"Loss Reserves and Rinancial Risk
The Rich and the Poor"

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The effect of including financial risk in actuarial reserve calculation is investigated. It is demonstrated that the requirements are quite sensitive towards the investment profile.

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1. Introduction

Insurance is transfer of risk from the customer to the company. The premium is paid in advance and claims covered later. Reserving is essential in insurance, and receives the primary attention of the actuary. Actuaries use statistical models to calculate expected loss for a given portfolio.[i] These funds are invested, and this creates financial risk in addition to the insurance risk. This article will address whether the actuary should include financial risk into the calculation of the total reserves. The exercise is not difficult today with modern powerful computers. The two risk processes are modelled separately, and their combined effect evaluated through Monte Carlo simulations, as for example in Daykin, Pentikainen and Pesonen.[ii]

This paper will demonstrate one method, and describe what effect financial risk has on the total reserve. We will describe the effect of different investment strategies on the total security of the company. All calculations are based on real data.

2. Method

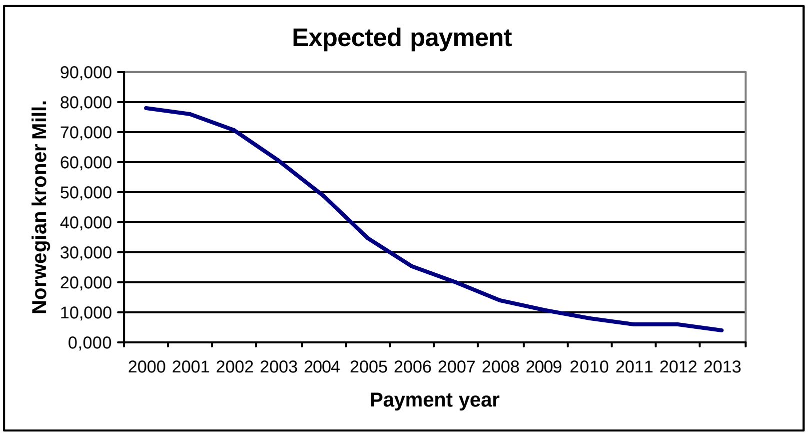
2.1. Loss reserves

In this example we look at the personal damage element of a motor portfolio. Total payments for this long-tailed portfolio will run over approximately 15 years. A factor analysis model for minimum loss reserves constructed by The Banking, Insurance and Securities Commission of Norway estimates the total reserves for the portfolio at 31.12.1999 to NOK 462 mill.[iii] Future payments pr. year as in figure 1:

Figure 1:

Expected payment pr. payment year for accident year < 2000

2000	78,085
2001	75,854
2002	70,897
2003	60,697
2004	48,861
2005	34,791
2006	24,908
2007	19,851
2008	13,894
2009	10,364
2010	7,654
2011	6,229
2012	5,841
2013	4,077



2.2. Financial Models

Bonds yield a fixed interest rate pr. year, and the contracts run over an agreed period of time. We have not developed a model for the bond prices and we will in the reserve model below use a fixed interest rate of 5% pr. annum.

Modelling stock prices is more difficult. Stock prices fluctuate with time, and over a long period the value is expected to increase. In this model we see stock prices as a random walk process. The monthly change in stock value is described by historic values for the particular stock price. Stock prices are dependent on inflation, and hence future price changes will be larger than past changes. In order to use historic data we must log-transform the historic values, and treat the changes as factors. In this example we look at historic values from the New York Stock Exchange Composite index. Figure 2 describes the real index changes, and figure 3 describes the log-transformed index changes.

Figure 2:

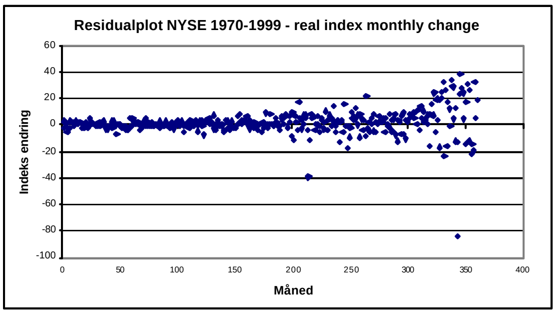
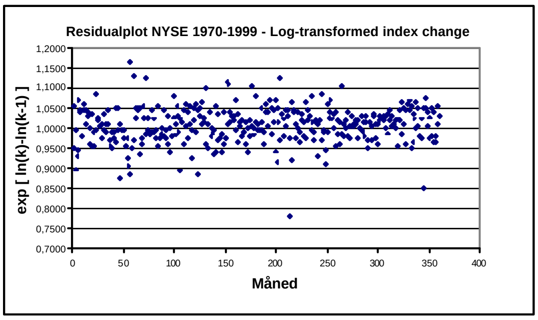


