A FRESH LOOK AT MATCHING ASSETS AND LIABILITIES
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The management and directors of pension organizations have long felt the need for an integrated policy in respect of investments and pensions. It is generally known that a division between the two disciplines is made on the tactical and the operational level within the organization. However, in order to support the strategic policy, it has become necessary to view these disciplines from an integral standpoint and to visualize the interactions between these disciplines.

Approach
First, a definition of the concept of ALM, or Asset Liability Management: we have defined this as controlling the constituent components of the assets and liabilities with the aim of matching the insurance and investment risks and optimizing the operating result.

This definition immediately brings forward the different view held by the work group in respect of the term ALM; the matching of asset and liabilities is, in our view, derived from the management of assets and liabilities. Matching can be striven for by policy makers in their capacity as such, but is not, however, elevated to become the ultimate policy objective. This view arises from the conviction that there is no single, unambiguous definition of the concept of "matching". There are, namely, several possible types of matching.
1. Matching of cash flows (terms)
2. Matching of interest elasticity (durations)
3. Matching of insurance and investment risks

In the further discussion on the problems and issues involved in matching, the latter type of matching is the type referred to. This choice will be examined in greater detail below.

**Assets**

Since the dawn of capital markets, many attempts have been made to conform the behaviour of the participants and the pricing mechanisms on these markets to a model or theory. Examples of this include technical analysis, fundamental analysis and the efficient market hypothesis. This latter theory states that an investment portfolio is compiled on the basis of return and risk. This conclusion, in particular, led to an entirely new and logical theory on the pricing mechanism in the financial markets: the Modern Portfolio theory.

This theory strives towards the quantification of the risk, which was traditionally merely a qualitative but not further specified concept, of an investment or a portfolio. The American H. Markowitz devised a formula based on the assumption that investors look for the highest expected rate of return with the smallest possible risk. He thereby introduced the notion of the efficient portfolio, i.e. a portfolio offering the maximum return feasible at a given risk level, or a portfolio within the minimum risk at a given level of return.
Markowitz thereby defined the concept of risk as the variance or standard deviation, as the case may be, of the expected rate of return. A portfolio may then be fully characterized by the two variables, expected rate of return and risk.

Markowitz subsequently proved that the return on a portfolio equals the weighted average of the return on the individual investments. The crucial point here is the fact that the risk for a portfolio does not equal the weighted average of the risk of the individual investments. This latter term must be adjusted by the correlations between the various forms of investment, which can be quantified by means of a correlation coefficient. The result is a set of efficient portfolios characterized by the fact that they offer either a minimum risk at a given rate of return, or a maximum return at a given level of risk. This set can be visualized in a risk-return diagram which demonstrates the well-known efficient frontier (figure 1).

Liabilities
The efficient frontier analyzed above represents a set of efficient portfolios. However, which portfolio is the most suitable for the pension fund in question?
OR : What is the maximum risk the fund is willing to run?
OR : What is the minimum rate of return to be achieved by a fund on its investments?
Efficient frontier

FIGURE 1
These questions should be answered based on the package of commitments of the fund. It so happens that if we are able to quantify the insurance risk, this will provide us with a guideline by means of which, together with the application of the Modern Capital Market theory, a match between assets and liabilities of the type described under point three can be realized: the matching of the insurance with the investment risk.

In our view, this type of matching must be founded on the risk for the entire insurance portfolio; as far as the measure of risk is concerned, our proposition is once again that the total does not add up to the sum of the component parts. Obviously, the question is how to qualify this so-called risk profile. The elaboration of this problem will be discussed below.

Elaboration

The difficulty in the elaboration of the approach selected was in quantifying the risk profile of the pension fund. A new term was introduced to define the basis on which to determine this, namely the buffer capital of the pension fund.

The underlying idea was to determine what the actual surplus capital of a fund was with which more risk could be run on the financial markets.
The basic equation for this ex post buffer capital is:

$$\text{Buffer capital} = \text{Total capital} - \text{Required capital}$$

As a result, the focus of the problem is shifted to determining the amount of the required capital of the fund.

Very soon it became apparent that this capital did not correspond with the well-known 4% dictated by the Verzekeringkamer, as the valuation of this liability item is made on a different principle than the valuation of the assets.

A choice was therefore made in favour of a valuation method of the liabilities, in which account is taken of business economic quantities. This resulted in the so-called actuarial valuation of the liabilities on a liquidation basis.

A business economic valuation of this kind entails, among other things, that:

- the fixed actuarial interest rate of 4% is abandoned, and that a switch is made to one or more actuarial interest rates which correspond with expected average market interest rates;
- the liabilities, or benefits accrued according to the regulations, are determined in such a way that anticipated future factors influencing these are taken into account.
Next to these business economic principles, the actuarial principles remain to be set forth according to the judgement of the actuary of the fund.

If the business economic value of the liabilities is known, the simple application of the above equation will yield the buffer capital. Obviously, the total capital will also be valued on a business economic basis (generally market value). An example as illustration: if, on the basis of the above economic appraisal, the required capital equals 96 at the end of the year, the total capital at the beginning of the year equals 100 and the stream of premiums and stream of benefits during the course of the year are equal, the minimum rate of return to be achieved so that the ex post buffer capital does not become negative: - (min) 4%.

