EXPERIENCE AND EXPOSURE RATING FOR PROPERTY PER RISK EXCESS OF LOSS REINSURANCE REVISITED

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ABSTRACT

Experience and exposure rating are traditionally considered to be independent but complementary methods for pricing property per risk excess of loss reinsurance. Strengths and limitations of these techniques are well-known. In practice, both methods often lead to quite different prices. In this paper we show that limitations of traditional experience rating can be overcome by taking into account historical profile information by means of exposure curves. For pricing unused or rarely used capacity, we propose to use exposure rating, calibrated on the experience rate of a working layer. We compare the method presented with more traditional methods based on the information which is generally available to the reinsurer.

KEYWORDS

Experience Rating, Exposure Rating, Property per Risk Excess of Loss Reinsurance, Exposure Curves.

1. INTRODUCTION

In this paper, we take the point of view of a reinsurer pricing property per risk excess of loss reinsurance based on the information which is generally available. In the literature dealing with this topic, a distinction is made between so-called experience rating (see e.g. Schmitter and Bütikofer [1998]) and exposure rating techniques (see e.g. Bernegger [1997] and Guggisberg [2004]). Traditionally, both techniques are considered to be independent, both with their advantages and disadvantages. The purpose of experience rating is to forecast the losses borne by the reinsurer based on historical claims information, possibly corrected for the current economic environment. Most of the traditional experience rating methods require that the relative portfolio composition remains constant over time, both regarding the risk types and the insured values of the risks. Mathematical models are often used to make extrapolations to price unused or rarely used capacity. These methods do not take into account the

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composition of the portfolio which generates exposure in the upper region of the reinsurance programme. Exposure rating methods take the profile of the portfolio as a starting point. In theory, this method should allow to perform pricing, even if no loss experience is available. In practice, exposure pricing also has its limitations.

We discuss the two traditional methods together with their limitations. The purpose of this paper is to show how it is possible to bring both methods together. The idea of using exposure rating techniques in order to come to a better experience rating method was already proposed by Mata and Verheyen [2005], but the techniques these authors use are different from the methodology we propose in this paper. For experience rating, we illustrate the necessity of taking detailed historical exposure information into account. We obtain an experience rating method based on historical profile information and exposure curves, which allow to determine more reliable measures for the evolution of the claim frequency and severity above different thresholds. For pricing unused or rarely used capacity, we use exposure rating calibrated on the experience of a working layer. This technique is being used in practice to calibrate exposure rating (see e.g. Snoussi et al. [2008]) and is referred to by Riegel [2010] as burning cost-adjusted exposure rating.

The rest of the paper is organized as follows. In section 2, we discuss the information which is commonly available for pricing property per risk excess of loss reinsurance. In section 3 and 4, we summarize some commonly used traditional experience rating methods and the basis of traditional exposure rating as well as the limitations of these methods. In section 5, we combine the strengths of experience and exposure rating in one method. We explain how the techniques presented can be parameterized in section 6. In section 7, we develop a real-life numerical example and compare the methods presented with different more traditional approaches. In this section, we also summarize the methods presented above by providing a step by step procedure to apply and compare the methods presented in practice. We conclude in section 8.

2. Available Information and Reinsurance Programme

Assume that for accident year $t \in \{1, \ldots, T-1\}$, the reinsurer receives the historical claims above a certain threshold A_t. Let the claims in year t be denoted by $\{C_{t,1}, \ldots, C_{t,n_t}\}$, where $t \in \{1, \ldots, T-1\}$ and n_t denotes the number of claims in year t. Assume that for each accident year $t \in \{1, \ldots, T-1\}$, we dispose of a profile with a structure as presented in table 1, in which B_t is the number of rows in the profile in year t. Assume we can also obtain an estimation of the profile for year T.

It is quite common to speak of the rows in the profile as "bands". The average insured value in a band is equal to $ASI_{t,b_t} = TSI_{t,b_t}/N_{t,b_t}$, for $t \in \{1, \ldots, T\}$ and $b_t \in \{1, \ldots, B_t\}$, with the convention that $ASI_{t,b_t} = 0$ if $N_{t,b_t} = 0$. Quite some cedents do not only give one profile of their entire portfolio. Often, the

TABLE 1

PROFILE FOR YEAR t, $t \in \{1, \ldots, T\}$.

Lower Bound	Upper Bound	Premium	Number of Risks	Total Sum Insured
$LB_1 = 0$	$UB_{t,1}$	$P_{t,1}$	$N_{t,1}$	$TSI_{t,1}$
$LB_{t,2} = UB_{t,1}$	$UB_{t,2}$	$P_{t,2}$	$N_{t,2}$	$TSI_{t,2}$
$LB_{t,3} = UB_{t,2}$	$UB_{t,3}$	$P_{t,3}$	$N_{t,3}$	$TSI_{t,3}$
$LB_{t,B_t} = UB_{t,B_t-1}$	UB_{t,B_t}	P_{t,B_t}	N_{t,B_t}	TSI_{t,B_t}
0	UB_{t,B_t}	$P_t = \sum_{b_t=1}^{B_t} P_{t,b_t}$	$N_t = \sum_{b_t=1}^{B_t} N_{t,b_t}$	$TSI_t = \sum_{b_t=1}^{B_t} TSI_{t,b_t}$

reinsurer receives profiles for different risk types such as simple risks, commercial risks, industrial risks, etc.. What is understood by these risk types may vary case by case. In this paper, we assume for simplicity that we dispose only of one profile for the entire portfolio. All results can easily be generalized to cases where more detailed information is available.

Suppose we want to price an excess of loss reinsurance programme covering fire on a per risk basis in year T with the structure as given in table 2. The first layer of this programme will provide coverage for the portion of all losses between D_1 and D_2. The second layer will provide coverage for the portion of all losses between D_2 and D_3, and so on.

TABLE 2

REINSURANCE PROGRAMME

Layer	Priority	Limit
XL_1	D_1	D_2
XL_2	D_2	D_3
XL_3	D_3	D_4

The fact that the programme works on a per risk basis means that the losses on different risks can not be aggregated before application of the reinsurance programme. In practice, reinsurers give only a limited number of reinstatements of the capacity they offer. If R_j reinstatements are given on a layer XL_j, this means that the cedent can use the capacity of the layer $R_j + 1$ times. Reinstatements can be free or (partially) payable. Some layers may be subject to an annual aggregate deductible (AAD). If there is an AAD working on a layer XL_j, then the first losses in the layer XL_j fall in the retention of the cedent, until the total amount of losses in the layer exceeds the AAD. The objective of this paper is not to discuss how such clauses can be priced. The interested reader is referred to Sundt [1991], Walhin et al. [2001], Walhin [2001] and Hürlimann [2005].

We suppose we want to make an as-if statistic for year T based on the claims information of the preceding years. This means that we want to adapt the claim severity and frequency which was observed in the past to the current economic conditions and exposure. Our interest in this paper goes mainly to determining the average cost of the claims in the layers of the reinsurance programme.

3. Traditional Experience Rating

3.1. Burning Cost

The burning cost is probably the most simple and widely known tool for pricing excess of loss reinsurance. It simply compares the historical reinsured losses on a given portfolio with the corresponding cedent's premium. The burning cost of layer XL_j, based on the past premiums, is calculated as:

$$BC_j = \frac{\sum_{t=1}^{T-1} \sum_{k_t=1}^{n_t} \min(D_{j+1} - D_j; \max(0; C_{t,k_t} - D_j))}{\sum_{t=1}^{T-1} P_t}. \tag{1}$$

Past claims are under current conditions expected to be more expensive because the costs of reconstruction increase with time. Therefore, if the same portfolio is underwritten, we should expect the frequency of the losses exceeding a certain threshold to increase with time. Furthermore, when the composition of a portfolio changes, this may have an impact on the claims distribution (above a given threshold). To take into account changes in the costs for reconstruction, we can work with a standard claims index. This index is in practice often based on the index of construction prices. Assume that the claims index is given by $\{CI_1, \ldots, CI_T\}$. The indexed claims are then defined and denoted by[1]:

$$C^I_{t,k_t} = C_{t,k_t} \frac{CI_T}{CI_t}, \; t \in \{1, \ldots, T-1\} \text{ and } k_t \in \{1, \ldots, n_t\}. \tag{2}$$

In some markets, it is a practice to automatically adapt the insured values of a risk due to evolutions in the construction prices. In other markets, some ceding companies apply such automatic indexation as well. For the remainder of this paper, we will assume that insured values are automatically indexed for changes in the cost of reconstruction. For portfolios where no such automatic indexation is applied, slight changes in the methods presented may be needed.

[1] The superscript I in equation (2) and in following equations indicates that we take into account indexation.

Note that if we take into account indexation, this means that in year T, we can estimate the claims distribution above the level:

$$A_{min}^{I} = \max(A_t^I)_{t \in \{1,\ldots,T-1\}}, \tag{3}$$

where for all $t \in \{1, \ldots, T-1\}$:

$$A_t^I = A_t \frac{CI_T}{CI_t}.$$

Below A_{min}^I, we do not have a full view on the claims information.

The premium which is asked for a certain risk also evolves through time. Premium can evolve due to several reasons. It usually changes because the insured values change due to indexation but it can also change because the cedent adapts its tariff. The tariff is also sometimes referred to as the premium rate. Property markets, and in particular the market of large commercial and industrial risks, are sometimes very sensitive to cyclical effects.

Changes of the premium due to increased insured values indicate that the exposure has increased. This can be taken into account by indexing the claims. To avoid that we take into account twice the effect of indexation of the insured values, the increase of the premium due to indexation should be neutralized when calculating a burning cost. Tariff variations on a stable portfolio on the other hand do not affect the exposure. Note that this is not necessarily true for a changing portfolio: if more dangerous risks are underwritten, the exposure increases and the tariff should increase as well. Therefore, the approach explained below has its limitations for changing portfolios. The overall[2] tariff in year t is defined as:

$$T_t = P_t / TSI_t, \; t \in \{1,\ldots,T\}. \tag{4}$$

If the tariff increases, we should give the past premiums more weight by indexing them and vice versa if the tariff decreases. Therefore, we correct the past premium for changes in the insured values and for changes in the tariff with a premium index which is equal to:

$$PI_t = T_t CI_t, \; t \in \{1,\ldots,T\}. \tag{5}$$

This means that we index past premiums as follows to obtain the indexed premium for year t:

$$P_t^I = P_t \frac{PI_T}{PI_t}, \; t \in \{1,\ldots,T-1\}. \tag{6}$$

[2] The word "overall" in this context indicates that we look at the tariff for the entire portfolio and not only for the risks which generate exposure.

