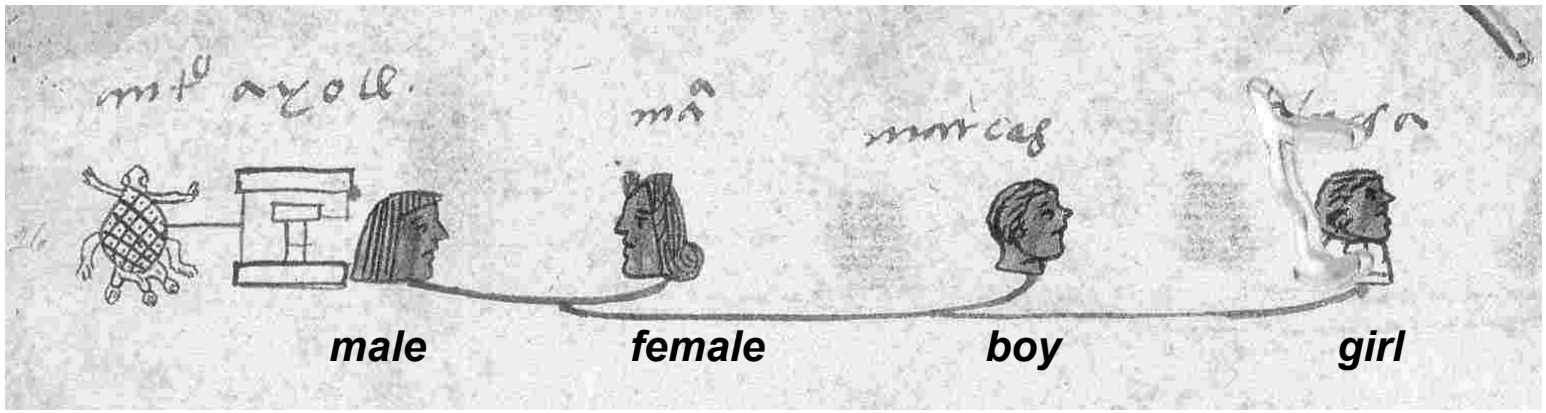


P3: Tribute Base

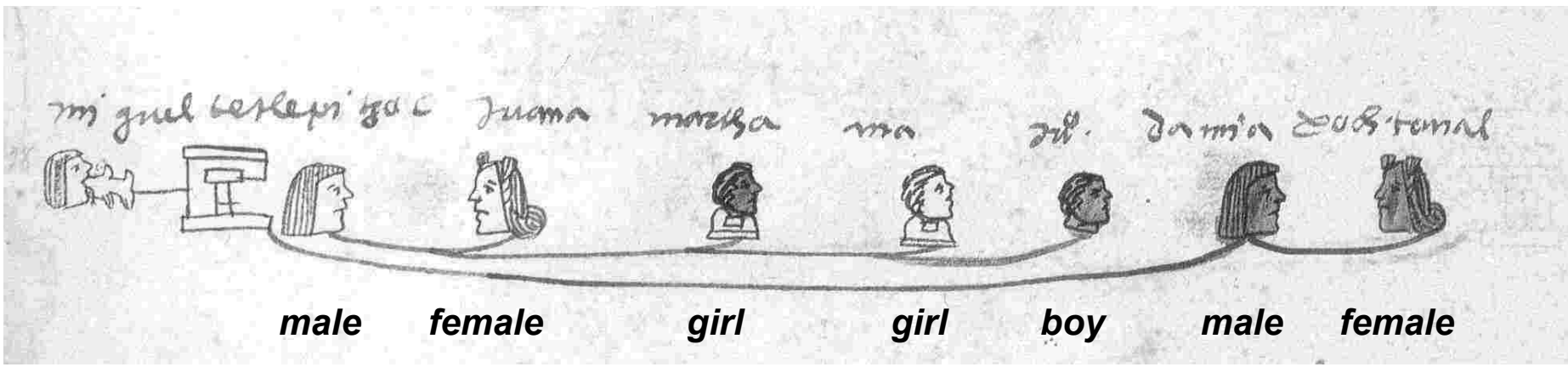


C. SantaMaríaAsunción

P1: Census



Simple Family Unit



Compound Household

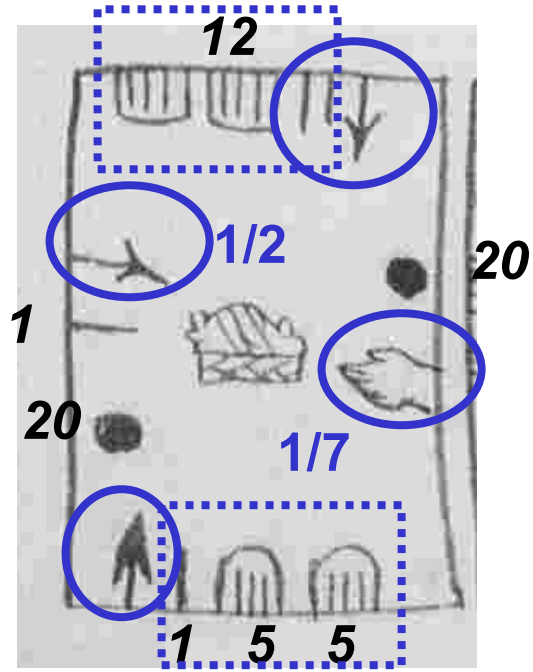
Household Demographic Data

P2: Land Plots

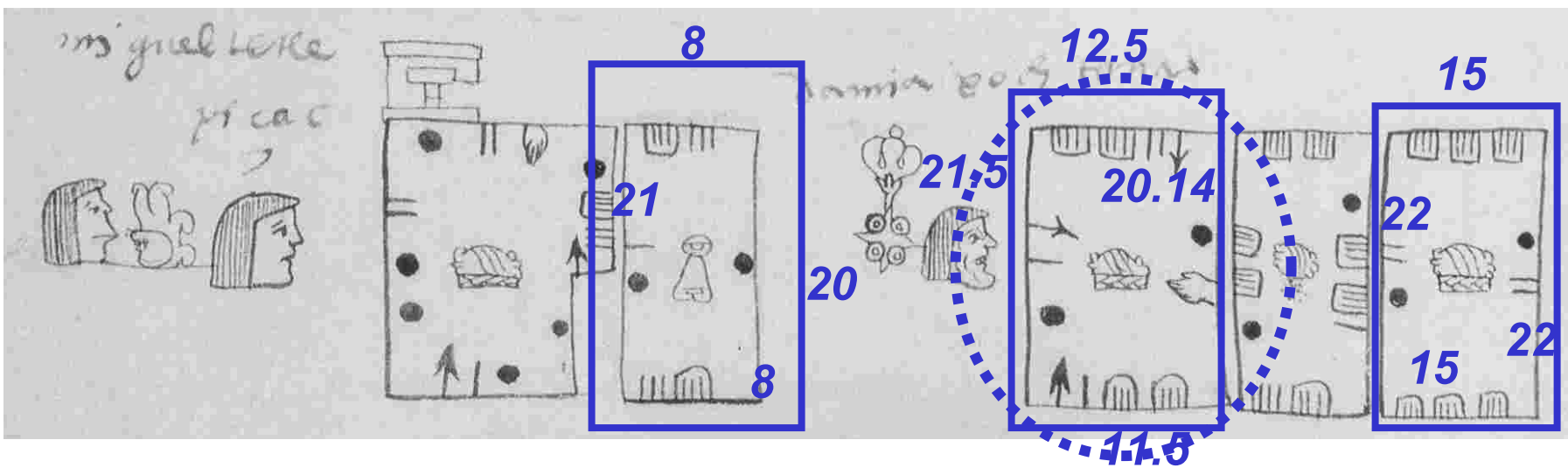
Additive

Vigesimal

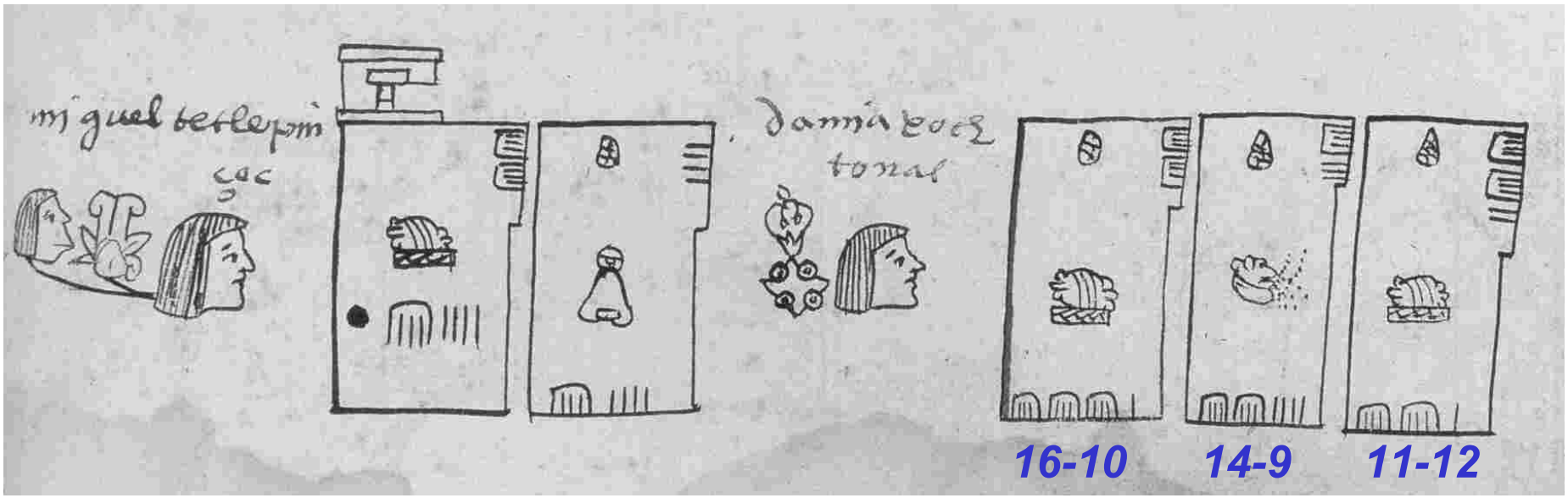
Ordered

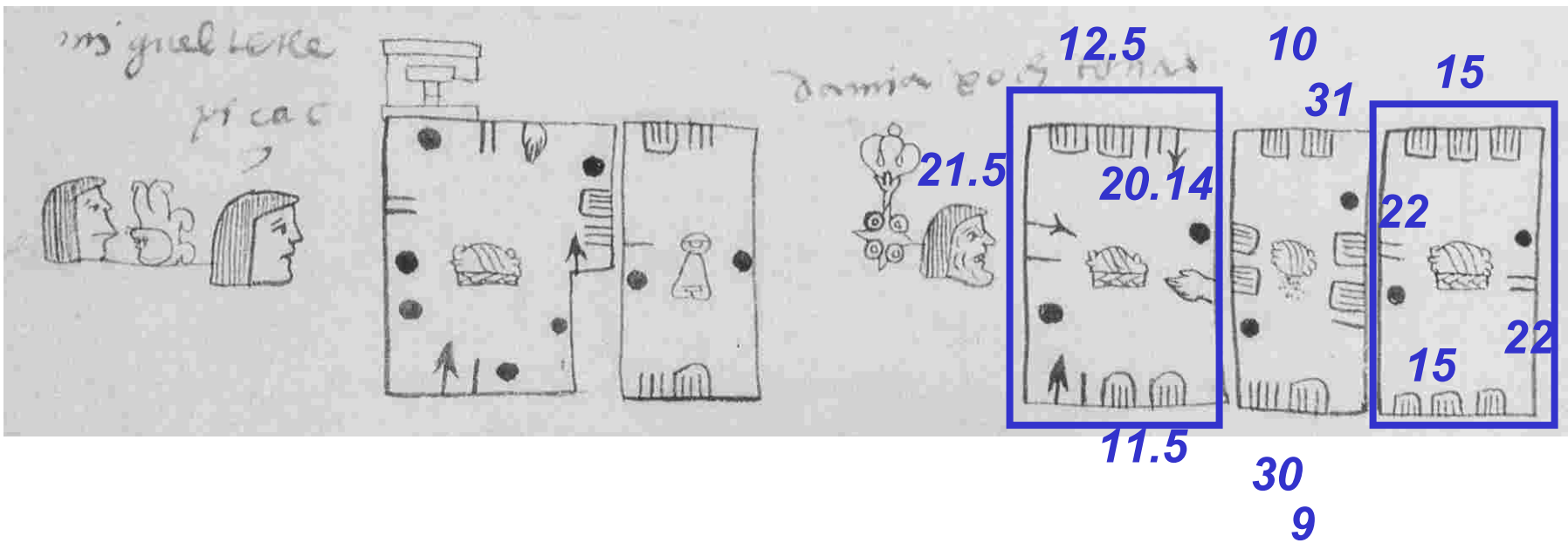


“Perimeteral Measures”

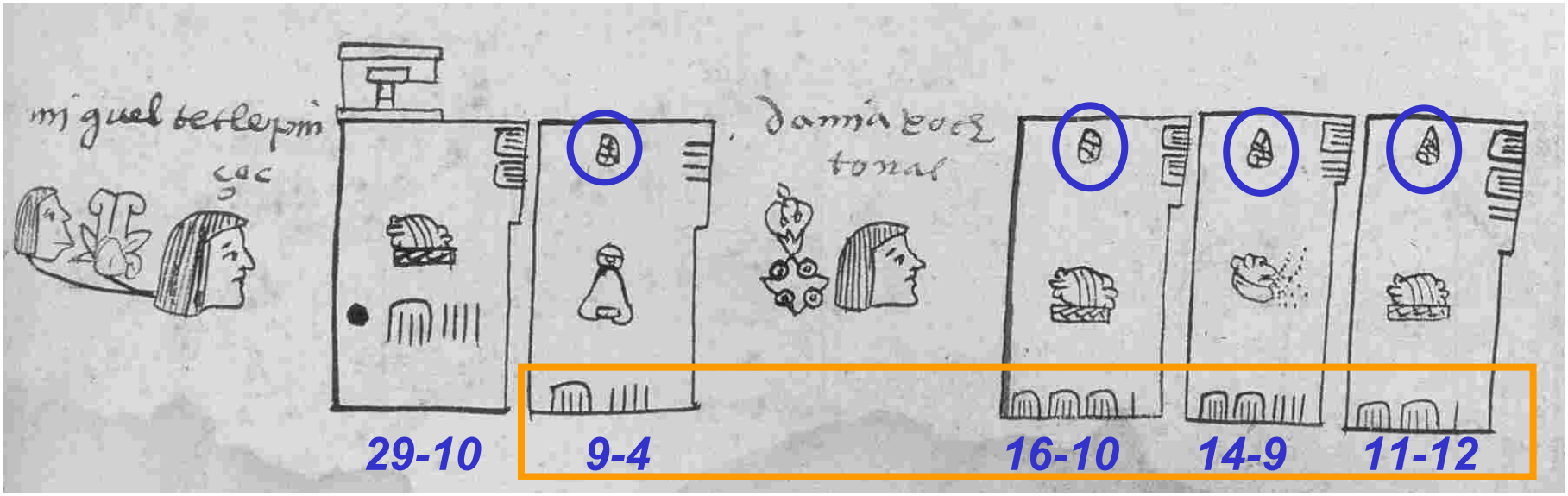


P3: Tribute Base

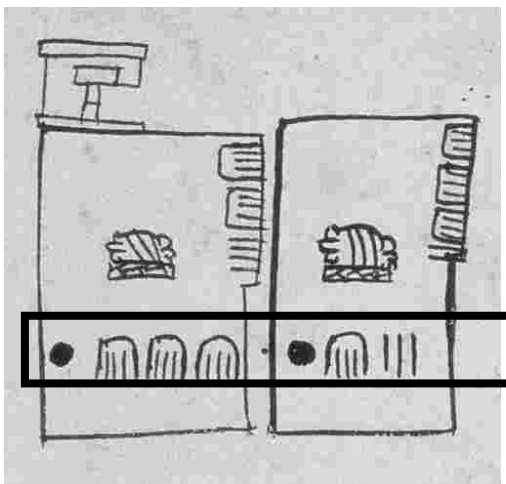


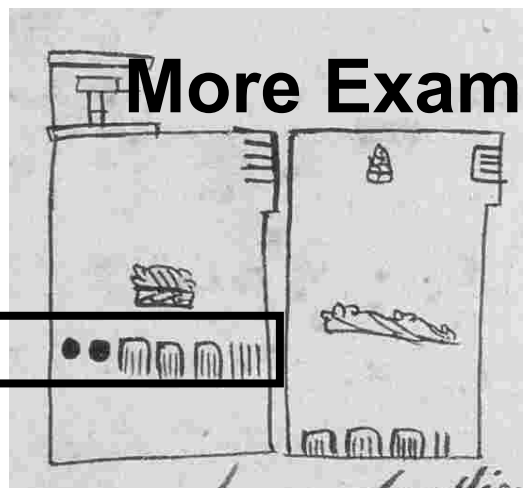


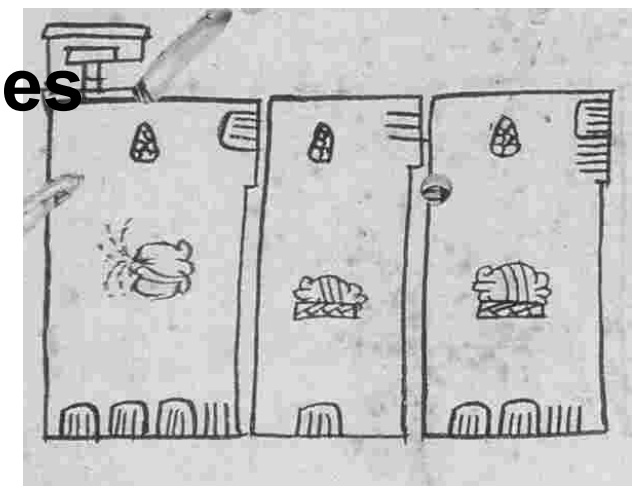
P3: Tribute Base



More Examples







P3: Tribute Base

AREA

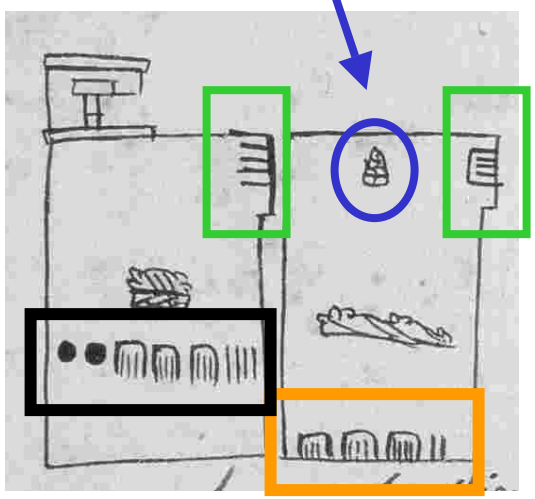
Additive

Vigesimal

Ordered

Fractions

“Place Holder”



Third Position:

Second Position:

First Position:

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   - Calendrics:
     - Calendrical Conventions