Figure 3:



The log-transformed residuals in figure 3 are evenly spread out over the whole period, and we use these data for simulation. The model describing NYSE composite index for stock price S_k at month k:

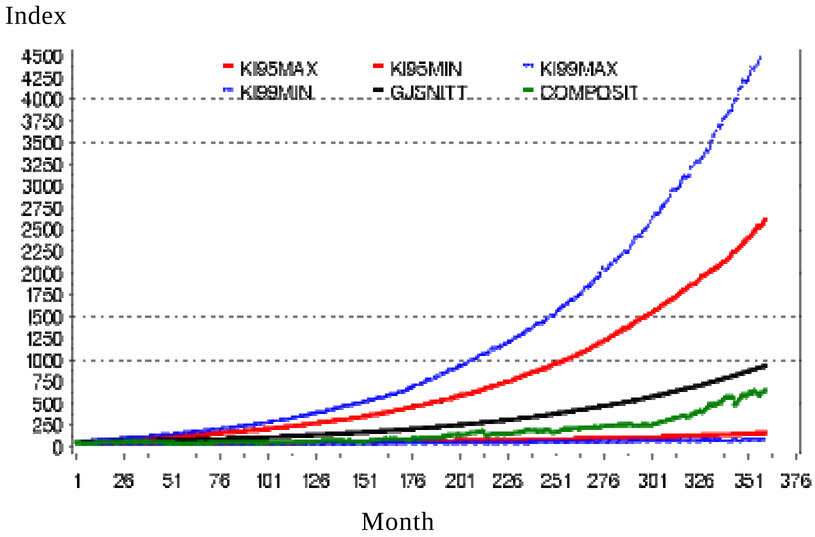
$$\ln S_k = \ln S_{k-1} + e_k \quad , k=1 \text{ to } 360$$

$$S_0 = 50$$

where $e_k = \ln m_k - \ln m_{k-1}$ describes the log-transformed value of monthly stock price change m at time k. The change in stock price is described by

$$S_k = S_{k-1} \cdot e^{e_k}$$

We draw values at random from historic stock price changes and repeat this process for 10000 iterations. From the 10000 possible stock price developments we sort the outcomes and find the value for the lower 99% confidence interval. The simulated NYSE composite index as describes in figure 4:



The model overestimates the expected NYSE index. The expected value is 950, while the observed index was 650 pr 31.12.1999.

This model can be applied to a single stock, or an index. In later examples we will use regional data from Morgan Stanley Capital Index. [iv]

2.3. Total reserve

In the reserve model we do not take investment in real estate and cash holdings into account. We simplify by only investing the reserve in stocks or bonds.

We expect the monthly return on bonds:

$$r_t = \exp(\log(1 + r_a)/12) - 1.$$

where r_a is the annual interest rate.

Model:

Bond rate: r
Historic index value: m_t
Stock development: S_t
Bond development: B_t
Claims payments: X_t
Reserve development: R_t

The reserves will develop over time t:

