The cost of target capital in the valuation of life annuity business

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Abstract

The classical actuarial approach to the valuation of a life portfolio comes from the embedded value framework, under which the value of in-force business is given by the present value of future industrial profits net of the cost of capital, the amount of the latter being a function of the discount rate.

In recent years, the adoption of market-consistent valuations of the insurance business has been advocated, mainly because of the lack of transparency of the classical model in setting the discount rate, joint to its inadequacy in reflecting properly the cost of the risks encumbering the portfolio. A market-consistent value usually acknowledges a reward for shareholders' capital as long as the market does, i.e. if the risk is systematic or undiversifiable.

Aim of this paper (which actually represents a preliminary study) is to investigate how the cost of shareholders' capital can be assessed referring to a portfolio of immediate life annuities, and allowing in particular for uncertainty in future mortality trends, namely for longevity risk. To this purpose a link between traditional valuations and market-consistent valuations is proposed, whence a proper risk discount rate can be obtained.

Keywords

Target capital, capital allocation, solvency, embedded value, market-consistent value, cost of capital, risk discount rate, longevity risk

1. Introduction

The classical actuarial approach to the valuation of a life portfolio comes from the embedded value framework, under which the value of in-force business is given by the present value of future industrial profits net of the cost of shareholders' capital, the amount of this cost being a function of the discount rate. See, for example, Turner (1978), Burrows and Whitehead (1987), LAVMWP (2001), Olivieri and Pitacco (2005).

A number of papers have been devoted to problems concerning the choice of the discount rate in the valuation process; usually it is meant that such a rate includes a proper reward for the risk inherent in the flows to be discounted and therefore it is referred to as a “risk discount rate”. The reader can refer, for example, to Turner (1978), Burrows and Whitehead (1987), Pemberton *et al.* (2000). In particular, Sherris (1987), Coleman *et al.* (1992) and Burrows and Lang (1997) discuss the use of the CAPM for determining the risk discount rate to be used in the life insurance valuation process.

In recent years, the adoption of market-consistent valuations of the insurance business has been advocated, mainly because of the lack of transparency of the classical model in setting the discount rate, joint to its inadequacy in reflecting properly the cost of the risks encumbering the portfolio. See, for example, CFO Forum (2004a, 2004b) and Tillinghast-Towers Perrin (2004). A market-consistent value usually acknowledges a reward for capital as long as the market does, namely if the risk is systematic or undiversifiable.

Shareholders' capital must be allocated to each portfolio, aiming at increasing the assets backing the portfolio liabilities and hence facing the risk of a poor experience, because of mortality, yield from investment, expenses, etc. Capital allocation should be the result of calculations worked out via an appropriate "internal model" (see, for example, Brender (2002)) whereas, in practice, only the legal requirements - typically concerning the solvency margin - are frequently accounted for. Whatever the allocation policy may be, the cost of capital depends on the capital actually assigned to the portfolio.

Risks affecting insurance portfolios and capital allocation policies have been analyzed by many Authors, from both a theoretical and a practical point of view. For useful insights into this topic, the reader can refer to the report by the IAA (2004).

When life annuity portfolios are concerned, special attention should be devoted to the "longevity risk". As is well known, observations of past mortality suggest to adopt projected mortality models for the actuarial appraisal of annuities (and other living benefits), i.e. to use mortality assumptions including a forecast of future mortality. Notwithstanding, whatever hypothesis is assumed, the future mortality trend is random, whence an uncertainty risk arises. When this risk mainly refers to mortality trend at old ages, it is usually called longevity risk.

Unlike the risk of random fluctuations in mortality, the longevity risk is a "systematic" risk, namely a risk of systematic deviations from the expected mortality (see, for example, Olivieri (2001)). Hence, it cannot be pooled, i.e. diversified increasing the portfolio size. Appropriate capital allocation strategies, driven by an adequate target capital, must be determined in order to manage the longevity risk (for example, see Olivieri and Pitacco (2003)). Clearly, capital allocation should coexist with other technical tools, including an appropriate pricing for life annuity products, traditional reinsurance arrangements, alternative risk transfers via modern financial instruments, e.g. longevity bonds. The latter topic is dealt with, for example, by Blake and Burrows (2001), Cairns et al. (2004), Lin and Cox (2005).