     - Maya Numerical Notation
     - Maya Astronomy
3. Final Remarks

Calendrics

- Observation of Cyclical Astronomical phenomena:
  - Equinox/solstice cycles
  - Cenital transit
  - Eclipses (moon /sun cycles)
  - Ecliptic constelations (“Zodiac”)
  - Venus and Mars synodic cycles
- **Time = combination of cycles**
- Two basic calendars:
  - 260 day “ritual” calendar (“*Tzolkin”*)
    - 20 “day bearers” of 13 day “weeks”
  - 365 day solar calendar (*Haab*)
    - 18 “months” of 20 days + 5 extra days
- Other time-keeping conventions
    - Calendar Round
    - Long Count
    - Distance Numbers

Tzolkin (260-day calendar)

- 20 “day symbols”
- 13 numerals
  - Use numbers 1-13
  - Move both each increment

Olmec invention?
Zapotec (piye)
Maya (“tzolkin”)
Aztec (tonalpohualli)

Madrid 75-6

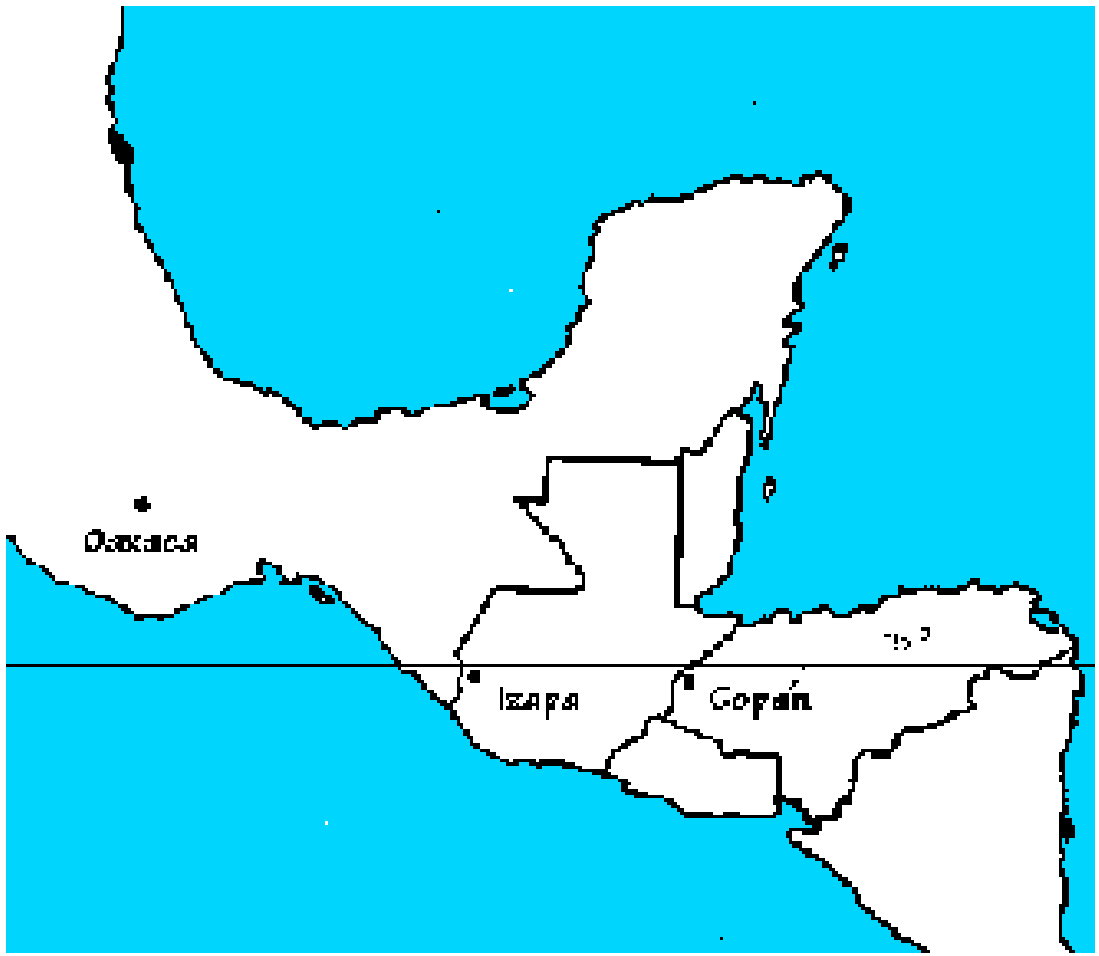
Tzolkin Day Glyphs

They appear only with numbers ONE to 13

Tonalpohualli

Borbonicus XIV

Tzolkin (260-day calendar)

