DISCRETE-TIME RISK MODELS BASED ON TIME SERIES FOR COUNT RANDOM VARIABLES

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ABSTRACT

In this paper, we consider various specifications of the general discrete-time risk model in which a serial dependence structure is introduced between the claim numbers for each period. We consider risk models based on compound distributions assuming several examples of discrete variate time series as specific temporal dependence structures: Poisson MA(1) process, Poisson AR(1) process, Markov Bernoulli process and Markov regime-switching process. In these models, we derive expressions for a function that allow us to find the Lundberg coefficient. Specific cases for which an explicit expression can be found for the Lundberg coefficient are also presented. Numerical examples are provided to illustrate different topics discussed in the paper.

KEYWORDS

Discrete-time risk model; Poisson MA(1) process; Poisson AR(1) process; Markov Bernoulli Process; Markovian Environment; Lundberg Coefficient.

1. INTRODUCTION

We consider the portfolio of an insurance company in the context of a discrete time risk model allowing different possible temporal dependence structures. We define a sequence of identically distributed but not necessarily independent random variables (r.v.'s) $\underline{W} = \{W_k, k \in \mathbb{N}^+\}$ where the r.v. W_k represents the aggregate claim amount in period k, $k = 1, 2, \ldots$. The r.v. W_k is distributed as W with cumulative distribution function (c.d.f.) F_W and moment generating function (m.g.f.) M_W. Let $\underline{N} = \{N_k, k \in \mathbb{N}^+\}$ be defined as a discrete time claim number process. In an insurance context, N_k corresponds to the number of claims in period k. The aggregate claim amount r.v. W_k is defined as

$$W_k = \sum_{j=1}^{N_k} B_{k,j}, \tag{1}$$

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assuming that $\sum_{j=1}^{0} a_j = 0$. The claim amounts in period k, denoted $B_{k,1}$, $B_{k,2}, \ldots$, form a sequence of i.i.d. r.v.'s with c.d.f. F_B and independent of N_k. It implies that W_k follows a compound distribution with $E[W] = E[N]\,E[B]$ and $M_W(r) = P_N(M_B(r))$, where $P_N(s)$ is the probability generating function (p.g.f.) of N. We assume that the m.g.f. of B, denoted $M_B(r)$, exists. The premium income per period is designated by π and satisfies the usual solvency condition $\pi > E[W]$. The strictly positive relative risk margin is $\eta = \frac{\pi}{E[W]} - 1$.

Let $\underline{U} = \{U_k, k \in \mathbb{N}\}$ be the surplus process of the insurance portfolio where U_k corresponds to the surplus level at time $k \in \mathbb{N}$. The dynamic of the surplus process is given by

$$U_k = U_{k-1} + \pi - W_k = u - \sum_{j=1}^{k} W_k + \pi k = u - S_k + \pi k,$$

for $k \in \mathbb{N}^+$ and initial surplus $U_0 = u$. We define $\underline{S} = \{S_k, k \in \mathbb{N}\}$ as the accumulated aggregate claim amount process with $S_k = W_1 + \ldots + W_k$ and $S_0 = 0$. We denote by the r.v. T the time of ruin where

$$T = \begin{cases} \inf\limits_{k \in \{1,2,3,\ldots\}} \{k, U_k < 0\}, & \text{if } U_k \text{ goes below 0 at least once} \\ \infty & , \text{ if } U_k \text{ never goes below 0} \end{cases}$$

or

$$T = \begin{cases} \inf\limits_{k \in \{1,2,3,\ldots\}} \left\{k, \sum_{j=1}^{k} W_k - \pi k > u\right\}, & \text{if } \sum_{j=1}^{k} W_k - \pi k \text{ exceeds } u \text{ at least once} \\ \infty & , \text{ if } \sum_{j=1}^{k} W_k - \pi k \text{ never goes above } u. \end{cases}.$$

The infinite time ruin probability is given by $\psi(u) = \Pr(T < \infty \mid U_0 = u)$ and, when certain conditions are satisfied, we have the asymptotic Lundberg-type result

$$\lim_{u \to \infty} -\frac{\ln(\psi(u))}{u} = \rho,$$

where ρ is the Lundberg adjustment coefficient. Based on this asymptotic result and for large values of u, $\psi(u)$ can be approximated by

$$\psi(u) \simeq e^{-\rho u}. \tag{2}$$

Define the convex function

$$c_n(r) = \frac{1}{n} \ln\left(E\left[e^{r(S_n - n\pi)}\right]\right). \tag{3}$$

Using different approaches, Nyrhinen (1998) and Müller and Pflug (2001) have shown that the Lundberg adjustment coefficient ρ is the solution to

$$c(r) = \lim_{n \to \infty} c_n(r) = 0. \tag{4}$$

We recall that the adjustment coefficient is a measure of dangerousness of an insurance portfolio. Nyrhinen (1999b) has shown how to use the adjustment coefficient ρ in Monte Carlo approximations of ruin probabilities. The expression of $c_n(r)$ defined in (3) depends on the temporal dependence structure for $\underline{W}$.

In the classical discrete time risk model due to De Finetti (1957), it is assumed that $\underline{W} = \{W_k, k \in \mathbb{N}^+\}$ forms a sequence of i.i.d. r.v.'s (see e.g. Bühlmann (1970), Gerber (1979) and Dickson (2005)). Some papers consider various models with temporal dependence. Gerber (1982) examines the estimation of ruin probabilities in a linear (Gaussian) risk model. Promislow (1991) derives upper bounds for a similar risk model. Christ and Steinebach (1995) propose an empirical-moment generating function type estimator of the adjustment coefficient in the risk model introduced by Gerber (1982). Yang and Zhang (2003) derive both exponential and non-exponential upper bounds for the infinite-time ruin probability in an extension to the model of Gerber (1982) with interest. In a multivariate extension to Yang and Zhang (2003), Zhang et al. (2007) obtain a Lundberg-type inequality for the ruin probability in a discrete-time model with dependent classes of business based on a multivariate first-order autoregressive time-series model and assuming a constant interest rate. Nyrhinen (1998, 1999a,b) derive Lundberg-type asymptotic results for the case of dependent claims with light tails using results from large deviation theory. Müller and Pflug (2001) obtain the same result using Markov inequalities. A special case of the classical discrete time risk model is the compound binomial risk model which was first proposed by Gerber (1988a, b) and further examined e.g. by Shiu (1989), Willmot (1993) and Dickson (1994). In the last decade, contributions such as Yuen and Guo (2001) and Cossette et al. (2003, 2004a, b, c) have considered temporal dependence within the compound binomial risk model.

In their paper, Müller and Pflug (2001) apply the result in (2) with (3) and (4) within notably the classical discrete time risk model and linear risk models considered by Gerber (1982) and Promislow (1991). However, the linear risk models such as the Gaussian AR(1) and ARMA(p,q) may be less applicable in the context of risk theory. As stated in almost all actuarial textbooks, compound distributions are the corner stones of several risk models in risk theory.

In this paper, we examine risk models based on compound distributions assuming time series models for count data as specific temporal dependence structures for $\underline{N} = \{N_k, k \in \mathbb{N}^+\}$. Time series of counts arise in many different contexts such as counts of cases of a certain disease, counts of price changes, counts of injuries in a workplace, etc. In our paper, the following types of model for time series of counts will be considered:

– *Models based on thinning*. This category of models includes the integer value moving average (INMA), integer value autoregressive (INAR), integer moving average autogressive models (INARMA). These models are based on appropriate thinning operations which replace the scalar multiplications by a fraction in the Gaussian ARMA framework of time series with continuous data (see e.g. Al-Osh and Alzaid (1987, 1988), Mckenzie (1986, 1988, 2003), and Joe (1997)). Quddus (2008) and Gourieroux and Jasiak (2004) apply the class of INAR models for the time series analysis of car accident count data. Freeland (1998) and Freeland and McCabe (2004) analyze a collection of time series of claim counts at the Worker's Compensation Board of British Columbia. Empirical studies of the INMA model include notably the one of Brannas, Hellström and Nordström (2002) on the tourism demand and the one of Brannas and Quoreshi (2004) on the number of transactions in stocks. Kremer (1995) adapts the theory of INAR processes to the context of IBNR-predictions.
– *Models based on Markov chains*. The discrete time process $\underline{N}$ is itself a Markov chain of order 1 or more (see Mackenzie (2003) and references therein). Markov chains can be used to deal with count data in time series. This approach is reasonable when there are very few possible values for $\underline{N}$. When the state space of $\underline{N}$ becomes too large, these models loose tractability. A good example is the Markov Bernoulli process on which is based the compound Markov binomial model proposed by Cossette et al. (2003, 2004a). Arvidsson and Francke (2007) fit the compound Markov binomial model to all risk insurance data from the insurance company Folksam.
– *Models based on a specific conditional distribution with stochastic parameters*. The dependence structure is based on an underlying process such as an ARMA time series or a hidden discrete time Markov chain defined on a finite time space (see e.g. Zeger (1988), Heinen (2003), and Jung et al. (2006) and references therein). When the underlying process is a hidden discrete time Markov chain, these models may be also called Markov regime switching models or risk models defined in a Markovian environment. Examples of the conditional distributions are the Poisson, the binomial or the negative binomial distributions. Malyshkina, Mannering and Tarko (2009) explore two-state Markov switching count data models to study accident frequencies.