As mentioned above, cost of shareholders' capital is a key element in the portfolio valuation process. Aim of this paper is to investigate how the cost of capital can be assessed referring to a portfolio of life annuities and allowing in particular for the need of capital facing the longevity risk. The investigation is a preliminary study, in particular as far as the pricing of longevity risk is concerned.

The paper is organized as follows. In Section 2 the traditional approach to portfolio valuations and the market approach are briefly described and compared. Section 3 focusses on capital allocation for solvency purposes and the notion of target capital. The survival model, allowing for longevity risk, adopted to represent annuitants' mortality is described in Section 4. Portfolio valuation, accounting for longevity risk and the related capital allocation, is addressed in Sections 5 and 6. Numerical results are presented in Section 7. Some remarks in Section 8 conclude the paper.

2. Portfolio valuation: the traditional vs the market approach

The traditional approach to portfolio valuation is based on the so-called "value of the in-force business" (VIF), defined as the present value of future distributable earnings calculated with a given Risk Discount Rate (RDR), net of the amount of shareholders' capital currently within portfolio assets. The distributable earning related to a given period, say a year, is defined as the flow from the portfolio to the insurance company (or vice versa) such that portfolio assets are equal to a given level, viz the mathematical reserve plus the target capital (for a definition of target capital, see Section 3).

According to this definition, the VIF at time t, VIF_t, is given by

$$VIF_t = \sum_{h=t+1}^{n} K_h \, v_\rho(t,h) - M_t \tag{2.1}$$

where K_h denotes the distributable earning in year h, $v_\rho(t,h)$ is the discount factor based on the annual RDR's $\rho_{t+1}, \rho_{t+2}, \ldots, \rho_h$, and M_t is the shareholders' capital contributing to the portfolio assets at time t.

It is possible to show (for example, see Olivieri and Pitacco (2005)) that, under reasonable hypotheses, an alternative (equivalent) expression for the VIF is as follows:

$$VIF_t = \sum_{h=t+1}^{n} U_h^I \, v_\rho(t,h) - \sum_{h=t+1}^{n} M_{t-1} \left(\rho_h - i_h^*\right) v_\rho(t,h) \tag{2.2}$$

where U_h^I denotes the "industrial profit" (an example referred to an annuity portfolio is provided by formula (5.2)), i_h^* is the estimated yield from investments, and $\left(\rho_h - i_h^*\right)$ represents the "risk premium" for one monetary unit of shareholders' capital (usually constant over time) rewarding shareholders for the risks encumbering the life portfolio (all but the market risk, whose reward is already embedded in i_h^*). The first term on the right-hand side of equation (2.2) is usually called "present value of future profits" (PVFP), whereas the second term represents the "cost of capital", i.e. the present value of risk premiums required on the allocated capital. Hence, equation (2.2) can also be written as follows:

$$VIF_t = PVFP_t - CC_t \tag{2.2'}$$

It is worth stressing some features of the traditional valuation model, represented by equations (2.1) to (2.2'). First, a deterministic approach is adopted for the valuation of future cash flows. The calculation of main quantities, e.g. the distributable earnings K_h in formula (2.1), is commonly based on the best-estimate scenario. Conversely, in latest years stochastic models are frequently used for the evaluation of options and guarantees embedded in life insurance products.

Secondly, in the traditional valuation model, portfolio riskiness could be allowed for mainly through the RDR's ρ_h, which as mentioned should account for various risks inherent in the portfolio itself (mortality risk, investment risk, etc.), but also for inefficiencies in managing the portfolio, etc. However, the RDR's are usually chosen according to current market practice, whence they are not specifically tailored to portfolio features.