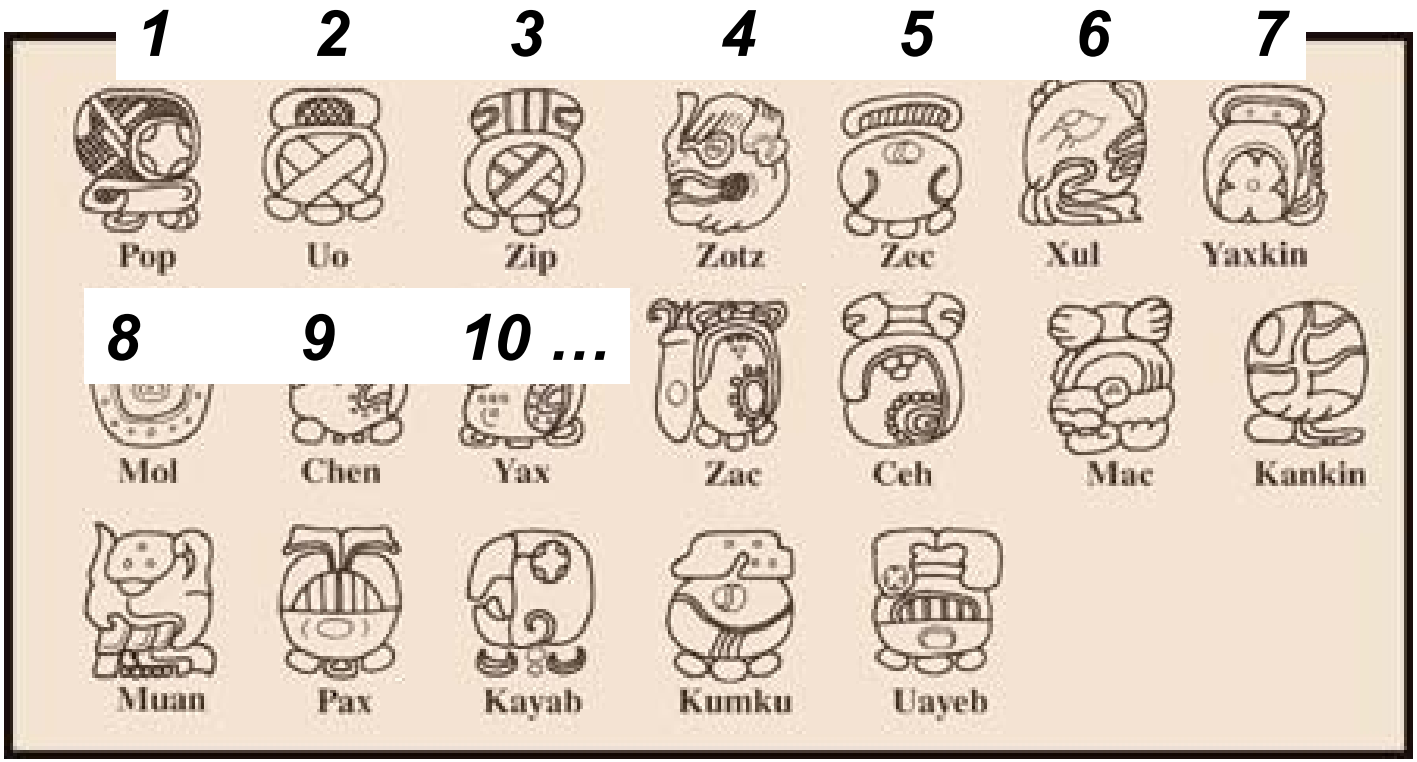


Mälstrom
Zenithal-Passage Hypothesis

Haab (365-day calendar)

- **18** 20-day months
  - 0-19 Numerals
  - 18 “Month” symbols (used sequentially)
  - Reverse lexicograhical ordering
- **1** 5-day month
  - This month continues the next year
  - The 366th day may begin only with one of four “month” symbols (1, 6, 11, 16) “*year bearers*”

Haab Month Symbols

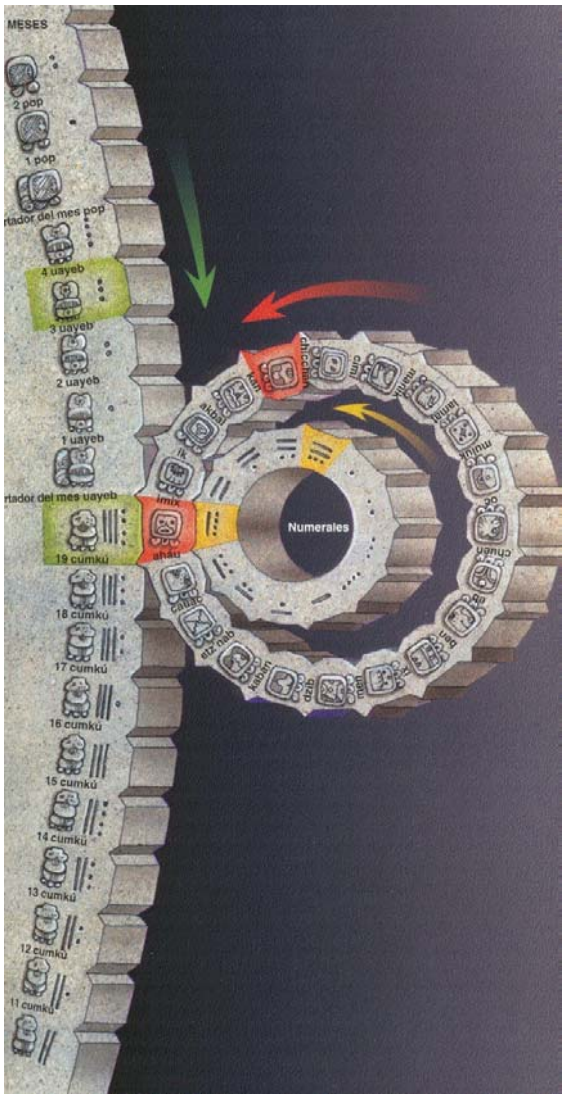


Numbers 0 (“seating glyph”) to 19 precede each number glyph

Other Time-keeping Conventions

- Calendar Round (one date, two calendars)
  - Combination of *Tzolkin* & *Haab*
- Long Count
  - Number of days elapsed since “origin”
- Distance Numbers
  - Number of days elapsed
    since a given date (in the same text)
- Lunar Series
  - Day of lunation, number of lunation (1-6) and deity in a six-lunations period, length of lunation. All this for a given date

Calendar Round Dates (in a 52 year cycle)



- Combination of *Haab* and *Tzolkin* cycles
  - *Haab* = 365 days
  - *Tzolkin* = 260 days

- Calendar Round = lcm(365,260) = 18980 days
  - CR = 52 *Haab*
  - CR = 73 *Tzolkin*

Long Count Dates

- Distance (in days) between a date of the “calendar round” and a fixed (“origin”) date
  - Maya “origin”: *4Ahau*, *8Cumhu* (Aug 13, 3113 BC)
  - Earliests long count dates known:
    - Chiapa de Corzo (36 BC)
    - Tres Zapotes (32 BC)
    - Tikal (292 AD) full positional, without zero
    - Uaxactun (357 AD) badly preserved, uses two zeroes

Calendrical Notation

Tzolkin Date

(Earliest known example of a the use of a number glyph)

Day glyph (“flint”)

Number glyph (“1”)

Zapotec
San José del Mogote, Oaxaca.
Estela 1
(300 BC)

Calendrical Notation
(Early use of number / day-sign combinations)

year

4 wind
(haab fragment)

1 flint (tzolkin)

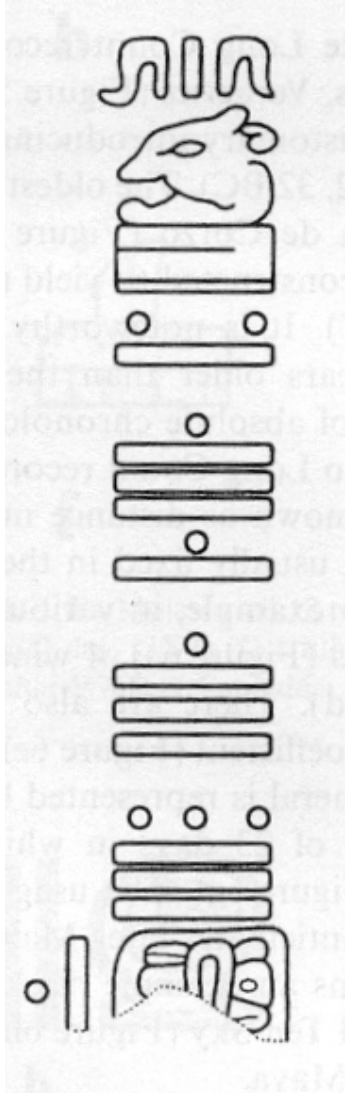
Zapotec
Monte Albán, Oaxaca
South Platform
Stelae 12, 13
(AD 300)

Calendrical Notation
(Earliest complete “long count” calendric date)

Olmec
Tres Zapotes, Veracruz
Estela C (fragment)
(32 BC)

Calendrical Notation
(Earliest full "long count" date)

Olmec
Tres Zapotes, Veracruz
Estela C (fragment)
(32 BC)



ISIG

(1) Uo (haab)

7.

16.

6.

16.