Other models such as models based on copulas, where the marginals are fixed and the dependence structure is based on a copula (see e.g. Joe (1997) and Frees and Wang (2006)), could have been considered. A review on time series models for count data can be found in the survey of McKenzie (2003), the monographs of Cameron and Trivedi (1998) and Kedem and Fokianos (2002). All examples considered for $\underline{N}$ in this paper satisfy the constraints on the process $\{W_k - \pi, k \in \mathbb{N}^+\}$ given in Müller and Pflug (2001).

The paper is structured as follows. In the next three sections, we present risk models based on compound distributions assuming for $\underline{N}$ a Poisson MA(1) process, Poisson AR(1) process, Markov Bernoulli process, and a Markov switching regime process. For each model, we examine its properties and derive explicit expressions for $c_n(r)$ and $c(r)$. Specific cases for which an explicit expression can be found for ρ are also presented. Numerical examples are provided to illustrate different topics discussed in the paper.

2. Models based on thinning

We begin this section by introducing the operator "$\circ$" used in models based on thinning. Let M be a non-negative integer-valued random variable and $\alpha \in [0,1]$. The $\circ$-operation of α on M is referred to as the binomial thinning of M and is defined as

$$\alpha \circ M = \sum_{i=1}^{M} Y_i,$$

where $\{Y_i, i = 1, 2, ...\}$ is a sequence of i.i.d Bernoulli r.v's with mean α and independent of M.

2.1. Risk Model – Poisson MA(1)

2.1.1. *Definitions and properties*

Now let us consider a Poisson MA(1) process for $\underline{N} = \{N_k, k \in \mathbb{N}^+\}$ whose dynamic is defined as

$$N_k = \alpha \circ \varepsilon_{k-1} + \varepsilon_k, \quad k = 1, 2, ..., \tag{5}$$

where $\underline{\varepsilon} = \{\varepsilon_k, k \in \mathbb{N}\}$ is a sequence of i.i.d. r.v.'s following a Poisson distribution with mean $\frac{\lambda}{1+\alpha}$ and $\alpha \in [0,1]$. Also,

$$\alpha \circ \varepsilon_{k-1} = \sum_{j=1}^{\varepsilon_{k-1}} \delta_{k-1,j}, \quad k = 1, 2, ..., \tag{6}$$

where $\{\delta_{k-1,j}\}$ is a sequence of i.i.d. Bernoulli r.v.'s with mean α. The sequences $\{\delta_{k,j}, j = 1, 2, ...\}$ (for $k = 1, 2, ...$) are assumed independent for different periods k. Given these distribution assumptions, the r.v. $\alpha \circ \varepsilon_{k-1}$ is Poisson with mean $\frac{\lambda\alpha}{1+\alpha}$. From (5) and (6), we have

$$\begin{aligned}
N_1 &= \varepsilon_1 + \sum_{j=1}^{\varepsilon_0} \delta_{0,j}, \\
N_2 &= \varepsilon_2 + \sum_{j=1}^{\varepsilon_1} \delta_{1,j}, \\
&\dots \\
N_k &= \varepsilon_k + \sum_{j=1}^{\varepsilon_{k-1}} \delta_{k-1,j}.
\end{aligned}$$

As stated in Al-Osh and Alzaid (1987), the marginal distribution of the model (5) is uniquely determined by the distribution of ε_k. Hence, N_k is Poisson distributed with mean

$$\begin{aligned} E[N_k] &= E[\alpha \circ \varepsilon_{k-1} + \varepsilon_k] \\ &= \frac{\lambda\alpha}{1+\alpha} + \frac{\lambda}{1+\alpha} \\ &= \lambda \end{aligned}$$

and (5) generates a stationary process with a Poisson(λ) marginal distribution. If $\alpha = 0$, the behavior of N_k is solely explained by ε_k, which means that the claim number r.v.'s are independent from one period to the other. If $\alpha = 1$, the r.v. N_k is equally affected in its behavior by the r.v.'s ε_k and ε_{k-1}. The number of claims N_k in period k is therefore mainly due to the new arrivals between $k-1$ and k, and a proportion of the new arrivals between $k-2$ and $k-1$ defined by the thinning procedure.

One can also use the p.g.f. to identify the distribution of N_k

$$\begin{aligned} P_{N_k}(r) &= E[r^{N_k}] = E[r^{\varepsilon_k}]E\left[r^{\sum_{j=1}^{\varepsilon_{k-1}}\delta_{k-1,j}}\right] = e^{\frac{\lambda}{1+\alpha}(r-1)}e^{\frac{\lambda}{1+\alpha}((1-\alpha)+\alpha r-1)} \\ &= e^{\frac{\lambda}{1+\alpha}((1+\alpha)r-(1+\alpha))} = e^{\lambda(r-1)}. \end{aligned}$$

The autocorrelation function of $\underline{N}$ is

$$\gamma_N(h) = \begin{cases} \dfrac{\alpha}{1+\alpha}, & h = 1 \\ 0, & h > 1 \end{cases}$$

(see McKenzie (1988)) which implies that $\gamma_N(1) \in [0,0.5]$. Therefore,

$$Cov(W_k, W_{k+h}) = \lambda\gamma(h)E[B]^2 = \begin{cases} \dfrac{\alpha\lambda E[B]^2}{1+\alpha}, & h = 1 \\ 0, & h > 1 \end{cases}.$$

Also from McKenzie (1988), the expression for the joint mass probability function of (N_k, N_{k-1}) is given by

$$\Pr(N_k = n_k, N_{k-1} = n_{k-1}) = \\ = e^{-\left(2-\frac{\alpha}{1+\alpha}\right)\lambda} \sum_{j=0}^{\min(n_k;n_{k-1})} \frac{\left(\dfrac{\alpha}{1+\alpha}\right)^j \left(1-\dfrac{\alpha}{1+\alpha}\right)^{n_k+n_{k-1}-2j} \lambda^{n_k+n_{k-1}-j}}{j!\,(n_k-j)!\,(n_{k-1}-j)!},$$

for $n_k, n_{k-1} \in \mathbb{N}$. See e.g. McKenzie (1988, 2003) for other properties of the Poisson MA(1) model.

2.1.2. *Expression for $c(r)$*

We derive the expression of the function $c(r)$ in a risk model which considers a Poisson MA(1) process for the dependence structure of the number of claims. As previously mentioned, the solution to $c(r) = 0$ is the adjustment coefficient which enables us to examine the riskiness of the surplus process.

Proposition 1. The expression for $c(r)$ is given by

$$c(r) = \frac{\lambda(1-\alpha)}{1+\alpha} M_B(r) + \frac{\lambda\alpha}{(1+\alpha)} M_B^2(r) - \frac{\lambda}{1+\alpha} - r\pi. \tag{7}$$

Proof. The m.g.f. of S_n is expressed as

$$\begin{aligned} E[e^{rS_n}] &= E\left[e^{r(W_1 + \ldots + W_n)}\right] \\ &= M_{W_1,\ldots,W_n}(r,\ldots,r). \end{aligned} \tag{8}$$

Let the joint p.g.f. of $(N_1, \ldots, N_n)$ be given by

$$P_{N_1,\ldots,N_n}(t_1,\ldots,t_n) = E[t_1^{N_1} t_2^{N_2} \ldots t_n^{N_n}].$$

The expression for the multivariate m.g.f. $M_{W_1,\ldots,W_n}(r_1, \ldots, r_n)$ of $(W_1, \ldots, W_n)$ is defined in terms of $P_{N_1,\ldots,N_n}(t_1, \ldots, t_n)$ and the m.g.f. of B

$$M_{W_1,\ldots,W_n}(r_1,\ldots,r_n) = P_{N_1,\ldots,N_n}(M_B(r_1),\ldots,M_B(r_n)). \tag{9}$$

The expression for $P_{N_1,\ldots,N_n}(t_1, \ldots, t_n)$ is given by

$$\begin{aligned} E\left[t_1^{N_1} t_2^{N_2} \ldots t_n^{N_n}\right] &= E\left[t_1^{\varepsilon_1} t_1^{\sum_{j=1}^{\varepsilon_0}\delta_{0,j}} t_2^{\varepsilon_2} t_2^{\sum_{j=1}^{\varepsilon_1}\delta_{1,j}} \ldots t_n^{\varepsilon_n} t_n^{\sum_{j=1}^{\varepsilon_{n-1}}\delta_{n-1,j}}\right] \\ &= E\left[t_1^{\sum_{j=1}^{\varepsilon_0}\delta_{0,j}}\right] E\left[t_1^{\varepsilon_1} t_2^{\sum_{j=1}^{\varepsilon_1}\delta_{1,j}}\right] \ldots E\left[t_{n-1}^{\varepsilon_{n-1}} t_n^{\sum_{j=1}^{\varepsilon_{n-1}}\delta_{n-1,j}}\right] E\left[t_n^{\varepsilon_n}\right], \end{aligned} \tag{10}$$

where

$$E\left[t_1^{\sum_{j=1}^{\varepsilon_0}\delta_{0,j}}\right] = e^{\frac{\lambda}{1+\alpha}((1-\alpha)+\alpha t_1 - 1)} = e^{\frac{\lambda\alpha}{1+\alpha}(t_1 - 1)} \tag{11}$$

$$E\left[t_n^{\varepsilon_n}\right] = e^{\frac{\lambda}{1+\alpha}(t_n - 1)}, \tag{12}$$

and

$$
\begin{aligned}
E\left[t_{n-1}^{\varepsilon_{n-1}} t_n^{\sum_{j=1}^{\varepsilon_{n-1}} \delta_{n-1,j}}\right] &= E_{\varepsilon_{n-1}}\left[E\left[t_{n-1}^{\varepsilon_{n-1}} t_n^{\sum_{j=1}^{\varepsilon_{n-1}} \delta_{n-1,j}} \,\middle|\, \varepsilon_{n-1}\right]\right] \\
&= E_{\varepsilon_{n-1}}\left[t_{n-1}^{\varepsilon_{n-1}} E\left[t_n^{\sum_{j=1}^{\varepsilon_{n-1}} \delta_{n-1,j}} \,\middle|\, \varepsilon_{n-1}\right]\right] \\
&= E_{\varepsilon_{n-1}}\left[t_{n-1}^{\varepsilon_{n-1}} (1-\alpha+\alpha t_n)^{\varepsilon_{n-1}}\right] \\
&= E_{\varepsilon_{n-1}}\left[\left(t_{n-1}(1-\alpha+\alpha t_n)\right)^{\varepsilon_{n-1}}\right] \\
&= e^{\frac{\lambda}{1+\alpha}((t_{n-1}(1-\alpha+\alpha t_n))-1)} \\
&= e^{\frac{\lambda}{1+\alpha}((1-\alpha)t_{n-1}+\alpha t_{n-1} t_n-1)}.
\end{aligned} \tag{13}
$$