The market approach to portfolio valuations aims at overcoming some weak points of the traditional actuarial model. It resorts to a "risk-neutral" valuation approach, according to which the discount factor is based on the risk-free rate. It follows, in particular, that the term CC_t (see equation (2.2')) is not accounted for.

It is worth noting that, according to this approach, only undiversifiable risks (in particular systematic risks) are rewarded. In practice, risks with a market evidence can be “easily” accounted for. The value of the portfolio to the insurance company is anyhow affected also by:

(a) systematic risks with no market evidence;

(b) inefficiency in managing the portfolio (for example, pooling risks not fully diversified);

(c) agency costs.

The longevity risk belongs to the class (a) above. In Sections 5 and 6 we describe a possible approach to the problem of including longevity risk in the cost of capital.

3. Solvency and target capital

Shareholders' capital must be assigned to each insurance portfolio, with the goal of a high probability of meeting the relevant obligations, namely aiming at solvency. Solvency assessment, if worked out on a sound basis (e.g. via an "internal model"), requires an appropriate evaluation of the risks borne by the insurer. In particular, mortality risks and market risks should be accounted for.

The result of the solvency assessment procedure, applied at time t, is the "target capital" M_t^{target}, which should be allocated to the portfolio. An amount of assets equal to the sum of the mathematical reserve and the target capital should ensure that the insurer's obligations will be met (over an assigned time horizon) with a given (high) probability.

The total amount of assets required in order to meet future obligations is also called the "solvency reserve". This amount can be globally determined, via an appropriate stochastic model, looking at the probability of meeting obligations over the stated time horizon, and hence disregarding, a priori, the concept of mathematical reserve (for example, see Olivieri and Pitacco (2003)). Splitting the solvency reserve into mathematical reserve and target capital is then a matter of regulation (which may require, for example, a minimum mathematical reserve for a given portfolio).

A (simplified) procedure for calculating the target capital facing the longevity risk in a portfolio of life annuities is presented in Section 5.

As regards the amount of shareholders' capital allocated to a portfolio, further constraints may come from requirements other than the solvency assessment worked out by the insurance company and leading to the target capital M_t^{target}. In particular, a minimum capital allocation, M_t^{leg}, may be required by law (e.g. the "required solvency margin", according to the European legislation). Moreover, a minimum capital allocation M_t^{rating} may arise from rating criteria. Clearly, the shareholders' capital, $M_t^{(r)}$, to allocate in order to meet the three requirements is given by

$$M_t^{(r)} = \max\left\{M_t^{\text{target}}, M_t^{\text{leg}}, M_t^{\text{rating}}\right\} \tag{3.1}$$

In what follows we assume the simplified requirement

$$M_t^{(r)} = \max\left\{M_t^{\text{target}}, 0.04\, V_t\right\} \tag{3.1'}$$

i.e. we disregard rating requirements, and assume the required margin calculated according to the European legislation. As far as the calculation of M_t^{target} is concerned, we adopt alternatively parameters based on the Solvency 2 project and on internal rules of conduct (see Section 7).

4. The survival model

In what follows, we refer to a portfolio of immediate life annuities, initially consisting of a given number N_0 of annuitants, all aged x_0. We assume that the portfolio is homogeneous in terms of entry time, age (hence it consists of a "cohort"), annual amount, etc. For simplicity, we disregard expenses and consider unitary annual benefits, to be paid at the end of each year.

As regards the survival model, we assume that the random lifetime of a generic annuitant can be described by the Weibull distribution, namely with probability density function given by

$$f_0(t \mid \alpha, \beta) = \frac{\alpha}{\beta}\left(\frac{t}{\beta}\right)^{\alpha-1} e^{-\left(\frac{t}{\beta}\right)^{\alpha}} \tag{4.1}$$

where the parameters α, β (common to all annuitants) are unknown, consistently with the uncertainty in future mortality trend. The assumed (discrete) probability distribution of the parameters is given in Table 4.1. We point out that the best estimate (BE) scenario, namely the scenario with the highest probability, is given by $\alpha = 9.15,\ \beta = 85.2$. In any case the maximum duration of life has been set equal to 120 years.