$$B_t = (1+r)B_{t-1}$$

$$\log S_t = \log S_{t-1} + e_t$$

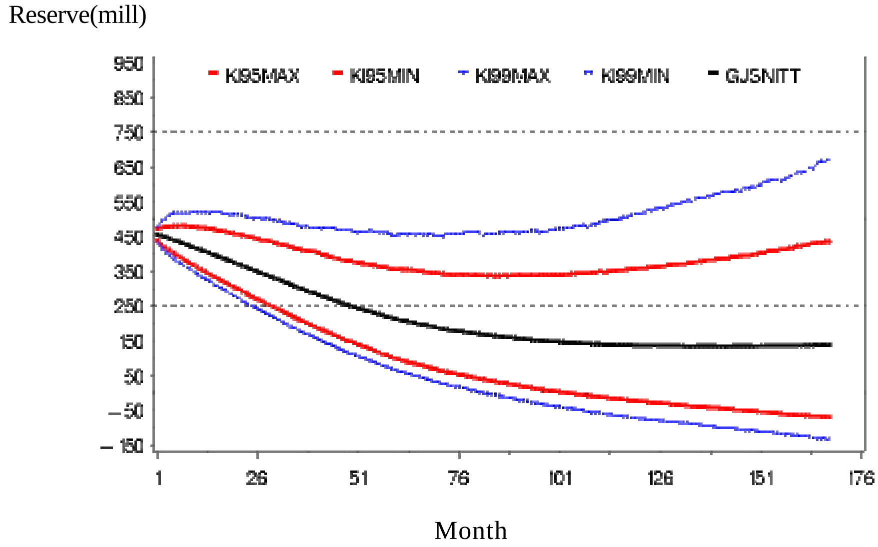
$$e_t = \ln m_t - \ln m_{t-1}$$

$$R_t = B_t + S_t - X_t$$

and run for 168 months. We calculate 10000 possible reserve outcomes, sort the outcomes and create confidence intervals.

Figure 5:

Simulated reserve – 14 years
Great Britain - 5% inerest rate, 3,5% inflation – 10000 iterations
35% stocks – 65% bonds



Development for a portfolio invested in Great Britain has an expected return of NOK 140 mill. Lower 99% confidence interval is NOK –132 mill. This is not an acceptable investment strategy for the company.

2.4. Optimum criteria

In this model we use value at risk as an optimum criteria. The company and regulators require a minimum probability of ruin. The reserve model describe the 99% confidence interval which applies well to value at risk. Value at risk is defined:

$$P(\Delta S_t < VaR) = a$$

where ΔS_t is the change in the portfolio value, VaR is the loss limit and a the percentile. The company must optimise its reserve development with the restraint of 99% confidence interval greater than 0 at the end of the investment period.

We simulate the reserve development for each scenario, and measure the value at risk at the end of the investment period. If the 99% confidence interval is lower than 0 at the end of the investment period, we have to increase the share invested in bonds. We repeat this process 8-15 times to find the optimal share invested in each financial object.

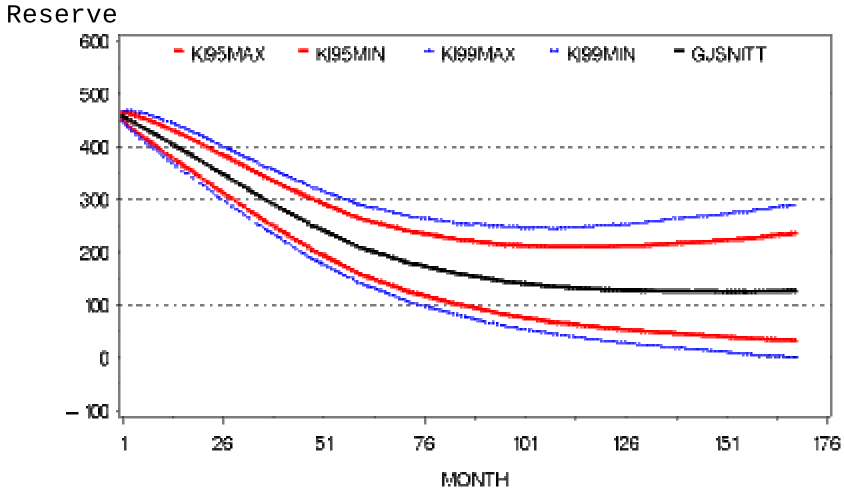
3. Results

3.1. Development of the reserves

Runoff portfolio: In this scenario we invest the reserve in 4 different markets. We have simplified the optimization problem by investing q% of the reserve in stocks, and thereby investing q/4% in each market. By investing the portfolio in Great Britain, United States, Norway and France we get the following result:

Figure 6:

Simulated reserve – 14 years
Great Britain, United States, Norway and France
5% inerest rate, 3,5% inflation – 10000 iterations
30% stocks – 70% bonds



The expected return on the portfolio is NOK 120 mill. and it is optimal to invest 30% of the reserve in stocks. The remaining 70% is invested in bonds and return 5% pr annum.