We then obtain the following burning cost taking into account indexation of both premium and claims:

$$BC_j^I = \frac{\sum_{t=1}^{T-1}\sum_{k_t=1}^{n_t} \min(D_{j+1} - D_j;\ \max(0; C_{t,k_t}^I - D_j))}{\sum_{t=1}^{T-1} P_t^I} \tag{7}$$

The total estimated cost TC_j^I for layer XL_j in year T is then equal to $BC_j^I\, P_T$. Now let $I_{C_{t,k_t}^I > D_j}$ be an indicator variable which is equal to 1 if C_{t,k_t}^I is larger than D_j and 0 otherwise. Then TC_j^I can be written as:

$$TC_j^I = \lambda_j^I\, AV_j^I,$$

where

$$\lambda_j^I = \frac{P_T \sum_{t=1}^{T-1}\sum_{k_t=1}^{n_t} I_{C_{t,k_t}^I > D_j}}{\sum_{t=1}^{T-1} P_t^I} \tag{8}$$

denotes the estimated number of claims in layer XL_j in year T and

$$AV_j^I = \frac{\sum_{t=1}^{T-1}\sum_{k_t=1}^{n_t} \min(D_{j+1} - D_j;\ \max(0; C_{t,k_t}^I - D_j))}{\sum_{t=1}^{T-1}\sum_{k_t=1}^{n_t} I_{C_{t,k_t}^I > D_j}}$$

denotes the estimated average cost of the claims hitting layer XL_j. λ_j^I can be interpreted as the estimated frequency of the claims exceeding the priority D_j. Under the assumption that the annual number of claims follows a Poisson distribution with measure of exposure P_t^I, $t \in \{1, \ldots, T\}$, it can be proven that (8) is the maximum likelihood estimator for the expected number of claims in year T (see e.g. Klugman et al. [2008]). AV_j^I can be interpreted as the average claim severity in the layer XL_j.

The burning cost as defined above has important limitations:

1. For layers which have not been hit in the past, the burning cost is zero. For layers with limited claims experience, the burning cost may provide unreliable results due to statistical uncertainty.
2. The burning cost does not take into account detailed evolutions in the portfolio. It only looks at the total (indexed) premium as a measure of exposure. In cases where the composition of the portfolio changes (e.g. relatively more or less big risks), the total premium of the portfolio may not be a reliable measure.

In the remainder of this paper, we will explain how to deal with these limitations.

3.2. Experience Rating Based on the Collective Risk Model

Working within the framework of the collective risk model (see Klugman et al. [2008]) is useful for several reasons:

- It allows to calculate a price for unused capacity.
- It allows to obtain a full distribution of the aggregate claims in any given layer.

This model is characterized by a frequency and severity distribution. In traditional experience rating based on this model, the severity distribution is fit on a set of historic claims which are corrected based on a claims index (e.g. construction prices). In reinsurance, the Compound Poisson-Pareto model is very popular in practice (see e.g. Schmitter and Bütikofer [1998]). In this model, the claims frequency is modelled using a Poisson distribution and the severity is modelled using a Pareto distribution.

There are two major drawbacks with these approaches:

1. The historic claims are simply corrected based on a claims index. This does not take into account the real evolution in the underlying portfolio. If the ceding company writes more big risks now, we may also expect a bigger potential for large losses than in the past.
2. When we fit a distribution on the claims experience and use it to price a region of the programme on which there are no or limited losses, we do not take into account the actual portfolio information in that region. Furthermore, when claims experience is limited, results of such extrapolations may be highly influenced by extreme observations, which have or have not occurred in the claims history by coincidence.

Exposure rating and the techniques presented further in the paper will allow to deal with these limitations.

4. Traditional Exposure Rating

4.1. Exposure Curves

Let Y be the random variable describing the loss for a risk with insured value M, given that there is a loss. The degree of damage X is defined as Y/M. Let D be a deductible and define d as D/M. Let $L(d) = \mathbb{E}[\min(d, X)]$ denote the limited expected value function for the risk. If the cedent buys non-proportional reinsurance with a deductible D, then the average retained loss for the risk with insured value M is equal to $L(d)M$. The exposure curve associated with this risk is then denoted and defined by:

$$G(d) = \frac{L(d)}{L(1)} = \frac{\int_0^d (1 - F_X(x))\,dx}{\int_0^1 (1 - F_X(x))\,dx}$$

where $F_X(x)$ denotes the distribution function of X. The exposure curve has a very simple interpretation: $G(d)$ represents the portion of the (pure) premium which is needed to cover the portion of all losses truncated to a degree of damage of d. As is explained in Bernegger [1997], if the exposure curve for a risk is given, its distribution function can be derived from:

$$F_X(d) = \begin{cases} 1 & \text{if } d = 1, \\ 1 - \frac{G'(d)}{G'(0)} & \text{if } 0 \leq d < 1, \end{cases} \tag{9}$$

where $F_X(0) = 0$ and $G'(0) = 1/\mathbb{E}[X]$. This means that the distribution function of a risk and its exposure curve are equivalent representations.

4.2. Exposure Rating Based on a Profile

Assume we want to price a layer with priority D_j and limit D_{j+1} for a portfolio with a profile in year T as described in table 1. In all bands $b_T \in \{1, \ldots, B_T\}$, calculate the ratios:

$$r_{j,b_T} = D_j / ASI_{T,b_T} \quad \text{and}$$
$$s_{j,b_T} = D_{j+1} / ASI_{T,b_T}.$$

Denote for all $b_T \in \{1, \ldots, B_T\}$ the exposure curve corresponding to the risks of band b_T as $G_{b_T}(d)$. We assume that $G_{b_T}(d) = 1$ if $d > 1$. Then $G_{b_T}(r_{j,b_T}) P_{T,b_T}$ corresponds to the part of the gross premium from band b_T, needed to cover all losses arising from risks in band b_T for which the degree of damage is limited to r_{j,b_T}. Similarly, $(1 - G_{b_T}(r_{j,b_T})) P_{T,b_T}$ corresponds to the part of the gross premium needed to cover the part of these losses exceeding a degree of damage r_{j,b_T}. If r_{j,b_T} is larger than 100%, then it is normally not possible to have losses above D_j from risks with an insured value of ASI_{T,b_T} and $(1 - G_{b_T}(r_{j,b_T}))$ $P_{T,b_T} = 0$. The part of the gross premium P_{T,b_T} needed to cover the part of the losses between a degree of damage of r_{j,b_T} and s_{j,b_T}, arising from risks with an insured value of ASI_{T,b_T}, is equal to $(G_{b_T}(s_{j,b_T}) - G_{b_T}(r_{j,b_T})) P_{T,b_T}$. The total gross premium needed to cover all losses between D_j and D_{j+1} for the portfolio in year T is given by:

$$EX_j = \sum_{b_T=1}^{B_T} (G_{b_T}(s_{j,b_T}) - G_{b_T}(r_{j,b_T}))\, P_{T,b_T}. \tag{10}$$

In equation (10), we define a portion of the premium. In order to make an estimation of the average losses inside this layer, we need to estimate a loss ratio. This can be obtained by estimating e.g. a fixed loss ratio LR for all the bands in the profile. Then, $EX_j LR$ is equal to the average total cost of the claims in layer XL_j. We refer to Guggisberg [2004] for a step-by-step illustration of how to apply exposure rating in practice.

4.3. Limitations of Traditional Exposure Rating

Although the exposure rating methodology seems theoretically sound, "pure" exposure rating has some limitations:

1. To be able to apply the pure exposure rating, we need an estimate of a loss ratio. In practice, this information is not always available. Furthermore, it is not obvious that the loss ratio in all the bands of the profile should be the same.
2. The gross premium used for exposure rating may contain parts meant to cover other losses than only the fire losses per risk. The premium could for instance contain parts which are meant to cover natural perils such as storm or flood, or other guarantees. It is not always possible to subtract the premium for these covers since the reinsurer does normally not have full insight in the pricing applied by the cedent.
3. In order to estimate full exposure curves, many data are needed for a big set of comparable risks. These data are often difficult to obtain in practice. Therefore, it is common to rely on standard curves to apply exposure rating. This creates model uncertainty.
4. Pure exposure rating does not take into account the claims information which is available to determine the cost of a given layer. In regions of the programme where there are sufficient claims, it is useful to take this information into account to reduce model uncertainty.

A possible solution to these limitations will be presented in section 5.2.

4.4. Link with Collective Risk Model

The purpose of this section is to show that there is a link between exposure rating and the collective risk model. We suppose that we are working with a profile for year T. The same reasoning can be developed for profiles from the past years $t \in \{1, \ldots, T-1\}$. We introduce the following notations:

- Let $R_{T, b_T, n_{T, b_T}}$ denote the n_{T, b_T}th risk in band b_T in the profile of year T, where $n_{T, b_T} \in \{1, \ldots, N_{T, b_T}\}$.
- For all $n_{T, b_T} \in \{1, \ldots, N_{T, b_T}\}$ and $b_T \in \{1, \ldots, B_T\}$, let $NL_{T, b_T, n_{T, b_T}}$ denote the number of losses on the risk $R_{T, b_T, n_{T, b_T}}$ in band b_T. We assume that

$$NL_{T, b_T, n_{T, b_T}} \overset{\text{i.i.d.}}{\sim} NL_{T, b_T} \sim Poisson(q_{T, b_T}), \tag{11}$$

  i.e. we assume that the number of losses on each of the different risks in a given band b_T has the same and independent Poisson distribution with an expected number of losses equal to q_{T, b_T}.

- For all bands $b_T \in \{1, \ldots, B_T\}$, let $M_{T,b_T} = \sum_{n_{T,b_T}=1}^{N_{T,b_T}} NL_{T,b_T,n_{T,b_T}}$ denote the number of losses arising from the risks in band b_T.
- For all $n_{T,b_T} \in \{1, \ldots, N_{T,b_T}\}$ and $b_T \in \{1, \ldots, B_T\}$, we will use the index $m_{T,b_T,n_{T,b_T}} \in \{1, \ldots, NL_{T,b_T,n_{T,b_T}}\}$ to enumerate the $m_{T,b_T,n_{T,b_T}}$th loss on the n_{T,b_T}th risk from band b_T, given that $NL_{T,b_T,n_{T,b_T}} > 0$.
- For all $n_{T,b_T} \in \{1, \ldots, N_{T,b_T}\}$, $b_T \in \{1, \ldots, B_T\}$ and $m_{T,b_T,n_{T,b_T}} \in \{1, \ldots, NL_{T,b_T,n_{T,b_T}}\}$, given that $NL_{T,b_T,n_{T,b_T}} > 0$, we denote with $Z_{T,b_T,n_{T,b_T},m_{T,b_T,n_{T,b_T}}}$ the random variable describing the $m_{T,b_T,n_{T,b_T}}$th loss on the n_{T,b_T}th risk from band b_T. We assume that:

$$Z_{T,b_T,n_{T,b_T},m_{T,b_T,n_{T,b_T}}} \overset{\text{i.i.d.}}{\sim} Z_{T,b_T}, \tag{12}$$

  i.e. we assume that all the losses on all risks in a given band b_T have the same and independent distribution.

Note that in reality, assumptions (11) and (12) will of course not be true. However, for applications in practice and especially in the field of reinsurance, this is an acceptable assumption since no information is generally available to the reinsurer to be able to estimate a frequency and severity distribution for each of the risks within a certain band of a portfolio separately. As a reinsurer it is therefore common to group risks by band and to suppose that they have the same properties. As we will explain in section 6.2.1 it is possible to split up the bands of a profile in more detail to be able to take differences between the risks due to their difference in size into account.

Note that the assumptions (11) and (12) are not crucial. The methods presented can be generalized using similar techniques as those used in the paper in the case more detailed information would be available.