Some features of the mortality scenarios result from Tables 4.2 and 4.3. In particular, Table 4.2 shows the relation between the parameters and the location of the Lexis point. So, an appropriate choice of the parameter values allows us to represent the so-called "expansion" phenomenon (for example, see Olivieri (2001)), whereas the randomness in future expansion trend is expressed by the parameter uncertainty and the relevant probability distribution (see Table 4.1).

α \ β	**82**	**83.5**	**85.2**	**87**	**89**	
7	0.01033	0.03155	0.04352	0.02287	0.00200	0.11028
8	0.00933	0.03055	0.04832	0.02582	0.00600	0.12003
9.15	0.00833	0.02955	0.39708	0.02828	0.00500	0.46825
10.45	0.00733	0.02755	0.11204	0.02701	0.00400	0.17794
12	0.00633	0.02855	0.06301	0.02461	0.00100	0.12351
	0.04165	0.14777	0.66397	0.12859	0.01801	1

Table 4.1 - Probability distribution of the Weibull parameters

α \ β	**82**	**83.5**	**85.2**	**87**	**89**
7	80.214	81.681	83.344	85.105	87.062
8	80.643	82.118	83.790	85.560	87.527
9.15	80.969	82.450	84.129	85.906	87.881
10.45	81.214	82.700	84.384	86.167	88.147
12	81.408	82.897	84.584	86.371	88.357

Table 4.2 - Modal age at death (Lexis point)

α \ β	**82**	**83.5**	**85.2**	**87**	**89**
7	82.599	90.181	99.035	108.646	119.473
8	69.555	76.119	83.758	92.042	101.422
9.15	58.894	64.518	71.013	77.999	85.857
10.45	50.135	54.895	60.337	66.126	72.563
12	42.406	46.326	50.749	55.389	60.477

Table 4.3 - Variance of the lifetime

Conversely, Table 4.3 shows the relation between the Weibull parameters and the variance of the random lifetime. To this regard, an appropriate choice of the parameter values allows us to represent the so-called "rectangularization" phenomenon (i.e. the concentration of deaths around the expected lifetime), whereas the randomness in future rectangularization trend is expressed, as above, by the parameter uncertainty.

5. Allowing for longevity risk: the traditional approach

We assume that the only risk perceived by the market is the longevity risk, i.e. a risk of systematic deviations from the expected number of survivors. Conversely, the risk of random fluctuations in mortality is assumed as fully diversified by the insurer, whence no reward is allowed for. Further, we assume that there is no financial risk, so that the yield from investment is the risk-free rate i (constant, for brevity). Finally, we disregard any expense or transaction cost.

Information asymmetries between the insurer and the annuitants concern both information held by annuitants only (whence the adverse selection risk arises) and information available to the insurer only (with regard to the description of the future mortality trend). Overall, such asymmetries lead to a safety loading embedded into the annuity single premium. We assume that all agents on the market (but the annuitants) hold the same information. Hence the same mortality model, featured by the same parameters, is adopted by all agents.

The insurer's income at time 0 is given by the single premiums paid by the annuitants, whose total amount is $N_0 V_0$, where N_0 (as already mentioned) is the initial size of the portfolio and V_0 is the individual mathematical reserve at time 0, calculated according to a mortality table embedding a safety loading with respect to the (projected) BE mortality assumption. In formal terms, we assume that V_0 is such that

$$\frac{V_0}{V_0^{[\mathrm{BE}]}} - 1 = \gamma \tag{5.1}$$

where $V_0^{[\mathrm{BE}]}$ denotes the mathematical reserve based on the BE mortality assumption and γ is the given safety loading for a monetary unit of premium.

As a unitary annual amount is assumed for all annuities, the random outflows of the insurer are given by $N_1, N_2, ..., N_n$, where N_t denotes the random number of annuitants alive at time t, i.e. at age $x_0 + t$; $n = \omega - x_0$ is the maximum residual lifetime of an individual aged x_0.