18

6 Edznab
(Tzolkin)

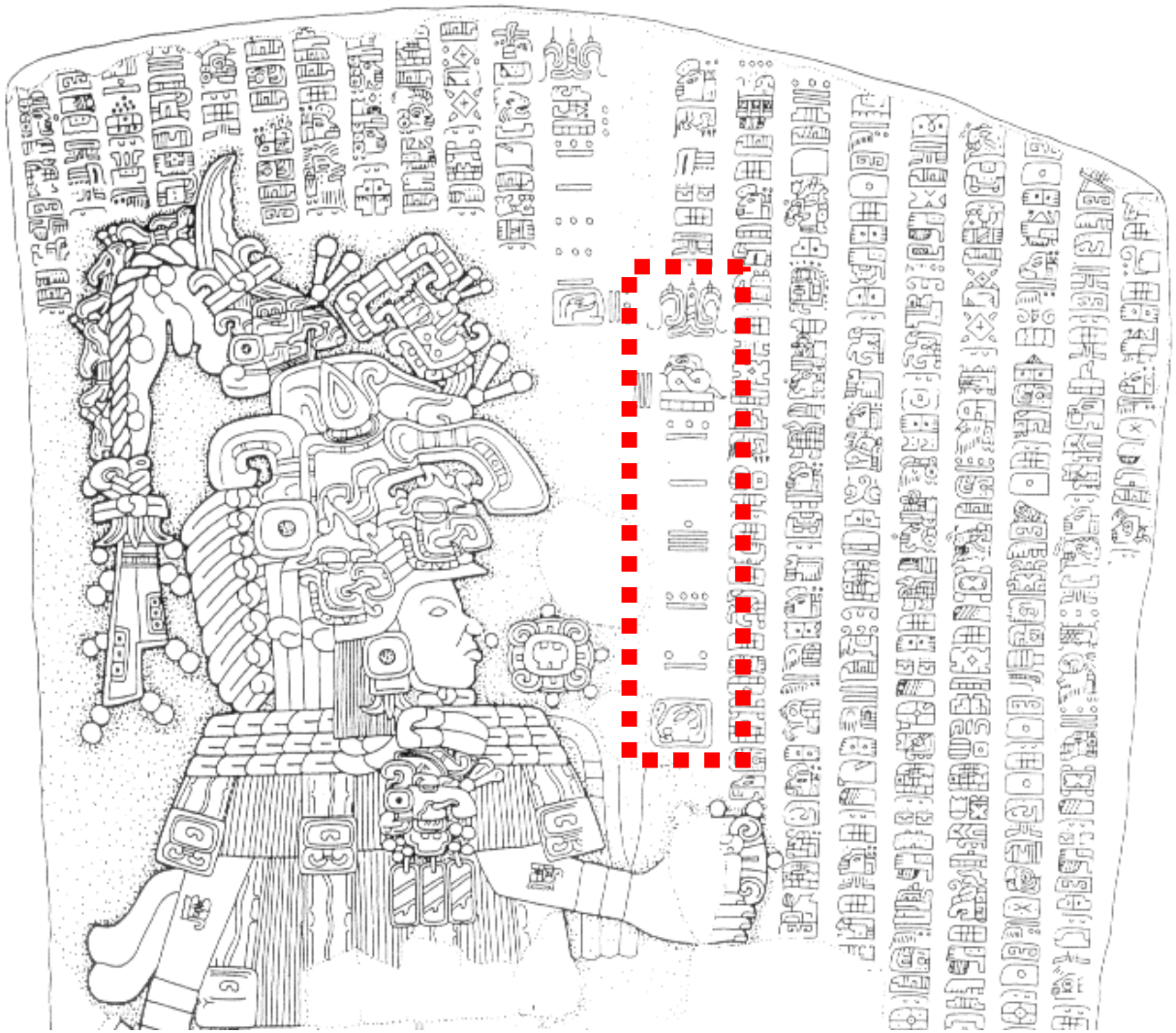
Bar & Dot Notation

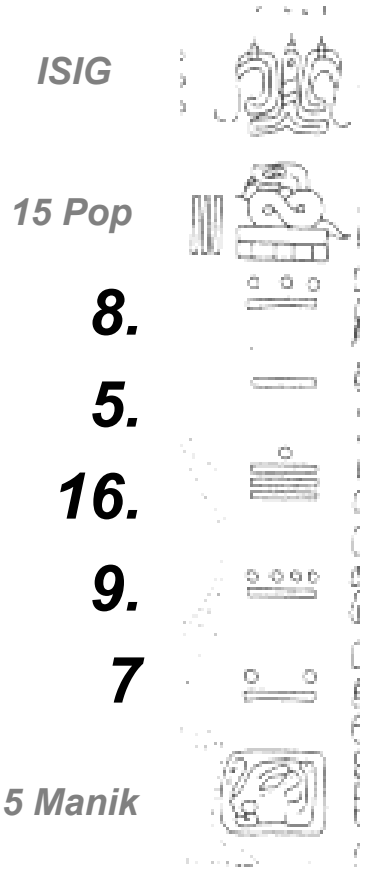
Bar = 5 units
Dot= 1 unit

Vigesimal

Positional Value

Calendrical Notation
(Early “long count” inscription)





Epi-Olmec
La Mojarra, Veracruz
(AD 100)

Calendrical Notation
(Early “distance number” inscription)

20.

3

Epi-Olmec
La Mojarra, Veracruz
(AD 100)

Long Count (5125 years)

Date=number of days from “origin”

Maya
Quiriguá, Stela C
AD 800

Unit's name	Time period	Glyph
Baktun	144,000 days	
Katun	7,200 days	
Tun	360 days	
Uinal	20 days	
Kin	1 day	

Classic Period Inscription

Palenque Tablero de los 80 glifos

Calendrical Arithmetic

- P1: Given two dates calculate the distance between them
- P2: Given a date (e.g.,in CR), obtain the equivalent (e.g., in LC)
- P3: Given a date, find other dates with “interesting” relevant features (astronomical events, cycles, nearest date such that …)

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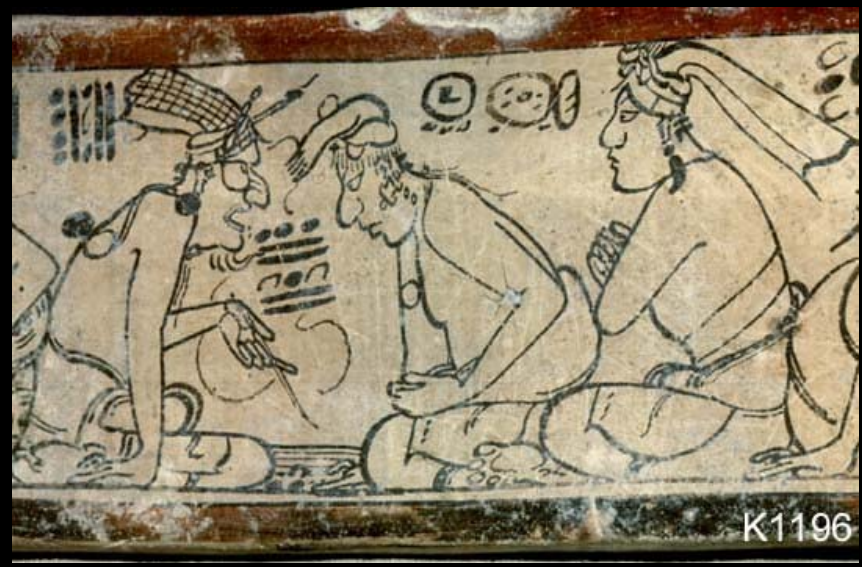


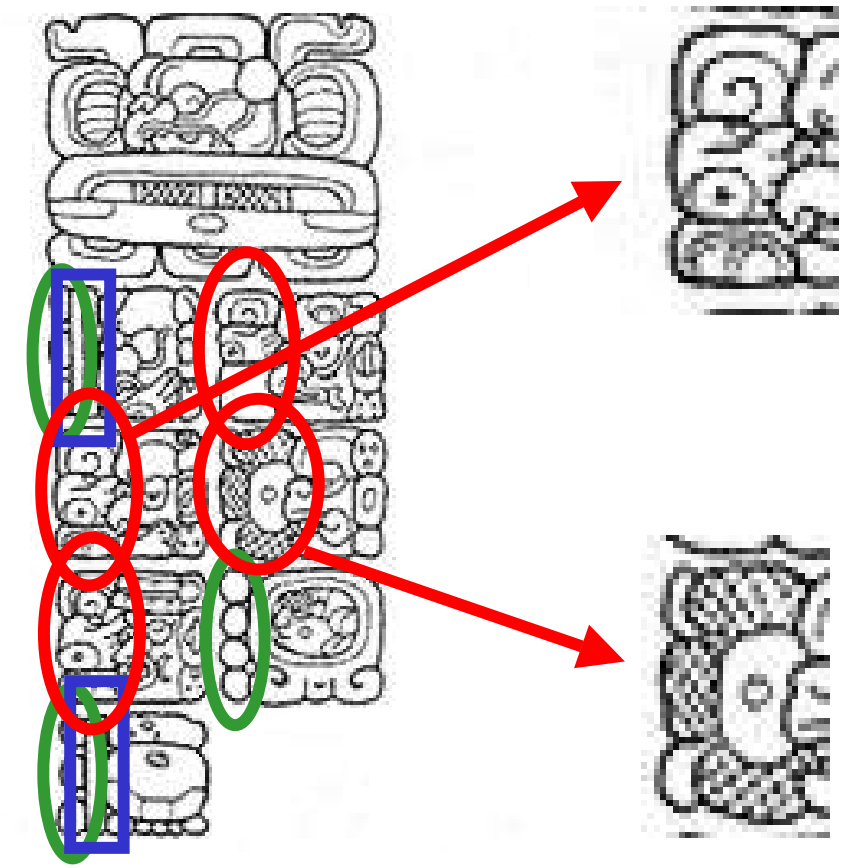
Two Facts about Maya Positional Notation

1. All known examples of native positional Mayan numbers are calendrical or astronomical examples.

2. All known Mayan numbers in positional notation use at most five positions

Brickner & Brickner, p193

Numeric Icons in Maya Inscriptions



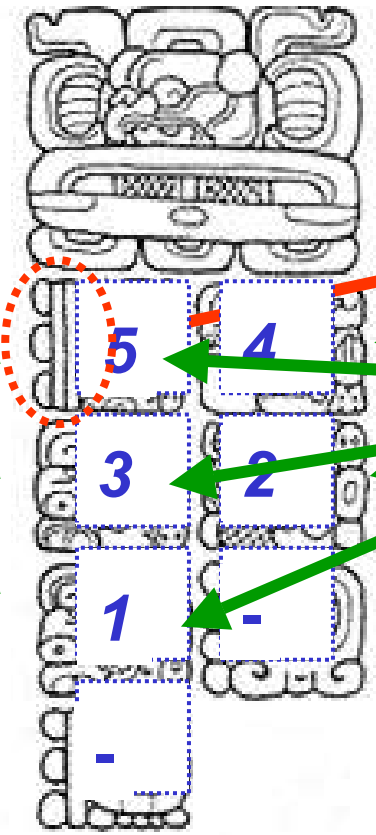
5 units (*"bar"*)

1 unit (*"dot"*)

Zero units,
full count,
place holder

Maya
Quiriguá, Stela C
AD 800

Notational Conventions in Maya Inscriptions



Additive (bar & dot)

13=3+5+5

Multiplicative (time units)

Value × Calendric unit

Positional?