As explained in Kaas et al. [2001], if $S_1, S_2, \ldots, S_m$ are independent compound Poisson random variables with Poisson parameter λ_i and claims distribution P_i, with $i \in \{1, \ldots, m\}$, the sum $S = \sum_{i=1}^m S_i$ will still be compound Poisson distributed with Poisson parameter

$$\lambda = \sum_{i=1}^m \lambda_i \text{ and claims distribution}$$
$$P = \sum_{i=1}^m \frac{\lambda_i}{\lambda} P_i.$$

By using this property a first time together with assumption (11) and (12), it is possible to see that the total loss amount on the risks $n_{T,b_T} \in \{1, \ldots, N_{T,b_T}\}$ in a given band $b_T \in \{1, \ldots, B_T\}$ follows a compound Poisson model with Poisson parameter $\mathbb{E}[M_{T,b_T}] = N_{T,b_T}\, q_{T,b_T}$ and claims distribution $F_{Z_{T,b_T}}$.

By using this property a second time, we can see that the total loss amount under this model also follows a compound Poisson model with

– Poisson parameter $\sum_{b_T=1}^{B_T} \mathbb{E}[M_{T,b_T}]$ and

– claims distribution F_{Z_T} where for all $z \in]0, ASI_{T,B_T}]$:

$$F_{Z_T}(z) = \frac{1}{\sum_{b_T=1}^{B_T} \mathbb{E}[M_{T,b_T}]} \sum_{b_T=1}^{B_T} \mathbb{E}[M_{T,b_T}] F_{Z_{T,b_T}}(z)$$

$$= \frac{1}{\sum_{b_T=1}^{B_T} \mathbb{E}[M_{T,b_T}]} \sum_{b_T=1}^{B_T} N_{T,b_T} q_{T,b_T} F_{X_{T,b_T}}(z/ASI_{T,b_T}) \text{ and where}$$

$F_{X_{T,b_T}}$ denotes the distribution of the degree of damage of the risks in band b_T in the profile in year T, given that there is a loss.

5. Combining Experience and Exposure Rating

In this section, we explain how experience and exposure rating can be combined to create a more reliable pricing method. For working layers, we propose to use experience rating, taking into account the evolutions in the underlying profile by using exposure rating techniques. For non-working layers, we propose to use exposure rating, calibrated on the loss experience of a well-chosen working layer. Throughout this section, we will assume that the parameters and distributions required to calculate the various results are known. In section 6 and 7, we will explain how these elements can be determined in practice.

5.1. Experience Rating based on Historic Profile Information

To build an experience rating method, two fundamental elements are needed: a reliable frequency measure to evaluate the claims frequency and the possibility to create representative as-if claims. As we will show further in this section, these two elements can be used to make burning cost calculations for working layers. We first explain how exposure curves can be used to make a reliable assessment of the evolution of the frequency and the severity of the claims above a certain threshold, using an example with one band, in which all the risks have the same insured value. We then explain how the frequency measure and as-if claims can be created in general, taking into account detailed evolutions in the underlying profiles, based on the information which is generally available to the reinsurer. We end this section by explaining how these two elements can be used to make burning cost calculations for working layers.

5.1.1. *Profile with one Band*

Suppose we have the evolution in a profile as given in table 3.

In what follows, we will assume for simplicity that all risks in year 1 have the same and independent loss distribution function and that all risks in year T have the same and independent loss distribution function. The hypothesis that

TABLE 3

EVOLUTION OF ONE BAND

Year	Nb	SI
1	1,000	200,000
T	1,200	250,000

the risks have the same distribution is not crucial. The methodology proposed can easily be generalized to profiles with multiple bands and risks with different exposure curves. Suppose that the construction price index increased with 10% between year 1 and year T and that we are interested in studying the evolution of the exposure above 100,000. Assume that we have all claims in year 1 above a threshold of 50,000. Hence, we have a list of claims: $\{C_{1,1}, \ldots, C_{1,n_1}\}$, which leads to a list of indexed claims: $\{C^I_{1,1}, \ldots, C^I_{1,n_1}\}$, where $C^I_{1,k_1} = 1.1\,C_{1,k_1}$, $k_1 \in \{1, \ldots, n_1\}$. Note that due to the indexation from year 1 to year T, we can only use the indexed claims above a minimum level of 55,000 in year T.

Frequency above a given threshold

Let $\bar{\lambda}_1^{100,000}$ denote the observed number of indexed claims above 100,000. Hence:

$$\bar{\lambda}_1^{100,000} = \sum_{k_1=1}^{n_1} I_{C^I_{1,k_1} > 100,000}\,.$$

Working within the framework of a collective risk model and using similar techniques as in 4.4, we can define

- M_1 as the random variable describing the number of losses in year 1 and
- $\bar{Z}_{1,i_1} \overset{\text{i.i.d.}}{\sim} \bar{Z}_1$, with $i_1 \in \{1, \ldots, M_1\}$, as the random variable describing the indexed loss amount for loss i_1 in year 1[3].

We then define $\bar{E}_1^{100,000}$ as the expected number of indexed claims larger than 100,000 in year 1. Using the properties of the collective risk model, we have that

$$\begin{aligned}\bar{E}_1^{100,000} &= \mathbb{E}[M_1]\,\mathbb{P}\left[\bar{Z}_{1,i_1} > 100,000\right] \\ &= 1,000\,q_1\,\mathbb{P}\left[X_1 > \frac{100,000}{1.1 \times 200,000}\right] \\ &= 1,000\,q_1\left[1 - F_{X_1}\left(\frac{5}{11}\right)\right],\end{aligned}$$

[3] The bar on top of the variable Z and subsequent variables indicates that indexation towards year T has been taken into account.

where q_1 is defined as the expected number of losses for each risk in the portfolio in year 1 and X_1 is the random variable describing the degree of damage of the risks in year 1, given that there is a loss. Similarly, in year T, we can define:

$$\begin{aligned} E_T^{100,000} &= \mathbb{E}[M_T]\ \mathbb{P}[Z_{T,i_T} > 100,000] \\ &= 1,200 q_T \mathbb{P}\left[X_T > \frac{100,000}{250,000}\right] \\ &= 1,200 q_T \left[1 - F_{X_T}\left(\frac{2}{5}\right)\right], \end{aligned}$$

where M_T is the number of losses in year T, $Z_{T,i_T} \overset{\text{i.i.d.}}{\sim} Z_T$ is the loss amount for loss i_T in year T, q_T is the expected number of losses for a risk in year T and X_T is the degree of damage of the risks in year T.

The claims frequency above the level 100,000 in year T, can then be estimated based on the observed claims frequency from year 1 and the evolution in the portfolio of risks as:

$$\hat{\lambda}_T^{100,000} = \bar{\lambda}_1^{100,000} \frac{E_T^{100,000}}{E_1^{100,000}}.$$

As-if claims above a given threshold

In order to create an as-if claim for year T based on the indexed claims, we have to take into account changes in the portfolio. Indeed, if the risks in year T would be the same as in year 1, they would now have an insured value of 220,000. However, the insured value of the risks is now assumed to be 250,000, implying that their distribution function may also have changed. For year 1, we define for all $z > 100,000$:

$$\begin{aligned} F_{\bar{Z}_{1,i_1}|\bar{Z}_{1,i_1} > 100,000}(z) &= \mathbb{P}[\bar{Z}_{1,i_1} \leq z \,|\, \bar{Z}_{1,i_1} > 100,000] \\ &= \frac{\mathbb{P}[\bar{Z}_{1,i_1} \leq z \cap \bar{Z}_{1,i_1} > 100,000]}{\mathbb{P}[\bar{Z}_{1,i_1} > 100,000]} \\ &= \frac{\mathbb{P}[220,000 X_1 \leq z \cap 220,000 X_1 > 100,000]}{\mathbb{P}[220,000 X_1 > 100,000]} \\ &= \frac{F_{X_1}(z/220,000) - F_{X_1}(5/11)}{1 - F_{X_1}(5/11)}. \end{aligned}$$

Similarly, for year T, we define for $z > 100,000$:

$$F_{Z_{T,i_T}|Z_{T,i_T}>100{,}000}(z) = \mathbb{P}[Z_{T,i_T} \leq z \,|\, Z_{T,i_T} > 100{,}000] = \frac{F_{X_T}(z/250{,}000) - F_{X_T}(2/5)}{1 - F_{X_T}(2/5)}.$$

The as-if claims above the threshold 100,000, taking into account indexation and evolutions in the portfolio of risks are then obtained by taking all claims C^I_{1,k_1}, for $k_1 \in \{1, \ldots, n_1\}$ for which $C^I_{1,k_1} > 100{,}000$ and applying the function $F^{-1}_{Z_{T,i_T}|Z_{T,i_T}>100{,}000} \circ F_{\bar{Z}_{1,i_1}|\bar{Z}_{1,i_1}>100{,}000}$, where:

$$F^{-1}_{Z_{T,i_T}|Z_{T,i_T}>100{,}000}(p) = \inf\{z \in \mathbb{R} \,|\, F_{Z_{T,i_T}|Z_{T,i_T}>100{,}000}(z) \geq p\} \text{ for all } p \in [0,1].$$

Using this function, we transform an indexed claim $C^I_{1,k_1} > 100{,}000$, for which the distribution function corresponds with a probability level $F_{\bar{Z}_{1,i_1}|\bar{Z}_{1,i_1}>100{,}000}(C^I_{1,k_1})$ in year 1, to a claim for which the distribution corresponds with the same level in year T. Figure 1 illustrates this procedure in case $C^I_{1,k_1} = 200{,}000$.

Note that typical distribution functions for the degree of damage to a property usually show a jump at the end, corresponding with the total loss probability of that property. In case we look at the loss distribution above a high threshold, the size of this jump will become more important. This corresponds with the intuition that, given that a loss is above a high portion of the insured value of a risk, it is likely that the loss will be total. As we can see in figure 1, there are different options to map a total loss from year 1 towards a loss in year T. Our interest goes mainly to the general case in which we have a profile with risks of many different sizes. The size of the jumps will then become very

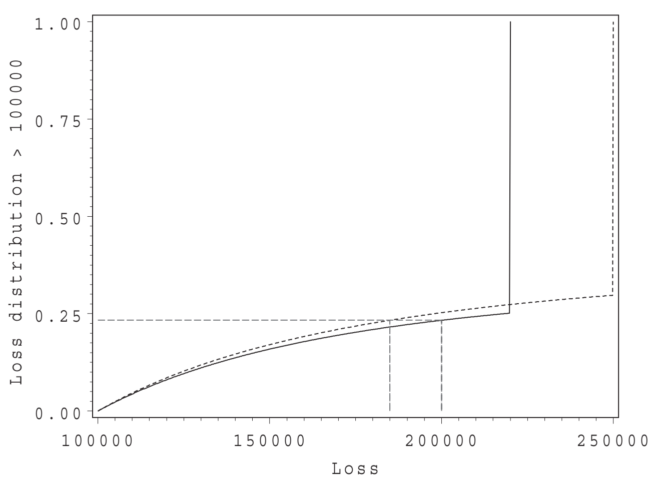

FIGURE 1: Example of calculation as-if claims based on profile evolution.
The full line corresponds with $F_{\bar{Z}_{1,i_1}|\bar{Z}_{1,i_1}>100{,}000}$ and the dashed line corresponds with $F_{Z_{T,i_T}|Z_{T,i_T}>100{,}000}$.

small. As we will see in section 7, it is possible to use good approximation methods for inverting the distribution functions in practice.