According to the traditional actuarial approach, the value of the portfolio to the insurer can be calculated as follows. First, the expected industrial profit, conditional on the BE scenario, is given by

$$U_t^I = N_{t-1}^{[\mathrm{BE}]} V_{t-1} (1+i) - N_t^{[\mathrm{BE}]} - N_t^{[\mathrm{BE}]} V_t \tag{5.2}$$

where $N_t^{[BE]}$ denotes the expected number of annuitants alive at time t under the BE mortality scenario. Then, the (traditional) VIF at time 0 is given (see formula (2.2)) by

$$VIF_0^T(\rho) = \sum_{t=1}^{n} U_t^I (1+\rho)^{-t} - \sum_{t=1}^{n} M_{t-1} (\rho - i)(1+\rho)^{-t} \tag{5.3}$$

which clearly depends on the RDR ρ (which is constant, due to the assumption on the risk-free rate), as stressed by the notation $VIF_0^T(\rho)$.

In order to calculate $VIF_0^T(\rho)$, a capital allocation policy must be chosen. In what follows we assume that the shareholders' capital M_t, $t = 0,1,\dots,n$, contributing to the portfolio assets is equal at any time to the required capital $M_t^{(r)}$ determined according to (3.1'). The target capital M_t^{target} is calculated via a stochastic model, with the following structure.

Let A_h denote the assets at time h facing portfolio liabilities. Assume that no shareholders' capital flow after a starting time t affects the level of portfolio assets. Hence, starting from a given initial amount A_t, the (random) evolution of assets is described by the following recursion

$$A_h = A_{h-1}(1+i) - N_h \qquad h = t+1,\, t+2, \dots \tag{5.4}$$

We assume the following solvency target:

$$\Pr\left\{ \bigcap_{h=t+1}^{T+t} \left(A_h \geq N_h V_h \right) \right\} = 1 - \varepsilon \tag{5.5}$$

where T is the time horizon, V_h is the individual mathematical reserve at time h, and ε is the accepted "ruin probability" (whence $1-\varepsilon$ can be interpreted as the "degree of solvency"). From equation (5.5) via a stochastic simulation procedure, the amount A_t, and hence the capital M_t^{target}, can be determined.

It is worth stressing that, in what follows, we assume that randomness in the numbers N_h is due to longevity risk only, whilst we disregard random fluctuations in mortality.

6. Allowing for longevity risk: the market approach

Turning to the market approach, first we have to assume some hypotheses coherent with a risk neutral valuation. To this purpose, we assume that the insurer transfers the longevity risk to a reinsurer through a swap-like arrangement. Let RP denote the reinsurance premium, paid by the cedant at time 0. According to the reinsurance arrangement, in each year the reinsurer pays to the insurer the random amount $N_t - N_t^{[BE]}$ if positive; otherwise the amount $N_t^{[BE]} - N_t$ is paid by the insurer to the reinsurer. It follows that the net annual outflow of the insurer is

$$N_t - \left(N_t - N_t^{[\mathrm{BE}]}\right) = N_t^{[\mathrm{BE}]} \tag{6.1}$$

Thus, the annual net flows of the insurer are known, whence the discount rate to be used for the valuation must be the risk-free rate i. The (market) VIF of the portfolio is then given by

$$VIF_0^M = \sum_{t=1}^{n} U_t^I (1+i)^{-t} - \sum_{t=1}^{n} M_{t-1} (i - i) (1+i)^{-t} - RP \tag{6.2}$$

From (5.2) it turns out

$$VIF_0^M = N_0 V_0 - \sum_{t=1}^{n} N_t^{[\mathrm{BE}]} (1+i)^{-t} - RP \tag{6.3}$$

where

$$N_0 V_0 - \sum_{t=1}^{n} N_t^{[\mathrm{BE}]} (1+i)^{-t} = CF_0 \tag{6.3'}$$

represents the present value at time 0 of the insurer's cash flows (certain in the presence of the swap-like arrangement; expected according to the BE scenario otherwise).