The views of the investor are expressed in similar fashion by his/her ideas regarding the expected rate of return, variances and co-variances of the investment objects, enabling the efficient frontier to be determined according to the standards of the fund.

It goes without saying that in determining the efficient frontier and the buffer capital, it is advisable to take the same expectations for the future into account.

How can we translate this buffer capital into specific investment quantities such as rate of return and risk in order to achieve a matching of assets and liabilities?
To this end, we operate on the likely presumption that the management board of the pension fund would probably not wish to see the current buffer capital held by the fund disappear within a year; hence, only a very small chance of losing the entire buffer capital is acceptable.

Let us assume, merely as illustration, that the chance of this being exceeded is equal to 0.5%.

This implies that investments with a chance of 0.5% may not yield less than, in our example, -4%: at this rate of return the negative return on investment will have led to the disappearance of exactly the amount of the total buffer capital. The difference between the total capital and the required capital of the pension fund will consequently equal nil.

If the investments should show a lower return than the aforesaid -4%, the pension fund will develop solvency problems; the total capital available will in that case be less than the capital which, on the basis of the business economic principles applied, is required to be able to meet the commitments entered into, now and in the future.

In this way, we have more or less implicitly translated the risk profile of the pension fund into the quantities used by Markowitz to explain the workings of the capital market, i.e. return and risk:

- the buffer capital is an indication of the return minimally to be realized on the investments. In our example, this minimum return is -4%.

- the risk of exceeding the limit stated indicates the risk of losing the buffer capital which is considered acceptable. In our example, this chance was set at 0.5%.
With the help of these quantities and a few basic statistical concepts, this risk profile of the pension fund may be represented by a straight line in the diagram of the efficient frontier, as shown in figure 2*). The efficient frontier is the line connecting all efficient investment portfolios with one another; the straight line delimits half a field containing the combinations of return and risk which may be, given the buffer capital of the pension fund and the maximum chance of loss, termed acceptable for the pension fund. This field is to the left of the straight line. The optimum investment portfolio for the pension fund in question is found at the intersection of the two lines; the choice of an investment portfolio with these return-risk features means an optimization of the investment result within the boundary condition, i.e. the straight line, set by the risk profile of the pension fund.

Any modifications in this risk profile due to changes in the buffer capital and/or a change in the maximum chance of loss will lead, in practice, to a translation or rotation respectively of the straight line and hence to a shift in the point of intersection.

The problem of matching the insurance risks with the investment risk is thus reduced to finding the point of intersection between the efficient frontier and the aforementioned straight line. This is a non-linear (i.e. quadratic) programming problem which can be solved relatively simply by means of some mathematical arithmetic (fig. 2)

*) For the derivation of the equation of the frontier, see the addendum at the conclusion of this article.
Efficient frontier
(optimum investment portfolio)

FIGURE 2
Conclusions
Can we state that the application of this approach will lead to the realization of the formulated objective? In our opinion, the answer is in the affirmative. In the approach discussed the interaction between assets and liabilities is made explicit; a further distillation of the application of this method will clearly demonstrate the influences of the various endogenous and exogenous variables on the composition of the assets and liabilities in a number of scenarios. These scenarios can yield insight into the policy and view of the future of both the actuary and the investor: the investor will be required to give opinions on future expected rates of return and the covariances of the investment objects. By contrast, the actuary will have to state his views on the developments in the pool of insured, the bases for the lump sum payments and reserves and the risk profile of the fund. In mutual consultation, expectations with respect to variables such as inflation and market interest rates must be formulated. These variable input data can offer the management a large degree of flexibility in testing and quantifying his/her policy.

Final comments
The approach selected has been worked out for practical purposes in the form of a computer model, which recently became operational. Tests performed on existing funds yielded surprising and promising results. A noteworthy point is that these results led directly to discussions between the investor and actuary, which was one of the motives behind the choice of this approach.
These successful reports have not led us to stop the further development of the model. On the contrary, new features such as simulation possibilities, various standard scenarios and other types of matching (cash-flow, interest-elasticity) are currently being worked on. This, in response to wishes of investors and actuaries who have seen and have become enthusiastic about the computer model. Which is precisely what we had hoped to achieve.
Addendum

R = Return on the investment portfolio

(1) \( P(R < -4\%) \leq 0.5\% \)

If we assume that the return \( R \) is normally distributed with expectations \( E(R) \) and standard deviation \( S(R) \), the following holds:

(2) \( P \left( \frac{R - E(R)}{S(R)} \leq Z(0.5\%) \right) \leq 0.5\% \)

in which \( Z \) equals the \( Z \) value of the standard normal distribution belonging to a left-sided probability of 0.5\%, i.e. 2.576.

Equation 3 follows from equation 1 and 2:

(3) \( \frac{-4 - E(R)}{S(R)} \leq -2.576 \)

From which it directly follows that:

(4) \( E(R) \geq -4 + 2.576 S(R) \)

in which \( E(R) = -4 + 2.576 S(R) \) represents the equation of the straight line indicating the risk profile of the pension fund in the risk-return diagram.