Canonical order

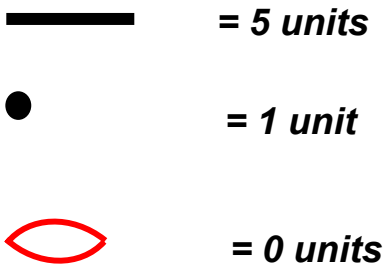
Zero (a Maya invention)

Maya
Quiriguá, Stela C
AD 800

Conjecture: Basic Mayan Positional Notation

$n_i \epsilon [0,19]$

Basic Symbols

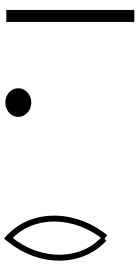


Whole Numbers

Position	Number	Value
5th. Position	n_4	$n_4 \times 20^4$
4th. Position	n_3	$n_3 \times 20^3$
3th. Position	n_2	$n_2 \times 20^2$
2nd. Position	n_1	$n_1 \times 20^1$
1st. Position	n_0	$n_0 \times 20^0$

Conjecture: Basic Mayan Positional Notation

Basic Symbols



= 5 units

= 1 unit

= 0 units

Whole Numbers

$n_i \in [0,19]$

Position	Number	Value	
5th. Position	n_4	$n_4 \times 20^4$	$n_4 \in [0,19]$
4th. Position	n_3	$n_3 \times 20^3$	$n_3 \in [0,19]$
3th. Position	n_2	$n_2 \times 20^2$	$n_2 \in [0,19]$
2nd. Position	n_1	$n_1 \times 20^1$	$n_1 \in [0,17]$
1st. Position	n_0	$n_0 \times 20^0$	$n_0 \in [0,19]$

Basic Mayan Calendrical Positional Notation

Basic Symbols

= 5 units

= 1 unit

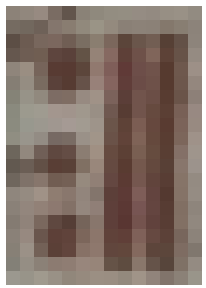
= 0 units

Whole Numbers

Position	Number	Value
5th. Position	n_4	$n_4 \times 20^3 \times 18$
4th. Position	n_3	$n_3 \times 20^2 \times 18$
3th. Position	n_2	$n_2 \times 20 \times 18$
2nd. Position	n_1	$n_1 \times 20^1$
1st. Position	n_0	$n_0 \times 20^0$

Numeric Icons in Maya Codices

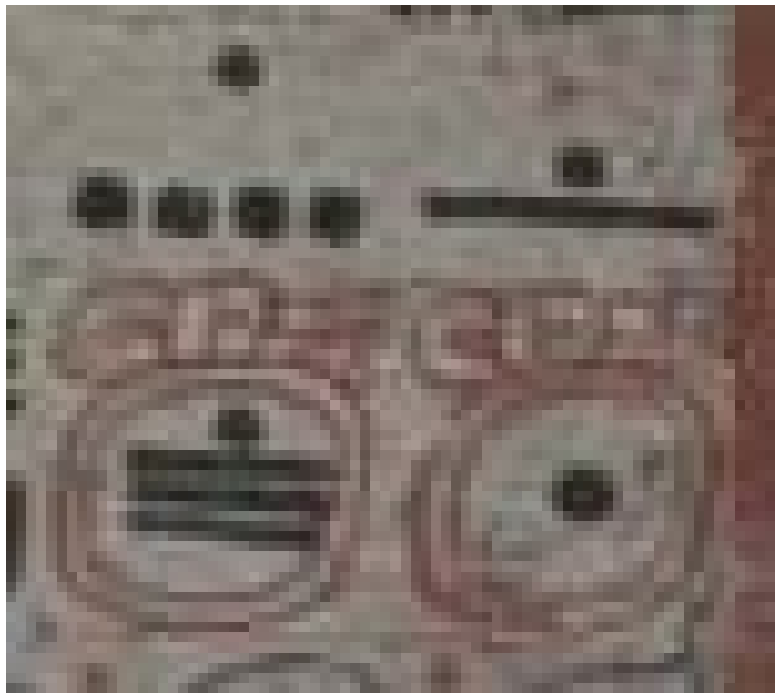
Bar & dot symbols



Symbols for zero

Symbol for twenty

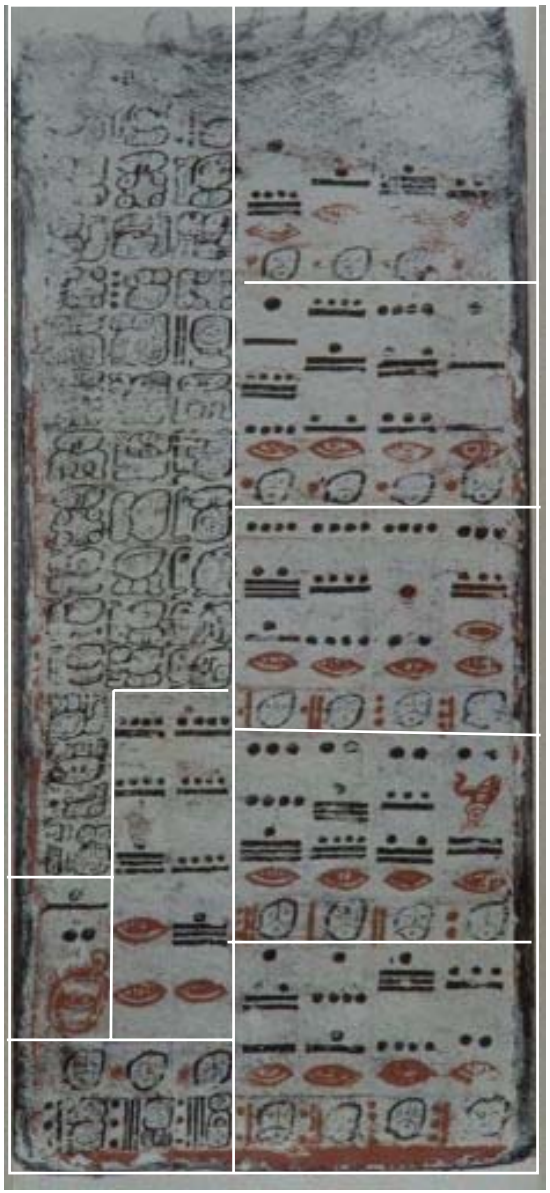
Deciphering Maya Numerical Notation: Negative (Ring) Numbers



... used for counting dates previous to the "origin"