As regards the position of the reinsurer, we first note that the reinsurer needs to counterbalance its risk. To this purpose, we assume that the reinsurer issues a bond with the following features:

- principal: 0;
- annual random coupon: $N_0 - N_t$ (i.e. equal to the random number of deaths in the cohort up to time t);
- price (at time 0): BP.

We point out that maybe the reinsurer cannot access directly the capital market, but intervention of a Special Purpose Vehicle (SPV) is required. This could lead to some transaction costs, not specifically addressed in this paper.

The annual net outflow of the reinsurer is given by

$$\left(N_t - N_t^{[\mathrm{BE}]}\right) + \left(N_0 - N_t\right) = N_0 - N_t^{[\mathrm{BE}]} \tag{6.4}$$

where the first term on the left-hand side denotes the annual flow paid to the insurer if positive (or received from the insurer if negative), while the second term denotes the annual flow to the market (i.e. to the buyer of the bond). The right-hand side of (6.4) shows that the annual outflow is known, whence the reinsurer's position is certain.

As regards the reinsurance premium and the bond price, bounds can be obtained from feasibility conditions. The feasibility of the overall situation for the insurer is described by the following condition:

$$N_0 V_0 - RP \geq \sum_{t=1}^{n} N_t^{[\mathrm{BE}]} (1+i)^{-t} \tag{6.5}$$

i.e. the net inflow of the insurer at time 0 must be greater or equal to the present value of the outflows (certain). Using (6.3), condition (6.5) can be simply written as

$$VIF_0^M \geq 0 \tag{6.5'}$$

Conversely, the feasibility of the overall situation of the reinsurer requires

$$RP + BP \geq \sum_{t=1}^{n} \left(N_0 - N_t^{[\mathrm{BE}]}\right)(1+i)^{-t} \tag{6.6}$$

i.e. the net inflow of the reinsurer at time 0 must be greater or equal to the present value of the outflows (certain).

Note that if conditions (6.5) and (6.6) are fulfilled in terms of a strict inequality, then they embed in particular a reward for transaction costs.

From (6.5) and (6.6), we find a lower and an upper bound for the reinsurance premium RP:

$$\sum_{t=1}^{n} \left(N_0 - N_t^{[\mathrm{BE}]}\right)(1+i)^{-t} - BP \leq RP \leq N_0 V_0 - \sum_{t=1}^{n} N_t^{[\mathrm{BE}]} (1+i)^{-t} \tag{6.7}$$

In order to ensure that the lower bound is actually less than the upper bound, i.e.

$$\sum_{t=1}^{n} \left(N_0 - N_t^{[\mathrm{BE}]}\right)(1+i)^{-t} - BP \leq N_0 V_0 - \sum_{t=1}^{n} N_t^{[\mathrm{BE}]} (1+i)^{-t} \tag{6.8}$$

a constraint on the bond price BP must be imposed, namely

$$BP \geq \sum_{t=1}^{n} N_0 \,(1+i)^{-t} - N_0\, V_0 \qquad (6.9)$$

Note that violation of this inequality would give rise to arbitrage opportunities with respect to the reinsurer.

Given the unavailability of a market for longevity risk, adoption of a risk-neutral probability raises a lot of issues. In what follows, we assume for the bond price a very naïf and basic rule, namely

$$BP = \sum_{t=1}^{n} \big(\mathrm{E}(N_0 - N_t) - \lambda\,\sigma(N_0 - N_t)\big)(1+i)^{-t} \qquad (6.10)$$

where the symbols E and σ denote the expected value and the standard deviation respectively, and λ represents the market price of risk. For an alternative approach, see Lin and Cox (2005).

We now turn to the main purpose of this work. For any given reinsurance premium RP, the (market) VIF at time 0 is given by expression (6.3). Assume that the traditional approach to portfolio valuation (expressed by formula (5.3)) leads, via an appropriate choice of the RDR ρ, to the same value of the VIF; hence

$$VIF_0^{T}(\rho) = VIF_0^{M} \qquad (6.11)$$

Condition (6.11) allows us to determine the "equivalent RDR", i.e. the RDR such that the traditional valuation coincides with the market one. As the only risk accounted for is the longevity risk, the resulting RDR expresses the riskiness of the portfolio due to uncertainty in future mortality trends.