Substituting (11), (12), and (13) into (10), we obtain

$$
\begin{aligned}
E\left[t_1^{N_1} t_2^{N_2} \dots t_n^{N_n}\right] = {} & e^{\frac{\lambda}{1+\alpha}((1-\alpha)+\alpha t_1-1)} e^{\frac{\lambda}{1+\alpha}((1-\alpha)t_1+\alpha t_1 t_2-1)} \cdots \\
& e^{\frac{\lambda}{1+\alpha}((1-\alpha)t_{n-1}+\alpha t_{n-1} t_n-1)} e^{\frac{\lambda}{1+\alpha}(t_n-1)}.
\end{aligned} \tag{14}
$$

Combining (14), (9) and (8), we have

$$
\begin{aligned}
E[e^{rS_n}] &= e^{\frac{\lambda}{1+\alpha}((1-\alpha)+\alpha M_B(r)-1)} e^{\frac{(n-1)\lambda}{1+\alpha}((1-\alpha)M_B(r)+\alpha M_B(r)^2-1)} e^{\frac{\lambda}{1+\alpha}(M_B(r)-1)} \\
&= e^{\frac{\lambda}{1+\alpha}\left\{(1+\alpha+(n-1)(1-\alpha))M_B(r)+(n-1)\alpha M_B(r)^2+1-\alpha-1-n+1-1\right\}} \\
&= e^{\frac{\lambda}{1+\alpha}\left\{(1+\alpha+(n-1)(1-\alpha))M_B(r)+(n-1)\alpha M_B(r)^2-\alpha-n\right\}} \\
&= e^{\frac{\lambda(n+\alpha)}{1+\alpha}\left\{\frac{(1+\alpha+(n-1)(1-\alpha))M_B(r)+(n-1)\alpha M_B(r)^2}{n+\alpha}-1\right\}} \\
&= e^{\frac{\lambda(n+\alpha)}{1+\alpha}\left\{\frac{(n(1-\alpha)+2\alpha)M_B(r)+(n-1)\alpha M_B(r)^2}{n+\alpha}-1\right\}}.
\end{aligned} \tag{15}
$$

After inserting (15) in (3), we obtain

$$
\begin{aligned}
c_n(r) &= \frac{1}{n}\ln\left(E\left[\exp(r(S_n-n\pi))\right]\right) \\
&= \frac{1}{n}\frac{\lambda}{1+\alpha}\left\{(n(1-\alpha)+2\alpha)\,M_B(r)+(n-1)\,\alpha M_B(r)^2-(n+\alpha)\right\}-r\pi \\
&= \frac{\lambda(1-\alpha)}{1+\alpha}M_B(r)+\frac{\lambda 2\alpha}{n(1+\alpha)}\,M_B(r)+\frac{\lambda\alpha(n-1)}{n(1+\alpha)}\,M_B^2(r)-\frac{(n+\alpha)\lambda}{n(1+\alpha)}-r\pi,
\end{aligned}
$$

which implies

$$
c(r) = \lim_{n\to\infty} c_n(r) = \frac{\lambda(1-\alpha)}{1+\alpha}\,M_B(r)+\frac{\lambda\alpha}{(1+\alpha)}\,M_B^2(r)-\frac{\lambda}{1+\alpha}-r\pi.
$$

□

Remark 2. *Given* (15), $S_n = \sum_{k=1}^{n} W_k$ *follows a compound Poisson distribution i.e. we can express* S_n *as*

$$S_n = \begin{cases} \sum_{j=1}^{M_n} C_j^{(n)}, & M_n > 0 \\ 0, & M_n = 0 \end{cases},$$

where M_n *has a Poisson distribution with mean* $\frac{\lambda(n+\alpha)}{1+\alpha}$ *and* $C_1^{(n)}, C_2^{(n)}, \ldots$ *is a sequence of i.i.d. r.v.'s distributed as* $C^{(n)}$ *with*

$$F_{C^{(n)}}(x) = \frac{(n(1-\alpha)+2\alpha)\,F_B(x) + (n-1)\alpha F_B^{*2}(x)}{n+\alpha}.$$

Note that F_B^{*n} *denotes the n-fold convolution of* F_B *for* $n \in \mathbb{N}^+$. *If* $\alpha = 0$, *then*

$$E[e^{rS_n}] = e^{n\lambda\{M_B(r)-1\}},$$

which corresponds to the m.g.f of the aggregate claim amount in the classical discrete time risk model.

2.1.3. *Impact of the parameter* α

To analyze the impact of the dependence parameter α, we take the derivative of $c(r)$ with respect to α and we obtain

$$\begin{aligned}
\frac{\partial c(r)}{\partial \alpha} &= -\frac{\lambda}{1+\alpha} M_B(r) - \frac{\lambda(1-\alpha)}{(1+\alpha)^2} M_B(r) + \frac{\lambda}{(1+\alpha)} M_B^2(r) \\
&\quad - \frac{\lambda\alpha}{(1+\alpha)^2} M_B^2(r) + \frac{\lambda}{(1+\alpha)^2} \\
&= -\frac{\lambda(1+\alpha)}{(1+\alpha)^2} M_B(r) - \frac{\lambda(1-\alpha)}{(1+\alpha)^2} M_B(r) + \frac{\lambda(1+\alpha)}{(1+\alpha)^2} M_B^2(r) \\
&\quad - \frac{\lambda\alpha}{(1+\alpha)^2} M_B^2(r) + \frac{\lambda}{(1+\alpha)^2} \\
&= -\frac{2\lambda}{(1+\alpha)^2} M_B(r) + \frac{\lambda}{(1+\alpha)^2} M_B^2(r) + \frac{\lambda}{(1+\alpha)^2} \\
&= \frac{\lambda}{(1+\alpha)^2} (M_B(r) - 1)^2 \geq 0.
\end{aligned}$$

If $\alpha < \alpha'$, it follows from above that the solutions ρ and ρ' to (7) are such that $\rho > \rho'$. This implies that the degree of dangerousness represented by the adjustment coefficient increases with the dependence parameter α. Given the structure of the model, when the dependence parameter increases, it becomes more likely that claims in period $k-1$ also lead to claims in period k, which increases the dangerousness of the portfolio.

The impact of the dependence parameter α on the Lundberg coefficient could have been studied using the supermodular order. However, after investigation, the proof of this inequality based on supermodular ordering remains an open problem.

2.1.4. *An explicit expression for the adjustment coefficient*

In the following proposition, we derive an explicit expression for the adjustment coefficient ρ in the case where the claim amount B is exponentially distributed.

Proposition 3. *Assume that $B \sim Exp(\beta)$ with mean $\frac{1}{\beta}$ and m.g.f. $M_B(r) = \frac{\beta}{\beta - r}$. Then, we have*

$$\rho = \frac{\beta}{2(1+\eta)}\left(2(1+\eta) - \frac{1}{1+\alpha} - \sqrt{4\frac{\alpha(1+\eta)}{1+\alpha} + \frac{1}{(1+\alpha)^2}}\right). \tag{16}$$

Proof. Here, the function $c(r)$ is

$$c(r) = \frac{\lambda(1-\alpha)}{1+\alpha}\frac{\beta}{\beta-r} + \frac{\lambda\alpha}{(1+\alpha)}\frac{\beta^2}{(\beta-r)^2} - \frac{\lambda}{1+\alpha} - r(1+\eta)\frac{\lambda}{\beta} = 0$$

which is equivalent to

$$(1-\alpha)\,\zeta\frac{\beta^2}{\beta-r} + \alpha\zeta\frac{\beta^3}{(\beta-r)^2} - \beta\zeta = r(1+\eta), \tag{17}$$

with $\frac{1}{1+\alpha} = \zeta$. Multiplying (17) by $(\beta - r)^2$, we obtain

$$(1-\alpha)\,\zeta\beta^2(\beta-r) + \alpha\zeta\beta^3 - \beta\zeta(\beta-r)^2 = r(1+\eta)(\beta-r)^2$$

leading to the equality

$$r((1+\eta)r^2 + (\beta\zeta - 2\beta(1+\eta))r + \eta\beta^2) = 0. \tag{18}$$

Solution to (18) leads to the desired result. □

Note that, when $\alpha = 0$ in (16), the adjustment coefficient becomes

$$\rho = \frac{\beta\eta}{1+\eta},$$

which corresponds to the adjustment coefficient when claim number r.v.'s are assumed to be independent.