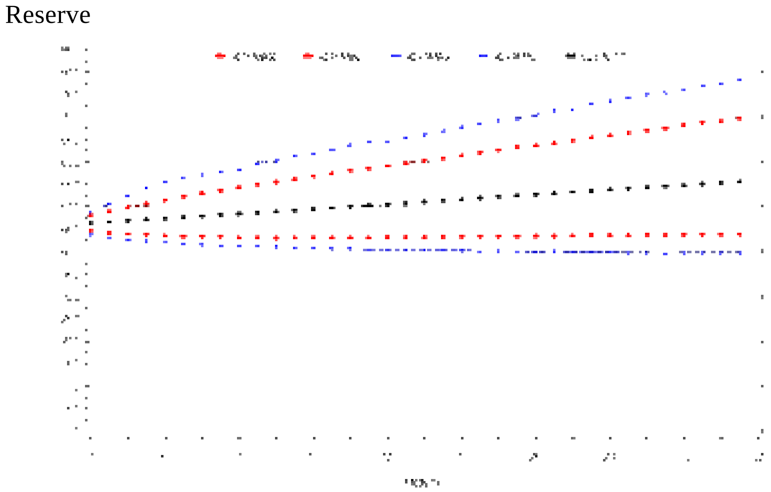
Running portfolio: In this scenario the company receives a yearly premium from its customers and pay claims for present and all past accident years. The model is similar to the runoff portfolio, but we have to estimate premium income and estimate claims payments for future development years.

Total reserve will develop over time: $R_t = P_t + B_t + S_t - X_t - Y_t$

P_t is estimated premium income, and Y_t is estimated claims paid in the present development year. Simulating a portfolio investment with 35% in stock and 65% in bonds for 10000 iterations give the following result:

Figure 7:

Simulated reserve – 3 years
Great Britain - 5% inerest rate, 3,5% inflation – 10000 iterations
35% stocks – 65% bonds



We have found the lower 99% confidence interval for this portfolio. The optimization problem for the portfolio is now more complex because we do not know the minimum required loss reserve in 3 years. The lower bound for the value at risk will be the estimated minimum reserve calculated by the appointed actuary and the investment strategy must be selected from the expected minimum reserve required.

3.2. Sensitivity

Interest rate and inflation: In the previous examples the interest rate is set to 5%, and yearly inflation is set to 3,5%. These levels have been chosen as expected values, but as we know the rates change over time. We have invested NOK 462 mill. in 4 markets and optimized the amount invested in stock for different combinations of interest rate and inflation. Table 1 below describes share invested in stock and expected return on the portfolio invested over 15 years.

Table 1:

	Inflation p.a.	Interest rate p.a.	Share invested in stock	Share invested in bond	Expected return mill. NOK	95% CI	99% CI
0,5% difference	2,0 %	2,5 %	20,6 %	79,4 %	64	18	1
	3,5 %	4,0 %	12,6 %	87,4 %	44	13	1
	5,0 %	5,5 %	9,1 %	90,9 %	37	11	1
1,5% difference	2,0 %	3,5 %	42,3 %	57,7 %	172	43	1
	3,5 %	5,0 %	30,0 %	70,0 %	128	34	1
	5,0 %	6,5 %	23,1 %	76,9 %	110	30	< 1
2,5% difference	2,0 %	4,5 %	53,8 %	46,2 %	255	58	< 1
	3,5 %	6,0 %	41,1 %	58,9 %	202	50	1
	5,0 %	7,5 %	32,9 %	67,8 %	178	46	1
3,5% difference	2,0 %	5,5 %	60,8 %	39,2 %	318	70	1
	3,5 %	7,0 %	48,9 %	51,1 %	265	63	< 1
	5,0 %	8,5 %	40,5 %	59,4 %	240	59	< 1

A high interest rate guaranties a high return on capital invested in bonds. This will give room for larger investments in stocks, which are more volatile but give a higher return. The inflation affects the future payments, with low inflation leading to relatively low future payments.

Markets: We want to describe the risk profile of the different regions. We invest the same portfolio in one stock and one bond in four different markets and observe the results. Table 2 describes the share invested in stock, bonds and the expected return.

