GLOBAL WARMING, EXTREME WEATHER EVENTS, AND FORECASTING TROPICAL CYCLONES: A MARKET-BASED FORWARD-LOOKING APPROACH

BY

CAROLYN W. CHANG, JACK S.K. CHANG AND KIAN GUAN LIM

ABSTRACT

Global warming has more than doubled the likelihood of extreme weather events, e.g. the 2003 European heat wave, the growing intensity of rain and snow in the Northern Hemisphere, and the increasing risk of flooding in the United Kingdom. It has also induced an increasing number of deadly tropical cyclones with a continuing trend. Many individual meteorological dynamic simulations and statistical models are available for forecasting hurricanes but they neither forecast well hurricane intensity nor produce clear-cut consensus. We develop a novel hurricane forecasting model by straddling two seemingly unrelated disciplines — physical science and finance — based on the well known price discovery function of trading in financial markets. Traders of hurricane derivative contracts employ all available forecasting models, public or proprietary, to forecast hurricanes in order to make their pricing and trading decisions. By using transactional price changes of these contracts that continuously clear the market supply and demand as the predictor, and with calibration to extract the embedded hurricane information by developing hurricane futures and futures option pricing models, one can gain a forward-looking market-consensus forecast out of all of the individual forecasting models employed. Our model can forecast when a hurricane will make landfall, how destructive it will be, and how this destructive power will evolve from inception to landing. While the NHC (National Hurricane Center) blends 50 plus individual forecasting results for its consensus model forecasts using a subjective approach, our aggregate is market-based. Believing their proprietary forecasts are sufficiently different from our market-based forecasts, traders could also examine the discrepancy for a potential trading opportunity using hurricane derivatives. We also provide a real case analysis of Hurricane Irene in 2011 using our methodology.

KEYWORDS

Global warming, Extreme weather events; Market-based hurricane forecasting; Calibration; Doubly-binomial tree with stochastic arrival intensity.

1. Introduction

Recent news in *Nature* (Schiermeier, 2011) has reported that human-induced global warming has more than doubled the likelihood of extreme weather events such as the 2003 European heat wave, the growing intensity of rain and snow in the Northern Hemisphere, and the increasing risk of flooding in the United Kingdom. How tropical cyclone activity will respond to global warming has also been a topic of much popular interest and scientific debate. This is especially true since Hurricane Katrina, a powerful category 5 storm, devastated the gulf coast of the United States in 2005. In light of the significant economic impact of global warming in general and on the insurance industry in particular, the global investment community has participated in the debate by developing new catastrophe risk management tools. New 10-day hurricane forecasting tools have been developed by global weather risk specialists like WSI and Guy Carpenter, and new catastrophe simulation models have been developed by highly skillful, multi-disciplinary-based specialist vendors like AIR, RMS and EqeCAT. In an effort to mitigate the costs of extreme weather events, i.e. by creating building codes, setting insurance premiums and planning for evacuations and relief efforts, federal agencies have also increased funding to finance weather research programs.

In this research, we focus on developing a new and novel hurricanes forecasting tool. Prevailing hurricane forecast models vary widely in structure and complexity[1]. While they have been increasingly successful in the forecasts of hurricane tracks, hurricane intensity forecasts are extremely difficult tasks and often offer conflicting and inconsistent results (e.g. Emanuel et al., 2004). This is because hurricanes are complex dynamical systems whose intensities at any given time are affected by a variety of physical processes, some of which are internal and others involve interactions between the storms and their environments. Since many of these processes are poorly understood, the forecasts of the intensity change of individual storms cannot be precise. Current computing powers are also limited in horizontal resolutions to compute hurricane eyes and eye-walls properly.