We now turn to the general case to explain how the above techniques can be used to determine a more reliable measure for the underlying claims frequency and to create as-if claims. Note that in practice the profile for year T is normally not known. In this section, we will assume for simplicity that this profile is known. In section 6, we will suggest how one can deal with this in practice.

5.1.2. *Estimation of Claim Frequency above a given Threshold*

We first define the observed frequency above a given threshold $D \geq A^I_{min}$ based on the indexed claims for year $t \in \{1, \ldots, T-1\}$ as:

$$\bar{\lambda}_t^D = \sum_{k_t=1}^{n_t} I_{C^I_{t,k_t} > D}.$$

Working within the framework of a collective risk model and using similar techniques as in section 4.4, we define

- M_{t,b_t} as the number of losses in year t arising from the risks in the band b_t and
- $\bar{Z}_{t,b_t,i_{t,b_t}} \overset{\text{i.i.d.}}{\sim} \bar{Z}_{t,b_t}$, where $i_{t,b_t} \in \{1, \ldots, M_{t,b_t}\}$, as the indexed loss amount for loss i_{t,b_t} from band b_t in year $t \in \{1, \ldots, T-1\}$.

For all $t \in \{1, \ldots, T-1\}$, we define $\bar{E}^D_{t,b_t}$ as the expected number of indexed claims larger than D in year t arising from risks in band b_t. Using the properties of the collective risk model and assuming that all risks in the same band have the same exposure curve, we have that:

$$\bar{E}^D_{t,b_t} = \mathbb{E}[M_{t,b_t}]\,\mathbb{P}\big[\bar{Z}_{t,b_t,i_{t,b_t}} > D\big] = N_{t,b_t} q_{t,b_t}\,\mathbb{P}\left[X_{t,b_t} > \frac{D}{ASI^I_{t,b_t}}\right],$$

where q_{t,b_t} is defined as the expected number of losses for each of the risks in band b_t in year t, X_{t,b_t} is the random variable describing the degree of damage of the risks in band b_t in year t (given that there is a loss) and ASI^I_{t,b_t} is the indexed average insured value of the risks in band b_t in year t. Defining $\bar{E}^D_t$ as the total expected number of indexed claims larger than D in year $t \in \{1, \ldots, T-1\}$, we obtain:

$$\bar{E}^D_t = \sum_{b_t=1}^{B_t} \bar{E}^D_{t,b_t} = \sum_{b_t=1}^{B_t} N_{t,b_t} q_{t,b_t}\,\mathbb{P}\left[X_{t,b_t} > \frac{D}{ASI^I_{t,b_t}}\right]. \tag{13}$$

Similarly, we can define the expected number of claims larger than D in year T as:

$$E_T^D = \sum_{b_T=1}^{B_T} N_{T,b_T}\, q_{T,b_T}\, \mathbb{P}\left[X_{T,b_T} > \frac{D}{ASI_{T,b_T}}\right], \tag{14}$$

where q_{T,b_T} is defined as the expected number of losses for each of the risks in band b_T in year T and X_{T,b_T} is the random variable describing the degree of damage for each of the risks in band b_T in year T (given that there is a loss).

As explained in Klugman et al. [2008], under the assumption that the annual number of claims above the threshold D in year t follows a Poisson distribution with measure of exposure $\bar{E}_t^D$, $t \in \{1, \ldots, T\}$, the maximum likelihood estimator for the expected number of claims above the priority D in year T can be written as:

$$\hat{\lambda}_T^D = \frac{E_T^D}{\sum_{t=1}^{T-1} \bar{E}_T^D} \sum_{t=1}^{T-1} \bar{\lambda}_t^D. \tag{15}$$

We can also write:

$$\hat{\lambda}_T^D = \sum_{t=1}^{T-1} \omega_t^D\, \bar{\lambda}_t^D\, \frac{E_T^D}{\bar{E}_t^D},$$

where the weights ω_t^D are defined by:

$$\omega_t^D = \frac{\bar{E}_t^D}{\sum_{s=1}^{T-1} \bar{E}_s^D}.$$

Using this notation, we can see that the observed number of indexed claims above the threshold D is corrected for each year based on the ratio of the measure of exposure in year T and year $t \in \{1, \ldots, T-1\}$, $E_T^D / \bar{E}_t^D$. Each term receives a weight according to the weight of the measure of exposure in year t over the entire period for which claims were observed.

Note: In order to obtain the parameters q_{t,b_t} and the distribution functions of the degree of damage X_{t,b_t} which are required in the above formulas, it will be necessary to choose an exposure curve for the risks in a given band $b_t \in \{1, \ldots, B_t\}$ in a given year $t \in \{1, \ldots, T\}$. In section 6 we present methods that allow to make these choices.

5.1.3. *Creating As-If Claims above a given Threshold*

To create as-if claims above a given threshold $D > A_{min}^I$, we have to take into account the following two effects:

- Indexation of the insured values and claims.
- Evolutions in the indexed profiles above D.

The first effect is taken into account by indexing the claims with the construction price index. The second effect can be taken into account as follows. Let $\bar{Z}_t$ denote the random variable describing the indexed claims from year t based on the entire profile of risks from year $t \in \{1, \ldots, T-1\}$. If $t = T$, we can denote this random variable both with Z_T or with $\bar{Z}_T$ because no indexation is required. Then consider for all $t \in \{1, \ldots, T\}$ the distribution:

$$F_{\bar{Z}_t|\bar{Z}_t > D}(z) = \mathbb{P}\left[\bar{Z}_t \leq z \,\middle|\, \bar{Z}_t > D\right] = \frac{\mathbb{P}[\bar{Z}_t \leq z \cap \bar{Z}_t > D]}{\mathbb{P}[\bar{Z}_t > D]}, \text{ for } z \geq D. \quad (16)$$

Hence, by making the same assumptions as in section 4.4 for all $t \in \{1, \ldots, T\}$, we can write for all $t \in \{1, \ldots, T\}$ and $z > D$:

$$\begin{aligned}
\mathbb{P}[\bar{Z}_t \leq z \cap \bar{Z}_t > D] &= \frac{1}{\sum_{b_t=1}^{B_t} \mathbb{E}[M_{t,b_t}]} \sum_{b_t=1}^{B_t} \mathbb{E}[M_{t,b_t}] \mathbb{P}[\bar{Z}_{t,b_t,i_{t,b_t}} \leq z \cap \bar{Z}_{t,b_t,i_{t,b_t}} > D] \\
&= \frac{1}{\sum_{b_t=1}^{B_t} N_{t,b_t} q_{t,b_t}} \sum_{b_t=1}^{B_t} N_{t,b_t} q_{t,b_t} \mathbb{P}[X_{t,b_t} ASI^I_{t,b_t} \leq z \cap X_{t,b_t} ASI^I_{t,b_t} > D] \\
&= \frac{1}{\sum_{b_t=1}^{B_t} N_{t,b_t} q_{t,b_t}} \sum_{b_t=1}^{B_t} N_{t,b_t} q_{t,b_t} \left[F_{X_{t,b_t}}\left(\frac{z}{ASI^I_{t,b_t}}\right) - F_{X_{t,b_t}}\left(\frac{D}{ASI^I_{t,b_t}}\right) \right]
\end{aligned}$$

and that

$$\mathbb{P}[\bar{Z}_t > D] = \frac{\bar{E}_t^D}{\sum_{b_t=1}^{B_t} \mathbb{E}[M_{t,b_t}]} = \frac{\bar{E}_t^D}{\sum_{b_t=1}^{B_t} N_{t,b_t} q_{t,b_t}}.$$

For all $t \in \{1, \ldots, T-1\}$ and $k_t \in \{1, \ldots, n_t\}$, the distribution function, corresponding with an indexed claim $C^I_{t,k_t} > D$, corresponds with a level:

$$F_{\bar{Z}_t|\bar{Z}_t > D}(C^I_{t,k_t}).$$

A claim corresponding with the same level in the distribution function for year T, taking into account the portfolio evolution between year t and year T, would then be equal to:

$$C^{as-if,D}_{t,k_t} = F^{-1}_{Z_T|Z_T > D}(F_{\bar{Z}_t|\bar{Z}_t > D}(C^I_{t,k_t})). \quad (17)$$

Note that $C^{as-if,D}_{t,k_t}$ is only defined if $C^I_{t,k_t} > D$. Similar continuity issues as discussed in section 5.1.1 may occur in this general case, although the magnitude of the jumps will be a lot less pronounced if we are dealing with a full portfolio of risks. Practical solutions to deal with this will be proposed in section 6.

5.1.4. *Burning Cost for Working Layers taking into account Detailed Portfolio Evolutions*

A working layer is characterized by the fact that there is sufficient claims experience in the layer to allow to price it based on claims experience. We define a level L up to which we judge that the layer is working. Ideally, we would like to price as much as possible using experience rating. In practice, it is necessary to deal with the information which is available. The higher the layer chosen, the more experience rating results are influenced by statistical uncertainty. Measuring the impact of statistical uncertainty on burning cost calculations is difficult in practice. However, it is possible to verify the above assertion based on simulation by assuming a collective risk model and calculating the difference between the theoretical and observed total cost in lower and higher layers. This can also be understood based on the following intuitive argument. In case a working layer is exceeded by 5 claims over the pricing period, adding a 6th claim which exceeds the layer will increase the results with 20% in the part of the layer where there were only 5 claims. The effect on the lower parts of the layer (where there are more claims) will be less than 20%. When a higher (part of a) layer is only exceeded by one claim, adding a second claim which exceeds the layer will increase the results in this part of the layer with 100%.

In order to apply the techniques presented above, it is necessary to choose a threshold D above which the frequency is estimated and as-if claims are created. This choice is a compromise between reliability and representativeness. On the one hand, the more claims are observed historically above the threshold, the more reliable the estimation of the claims frequency above that threshold will be. On the other hand, it is useful to choose it as close as possible to the programme which has to be priced, such that the level above which we use the evolution of the measure for the frequency reflects as much as possible the actual evolution in the layer we have to price. If possible (i.e. if there is sufficient claims experience), the threshold can be fixed at the level of the lowest layer which needs to be priced.

Taking into account historical profile evolutions above the threshold D to estimate the evolution of the claims frequency and severity, we can calculate total cost in a working layer XL_j for which $D \leq D_j$ and $D_{j+1} \leq L$, where L denotes the level up to which we judge that layers are working:

$$TC_j^D = \frac{E_T^D}{\sum_{t=1}^{T-1} \bar{E}_t^D} \sum_{t=1}^{T-1} \sum_{k_t=1}^{n_t} \min(D_{j+1} - D_j;\ \max(0; C_{t,k_t}^{as-if,D} - D_j))\, I_{C_{t,k_t}^I > D}. \tag{18}$$

We can split TC_j^D in a frequency and severity part by writing:

$$TC_j^D = \hat{\lambda}_T^D \frac{\sum_{t=1}^{T-1}\sum_{k_t=1}^{n_t} \min(D_{j+1} - D_j;\ \max(0; C_{t,k_t}^{as-if,D} - D_j))\, I_{C_{t,k_t}^I > D}}{\sum_{t=1}^{T-1} \bar{\lambda}_t^D},$$

where $\hat{\lambda}_T^D$ is defined as in equation (15) and

$$\frac{\sum_{t=1}^{T-1}\sum_{k_t=1}^{n_t} \min(D_{j+1} - D_j;\ \max(0; C_{t,k_t}^{as-if,D} - D_j))\, I_{C_{t,k_t}^I > D}}{\sum_{t=1}^{T-1} \bar{\lambda}_t^D}$$

is the average claims severity in the layer XL_j generated by claims above the threshold D.