To take a look at the impact of α on ρ, we differentiate the expression derived for ρ in terms of α. We find

$$\frac{\partial \rho}{\partial \alpha} = \frac{\beta}{2(1+\eta)} \left(\frac{1}{(1+\alpha)^2} - \frac{1}{(1+\alpha)^2} \frac{\left(2(1+\eta) - \frac{1}{1+\alpha}\right)}{\sqrt{\left(2(1+\eta) - \frac{1}{1+\alpha}\right)^2 - 4(1+\eta)\eta}} \right). \tag{19}$$

Since

$$\frac{\left(2(1+\eta) - \frac{1}{1+\alpha}\right)}{\sqrt{\left(2(1+\eta) - \frac{1}{1+\alpha}\right)^2 - 4(1+\eta)\eta}} > 1,$$

for $\alpha \in [0, 1]$, we have $\frac{d\rho}{d\alpha} < 0$. Hence, the adjustment coefficient ρ decreases (as shown in Section 2.1.3) as the dependence parameter α increases.

Example 4. *We consider an insurance portfolio where the claim amount r.v. B has an exponential distribution with mean $\frac{1}{\beta} = 1$. The premium income π includes a relative risk margin η equal to 20%. In Table 1, we provide values of ρ computed with (16) for different values of α. Using (2), we approximate the infinite time ruin probability $\psi(u)$ by $e^{-\rho u}$ and, based on this approximation, we find the amount of initial surplus required to have an infinite time ruin probability of 1%.*

TABLE 1

VALUES OF THE LUNDBERG COEFFICIENT IN THE POISSON MA(1) RISK MODEL

α	0	0.25	0.5	0.75	1
ρ	0.1667	0.1396	0.1265	0.1186	0.1134
$u = -\rho^{-1} \ln(0.01)$	27.6310	32.9835	36.4174	38.8272	40.6162

Results in Table 1 clearly confirm that the adjustment coefficient ρ decreases as the dependence parameter α increases. The adjustment coefficient is a measure of dangerousness of the risk portfolio. As the adjustment coefficient decreases, the risk process becomes more dangerous. Based on the approximation of $\psi(u)$ by $e^{-\rho u}$, it means that the initial surplus which is required to have an infinite time ruin probability of 1% increases as the dependence parameter α increases. Therefore, we may conclude that if the dependence between the claim number r.v.'s is at a high (low) level then it requires a large (small) amount of initial surplus to satisfy an infinite time ruin probability of 1%. □

2.2. Risk Model – Poisson AR(1)

2.2.1. *Definitions and properties*

We suppose here that $\underline{N} = \{N_k, k \in \mathbb{N}^+\}$ is a Poisson AR(1) process where the r.v. N_1 has a Poisson distribution with mean λ and the autoregressive dynamic for $N_2, N_3, \ldots$ is given by

$$N_k = \varepsilon_k + \alpha \circ N_{k-1}, \tag{20}$$

for $k = 2, 3, \ldots$. We assume that $\underline{\varepsilon} = \{\varepsilon_k, k \in \mathbb{N}^+\}$ is a sequence of i.i.d. r.v.'s following a Poisson distribution with mean $(1-\alpha)\lambda$ where $\alpha \in [0,1]$. Following Joe (1997), the dependence structure of the Poisson AR(1) process can be represented as follows

$$\begin{aligned}
N_2 &= \sum_{i=1}^{N_1} \delta_{21i} + \varepsilon_2, \\
N_3 &= \sum_{i=1}^{N_1} \delta_{21i}\,\delta_{31i} + \sum_{i=1}^{\varepsilon_2} \delta_{31i} + \varepsilon_3, \\
&\ldots \\
N_k &= \sum_{i=1}^{N_1} \delta_{21i}\,\delta_{31i} \ldots \delta_{k1i} + \sum_{j=2}^{k-1} \sum_{i=1}^{\varepsilon_j} \prod_{l=j+1}^{k} \delta_{l,j,i} + \varepsilon_k \; (k = 3, 4, \ldots).
\end{aligned}$$

The r.v.'s $\varepsilon_2, \varepsilon_3, \ldots$ are i.i.d. and follow a Poisson distribution with mean $((1-\alpha)\lambda)$ and $\delta_{21}, \delta_{31}, \delta_{32}, \ldots, \delta_{k1}, \ldots, \delta_{k,k-1}$ are i.i.d. Bernoulli r.v's with mean α. Hence, given a sequence of i.i.d. Poisson r.v's $\underline{\varepsilon}$ with mean $(1-\alpha)\lambda$, the model given in (20) yields a stationary sequence of Poisson r.v.'s with mean λ. The model given by (20) may be interpreted in the context of the evolution of a population as the number of people at time k, N_k, being the sum of those who arrive in the interval $(k-1, k)$ and survive until time k, i.e. ε_k, and those who survive from time $(k-1)$ to k, *i.e.* $\alpha \circ N_{k-1}$. In an insurance context, the number of claims in period k, meaning N_k, can be viewed as the sum of the new claims during period k, and the claims of period $k-1$ leading to claims in period k.

As for a classical Gaussian AR(1) model, the autocorrelation function for $\underline{N}$ is equal to $\gamma_N(h) = \alpha^h$, for $h \geq 1$ (see McKenzie (1988)) with $\gamma_N(1) \in [0,1)$. The expression for the covariance between W_k and W_{k+h} corresponds to

$$Cov(W_k, W_{k+h}) = \lambda\alpha^h E[B]^2,$$

for $h \geq 1$. The joint p.m.f. of (N_k, N_{k-1}) is given by

$$\Pr(N_k = n_k, N_{k-1} = n_{k-1}) = e^{-(2-\alpha)\lambda} \sum_{j=0}^{\min(n_k; n_{k-1})} \frac{\alpha^j (1-\alpha)^{n_k + n_{k-1} - 2j} \lambda^{n_k + n_{k-1} - j}}{j!(n_k - j)!(n_{k-1} - j)!},$$

(see, e.g. McKenzie (1988)).

2.2.2. *Expression for* $c(r)$

The expression for $c(r)$ is provided in the following proposition.

Proposition 5. *Assuming that* $\alpha M_B(r) < 1$*, the expression for* $c(r)$ *is given by*

$$c(r) = \frac{(1-\alpha)^2 \lambda M_B(r)}{1-(\alpha M_B(r))} - (1-\alpha)\lambda - r\pi = \frac{\gamma^2 \lambda M_B(r)}{1-(\alpha M_B(r))} - \gamma\lambda - r\pi, \quad (21)$$

where $\gamma = 1 - \alpha$.

Proof. We have

$$S_n = W_1 + ... + W_n = \sum_{j=1}^{L_n} C_j$$

where

$$L_n = N_1 + N_2 + ... + N_n$$

and C_1, C_2, ... form a sequence of i.i.d. r.v.'s distributed as B.

Also, we have

$$\begin{aligned} M_{S_n}(r) &= P_{N_1, ..., N_n}(M_B(r), ..., M_B(r)) \\ &= E\left[M_B(r)^{N_1} ... M_B(r)^{N_n} \right] \\ &= E\left[M_B(r)^{N_1 + ... + N_n} \right] \\ &= E\left[M_B(r)^{L_n} \right] \\ &= P_{L_n}(M_B(r)). \end{aligned}$$

We need to find the expression for $P_{L_n}(t)$.

Let us develop the expressions for $P_{L_n}(t)$ for periods $n = 1, 2, 3, 4$. We have

$$P_{L_n}(t) = E\left[t^{N_1 + ... + N_n} \right] = E\left[t^{N_1} ... t^{N_n} \right].$$

For $n = 1$, we have

$$P_{L_n}(t) = E\left[t^{N_1} \right] = e^{\lambda(t-1)}.$$

For $n = 2$, we have

$$\begin{aligned} P_{L_n}(t) &= E\left[t^{N_1} t^{N_2} \right] \\ &= E\left[t^{N_1} t^{\sum_{i=1}^{N_1} \delta_{21i}} t^{\varepsilon_2} \right] \\ &= e^{\lambda(\gamma t + \alpha t^2 - 1)} e^{\gamma\lambda(t-1)}. \end{aligned}$$

For $n = 3$, we find

$$
\begin{aligned}
P_{L_n}(t) &= E\left[t^{N_1} t^{N_2} t^{N_3}\right] \\
&= E\left[t^{N_1} t^{\left\{\sum_{i=1}^{N_1} \delta_{21i} + \sum_{i=1}^{N_1} \delta_{21i}\, \delta_{31i}\right\}} t^{\varepsilon_2} t^{\left\{\sum_{i=1}^{\varepsilon_2} \delta_{32i}\right\}} t^{\varepsilon_3}\right] \\
&= E\left[t^{N_1} t^{\left\{\sum_{i=1}^{N_1} \delta_{21i} + \sum_{i=1}^{N_1} \delta_{21i}\, \delta_{31i}\right\}}\right] E\left[t^{\varepsilon_2} t^{\left\{\sum_{i=1}^{\varepsilon_2} \delta_{32i}\right\}}\right] E\left[t^{\varepsilon_3}\right] \\
&= e^{\lambda((\gamma^2 + \alpha\gamma)t + \alpha\gamma t^2 + \alpha^2 t^3 - 1)} e^{\gamma\lambda(+\alpha t^2 - 1)} e^{\gamma\lambda(t-1)}.
\end{aligned}
$$