We take a novel alternative market-based forecasting approach to prevailing physical and statistical models. Motivated by exploring the price discovery function of exchange-traded hurricane derivatives and the well-known observation

[1] Dynamical numerical and simulation models, using high-speed computers to solve the physical equations of motion governing the atmosphere, are the most complex. Statistical models, in contrast, use historical relationships between storm behavior and storm-specific details such as location and date to forecast and are simple to implement. Statistical-dynamical models blend both dynamical and statistical techniques by making a forecast based on established historical relationships between storm behavior and atmospheric variables provided by dynamical models. Trajectory models move a tropical cyclone along based on the prevailing flow obtained from a separate dynamical model. Finally, consensus models are created by combining the forecasts from a collection of other models.

from operational forecasting that model consensus is usually superior to any individual model, we employ transactional hurricane derivatives prices as a forward-looking market-consensus predictor to forecast the expected destructive power of a hurricane on the landfall date and its evolution over time upon news arrival.

Contemporaneous to the development of new catastrophe risk management tools is the designing and listing on exchanges of a new breed of hurricane derivatives that include hurricane futures and futures options contracts on the CME (Chicago Mercantile Exchange) in 2007, and hurricane futures contracts on the IEM (Iowa Electronic Markets) and EUREX (European Exchange) in 2006 and 2009, respectively. Needless to say one major function of these contracts is being an effective hedging vehicle for related parties, such as insurers/reinsurers and energy companies, to mitigate extreme weather exposures. However, another major economic function which cannot be overlooked is the price discovery capability of these contracts. Essentially, traders of these contracts utilize all available information, i.e. forecasting results from public or proprietary sources, to make judicious pricing and trading decisions, which on the aggregate induces market price changes through the interaction of supply and demand. Thus examining the transaction price changes of these contracts can convey market-consensus forecasts that aggregate most, if not all, individual forecasts. To name a few applications, the best way to forecast a frost in certain regions of the US is to follow the change of orange juice futures prices, while the best way to forecast the 30-day-ahead future volatility for stock, gold and oil prices is the market-based forward-looking VIX, implied out of option prices. In hurricane forecasting specifically, Kelly et al. (2009) has attempted to develop a market-based forecasting model using the IEM futures data to predict whether a hurricane will or will not make landfall in a given area. They find that futures price changes are more accurate than the NHC (National Hurricane Center) for storms more than five days from landfall (69% to 54%), but less accurate for storms two days or less from landfall (90% versus 100%). While the NHC blends 50 plus individual forecasting results for its consensus model forecasts using a subjective approach, our aggregate is market-based.

Specifically, we first adopt a doubly-binomial process with stochastic arrival intensity to model how news regarding a hurricane would arrive with a varying rate and upon news arrival how a hurricane futures price would respond to reflect changes in the prediction of the hurricane's power on the landfall date. This specification is consistent in the spirit with the doubly-binomial model of Gerber (1984, 1988) and others in the actuarial risk theory but more general in allowing for stochastic arrival intensity to reflect the recent work of Wu and Chung (2010) and Chang, Lin, and Yu (2011) who have demonstrated empirically that stochastic arrival rate is superior to constant arrival rate as a more adequate specification of the stochastic nature of hurricane arrivals. We then price hurricane futures options in this framework by extending the discrete-transaction-time option pricing methodology of Chang,

Chang, and Lu (2008)[2] to subsume stochastic arrival intensity. As Geman and Yor (1997) have suggested, unlike other pricing models in the catastrophe option pricing literature[3] that are developed in calendar time and do not incorporate information conveyed in transaction arrivals, this pricing methodology is unique in having the merit of illuminating the information conveyed by transactions, as market proxies of news/information arrivals. In this vein and for expositional convenience, we will use transaction arrivals, news arrivals, and information arrivals interchangeably in the rest of the paper.