Fösterman 62

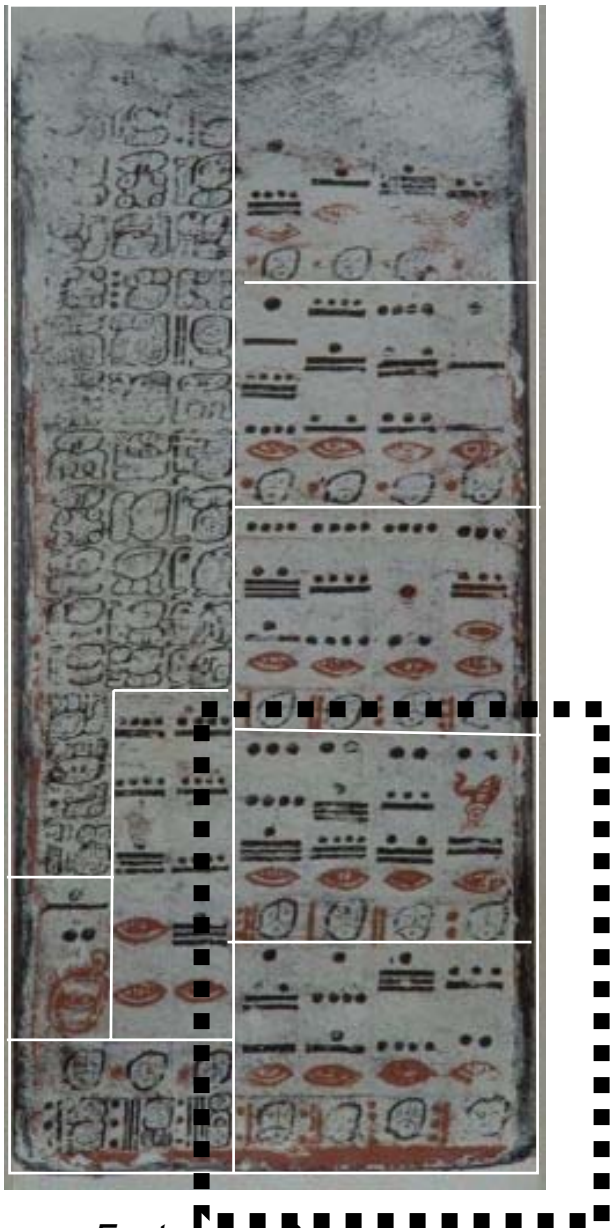
Reading a Page of the Dresden Codex

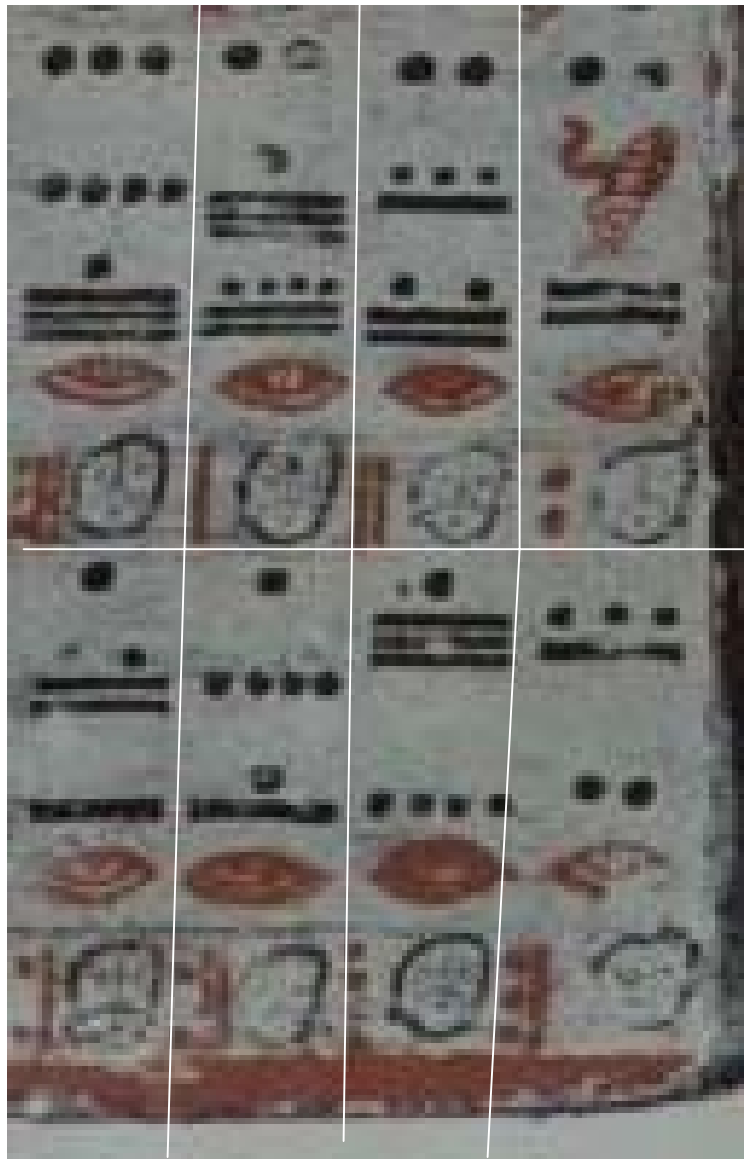


Basic Layout

Fosterman 24

Reading a Page of the Dresden Codex





Fosterman 24

Reading a Page of the Dresden Codex

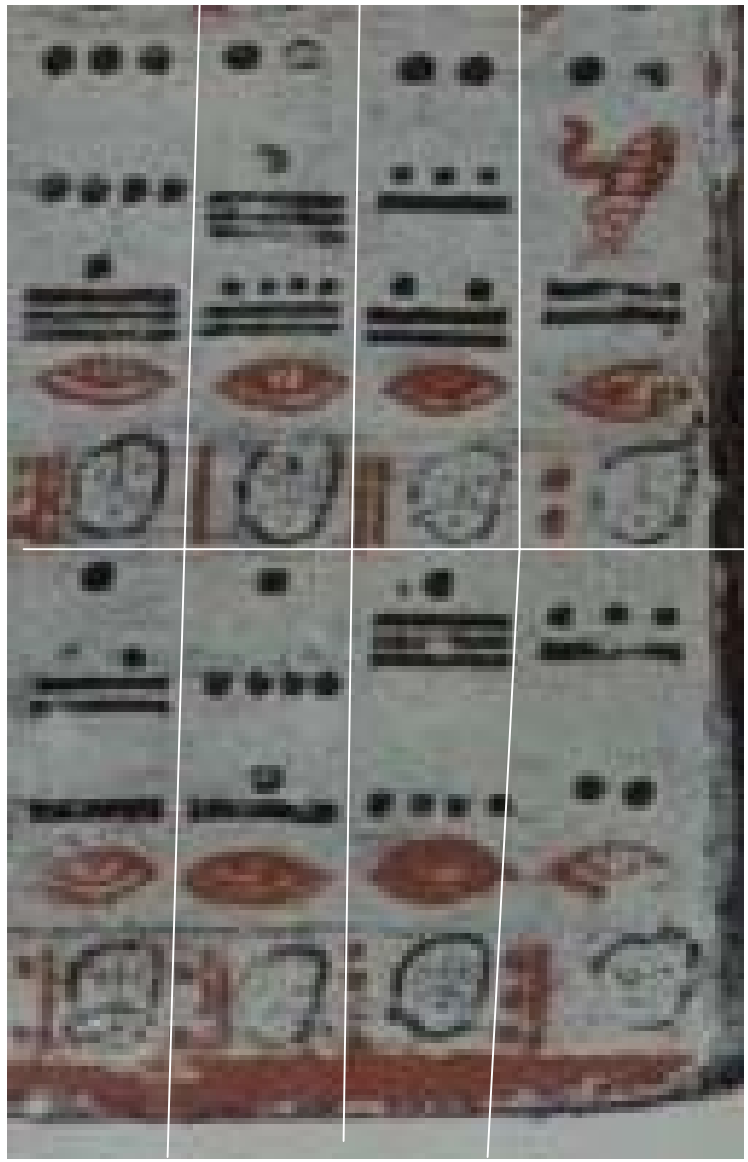
8A	7A	6A	5A
3	2	2	2
4	16	8	0
16	14	12	10
0	0	0	0

13 Ahau	5	10	2

4A	3A	2A	A
1	1		
12	4	16	8
8	6	4	2
0	0	0	0

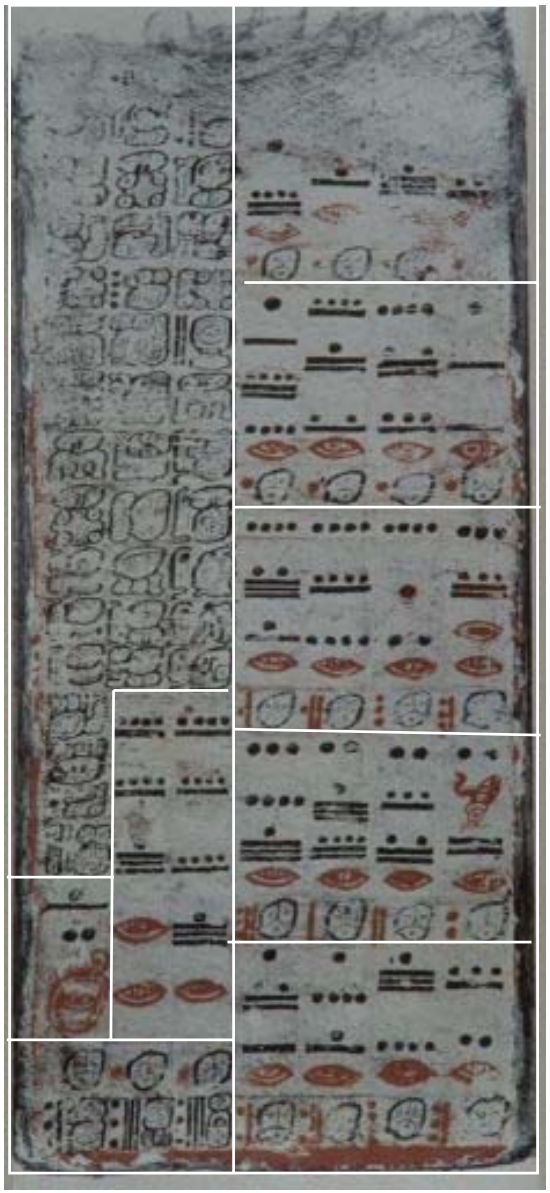
7 Ahau	12	4	9

A = 8.2.0 = 2920 = 5 x 584



Fosterman 24

Reading a Page of the Dresden Codex



260V 195V 130V 65V

Preface to the Venus Tables

Venus Cycle corrections

60V 55V 50V 45V

40V 35V 30V 25V

20V 15V 10V 5V

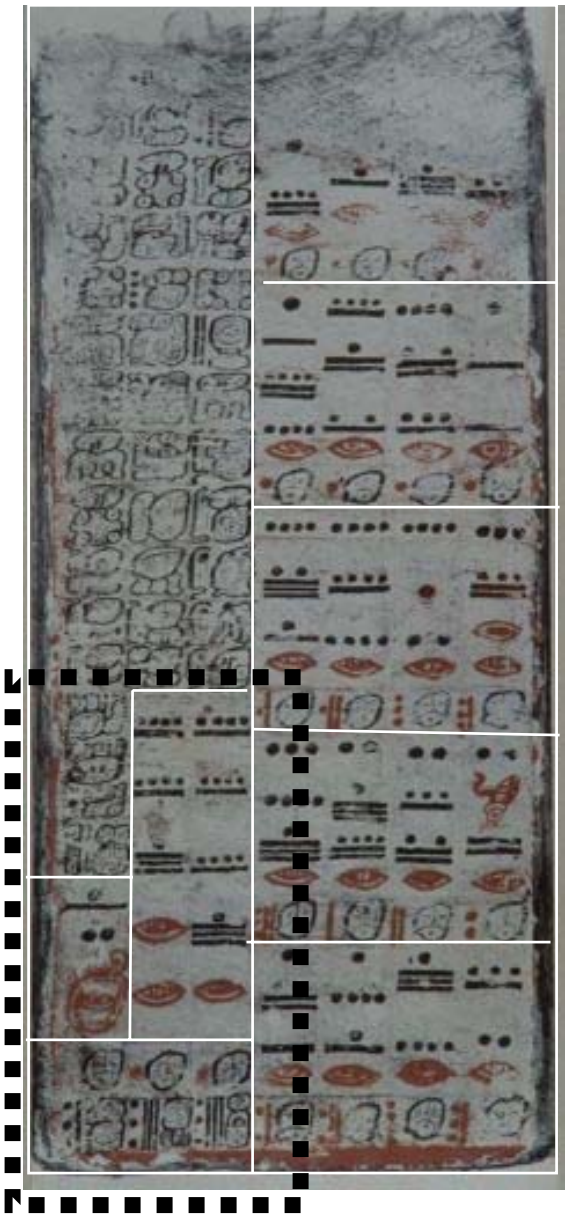
A = 2920
= 8 x 365
= 5 x 584

584 = Venus Synodic Rev.

4A 3A 2A A

Fosterman 24

Reading a Page of the Dresden Codex



Fosterman 24

Preface to the Venus Tables

Normal Base "origin"		*4Ahau 8 Cumhu*
Minus Ring Number and –B	*-6.2.0*	*1 Ahau 18 Kayab*
Distance no. on pre-zero day	*9.9.16.0.0*	
Terminal date, nrml.day no.	*9.9.9.16.0*	*1 Ahau 18 Kayab*

Ring Number: 6.2.0

9.9.16.0.0
9.9.9.16.0

4 Ahau 8 Cumhu
1 Ahau 18 Kayab
1 Ahau 18 Uo

Two Facts of Mayan Positional Notation

1. All known examples of native positional Mayan numbers are calendrical or astronomical examples.

Almost True

2. All known Mayan numbers [illegible]nal notation use at most five positions

NOT True

Brickner & Brickner, p193

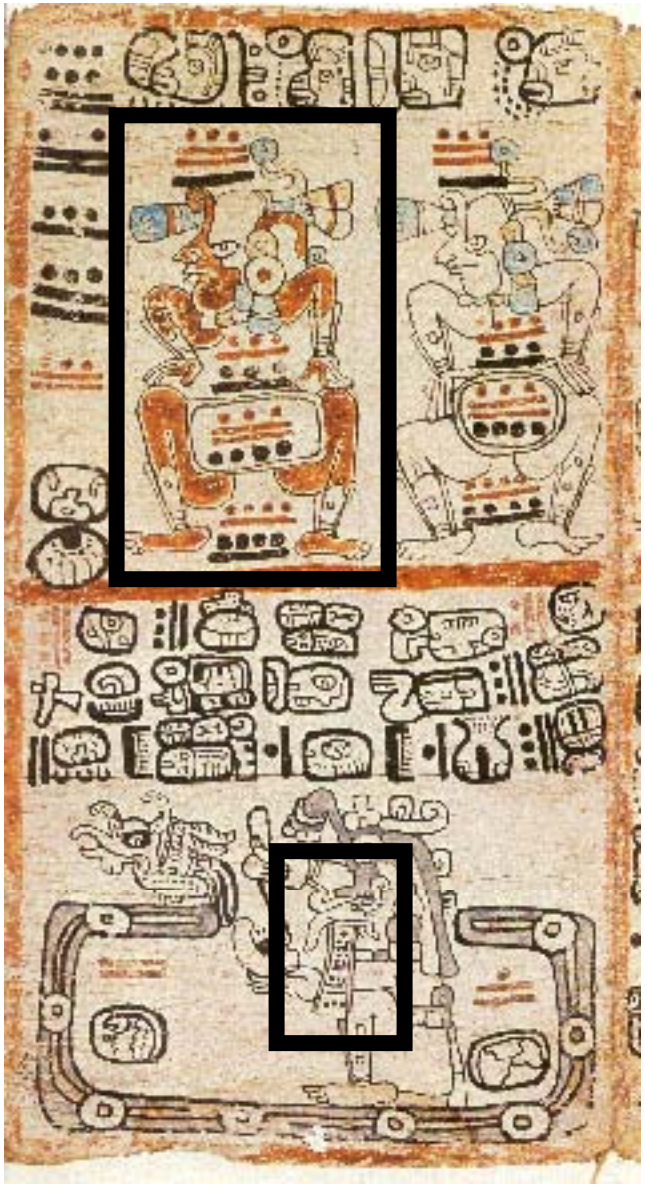
"...They have other very long counts, extended to infinity, counting twenty times 8000, or 160,000; then they multiply this 160,000 again by twenty, and so on until they reach an uncountable figure. They do all this counting on a flat Surface"