For $n = 4$, we begin with

$$
\begin{aligned}
P_{L_n}(t) &= E\left[t^{N_1} t^{N_2} t^{N_3} t^{N_4}\right] \\
&= E\left[t^{\{N_1 + \sum_{i=1}^{N_1} \delta_{21i} + \sum_{i=1}^{N_1} \delta_{21i} \delta_{31i} + \sum_{i=1}^{N_1} \delta_{21i} \delta_{31i} \delta_{41i}\}} t^{\{\varepsilon_2 + \sum_{i=1}^{N_1} \delta_{32i} + \sum_{i=1}^{\varepsilon_2} \delta_{32i} \delta_{42i}\}} t^{\{\varepsilon_3 + \sum_{i=1}^{\varepsilon_3} \delta_{43i}\}} t^{\varepsilon_4}\right] \\
&= E\left[t^{\{N_1 + \sum_{i=1}^{N_1} \delta_{21i} + \sum_{i=1}^{N_1} \delta_{21i} \delta_{31i} + \sum_{i=1}^{N_1} \delta_{21i} \delta_{31i} \delta_{41i}\}}\right] \\
&\quad \times E\left[t^{\{\varepsilon_2 + \sum_{i=1}^{\varepsilon_2} \delta_{32i} + \sum_{i=1}^{\varepsilon_2} \delta_{32i} \delta_{42i}\}}\right] E\left[t^{\{\varepsilon_3 + \sum_{i=1}^{\varepsilon_3} \delta_{43i}\}}\right] E\left[t^{\varepsilon_4}\right] \\
&= E\left[t^{\{N_1 + \sum_{i=1}^{N_1} \delta_{21i} + \sum_{i=1}^{N_1} \delta_{21i} \delta_{31i} + \sum_{i=1}^{N_1} \delta_{21i} \delta_{31i} \delta_{41i}\}}\right] \\
&\quad \times e^{\gamma\lambda((\gamma^2 + \alpha\gamma)t + \alpha\gamma t^2 + \alpha^2 t^3 - 1)} e^{\gamma\lambda(\gamma t + \alpha t^2 - 1)} e^{\gamma\lambda(t-1)}.
\end{aligned} \tag{22}
$$

Given (22), we need to find the expression for

$$
E\left[t^{\{N_1 + \sum_{i=1}^{N_1} \delta_{21i} + \sum_{i=1}^{N_1} \delta_{21i} \delta_{31i} + \sum_{i=1}^{N_1} \delta_{21i} \delta_{31i} \delta_{41i}\}}\right]
$$

and we obtain the following

$$
\begin{aligned}
&E\left[t^{\{N_1 + \sum_{i=1}^{N_1} \delta_{21i} + \sum_{i=1}^{N_1} \delta_{21i} \delta_{31i} + \sum_{i=1}^{N_1} \delta_{21i} \delta_{31i} \delta_{41i}\}}\right] \\
&= E\left[t^{N_1} E\left[t^{\{\sum_{i=1}^{N_1} \delta_{21i}\}} E\left[t^{\{\sum_{i=1}^{N_1} \delta_{21i} \delta_{31i}\}} E\left[t^{\{\sum_{i=1}^{N_1} \delta_{21i} \delta_{31i} \delta_{41i}\}} \,\middle|\, N_1, \delta_{2i}, \delta_{3i}\right] \middle|\, N_1, \delta_{21i}\right] \middle|\, N_1\right]\right] \\
&= E\left[t^{N_1} E\left[t^{\{\sum_{i=1}^{N_1} \delta_{21i}\}} E\left[t^{\{\sum_{i=1}^{N_1} \delta_{21i} \delta_{31i}\}} \prod_{i=1}^{N_1} \left(\gamma + \alpha t^{\{\delta_{21i} \delta_{31i}\}}\right) \middle|\, N_1, \delta_{21i}\right] \middle|\, N_1\right]\right] \\
&= E\left[t^{N_1} E\left[t^{\{\sum_{i=1}^{N_1} \delta_{21i}\}} E\left[\prod_{i=1}^{N_1} \left(\gamma t^{\{\delta_{21i} \delta_{31i}\}} + \alpha t^{\{2\delta_{21i} \delta_{31i}\}}\right) \middle|\, N_1, \delta_{21i}\right] \middle|\, N_1\right]\right]
\end{aligned}
$$

$$
\begin{aligned}
&= E\left[t^{N_1} E\left[t^{\{\Sigma_{i=1}^{N_1}\delta_{21i}\}} \prod_{i=1}^{N_1}\left(\gamma\left(\gamma+\alpha t^{\delta_{21i}}\right)+\alpha\left(\gamma+\alpha t^{2\delta_{21i}}\right)\right)\middle| N_1\right]\right] \\
&= E\left[t^{N_1} E\left[\prod_{i=1}^{N_1}\left(\gamma t^{\delta_{21i}}\left(\gamma+\alpha t^{\delta_{21i}}\right)+\alpha t^{\delta_{21i}}\left(\gamma+\alpha t^{2\delta_{21i}}\right)\right)\middle| N_1\right]\right] \\
&= E\left[t^{N_1} E\left[\prod_{i=1}^{N_1}\left(\gamma^2 t^{\delta_{21i}}+\alpha\gamma t^{2\delta_{21i}}+\gamma\alpha t^{\delta_{21i}}+\alpha^2 t^{3\delta_{21i}}\right)\middle| N_1\right]\right]. \qquad (23)
\end{aligned}
$$

Further manipulations lead to

$$
\begin{aligned}
&E\left[t^{N_1} E\left[\prod_{i=1}^{N_1}\left(\gamma^2 t^{\delta_{21i}}+\alpha\gamma t^{2\delta_{21i}}+\gamma\alpha t^{\delta_{21i}}+\alpha^2 t^{3\delta_{21i}}\right)\middle| N_1\right]\right] \\
&= E\left[t^{N_1}\prod_{i=1}^{N_1}\left(\gamma^2(\gamma+\alpha t)+\alpha\gamma(\gamma+\alpha t^2)+\gamma\alpha(\gamma+\alpha t)+\alpha^2(\gamma+\alpha t^3)\right)\right] \\
&= E\left[t^{N_1}\left(\gamma^2(\gamma+\alpha t)+\alpha\gamma(\gamma+\alpha t^2)+\gamma\alpha(\gamma+\alpha t)+\alpha^2(\gamma+\alpha t^3)\right)^{N_1}\right] \\
&= e^{\lambda((\gamma^3+2\alpha\gamma^2+\alpha^2\gamma)t+(\alpha\gamma^2+\alpha^2\gamma)t^2+\alpha^2\gamma t^3+\alpha^3 t^4-1)}. \qquad (24)
\end{aligned}
$$

Finally, combining (24) and (22), we obtain

$$
\begin{aligned}
P_{L_4}(t) &= E\left[t^{N_1}t^{N_2}t^{N_3}t^{N_4}\right] \\
&= e^{\lambda\{(\gamma^3+2\alpha\gamma^2+\alpha^2\gamma)t+(\alpha\gamma^2+\alpha^2\gamma)t^2+\alpha^2\gamma t^3+\alpha^3 t^4-1\}} \\
&\quad\times e^{\gamma\lambda\{(\gamma^2+\alpha\gamma)t+\alpha\gamma t^2+\alpha^2 t^3-1\}}\, e^{\gamma\lambda\{\delta t+\alpha t^2-1\}}\, e^{\gamma\lambda\{(t-1)\}}
\end{aligned}
$$

and

$$
\begin{aligned}
M_{S_4}(r) &= P_{L_4}(M_B(r)) \\
&= e^{\lambda\{(\gamma^3+2\alpha\gamma^2+\alpha^2\gamma)M_B(r)+(\alpha\gamma^2+\alpha^2\gamma)M_B(r)^2+\alpha^2\gamma M_B(r)^3+\alpha^3 M_B(r)^4-1\}} \qquad (25) \\
&\quad\times e^{\gamma\lambda\{(\gamma^2+\alpha\gamma)M_B(r)+\alpha\gamma M_B(r)^2+\alpha^2 M_B(r)^3-1\}}\, e^{\gamma\lambda\{\gamma M_B(r)+\alpha M_B(r)^2-1\}}\, e^{\gamma\lambda\{M_B(r)-1\}}.
\end{aligned}
$$

In (25), we observe that

$$
\begin{aligned}
(\gamma^2+\alpha\gamma) &= \gamma(\gamma+\alpha) = \gamma \\
(\alpha\gamma^2+\alpha^2\gamma) &= \gamma\alpha(\gamma+\alpha) = \alpha\gamma \\
(\gamma^3+2\alpha\gamma^2+\alpha^2\gamma) &= \gamma(\gamma+\alpha)^2 = \gamma \\
& \mathit{etc.}
\end{aligned}
$$