We shall develop our pricing model based upon the more comprehensive CME CHI (CME Hurricane Index) futures and futures options[4], where the CHI[5] is a numerical measure of the potential for damage from a hurricane. The CHI

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[2] It is the discrete-time counterpart of Chang, Chang, and Yu (1996), who have proposed a unique "randomized operational time" approach to price CAT futures options. This "randomized operational time" concept, originated in probability theory (Feller, 1971), is widely applied in systems and engineering fields. It dictates that a simple change of time scale will frequently reduce a general nonstationary process in the usual calendar-time scale to its stationary operational counterpart in a new time scale dictated by the nature of things. In the finance literature, Clark (1973) first applied this concept to subordinate stock returns to news arrivals with transaction arrivals being a market proxy, while in the insurance literature Chang, Chang, and Yu first applied this concept to subordinate CAT futures return to CAT futures transaction arrivals. This time-change transforms a calendar-time CAT option with stochastic volatility to an isomorphic transaction-time CAT option with random maturity (to reflect the randomness of transaction arrivals), which leads to a transaction-time option pricing formula as a risk-neutral Poisson sum of Black's (1976) prices over the option's maturity domain. It is parsimonious in requiring only two unobservable variables — the transaction arrival intensity and the per-transaction futures volatility.

[3] Extant approaches for pricing catastrophe (CAT hereafter) derivatives include Aase (2001), Cummins and Geman (1995), Chang, Chang, and Yu (1996), and Chang, Chang, and Lu (2008) for pricing CAT futures and futures options; Bakshi and Madan (2002), Aase (1999), Geman and Yor (1997), and Chang, Chang, and Lu (2010) for pricing CAT cash options; Lee and Yu (2002) and Loubergé, Kellezi, and Gilli (1999]) for pricing CAT bonds; Jaimungal and Wang (2006) for pricing CatEPut; and more recently, Wu and Chung (2010) for pricing catastrophe products with counterparty risk.

[4] There are two types of event-driven CHI futures contracts — the Eastern USA contract, and the CHI-Cat-In-A-Box contract that covers the major oil & gas production in the Gulf of Mexico. Two types of American-style call options are traded on these futures contracts — plain vanilla and binary. Payoff for the former is the in-the-money amount but \$10,000 for the latter. They trade as follows: at the beginning of each season, storm names are used from a list, starting with A and ending with Z, maintained by the World Meteorological Organization. In the event that more than 21 named events occur in a season, additional storms will take names from the Greek alphabet: Alpha, Beta, Gamma, Delta, and so on. Named hurricanes must make landfall in the Eastern U.S. (Brownsville, TX to Eastport, ME) for the Eastern USA contract and Galveston-Mobile area (95°30'0"W on the West, 87°30'0"W on the East, 27°30'0"N on the South, and the corresponding segment of the U.S. coastline on the North) for the CHI-Cat-In-A-Box contract, respectively, to have CHI values. Trading shall terminate at 9:00 A.M. on the first Exchange business day that is at least two calendar days following the dissipation or exit from the designated area of a named storm. All futures contracts remaining open at the termination of trading shall be settled using the reported respective CHI final value and CHI-Cat-In-A-Box final value (for the latter the maximum calculated CHI value while the hurricane is within the Box) by EqeCAT. In this research, we develop our forecasting model based upon the plain vanilla Eastern USA contract, because the CHI-Cat-In-A-Box final value is based upon the maximum calculated CHI value while the hurricane is within the Box. Extension of our model to the CHI-Cat-In-A-Box contract will be a future research topic.

[5] CHI is compiled by EqeCAT, a leading authority on extreme-risk modeling, using publicly available data from the National Hurricane Center.

incorporates not only a hurricane's maximum wind velocity as a measure of intensity but also the size (radius) of hurricane force winds and is a continuous measurement[6]. Moreover, the trading volume of these contracts has been growing rapidly with 32,600 contracts traded alone in 2008, a needed ingredient for enhancing market efficiency. The Eurex contracts, on the other hand, are settled based on actual insurance industry losses with a lengthy reporting period as compiled by ISO's Property Claim Services (PCS) unit. While the IEM contracts are based on tracking only — where a given hurricane makes its first landfall.