Landa
Relación de las Cosas de Yucatán
Sec.XXIV

Codex Madrid

Graz 73

1.
7.
2.
2.
12.
13.
3

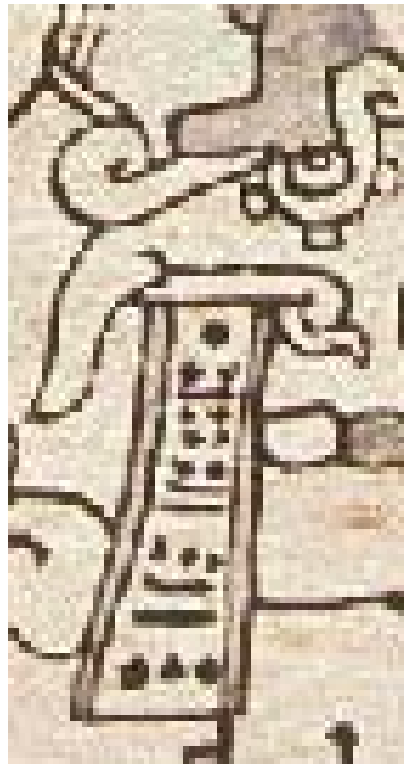


Graz 73

n

13.5.8.8.13.4.9.9

16’’986’985,789



d

1.7.2.2.12.13.3

86’741,063

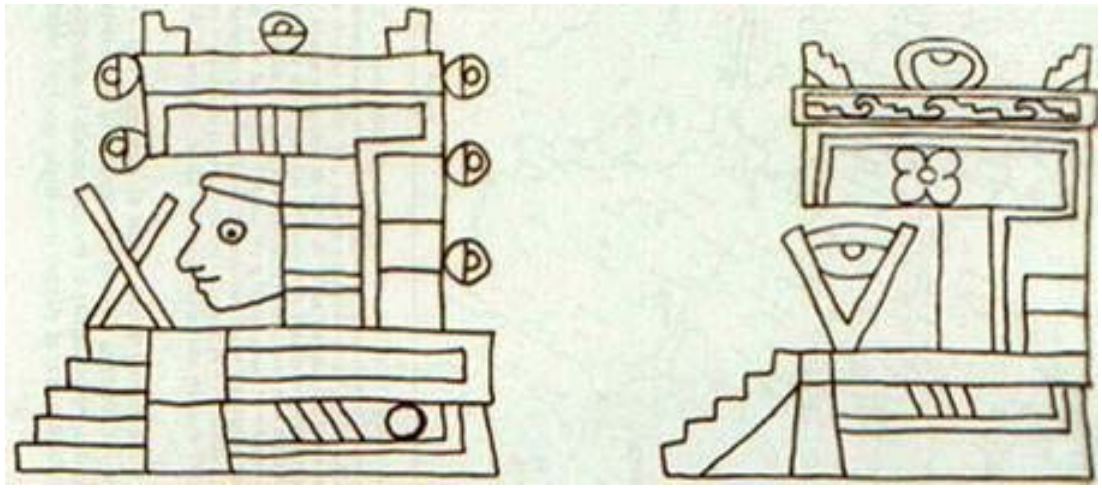
n/d=195.836

Contents

1. Domain of Discourse
2. Mathematical Applications:
   - Tribute Rolls
   - Cadastral Registers
   - Calendrics:
     - Calendrical Conventions
     - Maya Numerical Notation
     - Maya Astronomy
3. Final Remarks

Naked-eye Astronomy

Mayan Astronomical Cycles



1. Solar Cycles
2. Lunar Cycles
3. Eclipses
4. Venus
5. Mars
6. Ecliptic Constellations (Zodiac)

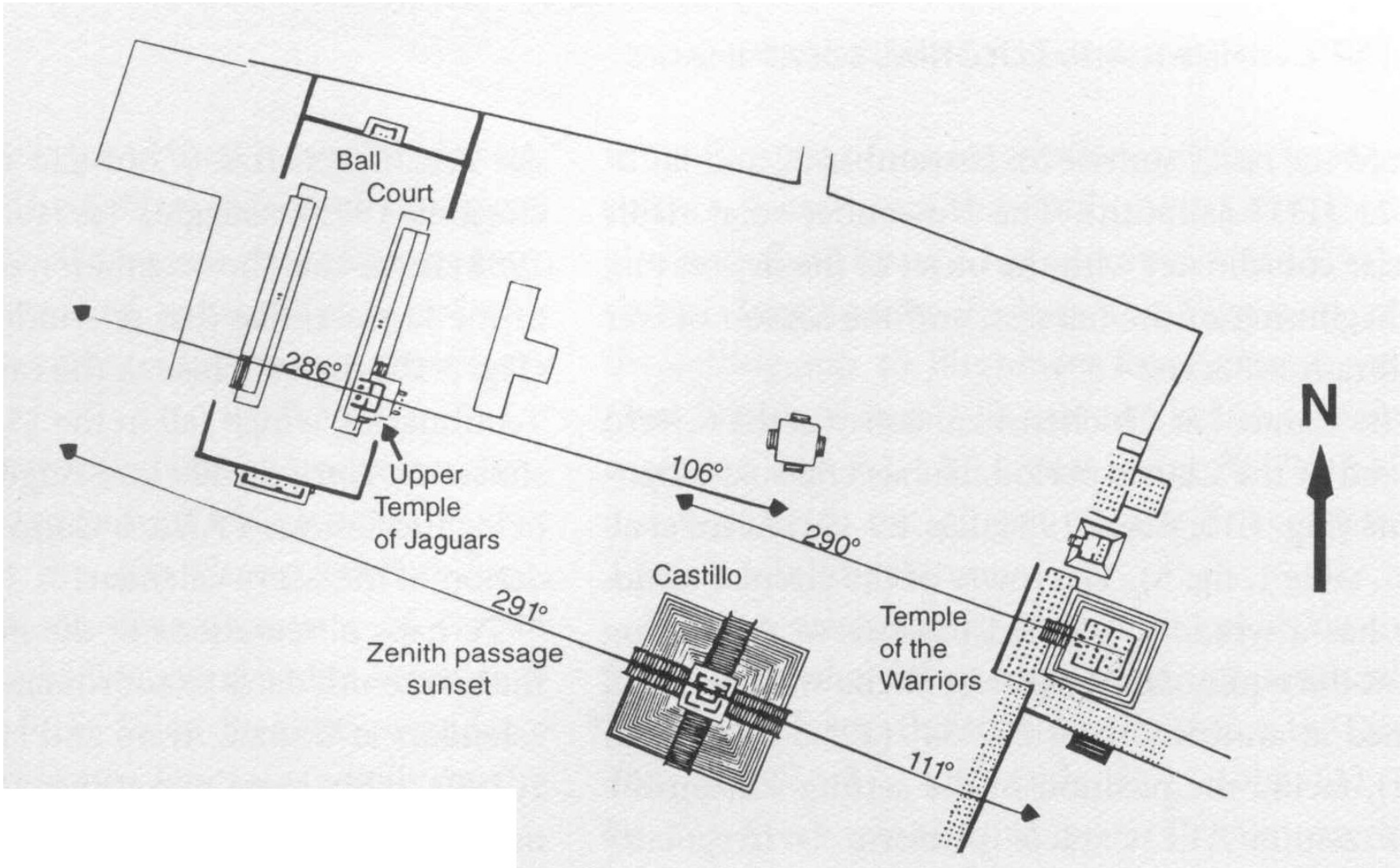
Solar Cycles

- Equinox / Solstice
  - Alignments in:
    - Uaxactún (Group E)
    - Chichen Itzá (Caracol, Castillo)
    - Dzibilchaltun,...

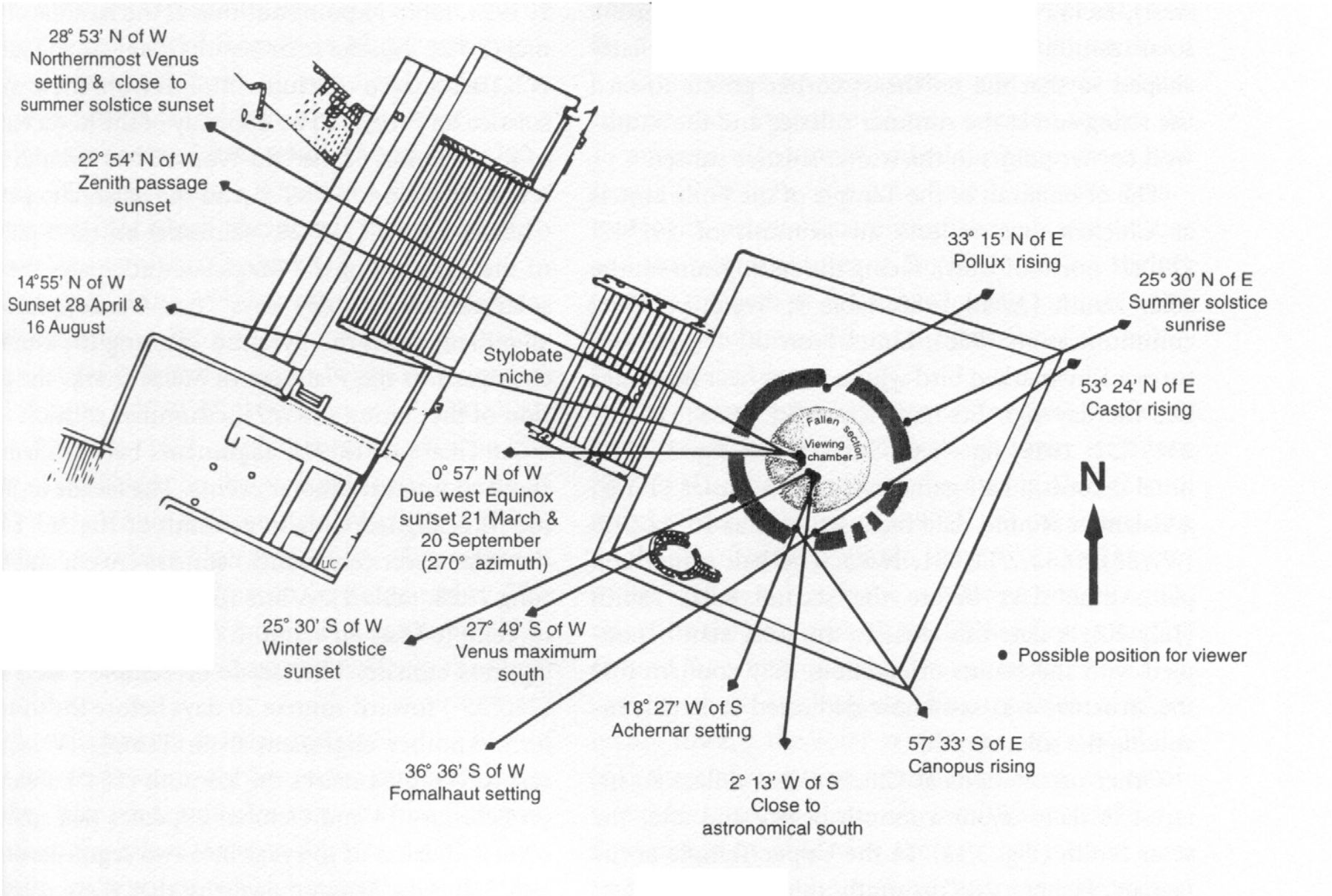
O.Casares: Dzibilchaltun

  - Shadow plays in:
    - Chichen Itzá (Castillo)
    - Uxmal (C. De las Monjas)