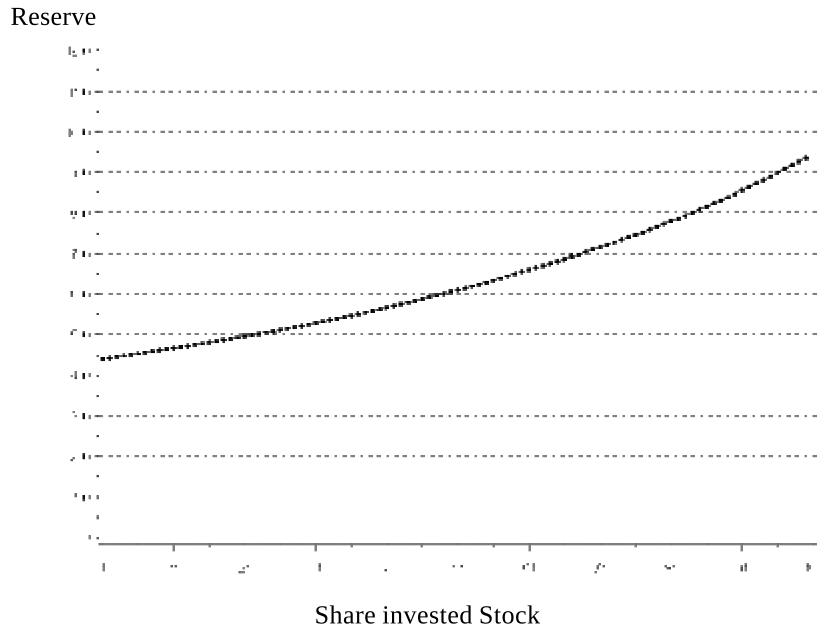
Table 2:

	Share invested in stock	Share invested in bond	Expected return mill. NOK	95% CI	99% CI
USA	13,8	86,2	74,4	161	1
Norway	7,8	92,2	69,5	147	1
Great Britain	9,7	90,3	73,4	160	1
France	9,2	90,8	69,5	158	1

The volatility in the US market is lower than the other 3 and the optimal investment strategy is to invest 13,8% of the reserves in stocks. This investment strategy generates the highest expected return after 14 years at NOK 74,4 mill. The Norwegian stock market is more volatile and the insecurity forces the investor to place a larger share of the reserve in bonds. The optimal strategy is to invest 7,8% of the reserve in stocks in order to guaranty a positive reserve after 15 years. The expected return is NOK 69,5 mill.

Minimum reserve: A different angle to the problem is to model the amount of capital needed for a particular investment strategy. Figure 8 describes the minimum reserve required for investments pr. percent invested in stocks. We have again invested NOK 462 mill. in Great Britain.

Figure 8:



The convex curve shows that the higher share invested in stocks, the higher the need for capital. We have shown that by investing the total reserve in bonds we need NOK 440 mill., which is less than the initial reserve. By increasing investments in stocks we must increase the financial buffer in order to meet the risk from stock investments.

When the investment strategy is selected directed on the percentiles far out in its tail the results tend to become sensitive towards variations in the assumptions and conditions. We see that effect clearly in table 1 where the expected return change quite a lot according to change in interest rate and inflation. Clearly the increased risk by investing heavily in the stock market leads to considerably higher requirements on the reserves as shown in figure 8. In order to achieve a high expected return one has to live with higher minimum capital requirements. This is a reflection of the fact that one has to be big in order to be able to exploit the most profitable investment opportunity.

4. Conclusion

Calculation of total reserves for an insurance company may include both insurance and financial risk. This seems to be standard actuarial work in both the United States and Britain, while actuaries in Norway have been much more reluctant to integrate financial consideration into their work. This may be slightly surprising since financial risk is essential to the total result.

A company with a limited capital basis will be unable to harvest good dividend on its investment. If exposed in a falling capital market, it will be forced to liquidate its stock portfolio at an inopportune time. A rich company will, on the other hand, be able to keep the stock and wait for the traditional lift of the market. This element is clearly a drive towards consolidation of the large actors in the financial markets. Bigger is better, and safer!

Endnotes:

[i] Greg Taylor, Loss Reserving, An Actuarial Perspective, 2000

[ii] Daykin, Pentikainen and Pesonen, Practical risk theory for actuaries, 1994

[iii] Modeller og metoder for beregning av det forsikringstekniske ansvaret og dets komponenter, The banking, Insurance and Securities Commission of Norway, 1991

iv Morgan Stanley Capital International: www.mscidata.com