By calibration through our pricing model using transactional futures option prices, we can imply out the values of a sufficient set of parameters to fully estimate the underlying transaction-time futures price process, which in turn enables us to develop our dynamic market-consensus forward-looking forecast as a multi-period transaction-time binomial tree, where news will arrive randomly at a varying speed and upon news arrival hurricane derivative prices will change to reflect the new information, leading to continuously updated forecasts about the hurricane's destructive power on the landfall date. In a related vein, traders, believing their proprietary forecasts are sufficiently different from our market-based forecasts, could examine the discrepancy to identify a potential trading opportunity using hurricane futures and futures options.

The rest of our paper is organized as follows: In Section 2, we develop our theoretical forecasting model starting from specifying the futures price process, developing a futures option pricing model, and ending with identifying the set of implied parameters necessary for constructing the forecasts. In Section 3, we run simulations to first show how to implement our pricing model and calibrate the futures price process, and then we illustrate how to forecast 1) the time-varying transaction arrival probability and the probability of the number of transaction arrivals prior to a hurricane's landfall, 2) the static probability distribution of the CHI value on the landfall date, and 3) the dynamic evolution of the predicted CHI value over news arrival as a multi-period transaction-time binomial tree. In section 4, we provide an example of application in the analysis of Hurricane Irene that occurred in August 2011. In section 5, we conclude the paper and discuss future research directions.

2. The Market-Based Forecasting Approach

We specify the hurricane futures price process as a doubly-binomial process with mean-reverting stochastic arrival intensity, risk-neutralize this process, make a time change from calendar time to transaction time to reduce computational

[6] The commonly used Saffir-Simpson Hurricane Scale (SSHS) classifies hurricane intensities in categories from 1 to 5 by considering only the velocity but not the radius of a hurricane, and thus cannot be used to measure the actual physical impact, making it less than optimal for use by the insurance community and the public at large. For example, Hurricane Katrina in 2005 was described as a weak category-4 storm at the time of its landfall but exerted significantly more physical damage than Hurricane Wilma, which at one point in its life was mentioned as the strongest storm on record.

complexity, price futures options in transaction time by using the no-arbitrage martingale methodology, and finally identify the set of implied parameters needed to calibrate the futures price process in order to implement our market-consensus forward-looking forecasting model.

2.1. The Hurricane Futures Price Process

We utilize the transactional CME CHI futures and futures option prices to predict the final destructive power of a hurricane on the landfall date and how this power would evolve throughout its lifetime upon news arrivals. As the underlying CHI index is physical in nature with its changes uncorrelated with changes in financial prices, we assume the futures price change bear no systematic risk[7]. This assumption implies that today's futures price embeds no risk premiums and thus should be a statistically unbiased forwarding-looking market predictor of the CHI value on the expected landfall date. On this basis, we specify the futures price process and construct the futures option pricing model.

Unlike usual news/transaction arrivals in financial markets that can be approximately continuous, hurricane news arrivals are sporadic, random and discrete, with arrival rate time-varying and exhibiting mean-reversion (see Wu and Chung (2010) and Chang, Lin, and Yu (2011) for recent evidence). To accommodate these features, we extend the doubly-binomial setup of Chang, Chang, and Lu (2008), which is rooted in Gerber's doubly-binomial model (1984, 1988), to incorporate a third process — a mean-reverting Ornstein-Uhlenbeck arrival intensity process[8].

In this setup, the first binomial variable is to determine if a transaction will arrive in the next calendar-time-period (period hereafter) and the second to determine if the corresponding futures price jumps up or down. Subordination collapses the two binomial processes onto to the following trinomial futures price changes:

$$\begin{tikzcd}[row sep=small] & uF \\ F \arrow[ur, dash] \arrow[r, dash] \arrow[dr, dash] & F \\ & dF \end{tikzcd}$$

- uF: with probability $g_t h$ one transaction arrives and the futures price jumps up at a gross rate u,
- F: with probability $1 - g_t$ no transaction arrives and the futures price stays the same,
- dF: with probability $g_t(1 - h)$ one transaction arrives and the futures price jumps down at a gross rate d,

