27th International Congress of Actuaries

ICA CANCUN 2002

The Immunisation of a Workers' Compensation Fund

Carlos Pereira da Silva (ISEG)
Joaquim Cadete (Univ. Católica)
João Pedro Nunes (CEMAF/ISCTE)

A New Millennium. A New Challenge for Actuaries

1. Plan of Presentation

1st Part

1. Workers' Compensation Insurance in Portugal
2. The Market Operating Account
3. The Model of Deterministic Immunisation
4. Empirical Application

2nd Part

5. Stochastic Approach - Models
6. Longstaff & Schwartz (1992)
7. Immunisation Rule
8. Model Estimation
9. Conclusions

2. The Workers' Compensation Social Insurance in Portugal

Aims:

- **compensation** for **loss of income** due to **temporary incapacity**
- **pension for life** in event of **permanent disability**

Those insured:

3rd-party **legal obligation** on **employers** to cover **all the work-force**

Also, the **self-employed** (since 1999)

Management:

Created by law in **1936.**

Managed by **insurance companies** since **1965.**

3. The Market Operating Account for Workers' Compensation Insurance

TABLE 1

Workers' Compensation in Portugal

	1990	1995	1996	1997
Premiums	**253**	**376**	**391**	**396**
Earned Premiums	99%	99,1%	99,1%	99,7%
Loading	80,3%	78,1%	81,3%	83,5%
Underwriting Margin	19,7%	21,0%	17,7%	16,2%
Operating Costs	40,2%	29,2%	30,4%	28,5%
Net Margin	-20,5%	-8,2%	-12,7%	-12,4%
Reinsurance Balance	0,2%	-0,2%	-0,1%	-0,1%
Op. Result	**-20,3%**	**-8,4%**	**-12,7%**	**-12,5%**
Fin. Income	**23,6%**	**17,0%**	**19,1%**	**27,3%**
Final Result	**3,3%**	**8,6%**	**6,3%**	**14,8%**
Technical Provisions	537.960.166	741.039.096	822.657.396	911.608.025
TP/Premiums	**2,13**	**1,97**	**2,10**	**2,30**

U:Millions Euros

4. The Model of Deterministic Immunisation of Worker's Compensation Pensions

Workers' Compensation pensions are sensitive to ***interest-rate movements*** and to ***longevity***:

$$a_x \le a_{\overline{n|}}, \quad \forall x$$

- where *n* equals the pensioner's life expectancy at age x

Effect on the annuity when:

- change in the mortality rate = change in the continuous interest rate

5. Redington Immunisation

As long as:

$$\frac{\partial VA}{\partial r} = \frac{\partial VL}{\partial r} \quad \textbf{(Duration)}$$

$$\frac{\partial^2 VA}{\partial r^2} > \frac{\partial^2 VL}{\partial r^2} \quad \textbf{(Convexity),}$$

where VA= the present value of the assets

VL= the present value of liabilities

r= the market interest rate

- The Fund is immunised.
- **BUT** in this situation, **arbitrage profits** could **occur**

$$S(r+\Delta r) \cong S(r) + S'(r)\Delta\delta + \frac{1}{2}S''(r)(\Delta r)^2$$

As $S''(r) > 0$ thus $S(r+\Delta r) > S(r)$.

6. Empirical Application

- Portfolio of a Portuguese insurance company (3% of the market)

Year	Cash-flow of assets	Present value	Cash-flow of liabilities	Present value
1	1.263.960	1.192.415	1.445.220	1.363.416
2	4.661.528	4.148.744	1.434.743	1.276.916
….				
….				
8	1.613.616	1.012.403	1.327.191	832.696
9	2.762.996	1.635.413	1.303.313	771.429
10	12.428.006	6.939.733	15.305.874	8.546.720
	Total	20.282.348		17.959.217
	Net value			2.323.136
	Dur.	**6,4**	**6,73**	
	Disp.	**12**	**10,88**	

U: Euros, Present Values at 6% interest rate

-The two conditions:

- **VA*DA>=VL*DL and**
- **DA > DL**

are fulfilled.

7. Stochastic Approach - *models*

Two *equilibrium*, *affine* and *non-negative* interest rate models, are compared:

- Only need to model discount factors (⇒ equilibrium framework);
- With explicit solutions (⇒ affine models); and
- Under realistic assumptions (⇒ square-root processes).

8. Stochastic Approach – *L&S (1992)*

Two-factor (r, v) model: Longstaff and Schwartz (1992).

– **Discount factors**:

$$P(r,v,\tau) = A^{2\gamma}(\tau)B^{2\eta}(\tau)e^{(k\tau + C(\tau)r + D(\tau)v)}$$

where $A(\tau), B(\tau), C(\tau), D(\tau)$

are model-dependent deterministic functions.

– **consistent with PCA results**.

9. Stochastic Approach - *immunisation*

- Immunisation rule: $$\left\| \frac{1}{VA} \frac{\partial VA}{\partial \underline{X}'} \cdot \sigma(\underline{X}) \right\|^2 = \left\| \frac{1}{VL} \frac{\partial VL}{\partial \underline{X}'} \cdot \sigma(\underline{X}) \right\|^2$$

where

VA (VL) is the present value of assets (liabilities), $\underline{X}$ is the vector of state variables, and matrix $\sigma(\underline{X})$ is its instantaneous variance.

- That is: adjust stochastic durations of assets $\left(\frac{1}{VA} \frac{\partial VA}{\partial \underline{X}'}\right)$ and of liabilities $\left(\frac{1}{VL} \frac{\partial VL}{\partial \underline{X}'}\right)$.