7. Numerical examples

Consider a portfolio initially consisting of $N_0 = 1000$ annuitants, aged $x_0 = 65$. The annual rate of interest is $i = 2.5\%$.

As regards the safety loading of the insurer, we assume (see (5.1)) $\gamma = 5\%$. Let $q_x^{[\mathrm{BE}]}$ denote the best estimate mortality rate at age x (derived from the Weibull distribution with the BE parameters, see Table 4.1), while the mortality rate used in the pricing and reserving basis, q'_x, is such that

$$\frac{q'_x}{q_x^{[\mathrm{BE}]}} = 87.246\%$$

The required capital at time t, $M_t^{(r)}$, is determined according to formula (3.1'), and the target capital at time t, M_t^{target}, is calculated aiming at portfolio solvency and adopting the approach described in Section 5 (see formula (5.5) in particular). We have assumed, alternatively, $T = 1, \varepsilon = 0.05\%$ (following Solvency 2 parameters) and $T = 5, \varepsilon = 0.5\%$ (thinking to internal parameters of the insurance company, reflecting its capital allocation politicy); we will refer to these two cases respectively as “target Solvency 2” and “internal target”. Table 7.1 quotes some values of the required capital $M_t^{(r)}$, under the two alternative assumptions; at each time, the calculation has been performed assuming that the current size of the portfolio is as expected under the BE scenario. Note that the internal rule of conducts (which refers to a longer time horizon) usually leads to a higher target capital than the legal requirement, due to the fact that longevity risk is a long term risk.

t	**target Solvency 2**	**internal target**
0	557.25	557.25
1	530.76	530.76
2	504.10	504.10
3	477.32	477.32
4	450.48	450.48
5	423.66	423.66
…	…	…
15	176.46	322.35
16	156.09	322.93
17	136.92	319.56
18	119.01	312.24
19	102.44	301.07
20	87.26	286.33
…	…	…
30	23.85	75.50
31	18.87	58.93
32	14.54	44.73
33	10.88	32.97
34	7.90	23.54
35	5.56	16.26
…	…	…

Table 7.1 - Required capital

The present value of the insurer's cash flows (see (6.3')) is $CF_0 = 0.66340$. From constraint (6.7) the upper bound for the reinsurance premium is $RP_{\max} = 0.66340$, whereas from constraint (6.9) the maximum market price of risk (see (6.10)) is $\lambda_{\max} = 0.86164$.

In Table 7.2 values of the minimum reinsurance premium, $RP_{\min}$, as it results from the lower bound in (6.7), are tabulated against some values of the parameter λ.

λ	$RP_{\min}$
0	0
0.5	384.96
0.86164	663.40

Table 7.2 - Minimum reinsurance premium

Finally, Table 7.3 and 7.4 present some results concerning the "equivalent RDR". For any given value of the reinsurance premium RP (consistent with the market price of risk λ), the resulting (market) VIF is reported. Assuming that the traditional VIF coincides with the market VIF (see (6.11)), the equivalent risk discount rate ρ can be calculated. Then, the positive term ($PVFP_0$) and the negative term (CC_0) of the VIF (according to expressions (2.2) and (2.2')) can be derived.

λ	RP	VIF_0
0	0	663.40
0.5	384.96	278.43
0.5	500.00	163.40
0.86164	663.40	0.00

Table 7.3 - Market price for risk, reinsurance premium, VIF

RP	VIF_0	Target Solvency 2			Internal target		
		RDR ρ	$PVFP_0$	CC_0	**RDR ρ**	$PVFP_0$	CC_0
0	663.40	2.500%	663.40	0	2.500%	663.40	0
384.96	278.43	5.326%	471.78	193.35	2.878%	631.87	353.43
500.00	163.40	6.746%	405.02	241.63	3.023%	620.30	456.91
663.40	0.00	9.700%	305.24	305.24	3.274%	601.07	601.07

Table 7.4 - Reinsurance premium, VIF, equivalent RDR and splitting of the VIF

Note that when a more severe target capital (as implied by the choice “internal target” compared to the choice “target Solvency 2”), a lower RDR follows. Actually the risk is absorbed by the higher amount of capital.