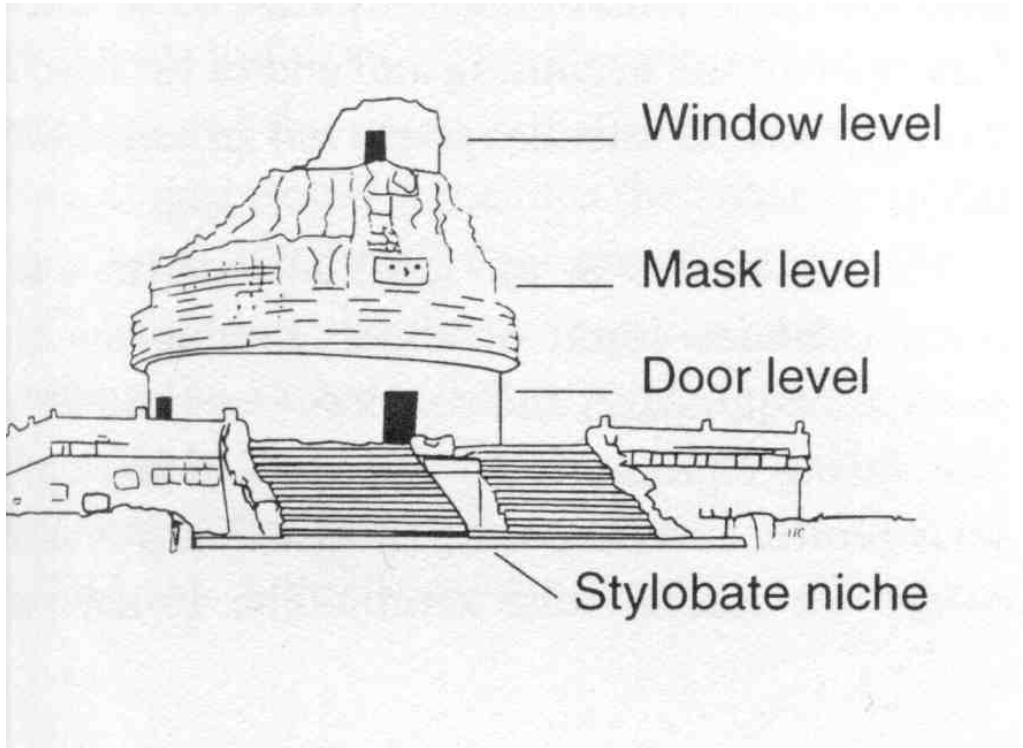
Solar Cycles
(Alignments in Chichén Itza)



Solar Cycles
(Alignments in Chichén Itzá)

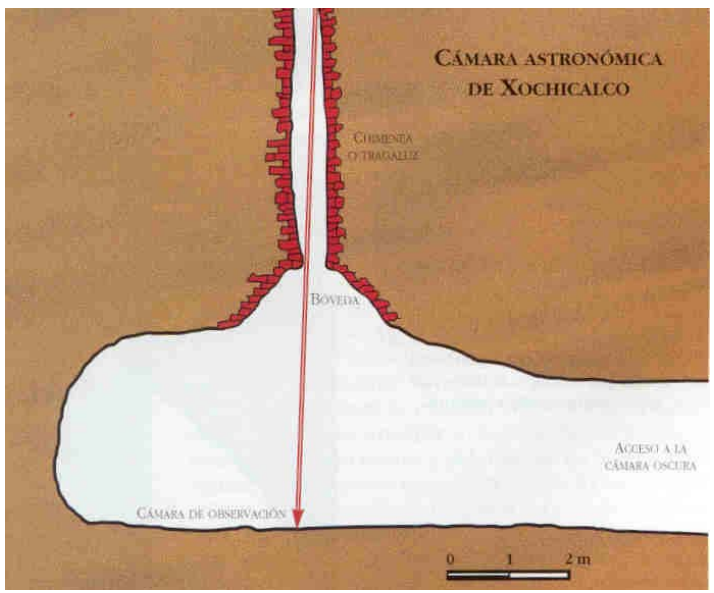


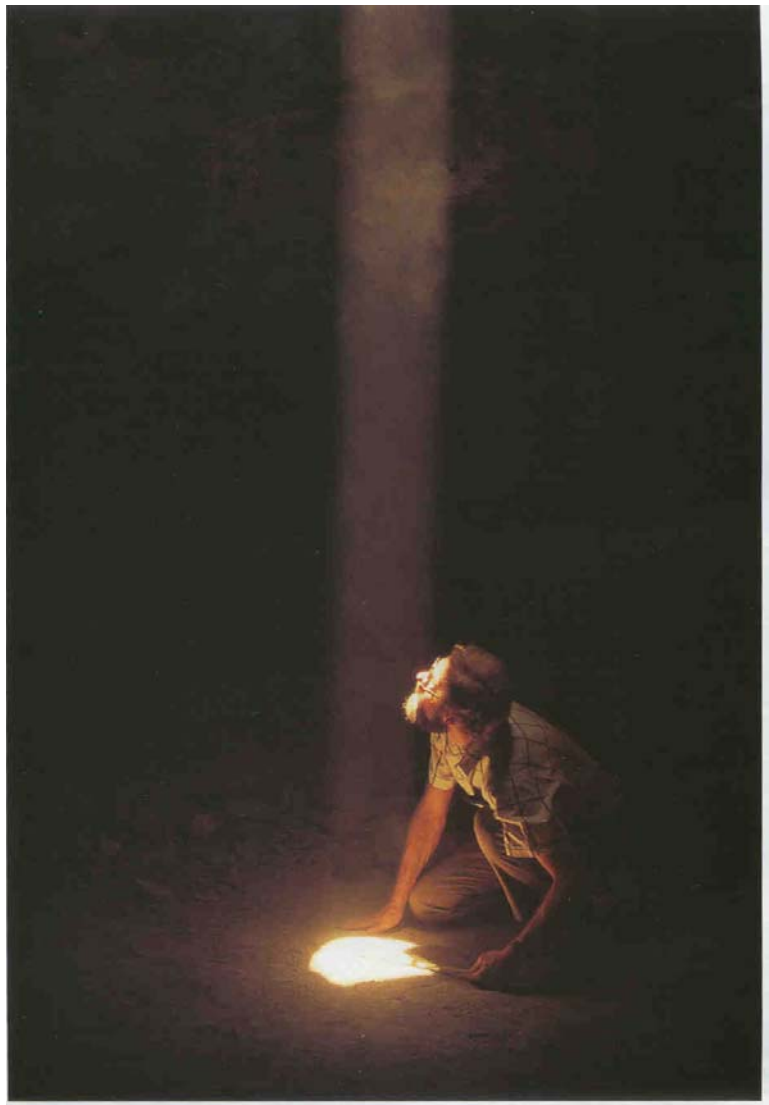
Solar Cyclesw
(Alignments in Chichén Itzá)



Solar Cycles

- Zenithal Transit (105-day period)
  - Copán, Izapa
  - Chichén Itzá (Caracol)
  - Teotihuacán, M. Albán
  - Xochicalco (2° correction)





Venus Cycles

- Venus Synodic Revolution = 584 days
  - Venus as Morning star: 263 days
  - Superior conjunction: 50 days
  - Venus as Evening Star: 263 days
  - Inferior Conjunction: 8 days
- 5 Venus Synodic Revolutions = 8 Haab
- Dresden pp.24-29
  - Conmensurates (VSR, Haab, Tzolkin)
  - i.e. lcm(584, 365, 260) = 37960 days = 584 x 5 x 13 x 8

Lunar Cycles

- 29.53059 days
  - Copan = 29.530201
  - Palenque = 29.530864

- “Complementary lunar series” in Long Count Dates (between Tzolkin and Haab dates):
  - 18 “months”, divided into “trimesters”(6months each)
  - Age of the Moon
  - Lunar month length (29 or 30 days)
  - Ocasional extra 30-d month

Eclipses

- Codex Dresden pp. 51-58. Tables for 33 years, or 405 lunar months with correction values.

Fosterman 52 (fragment)

Fosterman 54

Martian Cycles

- Martian Synodic Revolution = 780 days
- Martian Synodic Revolution = 3 Tzolkin
- Dresden pp. 43-45 (?)

Ecliptical Constellations

- 13 constellations in the Ecliptic

- 364 days = 13 constellations × 28 days

- Codex Paris
- Lintel, Las Monjas Annex (Chichen Itza)
- Throne, Governor Palace (Uxmal)

Contents

1. Domain of Discourse

2. Mathematical Applications

3. Final Remarks
   - An Overview
   - Open Questions

The Maya Scribe

"The *chilanes* were charged with giving all those in the community the oracles of the demon, ...casting lots in divination, ...They awarded them utmost respect"

Landa
Relación de las Cosas de Yucatán
Sec. XXVII

Scribes in a Maya Vase

Scribes in a Maya Vase


K5721

Mathematical Concepts in Mesoamerica

- Evidence of
  - Zero (Maya, *Aztec*)
  - Negative Numbers (Maya)
  - Fractions (*Aztec*)
  - Positional notation and other *conventions*
  - Modular Arithmetic (Maya)
  - Area Calculation (Aztec)

Mesoamerican Mathematics

- Open Topics
  - Geometric and numerical concepts in Architecture
  - Geometric and Topological concepts in art
  - Demographic and Geometric analysis of Cadastral Registers
  - Economic analysis of tribute
  - Algorithms
  - Metrics
  - Systematic scribal errors

- Hermeneutical Concerns
  - Internal coherence
  - Catachronism / anachronism
  - Iconic languages


K1523