10. Empirical Analysis – *model estimation*

- Cross-sectional fit to gross prices of German Govt. Bonds (with synthetic zero-coupons for short maturities),

- Through Non-linear Least Squares, and

- Subject to: r = O/N; v = 0.15%; admissibility restrictions.

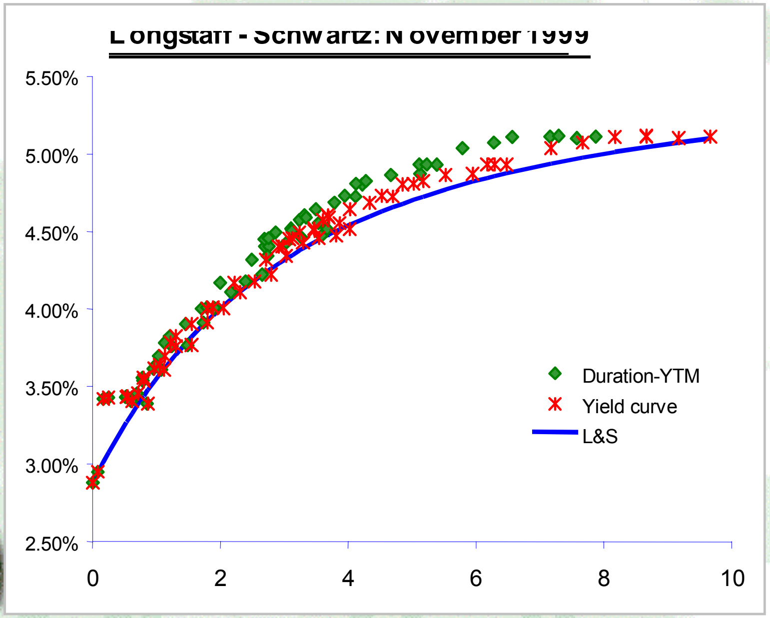
11. Empirical Analysis – *model estimation*

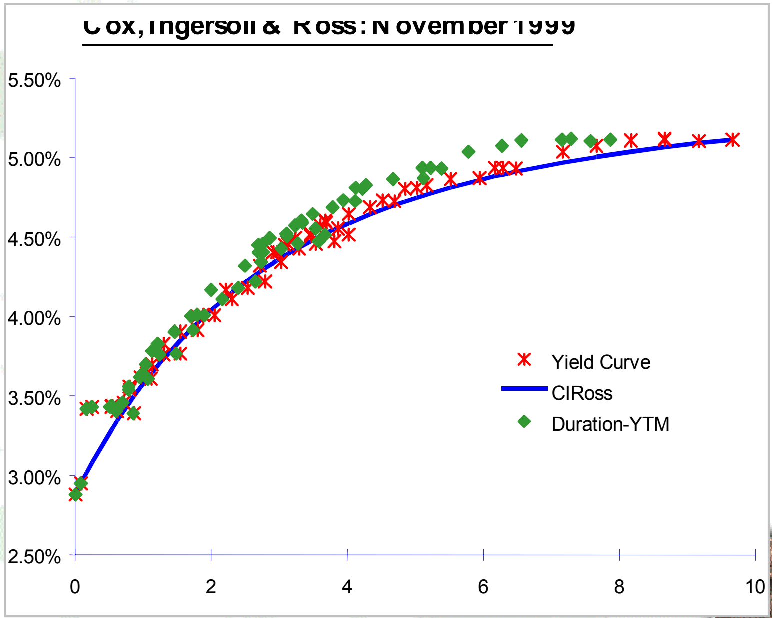
Longstaff & Schwartz

	03-Nov-99
r	0.0288
V	0.0015
α	0.0200
β	0.0621
γ	0.0000
δ	0.7558
η	0.5483
λ	-2.0560
ξ	2.6470

Cox, Ingersoll & Ross

	03-Nov-99
r	0.0288
$\bar{r}$	0.0533
σ	0.0015
α	0.6484





12. Empirical results

Longstaff & Schwartz (03-Nov-99)

time (years)	discount factor	cash flows from assets	assets' present value	cash flows from liabilities	liabilities' present value
1	0.96532	1,263,960	1,220,123	1,445,220	1,395,097
2	0.92381	4,661,528	4,306,366	1,434,743	1,325,430
3	0.88011	886,767	780,456	1,406,500	1,237,880
....					
....					
....					
7	0.71465	2,838,568	2,028,588	1,342,167	959,182
8	0.67772	1,613,616	1,093,575	1,327,191	899,460
9	0.64264	2,762,996	1,775,609	1,303,313	837,560
10	0.60935	12,428,006	7,573,018	15,305,874	9,326,650
	Total		**21,663,472**		**19,268,452**
Stochastic duration (years)			3.500		3.887

U: EUR

The value of the liabilities has increased about 8% in comparison with the deterministic model (90% of the value of assets)

13. Conclusions

• The value of the **liabilities** is significantly **higher** in the **stochastic approach**, causing a **mismatch of approximately 8%** in the case studied.

• The **durations** of **assets and liabilities** are considerably **lower with a stochastic model** ⇒ need to consider the **volatility** of the interest rates and its influence on the net value.

• The legal (mandatory) discount rate for insurance liabilities related to Workers' Compensation had to be revised **down**, in order to accommodate the environment of low interest rates induced by the **European Monetary Union** process (1992 – 1999).