8. Final remarks

We have described a possible approach to include the market price of risk in the embedded value framework. The pricing of longevity risk constitutes the main problem in the valuation process. More generally, the presence of non-traded risks require further research work.

The numerical results we have presented clearly depend on the assumed hypotheses and the values assigned to a number of parameters. In particular, the Weibull assumption, the parameter space and the relevant probability distribution used to represent possible future mortality trends clearly affect the results. Another important assumption concerns the expression of the market price of risk.

Nonetheless, in our opinion the proposed model can provide a useful tool for linking the traditional valuation approach to the market one, via an appropriate quantification of the riskiness due to uncertainty in future mortality trends.

Bibliography

Brender A. (2002), The use of internal models for determining liabilities and capital requirements, *North American Actuarial Journal*, **6** (2): 1-10.

Blake D., Burrows W. (2001), Survivor bonds: helping to hedge mortality risk, *The Journal of Risk and Insurance*, **68** (2): 339-348.

Burrows R.P., Lang J. (1997), Risk discount rates for actuarial appraisal values of life insurance companies, *Proceedings of the 7th International AFIR Colloquium*, Cairns (Australia): 283-307.

Burrows R.P, Whitehead G.H. (1987), The determination of life office appraisal values, *Journal of the Institute of Actuaries*, **114**: 411-465.

Cairns A.J.C., Blake D., Dowd K. (2004), *Pricing death: frameworks for the valuation and securitization of mortality risk*, Preprint, Actuarial Mathematics and Statistics, School of Mathematical and Computer Sciences, Heriot-Watt University.

CFO Forum (2004a), *European Embedded Value Principles*, Stichting CFO Forum Foundation.

CFO Forum (2004b), *Basis for Conclusions European Embedded Value Principles*, Stichting CFO Forum Foundation.

Coleman A.M., Edwards B.A., Torrance D.M. (1992), *Maintanable earnings and actuarial methods for valuing life insurance companies*, The Institute of Actuaries of Australia, Sessional Meeting 1992.

IAA (2004), *A global framework for insurer solvency assessment*, Report by the Insurance Solvency Assessment Working Party of the International Actuarial Association.

LAVMWP (Life Assurance Value Measurement Working Party) (2001), *Summary and comparison of approaches used to measure life office values*, The Staple Inn Actuarial Society, London.
Lin Y., Cox S.H. (2005), Securitization of mortality risks in life annuities, *The Journal of Risk and Insurance*, **72** (2): 227-252.
Olivieri A. (2001), Uncertainty in mortality projections: an actuarial perspective, *Insurance: Mathematics & Economics*, **29** (2): 231-245.
Olivieri A., Pitacco E. (2003), Solvency requirements for pension annuities, *Journal of Pension Economics & Finance*, **2** (2): 127-157.
Olivieri A., Pitacco E. (2005), *La valutazione nelle assicurazioni vita. Profili attuariali*, EGEA, Milano.
Pemberton J., Allen I., Brown D., Hardwick S., Shah N., Stevens A., Wilson C. (2000), *Classifying DCF models to explicate EVA*, The Staple Inn Actuarial Society, London.
Pitacco E. (2004), Survival models in a dynamic context: a survey, *Insurance: Mathematics & Economics*, **35** (2): 279-298.
Sherris M. (1987), On the risk adjusted discount rate for determining life office appraisal values, *Journal of the Institute of Actuaries*, **114**: 581-589.
Tillinghast-Towers Perrin (2004), *Market-Consistent Embedded Value*, United Kingdom.
Turner S. (1978), Actuarial appraisal valuations of life insurance companies, *Transactions of the Society of Actuaries*, **30**: 139-167.