Consequently, we deduce the following general form for M_{S_n} for $n = 2, 3, \ldots$

$$\begin{aligned} M_{S_n}(r) &= P_{L_n}(M_B(r)) \\ &= e^{\alpha\lambda\left\{\gamma M_B(r)\Sigma_{k=0}^{n}(\alpha M_B(r))^k + \alpha^n (M_B(r))^n - n\right\}} \\ &\quad \times e^{\gamma\lambda\left\{\gamma n M_B(r)\Sigma_{k=0}^{n}(\alpha M_B(r))^k + M_B(r)\Sigma_{k=0}^{n}(\alpha M_B(r))^k - M_B(r)\Sigma_{k=0}^{n}(k+1)(\alpha M_B(r))^k - n\right\}} \qquad (26) \\ &= e^{\alpha\lambda\left\{\gamma M_B(r)\frac{1-(\alpha M_B(r))^n}{1-\alpha M_B(r)} + \alpha^n (M_B(r))^n - n\right\}} \\ &\quad \times e^{\gamma\lambda\left\{\gamma n M_B(r)\frac{1-(\alpha M_B(r))^n}{1-\alpha M_B(r)} + M_B(r)\frac{1-(\alpha M_B(r))^n}{1-\alpha M_B(r)} - M_B(r)\left(-\frac{-n(\alpha M_B(r))^{n-1}}{(1-\alpha M_B(r))} + \frac{1-(\alpha M_B(r))^n}{(1-\alpha M_B(r))^2}\right) - n\right\}}. \end{aligned}$$

From (26), we find this expression for $c_n(r)$

$$\begin{aligned} & c_n(r) \\ = {} & \frac{1}{n}\ln\left\{E\left[e^{r(S_n-\pi)}\right]\right\} \\ = {} & \frac{\left\{\begin{array}{c} \alpha\lambda\left(\gamma M_B(r)\dfrac{1-(\alpha M_B(r))^n}{1-\alpha M_B(r)} + \alpha^n (M_B(r))^n - 1\right) \\ + \gamma\lambda M_B(r)\left(\gamma n \dfrac{1-(\alpha M_B(r))^n}{1-\alpha M_B(r)} + \dfrac{1-(\alpha M_B(r))^n}{1-\alpha M_B(r)} - \dfrac{n(\alpha M_B(r))^{n-1}}{(1-\alpha M_B(r))} - \dfrac{1-(\alpha M_B(r))^n}{(1-\alpha M_B(r))^2}\right) \\ - n\gamma\lambda - n\pi r \end{array}\right\}}{n}. \end{aligned} \qquad (27)$$

Assuming that $\alpha M_B(r) < 1$ and taking the limit of (27), we obtain the desired result

$$c(r) = \lim_{n\to\infty} c_n(r) = \frac{\gamma^2 \lambda M_B(r)}{1-(\alpha M_B(r))} - \gamma\lambda - r\pi.$$

□

Note that, given (26), S_n follows a compound Poisson distribution.

Combining (4) and (21), ρ is the strictly positive solution to

$$\frac{\gamma^2 M_B(r)}{1-(\alpha M_B(r))} - \gamma - r(1+\theta)E[B] = 0,$$

in which the parameter λ does not appear.

2.2.3. *Impact of the parameter* α

To have an idea of the influence of the dependence parameter α on the Lundberg coefficient, we take the derivative of $c(r)$ defined by (21) with respect to α

$$\frac{dc(r)}{d\alpha} = \frac{2(1-\alpha)\lambda M_B(r)}{1-(\alpha M_B(r))} + M_B(r)\frac{(1-\alpha)^2\lambda M_B(r)}{(1-(\alpha M_B(r)))^2} + \lambda \geq 0.$$

If $\alpha < \alpha'$, then the solutions ρ and ρ' to (21) are such that $\rho > \rho'$ since $\frac{dc(r)}{d\alpha} \geq 0$. It means that the degree of dangerousness represented by the adjustment coefficient increases with the dependence parameter α.

2.2.4. *An explicit expression for the adjustment coefficient*

A nice expression for the Lundberg coefficient is provided in the next proposition when the claim amount r.v. B follows an exponential distribution.

Proposition 6. *Assume that* $B \sim Exp(\beta)$ with mean $\frac{1}{\beta}$ *and m.g.f.* $M_B(r) = \frac{\beta}{\beta - r}$. *Then, we have*

$$\rho = \frac{\gamma\beta\eta}{1+\eta} = \frac{(1-\alpha)\beta\eta}{1+\eta}, \tag{28}$$

where $\alpha \in [0, 1)$.

Proof. When B follows an exponential distribution, (21) becomes

$$\frac{\gamma^2\lambda\frac{\beta}{\beta-r}}{1-\alpha\frac{\beta}{\beta-r}} - \gamma\lambda - r\pi = 0,$$

which can be rewritten as follows

$$\frac{\lambda\gamma^2\beta}{\gamma\beta - r} - \gamma\lambda - r\pi = \frac{\gamma^2\beta}{\gamma\beta - r} - \gamma - r(1+\eta)E[B] = 0,$$

or

$$\gamma^2\beta - \gamma^2\beta - \gamma(1+\eta)r + r\gamma + r^2(1+\eta)E[B] = 0.$$

After some rearrangements, we find the desired result. □

As expected, if $\alpha = 0$ (i.e. the independence case), the expression (28) for the Lundberg coefficient is reduced to

$$\rho = \frac{\beta\eta}{1+\eta}.$$

Example 7. *For an insurance portfolio, we assume that the claim amount r.v. B has an exponential distribution with mean* $\frac{1}{\beta} = 1$ *and that the premium income* π *includes a relative risk margin* η *of 20%. In Table 2, we apply (28) to compute values of* ρ *assuming different values of dependence parameter* α*. Using (2), we approximate the infinite time ruin probability* $\psi(u)$ *by* $e^{-\rho u}$ *and, based on this approximation, we find the amount of initial surplus required to have an infinite time ruin probability of 1%.*

TABLE 2

VALUES OF THE LUNDBERG COEFFICIENT IN THE POISSON AR(1) RISK MODEL.

α	0	0.25	0.5	0.75	0.995
ρ	0.1667	0.125	0.0833	0.04167	0.00083
$u = -\rho^{-1}\ln(0.01)$	27.6310	36.8414	55.2620	110.5241	5526.2042

Results in Table 2 illustrate the dramatic impact of the dependence parameter α *on the adjustment coefficient* ρ*. The risk process becomes more dangerous as the dependence (represented by the parameter* α*) between the claim number r.v.'s become more significant. Therefore, it requires a larger (smaller) amount of initial surplus to satisfy an infinite time ruin probability of 1% as the dependence parameter increases (decreases).* □

2.3. Comments

One can carry a similar analysis for a Poisson moving average or a Poisson autoregressive model of order greater than 1, and also for a Poisson autoregressive moving average process. Other marginals such as the negative binomial distribution could be considered.

3. MARKOV BERNOULLI PROCESS

3.1. Definitions and properties

We assume that the claim number process $\underline{N}$ is a Markov Bernoulli process i.e. $\underline{N}$ is a Markov chain with state space $\{0,1\}$ and with transition probability matrix

$$\underline{P} = \begin{pmatrix} 1-(1-\alpha)q & (1-\alpha)q \\ (1-\alpha)(1-q) & \alpha+(1-\alpha)q \end{pmatrix} = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix}, \tag{29}$$

where α can be seen as the dependence parameter, introducing a positive dependence relation between the claim numbers r.v.'s. In this risk model, at most one claim can occur over a period. The initial probabilities associated to $\underline{P}$ are

$$\Pr(N_0 = 1) = q = 1 - \Pr(N_0 = 0),$$

where $0 \le \alpha < 1$ and $0 < q < 1$. When α tends to 1, a period with a (no) claim will be likely followed by a period with a (no) claim. If $\alpha = 0$, the claim number process $\underline{N}$ becomes a sequence of i.i.d. r.v.'s.

The covariance between N_k and N_{k+h} is given by

$$\begin{aligned} Cov(N_k, N_{k+h}) &= \Pr(N_{k+h} = 1, N_k = 1) - \Pr(N_{k+h} = 1)\Pr(N_k = 1) \\ &= q\Pr(N_{k+h} = 1 \mid N_k = 1) - q^2 \\ &= q(1-q)\alpha^h, \end{aligned}$$

for $k = 1, 2, \ldots$ and $h = 0, 1, 2, \ldots$. Also, we know that $Cov(W_k, W_{k+h}) = E[B]^2$ $Cov(N_k, N_{k+h})$ for $k = 1, 2, \ldots$ and $h = 1, 2, \ldots$ which implies that

$$\begin{aligned} \gamma(h) &= \frac{E[B]^2 q(1-q)\alpha^h}{Var(B)q + E[B]^2 q(1-q)} \\ &= \frac{E[B]^2(1-q)}{Var(B) + E[B]^2(1-q)}\alpha^h, \end{aligned}$$

for $h = 1, 2, \ldots$. Hence, the autocorrelation function $\gamma(h)$ decreases exponentially. A special case of this model is examined in Cossette et al. (2003, 2004a, b). In the latter model, it is assumed that the premium rate is equal to 1 and the claim amount distribution is defined over $\mathbb{N}^+$. Here, the claim amount distribution is defined over $\mathbb{R}^+$.

3.2. Expression for $c(r)$

The next proposition gives the function $c(r)$ when $\underline{N}$ is a Markov Bernoulli process.

Proposition 8. *The expression for $c(r)$ is given by*

$$\begin{aligned} c(r) = \ln&\left\{(p_{00} + p_{11}M_B(r)) + \sqrt{(p_{00} + p_{11}M_B(r))^2 - 4M_B(r)(p_{00}p_{11} - p_{10}p_{01})}\right\} \\ &- \ln 2 - \pi r, \end{aligned}$$

where $(p_{00}p_{11} - p_{10}p_{01}) = \alpha$.