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[7] See Hoyt and McCullough, 1999, for empirical evidence and why this benefit of diversification is one major motivation for portfolio managers to invest in catastrophe products

[8] Chang, Chang, and Lu (2008) discussed how to extend their model to incorporate stochastic arrival intensity but did not implement it in their modeling, e.g. the futures price process specification, and there are errors in their discretization as will be discussed in footnote 9.

where F denotes the futures price at the beginning of the period, u and d denote the respective constant up and down gross jump sizes, g_t and $1-g_t$ denote the respective time-varying transaction arrival and no arrival probabilities, and h and $1-h$ denote the respective jump up and down probabilities upon an arrival. Unlike Chang, Chang, and Lu, our transaction arrival probability is time-varying while it is constant in their setup. Finally, we let R denote one plus the riskless rate over one period with the usual regularity condition that $u > R > d$ to prevent riskless arbitrage.

To specify g_t we assume j, the news arrival intensity, follows a mean-reverting Ornstein-Uhlenbeck process:

$$dj = \kappa_j(m_j - j)dt + \sigma_j dZ_j, \tag{1}$$

where κ_j denotes the speed of adjustment, m_j the long-run mean rate, σ_j^2 the instantaneous variance, and Z_j the standard Wiener process. The solution of Eq. (1) for the time-varying intensity is known to be

$$j(v) = m_j + \left(j(t) - m_j\right)e^{-\kappa_j(v-t)} + \sigma_j e^{-\kappa_j v}\int_t^v e^{\kappa_j s} dZ_j(s), \quad v \in \left[t, T\right], \tag{2}$$

where $j(t)$ denotes the current level of intensity. The expected intensity over a time period $T-t$ is determined via integration as

$$E\left(j(T-t)\right) = m_j \times (T-t) + \left[j(t) - m_j\right]H_j(T-t), \tag{3}$$

where $H_j(T-t) = \dfrac{1-e^{-\kappa_j(T-t)}}{\kappa_j}$.

Choosing n periods over an expected maturity of $T-t$ to implement discretization, we find the probability that news (transaction) will arrive in the next period to be

$$g_t = E_t\left(j\left(\frac{T-t}{n}\right)\right) = \left(m_j\left(\frac{T-t}{n}\right) + \left[j(t) - m_j\right]H_j\left(\frac{T-t}{n}\right)\right), \tag{4}$$

subject to the regularity condition that the parameters be appropriately chosen so that $g_t \leq 1$. It is determined by the long-run mean rate m_j, the deviation of the current level of intensity $j(t)$ from m_j, and that how this deviation persists.

To illustrate how this probability changes with the intensity parameters, we compare two scenarios as reported in Figure 1 below. With the long-run claim arrival intensity at 80 in an event quarter and the number of time steps at 30, we have $m_j = 80$ and $\Delta t = 0.0083$. We consider two scenarios: when the initial intensity is low at 60 and high at 100. In each scenario, we vary κ_j, the speed of adjustment toward the mean, from 2 to 30, and then compute the corresponding risk-neutral news-arrival probability for the next period. The results show that

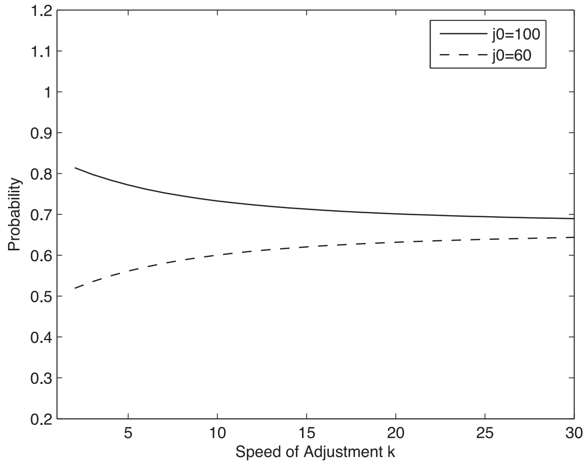


FIGURE 1: Examining the News-Arrival Probability in two Stochastic Arrival Intensity Scenarios.