A burning cost for the working layer XL_j is then obtained as:

$$BC_j^D = TC_j^D / P_T. \tag{19}$$

In contrast to section 3, where the estimated number of claims is obtained from the observed number of indexed losses using only the total indexed premium, the reader will notice that the estimated number of claims now takes into account the detailed characteristics of the history of profiles through the measures $\bar{E}_t^D$, with $t \in \{1, \ldots, T-1\}$, and E_T^D.

5.2. Calibrated Exposure Rating

The price for layers for which it is judged that sufficient and reliable claims experience is available can be estimated using equation (19). In order to price unused or rarely used capacity, we propose to use exposure rating with a calibration on the experience rate of a working layer. In this process, it is important to bear in mind the potential impact of statistical uncertainty. It is intuitive that this uncertainty is larger on higher layers, which are hit by fewer losses (see also in section 5.1.4). If we have determined a working layer WL with priority P and limit L, we can obtain a burning cost for a layer j with unused or rarely used capacity and for which $D_j > L$ as follows[4]:

$$BC_j^{(C)} = EX_j \frac{BC_{WL}^D}{EX_{WL}}, \tag{20}$$

where EX_j is defined by (10), BC_{WL}^D is defined by (19) and

$$EX_{WL} = \sum_{b_T=1}^{B_T} (G_{b_T}(L/ASI_{T,b_T}) - G_{b_T}(P/ASI_{T,b_T}))\, P_{T,b_T}.$$

[4] The superscript (C) in (20) denotes that we work with an exposure rating burning cost, calibrated on the experience rate for a working layer.

We will call BC_{WL}^{D} / EX_{WL} the calibration ratio. The motivation behind this method is that we compare the exposure rating result for a working layer on which we estimate the experience rate to be reliable with the exposure rate on the layer without reliable claims experience. This approach has the advantage of being "Loss-Ratio-independent". In traditional exposure rating as described in section 4.2, a loss ratio is used to transform the gross premium resulting from the pricing of the cedent into an estimated average loss. The adequate loss ratio may in practice not always be known. If we assume that the loss ratio on the working layer which is used to perform the calibration and the loss ratio of a higher layer we want to price are the same, then, since this loss ratio is present both in the numerator and denominator of (20), it disappears after calibration. This approach also has the advantage that in cases where not enough information is available to estimate a set of adequate exposure curves for the portfolio of a given cedent, we can work with a set of standard curves. Using equation (20), the results are calibrated on a layer on which we estimate claims experience to be sufficiently reliable. Through the calibration, a part of the uncertainty due to the fact that no cedent-specific exposure curve(s) can be estimated is eliminated.

In practice, the limit of the working layer which is chosen will not always match with the priority of a subsequent layer which has to be priced. In such cases, one can work as follows:

- Use calibrated exposure rating for all layers XL_j for which $D_j > L$.
- For layers for which $D_j < L$ and $D_{j+1} > L$, the layer is split in two parts:
  - The part between D_j and L is priced as in (19).
  - The part between L and D_{j+1} is priced as in (20).

6. Parameterization based on available information and Continuity Issues

The purpose of this section is to discuss how the distribution functions presented in section 5 can be parameterized based on the available information and to discuss how to deal with some continuity issues in practice.

6.1. Parameterization based on available information

In order to be able to apply the techniques proposed in section 5.1, we need the distribution of the degree of damage X_{t,b_t} and the the expected number of losses from ground up q_{t,b_t} for the risks in the bands $b_t \in \{1, \ldots, B_t\}$ for each year $t \in \{1, \ldots, T\}$. As explained in (9), the distribution function of the degree of damage can be derived from the exposure curve. In this section, we present a method to fix the exposure curve. It is also important to note that the profile for year T is normally not known since it still has to be underwritten by the insurer. We will first explain how one can deal with this limitation in practice.

6.1.1. *Profile for Year T*

There are different options to deal with the fact that the profile for year T is unknown at the time the pricing needs to be made.

- **Option 1:**
  Assume that the profile will be stable between year $T-1$ and year T. In this method, the frequency measure for year T can be calculated as:

$$E_T^D = \bar{E}_{T-1}^D \frac{P_T^I}{P_{T-1}^I}. \tag{21}$$

  The as-if claims are then calculated as:

$$C_{t,k_t}^{as-if,D} = F^{-1}_{\bar{Z}_{T-1}|\bar{Z}_{T-1}>D}(F_{\bar{Z}_t|\bar{Z}_t>D}(C_{t,k_t}^I)), \tag{22}$$

  where $t \in \{1, \ldots, T-1\}$, $k_t \in \{1, \ldots, n_t\}$ and $C_{t,k_t}^I > D$.

- **Option 2:**
  Make a projection for the profile for year T based on the historic evolution of the profiles and/or the expected evolution between year $T-1$ and T due to possible changes in the underwriting policy. This solution may be useful in case it is expected that the insurer will change its underwriting limit. In that case, there may be bigger risks in the profile next year, which could affect the distribution function of the loss above a given threshold. Judgement may be required in this option.

In the example in section 7, we will use the first option for simplicity.

6.1.2. *Exposure Curve*

For simplicity, we assume that all risks in the same band $b_t \in \{1, \ldots, B_t\}$ have the same exposure curve. Note that this may in reality not be true. Two industrial plants might have a comparable insured value but a different exposure curve because in one of them, dangerous (e.g. explosive) goods are stocked and in the other not. Secondly, we assume that the exposure curve is linked with the insured value of a risk. This allows to obtain a decreasing average degree of damage and total loss probability in function of the insured value of the risk. Note that we take into account that a risk of 1 million in year t may after 5 years e.g. be worth 1.15 million, due to indexation. This means that if in year t, a certain exposure curve is used for a risk of 1 million, in year $t+5$, it should be used for a risk with an insured value of 1.15 million.

In Bernegger [1997], a class of functions is defined which can be used for exposure rating. In Lampaert and Walhin [2005], this class of functions is used in order to study the optimality of proportional reinsurance on a property portfolio. In this model, to describe the exposure curve $G_{b,g}(X)$ of a risk X, we base ourselves on the MBBEFD[5] class of increasing and concave functions $G_{b,g}(d)$ (see Bernegger [1997]) on the unit interval with $G_{b,g}(0) = 0$ and $G_{b,g}(1) = 1$. These functions are defined as:

$$G_{b,g}(d) = \begin{cases} d & \text{if } g = 1 \text{ and } b = 0, \\ \frac{\ln(1+(g-1)d)}{\ln(g)} & \text{if } b = 1 \text{ and } g > 1, \\ \frac{1-b^d}{1-b} & \text{if } bg = 1 \text{ and } g > 1, \\ \frac{\ln(((g-1)b+(1-gb)b^d)/(1-b))}{\ln(gb)} & \text{if } b > 0 \text{ and } b \neq 1 \text{ and } bg \neq 1 \text{ and } g > 1. \end{cases}$$

By taking $G_X(d) = G_{b(c),g(c)}(d) = G_c(d)$, with

$$b(c) = e^{3.1-0.15(1+c)c} \text{ and} \tag{23}$$

$$g(c) = e^{(0.78+0.12c)c}, \tag{24}$$

we obtain a one-parameter subset of the MBBEFD exposure curves. If $b > 0$, $b \neq 1$, $bg \neq 1$ and $g > 1$, the average degree of damage $\mathbb{E}[X]$ of a risk with exposure curve $G_{b,g}(d)$ is equal to $\frac{\ln(gb)(1-b)}{\ln(b)(1-gb)}$. The total loss probability of a risk with exposure curve $G_{b,g}(d)$ is equal to $p = 1/g$. Under this model, the distribution of the degree of damage is equal to:

$$F_{b,g}(d) = \begin{cases} 1 & \text{if } d = 1, \\ 1 - \frac{1-b}{(g-1)b^{1-d}+(1-gb)} & \text{if } 0 \leq d < 1 \end{cases} \tag{25}$$

In table 4, we summarize the main characteristics of some well-known exposure curves based on this model.

TABLE 4

PROPERTIES OF SOME WELL KNOWN EXPOSURE CURVES

Name	c	p	$\mathbb{E}[X]$
Total-loss Distribution	0	100%	100%
Swiss Re 1	1.5	23.69%	34.85%
Swiss Re 2	2	13.00%	22.61%
Swiss Re 3	3	3.27%	8.72%
Swiss Re 4	4	0.65%	3.19%
Lloyd's Industrial Risks	5	0.10%	1.21%

[5] MBBEFD stands for Maxwell-Boltzmann, Bose-Einstein, Fermi-Dirac.

We refer to section 7 for more details on the parameters we use in practice.

6.1.3. *Expected Number of Losses*

To determine the expected number of losses produced by a risk, we can use different methods. If we obtain information from the ceding company which allows to estimate the q-parameters for the risks in different bands and years, we can of course use this. Extrapolations may need to be made for bands with only a few risks. Such detailed information is in practice rarely available to the reinsurer, which forces us to look for other solutions.

- **Option 1:**
  The easiest method is to use the same q parameter for all risks. With this assumption, after the choice of the exposure curve, all elements in equations (13) and (14) are known, except for the parameter q. The estimated frequency for year T, $\hat{\lambda}_T^D$, can then be calculated since the q-parameter is present both in the numerator and denominator of (15). This method is interesting due to its simplicity but may not be sufficiently detailed. In practice, we expect larger risks to have a larger q-parameter, even if measures of prevention for larger risks are often more important (see e.g. Benktander [1973]). This method could therefore underestimate the impact of the large risks on the evolution of the claim frequency.

- **Option 2:**
  A more advanced method to obtain the parameters q_{t,b_t}, $t \in \{1, \ldots, T\}$[6] and $b_t \in \{1, \ldots, B_t\}$ is based on the tariff. We have that:

$$T_{t,b_t} = \frac{P_{t,b_t}}{TSI_{t,b_t}}, \; t \in \{1,\ldots,T\} \text{ and } b_t \in \{1,\ldots,B_t\}.$$

  Now assume that P_{t,b_t} is determined using the expected value premium principle. Then, we have that:

$$P_{t,b_t} = q_{t,b_t} \mathbb{E}[X_{t,b_t}] TSI_{t,b_t}(1+\gamma_{t,b_t}), \; t \in \{1,\ldots,T\} \text{ and } b_t \in \{1,\ldots,B_t\}. \quad (26)$$

  γ_{t,b_t} is assumed to contain administration expenses, capital and/or reinsurance costs and other potential charges such as brokerage costs. Some of these charges will, relative to the insured value of the risk, be larger for small risks (e.g. administration expenses). Others may, relative to the insured value, be larger for larger risks (e.g. capital and/or reinsurance costs). Therefore, it is not unreasonable to assume that $\gamma_{t,b_t} = \gamma$ for all $t \in \{1, \ldots, T\}$ and $b_t \in \{1, \ldots, B_t\}$, i.e. we assume that γ_{t,b_t} is constant over time and for the

[6] Or $t \in \{1, \ldots, T-1\}$ in case we do not have / estimate a profile for year T (see section 6.1.1).

different bands. As such, $q_{t,b_t}(1+\gamma)$ can be obtained as $T_{t,b_t} / \mathbb{E}[X_{t,b_t}]$, $t \in \{1, \ldots, T\}$ and $b_t \in \{1, \ldots, B_t\}$. As $\mathbb{E}[X_{t,b_t}]$ usually decreases faster than the tariff in function of the insured value, q_{t,b_t} will be larger for larger risks. In the calculation of (15) and (16), the factor $(1+\gamma)$ will disappear.