Proof. According to Example 2 of Nyrhinen (1998), $c(r)$ is the natural logarithm of the maximal real eigenvalue of the matrix $\underline{M}(r)$

$$\underline{M}(r) = \begin{pmatrix} m_{00}(r) & m_{01}(r) \\ m_{10}(r) & m_{11}(r) \end{pmatrix}$$

whose entries are

$$m_{i0}(r) = p_{i0}e^{-\pi r}$$
$$m_{i1}(r) = p_{i1}M_B(r)e^{-\pi r},$$

for $i \in \{0, 1\}$ and some $r \geq 0$. The eigenvalues of $\underline{M}(r)$ are the solution to

$$\det(\underline{M}(r) - \zeta \times \underline{J}) = 0,$$

where $\underline{J}$ is the identity matrix. Therefore,

$$\underline{M}(r) - \zeta \times \underline{J} = \begin{pmatrix} p_{00}e^{-\pi r} - \zeta & p_{01}M_B(r)e^{-\pi r} \\ p_{10}e^{-\pi r} & p_{11}M_B(r)e^{-\pi r} - \zeta \end{pmatrix}$$

and its determinant is

$$h(\zeta) = \zeta^2 - (p_{00}e^{-\pi r} + p_{11}M_B(r)e^{-\pi r})\zeta + e^{-2\pi r}M_B(r)(p_{00}p_{11} - p_{10}p_{01}).$$

The maximal solution to $h(\zeta) = 0$ is

$$\zeta^*(r) =$$

$$\frac{(p_{00}e^{-\pi r} + p_{11}M_B(r)e^{-\pi r}) + \sqrt{(p_{00}e^{-\pi r} + p_{11}M_B(r)e^{-\pi r})^2 - 4e^{-2\pi r}M_B(r)(p_{00}p_{11} - p_{10}p_{01})}}{2}$$

$$= e^{-\pi r}\frac{(p_{00} + p_{11}M_B(r)) + \sqrt{(p_{00} + p_{11}M_B(r))^2 - 4M_B(r)(p_{00}p_{11} - p_{10}p_{01})}}{2}. \tag{30}$$

Taking the natural logarithm on both sides of (30) leads to the desired result. □

Example 9. *In the context of this particular risk model, we assume that the probability of occurrence of a claim q is equal to* 0.1 *and that the claim amount r.v. B has an exponential distribution with mean* $\beta^{-1} = 1$. *The relative security margin* η *is equal to 20%. In the following table, we provide values of the Lundberg coefficient for the dependence parameter* $\alpha = 0$, *0.25, 0.5, 0.75, 0.995. Approximating* $\psi(u)$ *by* $e^{-\rho u}$, *we find the amount of surplus such that* $\psi(u) = 1\%$.

As the dependence parameter α *becomes larger, the positive dependence relation between the claim numbers r.v.'s increases. It implies that the risk process*

TABLE 3

VALUES OF THE LUNDBERG COEFFICIENT IN THE MARKOV BERNOULLI RISK MODEL

α	0	0.25	0.5	0.75	0.995
ρ	0.175383924	0.133977918	0.091008226	0.046379157	0.000948
$u = -\rho^{-1}\ln(0.01)$	26.2576528	34.37260609	50.60169187	99.29396004	4857.774458

for the portfolio becomes riskier which requires a higher initial reserve to maintain an infinite-time ruin probability of 1%. □

4. Risk model defined in a Markovian Environment

4.1. Definitions and properties

We assume that the claim number process $\underline{N} = \{N_k, k \in \mathbb{N}^+\}$ is influenced by an underlying Marvovian environment represented by the time homogeneous Markov chain $\underline{\Theta}$ defined over the 2-state space $\{\theta_1, \theta_2\}$ with transition probabilities

$$p_{ij} = \Pr(\Theta_{k+1} = \theta_j \mid \Theta_k = \theta_i),$$

for $k \in \mathbb{N}^+$. We assume that the conditional p.m.f. of $(N_k \mid \Theta_k = \theta_j)$ $(k \in \mathbb{N}^+)$ is $f_{N\mid\Theta=\theta_j}$, the conditional c.d.f. is $F_{N\mid\Theta=\theta_j}$ and the corresponding conditional m.g.f. is $M_N^{(j)}(r)$. Assume that the conditional distribution of $(N_k \mid \Theta_k = \theta_j)$ is Poisson with mean $\lambda_j\,(j = 1, 2)$ with $\lambda_1 \leq \lambda_2$. The frequencies in the two different states are generated by separate counting processes. For example in Malyshkina, Mannering and Tarko (2009) they consider two different states of roadway safety to model vehicle accident frequencies. Other discrete distributions for $(N_k \mid \Theta_k = \theta_j)$ (e.g. negative binomial, binomial, etc.) could be considered.

We suppose that the transition probability matrix $\underline{P}$ of $\underline{\Theta}$ is

$$\underline{P} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} 1-(1-\nu)\chi & (1-\nu)\chi \\ (1-\nu)(1-\chi) & \nu+(1-\nu)\chi \end{pmatrix},$$

with $0 < \chi < 1$. To avoid negative transition probabilities, the parameter ν must satisfy the constraint $\left(-\frac{1-\chi}{\chi}, -\frac{\chi}{1-\chi}\right) < \nu < 1$. The stationary probabilities associated to $\underline{P}$ are

$$\Pr(\Theta_k = \theta_1) = \frac{p_{21}}{p_{12}+p_{21}} = 1-\chi,$$

$$\Pr(\Theta_k = \theta_2) = \frac{p_{12}}{p_{12}+p_{21}} = \chi.$$

Note that the claim amounts are assumed not to be affected by the Markovian process.

The expression for the conditional m.g.f. of W_k given $\Theta_k = \theta_j$ is given by

$$\begin{aligned} E[e^{rW_k} \mid \Theta_k = \theta_j] &= E[e^{N_k \ln(M_B(r))} \mid \Theta_k = \theta_j] \\ &= M_N^{(j)}(\ln(M_B(r))) = M_W^{(j)}(r) = e^{\lambda_j (M_B(r) - 1)}. \end{aligned}$$

4.2. Expression for $c(r)$

The function $c(r)$ is provided in the following proposition.

Proposition 10. *The expression for $c(r)$ is given by*

$$\begin{aligned} c(r) = \ln\Big\{ &\left(p_{11} M_W^{(1)}(r) + p_{22} M_W^{(2)}(r) \right) \\ &+ \sqrt{(p_{11} M_W^{(1)}(r) + p_{22} M_W^{(2)}(r))^2 - 4 M_W^{(1)}(r) M_W^{(2)}(r) (p_{11} p_{22} - p_{12} p_{21})} \Big\} \\ &- \ln 2 - \pi r, \end{aligned}$$

where $(p_{11} p_{22} - p_{12} p_{21}) = v$.

Proof. We define the matrix $\underline{M}(r)$ where the entries are

$$m_{ij}(r) = p_{ij} M_W^{(j)}(r) e^{-\pi r}$$

for $i, j \in \{1, 2\}$ and some $r \geq 0$. We find $c(r)$ in a similar way as in the previous section based on the eigenvalues of $\underline{M}(r)$. They are the solution to

$$\det(\underline{M} - \lambda \times \underline{J}) = 0,$$

where $\underline{J}$ is the identity matrix. Therefore,

$$\underline{M} - \zeta \times \underline{J} = \begin{pmatrix} p_{11} M_W^{(1)}(r) e^{-\pi r} - \zeta & p_{12} M_W^{(2)}(r) e^{-\pi r} \\ p_{21} M_W^{(1)}(r) e^{-\pi r} & p_{22} M_W^{(2)}(r) e^{-\pi r} - \zeta \end{pmatrix}$$

and its determinant is

$$\begin{aligned} h(\zeta) = \zeta^2 &- e^{-\pi r} (p_{11} M_W^{(1)}(r) + p_{22} M_W^{(2)}(r)) \zeta \\ &+ e^{-2\pi r} M_W^{(1)}(r) M_W^{(2)}(r) (p_{11} p_{22} - p_{12} p_{21}). \end{aligned}$$

The maximal solution to $h(\zeta) = 0$ is

$$\xi^*(r) = e^{-\pi r}\frac{(p_{11}M_W^{(1)}(r) + p_{22}M_W^{(2)}(r))}{2} + e^{-\pi r}\frac{\sqrt{(p_{11}M_W^{(1)}(r) + p_{22}M_W^{(2)}(r))^2 - 4M_W^{(1)}(r)M_W^{(2)}(r)(p_{11}p_{22} - p_{12}p_{21})}}{2}. \tag{31}$$

The desired result is obtained by taking the natural logarithm on both sides of (31). □

Remark 11. *We could have assumed* $\underline{\Theta}$ *to be defined over a* m-state *space* $\{\theta_1, \ldots, \theta_m\}$. *However, the expression for* $c(r)$ *would no longer be analytic.*

Example 12. *We assume that* $\lambda_1 = 1$, $\lambda_2 = 2$, $\chi = \frac{3}{4}$, *and that the claim amount follows an exponential distribution with mean* $\beta^{-1} = 1$. *The relative security margin* η *is equal to 50%. In Table 4, we compute the Lundberg coefficient for different values of the dependence parameter* $v = -0.25, 0, 0.25, 0.5, 0.75$. *Using the approximation* $\psi(u) \sim e^{-\rho u}$ *we compute the amount of surplus required to have an infinite time ruin probability of 1%.*

TABLE 4

VALUES OF THE LUNDBERG COEFFICIENT IN THE RISK MODEL DEFINED IN A MARKOVIAN ENVIRONMENT

v	–0.25	0	0.25	0.5	0.75
ρ	0.064595843	0.063320997	0.061352843	0.0578554	0.049905485
$u = -\rho^{-1}\ln(0.01)$	71.29205177	72.72737976	75.06042046	79.59793168	92.27783595

The dependence parameter v indicates the strength of the dependence relation between the claim number r.v.'s. As the parameter v increases, the risk process for the portfolio becomes riskier. Therefore, to meet an objective of an infinite-time ruin probability of 1%, one requires to set aside a higher initial reserve as v increases. □

5. WEAK DEPENDENCE PROPERTIES OF THE POISSON AR(1) MODEL

In Cossette et al. (2010), we give an estimation procedure for the adjustment coefficient for processes satisfying some weak dependence properties, such as the θ-dependence or η-dependence. For completeness, we first recall the definition of θ-dependence (see Dedecker et al. (2007) for a review on weak dependence). We then examine the weak dependence properties of the Poisson $AR(1)$ model more specifically.