We assume that the news arrival intensity follows a mean-reverting Ornstein-Uhlenbeck process and then we examine how the news-arrival probability in the next period, g_t, is affected by the deviation of the current level of arrival intensity from the long-run mean rate, $j(t) - m_j$, and the speed of adjustment, κ_j. With the long-run arrival intensity set at 80 and the number of time steps at 30 in an event quarter, we compute m_t as a function of κ_j ranging from 2 to 30 in two scenarios: when the initial intensity is low at 60 and high at 100.

1) in the low initial intensity case, as the speed of adjustment increases, clustering weakens and mean-reversion toward the higher mean strengthens, leading to increasing transaction-arrival probability, but 2) in the high initial intensity case, as the speed of adjustment increases, clustering weakens and mean-reversion toward the lower mean strengthens, leading to decreasing transaction-arrival probability.

Next the probability that news (transaction) will arrive in the second period is quickly checked as

$$
\begin{aligned}
g_{t+1} &= E_t\left(j\left(2\left(\frac{T-t}{n}\right)\right)\right) - E_t\left(j\left(\frac{T-t}{n}\right)\right) \\
&= \left(m_j \times 2\left(\frac{T-t}{n}\right) + \left[j(t) - m_j\right] H_j\left(2\left(\frac{T-t}{n}\right)\right)\right) \\
&\quad - \left(m_j \times 2\left(\frac{T-t}{n}\right) + \left[j(t) - m_j\right] H_j\left(\frac{T-t}{n}\right)\right),
\end{aligned} \tag{5}
$$

and in general, the probability that news (transaction) will arrive in the ith period where $i = 2, 3, \ldots$, can be calculated as $E_t\{j(i([T-t]/n))\} - E_t\{j([i-1]([T-t]/n))\}$[9].

[9] In their discussion to incorporate stochastic arrival intensity, Chang, Chang, and Lu (2008) did not properly discretize $E((j(T-t))$ (see their Eq. (25)) to obtain the time-varying news arrival probability as illustrated here in Eqs. (4) and (5).

2.2. Risk-Neutralization

As explained previously, since the arrival of CHI changes embeds no systematic risk, the news-arrival martingale probability over the next period denoted as m_t, should be equal to its physical counterpart g_t, as a function of the intensity parameters. It is thus attainable once the intensity parameter values are determined. Next to derive the price-change martingale probability by calibration, we apply the discrete-time no-arbitrage martingale pricing methodology. No-arbitrage dictates the following one-period martingale representation for the futures price:

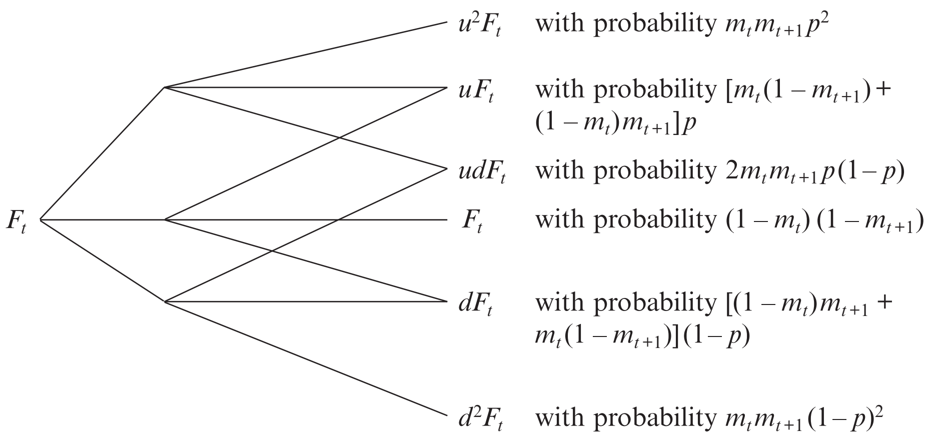
$$F = m_t puF + (1-m_t)F + m_t(1-p)dF, \tag{6}$$

where p and $1-p$ are the respective equivalent martingale probability measures for the asset price to move up and down; and m_t and $1-m_t$ are the respective equivalent martingale probability measures for news arrival and non-arrival.