In the application in section 7, we will work with the methodology proposed in option 2. One can see that the tariff has an important impact on the estimation of the parameters q_{t,b_t}. If the insurer decides to change the pricing method which is used, this may lead to tariff changes. If such changes are not related to changes in the nature of the risks and the number of losses that may be expected, it is useful to correct for the impact of the tariff changes. In practice, various methods can be used. Such methods can work both on a global profile level or on a detailed level (e.g. in function of the average insured value and the type of the risks). It is outside the scope of this paper to discuss methods for applying tariff corrections. For the remainder of this paper, we will use the tariffs in the profiles without corrections to calculate the measures for frequency and severity.

6.2. Continuity Issues

We identify two sources of continuity issues when using exposure rating and calculating as-if claims based on historic profile evolutions. The first type corresponds with the precision of the profile information which is available. The second type corresponds with the issues which are related to the total loss probability of a risk with a given insured value. This was already discussed in section 5.1.1 and is important in the calculation of the as-if claims given by (17), or given by (22) in case we do not have a profile for year T.

6.2.1. *Continuity Issues related to Precision of the Profiles*

The profiles given in appendix A have only a limited level of precision. In order to have more correct results when calculating exposure rating and as-if claims, we advise to split up the original bands in the profiles into more bands. In reality, risks take different values between the limits of the bands. If the level of detail of the original bands is not sufficient, this may have an important impact on the results.

Splitting up the bands can be done such that the tariff and average insured values of the original bands are conserved. In the example in section 7, we worked with a tolerance level of 5,000 for the bands below 2,000,000 and 25,000 for the bands above 2,000,000. This means that the original bands are split up into bands with a (maximum) length of 5,000 below the level of 2,000,000 and 25,000 above the level of 2,000,000. In the top bands of the profile, strictly applying the above precision levels can lead to some bands with less than 1 risk. Although it is not possible to have fractions of one risk in a profile in reality, tolerating that the number of risks is not fully integer is useful to achieve better quality exposure rating results. It is possible to adapt the tolerance levels to ensure that the bands contain at least 1 risk, which is useful to avoid that too detailed profiles are created in case this is not really needed. We refer to appendix B for a detailed description of the methodology. Sensitivity analysis on these precision levels have shown that the results obtained in section 7 would not be materially different if a more detailed split would be used.

6.2.2. *Continuity Issues related to the Total Loss Probabilities*

In order to calculate the as-if claims given by (22), we use the following methodology.

- First all the distribution functions given by (16) are calculated, up to a given precision. In the example in section 7, we use a precision of 1,000.
- Then for all the indexed claims above a chosen threshold, we calculate $F_{\bar{Z}_t|\bar{Z}_t>D}(C^I_{t,k_t})$ based on a linear interpolation between the values $F_{\bar{Z}_t|\bar{Z}_t>D}(R_{t,k_t})$ and $F_{\bar{Z}_t|\bar{Z}_t>D}(L_{t,k_t})$, where R_{t,k_t} and L_{t,k_t} are defined as the two loss levels which are just above and just below C^I_{t,k_t}. Hence, the difference between R_{t,k_t} and L_{t,k_t} is precisely 1,000.
- Finally, $F^{-1}_{\bar{Z}_{T-1}|\bar{Z}_{T-1}>D}(F_{\bar{Z}_t|\bar{Z}_t>D}(C^I_{t,k_t}))$[7] is calculated based on a linear interpolation between the loss levels R_{T-1,k_t} and L_{T-1,k_t} corresponding to the values $F_{\bar{Z}_{T-1}|\bar{Z}_{T-1}>D}(R_{T-1,k_t})$ and $F_{\bar{Z}_{T-1}|\bar{Z}_{T-1}>D}(L_{T-1,k_t})$ which are just above and just below $F_{\bar{Z}_t|\bar{Z}_t>D}(C^I_{t,k_t})$ in the distribution $F_{\bar{Z}_{T-1}|\bar{Z}_{T-1}>D}$.

Using this methodology, the maximal error we can make when calculating the inverse of $F_{\bar{Z}_{T-1}|\bar{Z}_{T-1}>D}$ in the process of creating the as-if claims is 1,000.

7. Numerical Application

In this section, we compare the following three methods:

- **TRM: Traditional method**
  For working layers, we use traditional experience rating using a burning cost with indexation of claims and premiums (see section 3.1).
- **HPE: Method based on historic profile evolutions**
  For working layers, we use experience rating taking into account the historic profile evolution (see section 5.1). Both the frequency measure and the as-if claims are based on the actual evolution in the profiles above a given level. For non-working layers, we use exposure rating calibrated on experience rating for working layer (see section 5.2).

[7] Or alternatively, $F^{-1}_{\bar{Z}_T|\bar{Z}_T>D}(F_{\bar{Z}_t|\bar{Z}_t>D}(C^I_{t,k_t}))$, in case we have / estimate a profile for year T.

- **PER: Pure exposure rating**
  Both for working and non-working layers, we use pure exposure rating with a loss ratio of 60%[8] for all bands (see section 4).

We first summarize the available information and the reinsurance programme which needs to be priced. Then, we give an overview of the main steps to be able to apply the 3 methods described above in practice. These steps are explained in detail in the subsequent sections.

7.1. Available Information and Reinsurance Programme

Assume that for year 6, the expected premium income P_6 is equal to 85,000,000 and we want to price the programme as given in table 5.

TABLE 5

REINSURANCE PROGRAMME

Layer	Priority	Limit
XL_1	500,000	1,500,000
XL_2	1,500,000	5,000,000
XL_3	5,000,000	15,000,000

Assume we dispose of the list of claims in excess of 300,000 for the period between year 1 and year 5 as given in table 6. In year 5, we only dispose of the claims up to the end of September.

TABLE 6

LIST OF CLAIMS IN EXCESS OF 300,000

Year 1	Year 2	Year 3	Year 4	Year 5
1,402,321	2,100,121	784,014	1,452,647	1,845,645
603,501	420,032	540,064	926,579	1,457,894
512,456	310,022	431,562	742,895	1,025,678
		365,742	526,157	625,678
		326,587	358,745	462,546
		321,546	347,984	378,954
			326,548	335,642

In table 13 in appendix A, we give the profiles between year 1 and year 5.

[8] This information is often available in the renewal information on an overall portfolio level. Sometimes, it is also available per band. For pure exposure rating, we choose to work with an overall loss ratio which is used for all bands. In case the loss ratio for all bands is statistically reliable, which is not always easy to judge in practice, it may be better to work with a different value per band. As mentioned above, calibrated exposure rating has the advantage that it becomes to an important extent "loss-ratio independent" (see section 5.2).

7.2. Overview of Calculation and Analysis Steps

In this section, we summarize the calculation and analysis steps to apply the methods described above in practice.

- **Step 1:** Calculation of claims and premium index.
- **Step 2:** Indexation of claims and profiles.
- **Step 3:** Analysis of portfolio evolution.
- **Step 4:** Choice of exposure curves.
- **Step 5:** Calculation of the measures for frequency for the experience rating methods.
- **Step 6:** Calculation of the as-if claims for the experience rating methods.
- **Step 7:** Calculation of prices on a set of working layers.
  In this step, the prices are calculated using the 2 experience rating methods and pure exposure rating.
- **Step 8:** Choice of the calibration ratio for exposure rating and experience rating level.
- **Step 9:** Pricing of the reinsurance programme and analysis of results.

7.3. Application

7.3.1. *Step 1: Calculation of Claims and Premium Index*

The claims index can be based on the evolution of the construction prices, which is usually a good indicator of the evolution of the reconstruction cost for a damaged property[9]. The profile history allows to calculate a tariff index as explained in (4)[10]. The claims and tariff index allow to calculate a premium

TABLE 7

EVOLUTION OF MAIN INDICES

Year	Construction prices	Tariff Index	Premium Index
1	100.0	100.0	100.0
2	101.3	103.0	104.3
3	103.7	101.3	105.0
4	109.3	100.9	110.2
5	113.3	101.3	114.8
6	120.0	102.0	122.4

[9] In many countries, there are market indices available which allow to estimate the past evolution of the construction prices. The estimation of the future evolution of the construction price index (between year $T-1$ and year T) can e.g. be obtained from the OECD.

[10] Since a profile for year T is normally not available, it will be necessary to estimate a tariff evolution between year $T-1$ and T. This can be obtained from expert judgement made by market specialists. Some cedents also discuss their tariff plans towards the future in the renewal information which is provided to the reinsurers.

index (see (5)). The premium index is used in the traditional experience rating method to inflate the premiums from the past (see (6)). The indexed premium from year $T-1$, P^I_{T-1}, is also used to estimate the evolution of the measure of frequency between year $T-1$ and T if option 1 is chosen to deal with the fact that there is no profile for year T (see (21) in section 6.1.1). We summarize these indices in our example in table 7.

7.3.2. *Step 2: Indexation of Claims, Profiles and Premiums*

In this step, the construction price index from table 7 is used to inflate the claims from table 6 using (2). The indexed claims will be used as as-if claims in traditional experience rating and will also be used as a basis for the as-if claims in the method using historic profile evolutions, before they are corrected to take into account the detailed evolutions in the profiles based on the functions given by (16).

To inflate the profiles from a given year, the premiums, insured values and the lower and upper bounds of each band are corrected (towards year 6 in our example) based on the evolution of the construction price index. All further calculations using profile information will be based on the indexed profiles. The indexed premium, which is needed for the burning cost calculation in the traditional method, is calculated using the premium index as explained in (6).

7.3.3. *Step 3: Analysis of Portfolio Evolution*

Table 8 shows the evolution of the total premium, number of risks and insured value from ground up and above the different priorities of the reinsurance programme between year 1 and 5. To make the comparison, each of the bands was split up into a lot of smaller subsequent bands conserving the average insured value and the tariff of the original bands, based on the methodology presented in Appendix B.

TABLE 8

EVOLUTIONS IN THE PROFILES ABOVE DIFFERENT THRESHOLDS

Threshold	Year 4 → Year 5			Year 3 → Year 4			Year 2 → Year 3			Year 1 → Year 2		
	Pr	Nb	SI	Pr	Nb	SI	Pr	Nb	SI	Pr	Nb	SI
0	4%	3%	4%	2%	0%	2%	–1%	–4%	0%	7%	4%	4%
500,000	11%	18%	9%	11%	3%	8%	12%	4%	10%	24%	16%	19%
1,500,000	7%	6%	6%	15%	10%	11%	18%	8%	14%	27%	20%	21%
5,000,000	13%	13%	11%	16%	14%	14%	33%	32%	28%	11%	5%	14%

As we see in table 8, the number of risks above 500,000 increases more than the total number of risks. Based on what we see in table 8, we may expect that the traditional experience rating method, which uses the (indexed) premium from ground up as a measure of exposure, will underestimate the claim frequency in the reinsurance programme.