5.1. θ-dependence

Let $\underline{X} = \{X_k, k \in \mathbb{N}\}$ be a stochastic process. The θ-dependence is defined in terms of the θ coefficients of the process $\underline{X}$ which are defined as follows.

Definition 14. *Let $\| \|_\infty$ denote the* sup *norm and $\|g\|_L = \max(\|g\|_\infty, lip(g))$ denote the Lipschitz norm of a Lipschitz function g (where $lip(g)$ denotes the Lipschitz constant of g). For $k \in \mathbb{N}$, we define coefficient $\theta(k)$ by*

$$\theta(k) = \sup \frac{\left|Cov(f(X_{i_1} \ldots, X_{i_u}), g(X_{j_1}, \ldots, X_{j_v}))\right|}{v\|f\|_\infty \|g\|_L},$$

where the supremum is taken over multi-indices $\mathbf{i} = (i_1, \ldots, i_u)$, $\mathbf{j} = (j_1, \ldots, j_v)$ such that

$$i_1 < \cdots < i_u \leq i_u + k \leq j_1 < \cdots < j_v$$

and all functions $f : \mathbb{R}^u \longrightarrow \mathbb{R}$, $g : \mathbb{R}^v \longrightarrow \mathbb{R}$ are bounded and satisfy the Lipschitz property, with respect to the following distance:

$$d(x, y) = \sum_{i=1}^{p} \left|x_i - y_i\right|, x = (x_1 \ldots, x_p), y = (y_1 \ldots, y_p).$$

A stochastic process X is said to be θ-dependent if the sequence $\{\theta(k), k \in \mathbb{N}\}$ is summable.

Because a Poisson $MA(1)$ model is 1-dependent, it is easily seen that it is θ-dependent. For the Poisson $AR(1)$ model some computations are required to show that we have the θ-dependence.

5.2. θ-dependence for the Poisson $AR(1)$ model

Here we show that the process $\underline{W} = \{W_k, k \in \mathbb{N}^+\}$ as defined in (1) is θ-dependent. We first need to bound the quantities $Cov(f(W_0), g(W_k))$, for bounded functions f and g, and $k \in \mathbb{N}^+$. A simple computation gives

$$\begin{aligned} &Cov(f(W_0), g(W_k)) \\ &\quad = \sum_{n,m \geq 1} \mathbb{E}\left(f\left(\sum_{j=1}^{n} B_{0,j}\right)\right) \mathbb{E}\left(g\left(\sum_{j=1}^{m} B_{k,j}\right)\right) \left[\Pr(N_0 = n, N_k = m)\right. \\ &\quad\quad \left. - \Pr(N_0 = n)\Pr(N_k = m)\right]. \end{aligned}$$

Thus, we are left to bound

$$\left[\Pr(N_0 = n, N_k = m) - \Pr(N_0 = n)\Pr(N_k = m)\right].$$

Following Al-Osh and Alzaid (1987), we know that

$$(N_0, N_k) \stackrel{\mathcal{L}}{=} \left(N_0, \alpha^k \circ N_0 + \sum_{i=0}^{k-1} \alpha^i \circ \varepsilon_{k-i}\right)$$

which implies

$$\Pr(N_0 = n,\, N_k = m) = \Pr(N_0 = n) \Pr\left(\alpha^k \circ n + \sum_{i=0}^{k-1} \alpha^i \circ \varepsilon_{k-i} = m\right),$$

where $\alpha^k \circ n$ denotes the sum of n i.i.d. Bernoulli r.v.'s with parameter α^k (α = dependence parameter). We have

$$\Pr(N_0 = n,\, N_k = m) - \Pr(N_0 = n) \Pr(N_k = m)$$

$$= \Pr(N_0 = n) \sum_{j=1}^{m} \left[\Pr(\alpha^k \circ n = j) - \Pr(\alpha^k \circ N_0 = j)\right] \Pr\left(\sum_{i=0}^{k-1} \alpha^i \circ \varepsilon_{k-i} = m - j\right).$$

For $j \geq 1$, we easily get

$$\Pr(\alpha^k \circ n = j) \leq \frac{\mathbb{E}\left([\alpha^k \circ n]^2\right)}{j^2} \text{ and } \Pr(\alpha^k \circ N_0 = j) \leq \frac{\mathbb{E}\left([\alpha^k \circ N_0]^2\right)}{j^2}.$$

Also,

$$\mathbb{E}\left([\alpha^k \circ n]^2\right) \leq n^2 \alpha^k \text{ and } \mathbb{E}\left([\alpha^k \circ N_0]^2\right) \leq \mathbb{E}(N_0^2) \alpha^k.$$

It follows that

$$\left|\sum_{n,m} \Pr(N_0 = n,\, N_k = m) - \Pr(N_0 = n) \Pr(N_k = m)\right|$$

$$\leq \alpha^k \sum_{n,m} \Pr(N_0 = n)\left[\left(n^2 + \mathbb{E}\left(N_0^2\right)\right) \sum_{j=1}^{m} \frac{1}{j^2} \Pr\left(\sum_{i=0}^{k-1} \alpha^i \circ \varepsilon_{k-i} = m - j\right)\right]$$

$$\leq \alpha^k \left[\sum_{n \geq 0} \left(n^2 + \mathbb{E}\left(N_0^2\right)\right) \Pr(N_0 = n) \sum_{j \geq 1} \frac{1}{j^2} \sum_{m \geq j} \Pr\left(\sum_{i=0}^{k-1} \alpha^i \circ \varepsilon_{k-i} = m - j\right)\right]$$

$$\leq \alpha^k 2\mathbb{E}\left(N_0^2\right) \sum_{j \geq 1} \frac{1}{j^2}.$$

Consequently, for any bounded functions f and g, we have that for some constant $C > 0$,

$$Cov(f(W_0), g(W_k)) \leq C \|f\|_\infty \|g\|_\infty \alpha^k.$$

A similar computation for

$$\Pr(N_{i_1} = n_1, \ldots, N_{i_u} = n_{i_u}, N_{j_1} = m_{j_1}, \ldots, N_{j_v} = m_{j_v}) \\ - \Pr(N_{i_1} = n_1, \ldots, N_{i_u} = n_{i_u} \Pr(N_{j_1} = m_{j_1}, \ldots, N_{j_v} = m_{j_v})$$

for multi-indices $i_1 < \cdots < i_u \le i_u + \chi \le j_1 < \cdots < j_v$ implies that we have, for some constant $C > 0$,

$$|Cov(f(W_{i_1}, \ldots, W_{i_u}), g(W_{j_1}, \ldots, W_{j_v}))| \le C\|f\|_\infty \|g\|_\infty \alpha^k.$$

Then, it follows that $\theta(k) \le C\alpha^k$ and the sequence $\{\theta(k), k \in \mathbb{N}\}$ is summable, meaning that the process $\underline{W} = \{W_k, k \in \mathbb{N}^+\}$ is θ-dependent. Consequently, the process $\{W_k - \pi, k \in \mathbb{N}^+\}$ associated to the risk model based on the Poisson $AR(1)$ process is also θ-dependent which implies that the estimation procedure proposed by Cossette et al. (2009) is applicable in the context of this risk model.

Let us recall the estimation procedure for the adjustment coefficient. For $r \in \mathbb{N}$, let

$$Y_r = \sum_{j=0}^{r} \left(X_j - \pi\right).$$

The function $\mathbb{E}(e^{tS_r})$ may be estimated by its empirical moment version: for $k \in \mathbb{N}$,

$$\widehat{M}_k^r(t) = \frac{1}{k} \sum_{i=0}^{k-1} e^{tZ_i^r},$$

where $Z_i^r = \sum_{j=1}^{r} X_{j+ir}$. Then we define $\hat{\rho}_r$ as the positive solution to $\frac{1}{r}\ln\left(\widehat{M}_k^r(t)\right) = 0$. We have that under the condition of θ weak dependence and on the existence of the adjustment coefficient ρ, $\hat{\rho}_r$ is a consistent estimator of ρ, provided we take $r = o(\ln k)$.

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