7.3.4. *Step 4: Choice of Exposure Curves*

We use the model presented in section 6.1.2. In table 9, we summarize the c-parameters used for the exposure curves in function of the insured values (c-values are based on indexation towards year 6). For insured values between the values given in table 9, we use a linear interpolation. For insured values less than or equal to 144,211, we use a c-parameter of 1.75, for insured values greater than or equal to 54,914,882, we use a c-parameter of 4.625.

TABLE 9

C-PARAMETERS IN FUNCTION OF THE INSURED VALUE (BASED ON INSURED VALUES IN YEAR 6 IN €)

SI	c
$\leq$ 144,211	1.75
288,422	2.25
721,054	2.75
1,442,108	3.2
10,982,976	3.9
$\geq$ 54,914,882	4.625

7.3.5. *Step 5: Calculation of the Measures for Frequency for the Experience Rating Methods*

In table 10, we give the evolution of the measures for frequency underlying the methods described in section 3 and section 5.1. We also give the same measures starting at 100 in year 1 in order to be able to compare the different measures easily. The measures for the method using historic profile evolutions are based on (13) and (14), in which we use (26) to calculate the parameters q_{t,b_t}, for $t \in \{1, \ldots, T-1\}$ and $b_t \in \{1, \ldots, B_t\}$, with a γ-parameter of 0.6 for all bands[11].

We observe that based on the indexed premium, the frequency measure increases with 10% between year 1 and year 6. When using the techniques from section 5.1, we estimate that the frequency measure above 500,000 increases with 67%. Above a level of 5,000,000, an increase of 90% is estimated. This shows the strength of the experience rating based on historic profile information and

[11] As long as the assumption that the loss ratio is similar for all bands which expose above the considered threshold is valid, the absolute level which is chosen has no impact on the results. If better information is available and reliable, this can of course be used.

TABLE 10

MEASURES FOR FREQUENCY

Year	Method section 3		Method section 5.1									
	Indexed premium		$\bar{E}_t^{500,000}$		$\bar{E}_t^{600,000}$		$\bar{E}_t^{700,000}$		$\bar{E}_t^{1,500,000}$		$\bar{E}_t^{5,000,000}$	
1	77,331,390	100	3.53	100	2.52	100	1.99	100	0.59	100	0.06	100
2	80,433,768	104	4.31	122	3.08	122	2.41	121	0.75	128	0.07	107
3	80,829,650	105	4.68	133	3.43	136	2.69	135	0.87	149	0.09	145
4	82,367,126	107	5.11	145	4.01	159	3.11	156	1.00	171	0.10	169
5	85,475,603	111	5.92	168	4.28	170	3.38	170	1.07	182	0.12	192
6	85,000,000	110	5.89	167	4.26	169	3.37	169	1.06	181	0.12	190

the importance of having a good view about the evolution of the underlying portfolios to be able to make reliable pricing for property per risk treaties.

7.3.6. *Step 6: Calculation of the As-If Claims for the Experience Rating Methods*

The basis to calculate the as-if claims above a given threshold in the method using historic profile evolutions are the functions which are defined in (16). In figure 2, we show these functions for year 1 and year 5 for the example at hand above a level of 500,000.

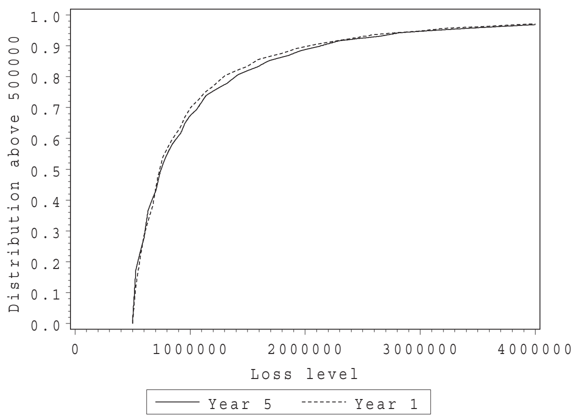

FIGURE 2: Distribution function for $F_{\bar{Z}_5|\bar{Z}_5>500,000}$ (full line) and $F_{\bar{Z}_1|\bar{Z}_1>500,000}$ (dashed line).

The distribution in year 5 is slightly more severe than the distribution in year 1. Therefore, we expect that losses above 500,000 will be slightly more severe in year 5 than in year 1. The impact of this portfolio change can be observed in figure 3, showing the as-if claims above 500,000. The grey bars correspond with the as-if claims used in traditional experience rating. The crossed bars correspond with the as-if claims based on (22).

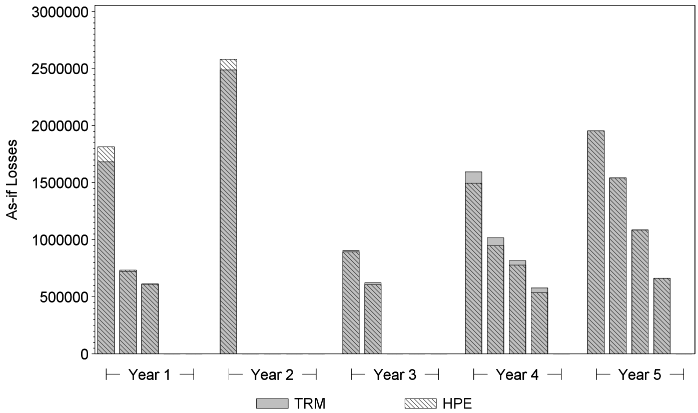

FIGURE 3: As-if claims above a level of 500,000 in the traditional method (TRM) and the HPE-method.

We see in figure 3 that the as-if claims taking into account the profile evolutions are a bit higher than those which take into account indexation with the construction price index only for year 1 and 2. The as-if claims for year 4 are lower than those which only take into account indexation based on the construction prices. Note that in table 8, we see that up to year 4, the average insured values above 500,000 have increased. Between year 4 and year 5, the average insured values above that level have considerably reduced. Again, the proposed method allows to take the actual evolution in the profiles better into account than the traditional method, in which all claims are indexed with the same index.

7.3.7. *Step 7: Calculation of Prices for a set of Working Layers*

In table 11, we summarize the results of the different methods on some candidates for working layers. For the TRM-method, the prices are based on (7). For the HPE-method, all prices are calculated based on (19), where $D = 500{,}000$. For the PER-method, the prices are based on (10). We also calculate a set of ratios between the different methods. All prices in this table are expressed in a percentage of the expected premium income for year 6[12].

In table 11, we observe that:

- The results for the HPE-method are about 18% above those for the traditional experience rating method. This is in line with the evolutions observed in table 8 and table 10. This ratio is stable for the different working layers.

[12] Traditionally, the price of reinsurance is expressed in a percentage of the premium. If the insurer writes more premium than estimated at the time of the pricing, the reinsurer will also receive more premium at the end of the year. We have shown that the premium may not always be an adequate measure for the exposure of the reinsurer.

TABLE 11

RESULTS FOR DIFFERENT WORKING LAYERS

Priority	Limit	Methods			Ratios		
		TRM	HPE	PER	TRM/PER	HPE/PER	HPE/TRM
500,000	1,500,000	1.956%	2.308%	2.740%	71.4%	84.2%	118.0%
500,000	1,750,000	2.169%	2.557%	3.020%	71.8%	84.7%	117.9%
600,000	1,500,000	1.598%	1.889%	2.174%	73.5%	86.9%	118.2%
600,000	1,750,000	1.811%	2.138%	2.453%	73.8%	87.1%	118.0%
700,000	1,500,000	1.312%	1.549%	1.735%	75.6%	89.3%	118.1%
700,000	1,750,000	1.525%	1.799%	2.015%	75.7%	89.2%	117.9%

- The results for the HPE-method are between 16% and 11% lower than the pure exposure rating results (based on a loss ratio assumption of 60%). In this example, this difference decreases with increasing priorities and limits for the working layers.
- The results for the traditional experience rating method are between 29% and 24% lower than the pure exposure rating results.

7.3.8. *Step 8: Choice of the Calibration Ratio for Exposure Rating and Experience Rating Level*

Based on the results in table 11, we have to determine a calibration ratio which will be used to calibrate exposure rating for non-working layers as in (20). We therefore use the HPE experience rating method. To determine the calibration ratio, it is important to analyze its sensitivity to the priority and the limit of different working layer options. The calibration ratios in table 11 are relatively stable. We choose a calibration ratio of 87% which is in the middle of the range and decide to use experience rating up to a level of 1,500,000. There are 5 claims reaching up to that level over the entire period. Layers above 1,500,000 are priced based on calibrated exposure rating.

7.3.9. *Step 9: Pricing of the Reinsurance Programme and Analysis of Results*

In table 12, we give the pricing results when using combined experience and exposure rating. In addition to the layers of the reinsurance programme, we also price a layer from 1,500,000 to 2,500,000, which is around the level of the largest as-if loss. Based on the retained rate, we also calculate the risk rate on line (RROL), which is equal to the expected amount of claims in one year in a layer, divided by the capacity of that layer. The inverse of the RROL corresponds to expected number of years we have to wait before the full capacity of a layer will have been used.

TABLE 12

PRICING RESULTS FOR COMBINED EXPERIENCE AND EXPOSURE RATING

Priority	Limit	HPE	PER	Retained	RROL	1 / RROL	Method
500,000	1,500,000	2.308%	2.740%	2.308%	196.2%	0.51	HPE
1,500,000	5,000,000	0.594%	1.502%	1.307%	31.7%	3.15	PER x 87%
5,000,000	15,000,000	0.000%	0.351%	0.305%	2.6%	38.51	PER x 87%
1,500,000	2,500,000	0.569%	0.819%	0.712%	60.5%	1.65	PER x 87%

We observe that:

- The burning cost based on the HPE-method is 0 on the top layer. On the second layer, it is considerably lower than the exposure rate. This could be due to the fact that the observation period was too short to have claims experience up to and above 5,000,000. This can be resolved by using calibrated exposure rating, which allows to take into account the composition of the profile in the regions without enough claims experience.
- The RROL on the second layer is 31.7%. Therefore, based on the current composition of the portfolio, we expect the capacity of this layer to be fully used once over a period of 3.15 years. Similarly, for the top layer, we expect this to happen over 38.51 years.
- The experience rate between 1,500,000 and 2,500,000 is about 20% lower than the retained rate obtained based on calibrated exposure rating. This indicates that, taking into account the exposure in the portfolio, the cedent may have been lucky in the past with the loss experience in this layer. The opposite can also happen, e.g. if due to bad luck, a ceding company suffers from a very large loss over a relatively short period of time.

8. CONCLUSION

In this paper, we have considered the pricing of property excess of loss reinsurance based on the information which is commonly available to the reinsurer. We discussed traditional experience and exposure rating methods, together with their limitations.

We have shown methods to overcome the different limitations mentioned using a combination of experience and exposure rating techniques if historical profile information is available. We propose an experience rating method in which the measure for frequency and the as-if claims are determined using the evolutions observed in the profiles. For pricing unused capacity, we use exposure rating calibrated on the experience rate for a working layer.

In a numerical example, we have shown how the different methods can be implemented and we have analyzed their impact in case the exposure above the priorities of the layers increases faster than from ground up. We paid special attention to the issue of statistical uncertainty. We went in detail over the available information and the hypotheses which can be taken when applying the methodology in practice.

We conclude that working on the basis of real exposure, i.e. taking into account the detailed evolutions in the past profiles, is essential in order to make a reliable estimation of the claims frequency and severity. Unfortunately, cedents are not always able to provide reliable or complete past information. We have shown that this leads to model uncertainty, since simplified approaches then need to be used. We have also argued that models based on exposure curves may be superior to traditional experience rating models for the pricing of property per risk reinsurance. Indeed, the latter models do not take into account the underlying profile and therefore have difficulties to deal with changes in the underlying exposure. Therefore both insurers and reinsurers should analyze current and past exposure based upon detailed profile information in order to analyze property per risk excess of loss treaties.

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APPENDIX A

Profile History for Year 1 to Year 5

TABLE 13

Profiles for year 1 to year 5 (Premiums and limits of bands are divided by 1.000 and total insured values are divided by 1.000.000).

Year		Year 5			Year 4			Year 3		
From	To	Pr	Nb	SI	Pr	Nb	SI	Pr	Nb	SI
1	15,000	80,142	325,210	76,110	74,169	314,317	70,705	69,341	314,236	65,857
1	100	4,595	67,310	3,580	4,644	66,599	4,090	4,846	70,004	4,310
100	200	24,441	136,097	22,654	24,512	139,350	22,307	26,490	151,097	23,810
200	300	26,076	91,240	21,759	23,185	82,112	19,475	20,317	71,458	16,927
300	400	6,435	15,672	5,359	5,308	12,930	4,408	4,121	9,990	3,345
400	500	2,057	3,808	1,786	1,694	3,234	1,446	1,454	2,772	1,267
500	600	1,181	1,778	1,006	1,036	1,584	903	939	1,451	818
600	700	936	1,201	800	862	1,088	709	681	969	637
700	800	776	890	662	671	756	553	589	701	529
800	900	651	681	591	554	612	514	456	535	445
900	1,000	502	530	498	507	490	474	411	452	432
1,000	1,250	1,185	1,072	1,154	1,118	1,025	1,133	975	943	1,046
1,250	1,500	1,041	840	1,126	942	768	1,033	818	703	948
1,500	1,750	908	629	1,000	893	635	1,017	704	505	832
1,750	2,000	992	537	994	857	489	899	580	395	751
2,000	2,500	1,230	681	1,474	1,123	623	1,382	957	540	1,202
2,500	3,000	1,028	497	1,323	913	439	1,172	809	365	1,011
3,000	4,000	1,507	583	2,049	1,420	545	1,959	990	456	1,545
4,000	5,000	850	309	1,395	708	276	1,209	740	287	1,250
5,000	6,000	676	220	1,222	733	222	1,213	562	190	1,048
6,000	7,000	720	171	1,135	494	139	881	411	125	809
7,000	8,000	581	136	1,021	411	110	843	349	94	714
8,000	9,000	369	86	743	351	85	720	270	52	436
9,000	10,000	280	53	499	345	56	533	186	39	372
10,000	12,500	697	115	1,325	482	88	982	442	74	868
12,500	15,000	428	70	955	409	62	852	244	38	506

Year		Year 2			Year 1		
From	To	Pr	Nb	SI	Pr	Nb	SI
1	15,000	68,536	327,295	64,013	63,163	313,973	60,757
1	100	5,141	75,304	4,592	4,297	65,398	3,848
100	200	28,354	163,077	25,009	27,249	160,308	24,695
200	300	19,477	68,701	15,950	18,318	68,688	15,982
300	400	3,786	9,167	3,094	3,770	9,826	3,393
400	500	1,404	2,629	1,207	1,291	2,567	1,133
500	600	902	1,442	783	825	1,282	722
600	700	653	962	637	607	899	572
700	800	530	699	516	467	630	479
800	900	490	532	448	377	468	393

Year		Year 2			Year 1		
From	To	Pr	Nb	SI	Pr	Nb	SI
900	1,000	421	458	437	314	408	384
1,000	1,250	791	848	915	586	673	732
1,250	1,500	737	665	890	515	550	731
1,500	1,750	663	481	778	488	383	614
1,750	2,000	455	338	622	330	264	495
2,000	2,500	806	509	1,113	557	397	867
2,500	3,000	621	340	911	483	279	748
3,000	4,000	929	445	1,500	729	366	1,248
4,000	5,000	719	292	1,264	590	247	1,096
5,000	6,000	270	97	535	302	106	583
6,000	7,000	186	64	410	246	71	450
7,000	8,000	192	51	386	211	49	365
8,000	9,000	128	46	388	163	31	268
9,000	10,000	316	46	435	92	20	193
10,000	12,500	346	63	709	228	41	475
12,500	15,000	222	35	484	130	21	290

APPENDIX B

Methodology to make Detailed Profiles

In order to simplify notations, we will explain the methodology for one band, for which we denote the upper bound as *UB*, the lower bound as *LB*, the premium in the band as *P*, the number of risks in the band as *N*, the total insured value of the band as *TSI* and the average insured value of the risks in the band as *ASI*. This can easily be generalized to a history of profiles with different bands. Our aim is to obtain a flexible method which allows to split this band into a series of bands such that:

- the distance between the lower and upper bound of the bands is always smaller than or equal to a given step θ,
- the average insured value of the risks in the original band is conserved and
- the tariff in the new bands is the same as in the original band.

The methodology consists of 2 steps. In the notations below, we will use two indices. The first index denotes the number of the step, the second index denotes the number of the band which is created in the step. It is important to note that these indices have a different meaning from the two indices which are used in the paper, where the first index denoted the year of the profile and the second the number of the band within that year.

- First we split the band into 2 bands, such that the average insured value of the two bands combined is equal to the original average insured value as follows:

id	$LB_{1,id}$	$UB_{1,id}$	$P_{1,id}$	$N_{1,id}$	$TSI_{1,id}$	$ASI_{1,id}$
1	LB	ASI	$TSI_{1,1} \times P/TSI$	$N_{1,1}$	$N_{1,1} \times ASI_{1,1}$	$(LB + ASI)/2$
2	ASI	UB	$TSI_{1,2} \times P/TSI$	$N_{1,2}$	$N_{1,2} \times ASI_{1,2}$	$(ASI + UB)/2$

where

$$N_{1,1} = N \frac{ASI_{1,2} - ASI}{ASI_{1,2} - ASI_{1,1}} \text{ and}$$

$$N_{1,2} = N - N_{1,1} = N \frac{ASI - ASI_{1,1}}{ASI_{1,2} - ASI_{1,1}}.$$

After this step, the average insured value of the 2 bands is equal to the average of the limits of the bands. This property allows to split the bands directly in the desired detail in the next step.
If one prefers to work always with bands with at least one risk, it is possible not to perform this step in case $N_{1,1}$ or $N_{1,2}$ is smaller than 1. In that case, it is possible that the distance between the lower and the upper bound of the band is bigger than θ. It may be useful to tolerate that $N_{1,1}$ or $N_{1,2}$ are smaller than 1 to obtain the desired precision on all bands.

- In case $N_{1,1} \leq 1$, it is not useful to further split the first band created after step 1 in the second step. The same holds for the second band created after step 1 in case $N_{1,2} \leq 1$. Now suppose that:

$$\begin{aligned} LB_{1,1} &= k\theta + \theta_1, \\ UB_{1,1} &= l\theta + \theta_2 = LB_{1,2} \text{ and} \\ UB_{1,2} &= m\theta + \theta_3, \end{aligned}$$

with $0 \leq \theta_1 < \theta$, $0 \leq \theta_2 < \theta$, $0 \leq \theta_3 < \theta$ and $k < l < m$. Then we can split band 1 in $l - k + 1$ bands and band 2 in $m - l + 1$ bands as follows:

id	$LB_{2,id}$	$UB_{2,id}$	$P_{2,id}$	$N_{2,id}$	$TSI_{2,id}$
1	$k\theta + \theta_1$	$(k+1)\theta$	$TSI_{2,1} \times P/TSI$	$N_{2,1}$	$N_{2,1} \times ASI_{2,1}$
2	$(k+1)\theta$	$(k+2)\theta$	$TSI_{2,2} \times P/TSI$	$N_{2,2}$	$N_{2,2} \times ASI_{2,2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$l-k$	$(l-1)\theta$	$l\theta$	$TSI_{2,l-k} \times P/TSI$	$N_{2,l-k}$	$N_{2,l-k} \times ASI_{2,l-k}$
$l-k+1$	$l\theta$	$l\theta + \theta_2$	$TSI_{2,l-k+1} \times P/TSI$	$N_{2,l-k+1}$	$N_{2,l-k+1} \times ASI_{2,l-k+1}$
$l-k+2$	$l\theta + \theta_2$	$(l+1)\theta$	$TSI_{2,l-k+2} \times P/TSI$	$N_{2,l-k+2}$	$N_{2,l-k+2} \times ASI_{2,l-k+2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$m-k+1$	$(m-1)\theta$	$m\theta$	$TSI_{2,m-k+1} \times P/TSI$	$N_{2,m-k+1}$	$N_{2,m-k+1} \times ASI_{2,m-k+1}$
$m-k+2$	$m\theta$	$m\theta + \theta_3$	$TSI_{2,m-k+2} \times P/TSI$	$N_{2,m-k+2}$	$N_{2,m-k+2} \times ASI_{2,m-k+2}$

where:

$$N_{2,id} = N_{1,1}\frac{UB_{2,id} - LB_{2,id}}{UB_{1,1} - LB_{1,1}} \quad \text{for } id \in \{1,2,\dots,l-k,\ l-k+1\},$$

$$N_{2,id} = N_{2,1}\frac{UB_{2,id} - LB_{2,id}}{UB_{1,2} - LB_{1,2}} \quad \text{for } id \in \{l-k+2,\dots,\ m-k+1,\ m-k+2\} \text{ and}$$

$$ASI_{2,id} = \frac{UB_{2,id} - LB_{2,id}}{2} \qquad \text{for } id \in \{1,2,\dots,m-k+2\}.$$

If one prefers to work always with bands with at least one risk, it is possible not to perform this step on the bands for which $id \in \{1,2, \dots, l-k, l-k+1\}$ and/or for which $id \in \{l-k+2, \dots, m-k+1, m-k+2\}$ in case $N_{2,id}$ for any of the bands in those sets would be smaller than 1. It may be useful however to allow that $N_{2,1}$, $N_{2,l-k+1}$ or $N_{2,m-k+2}$ are smaller than 1. In cases where θ_1, θ_2 and/or θ_3 are considerably smaller than θ, this may allow that the other bands have the desired precision with a number of risks bigger than or equal to 1. Note that θ_1, θ_2 and/or θ_3 can also be 0. In case $N_{2,id}$ as defined above is smaller than 1 for any of the other bands, it is useful to redefine the step size θ_u as follows:

$$\theta_u = \frac{UB_{1,u} - LB_{1,u}}{N_{1,u}} \quad \text{for } u = 1 \quad \text{and/or} \quad u = 2,$$

depending on whether $N_{2,id}$ is smaller than 1 for the bands which are originating from the first band after step 1 and/or from the second band after step 1. This ensures that the number of risks in those bands is always bigger than or equal to 1 and that we do not split bands using a too detailed resolution when this is not useful.

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