Solving and simplifying Eq. (6), we obtain the price-change martingale probability as:

$$p = \frac{1-d}{u-d}, \tag{7}$$

where u (= $\exp(\sigma_1)$) and d (= $1/u$) are the gross up/down jump rate of the futures price upon news arrival. The risk-neutral trinomial tree illustrated using two periods is thus



where news arrives with probability m_t in the first period per Eq. (4) but with probability m_{t+1} in the second period per Eq. (5), and upon the arrival, futures price either jumps up to uF_t with probability p or jumps down to dF_t with probability $1-p$.

2.3. Transaction-Time Option Pricing and Identification of the Implied Forecasting Parameters

Next we implement a stochastic time change from calendar time to transaction time to reduce the computational complexity by restoring the stationary binomial parent process, and then price the option in transaction time (see Geman, 2005, for the benefits of time and measure changes). Since the number of transaction arrivals in an n period trinomial tree (where $n \times$ intervals equal T) may vary from a minimum of zero to a maximum of n, assuming n is chosen to be sufficiently large, the restored binomial tree has a random number of time step, k, where $k \in [0, n]$. For example over two calendar-time periods we would have the following three possible transaction-time maturities:

1) $k = 0$ with probability $(1 - m_t)(1 - m_{t+1})$, where F_t does not change because no news arrives over two time periods. This probability is denoted M_0,
2) $k = 1$ with one news arrival and probability $[(1 - m_t)m_{t+1} + m_t(1 - m_{t+1})]$, where

$$F_t < \begin{array}{ll} uF_t & \text{with probability } p, \\ dF_t & \text{with probability } 1 - p. \end{array}$$

This probability is denoted M_1,

3) $k = 2$ with two consecutive news arrivals and probability $M_2 = m_t m_{t+1}$, where

$$F_t < \begin{array}{lll} & & u^2F_t \quad \text{with probability } m_t m_{t+1} p^2 \\ uF_t & < & \\ & & udF_t \quad \text{with probability } 2m_t m_{t+1} p(1-p) \\ dF_t & < & \\ & & d^2F_t \quad \text{with probability } m_t m_{t+1}(1-p)^2. \end{array}$$

In other words, our task now is to price an isomorphic option with random maturity in transaction time. We solve this problem by using the Euler equation as a conditional expectation over the transaction uncertainty. More specifically, the normalized price of an n-period call option can be solved as a random sum of the arrival-probability-weighted normalized prices of $n + 1$ k-transaction-time fixed-maturity options (denoted as C_k):

(8) $$\frac{C(n)}{B_T} = \sum_{k=0}^{n} M_k \frac{C_k}{B_T}, \text{ which simplifies to}$$

(9) $$C(n) = \sum_{k=0}^{n} M_k C_k,$$

where B_T is the price of the matching bond, M_k is the transaction-arrival martingale probability measure of k transaction arrivals in n periods as illustrated above in the two-period case using Eqs. (4) and (5), and C_k is the transaction-time

American binomial futures call price with maturity k. M_k is the probability of k news arrival in whatever sequence over n periods. We will discuss further the determination of M_k and C_k in the next section.

By Eq. (5), it is seen that given the intensity arrival parameters $j(t)$ (the current intensity level), κ_j (the speed of intensity adjustment parameter), and m_j (the long-run mean intensity level), we can determine m_{t+i}, the time-varying probability of transaction arrival in the future periods. By Eq. (7), it is seen that σ_1 (the per transaction arrival futures volatility) determines u, the rate the futures price would jump up upon transaction arrival, which in turn determines p, the probability that the futures price would jump up upon transaction arrival. Therefore, Eq. (9) links the option value to $j(t)$, κ_j, m_j, and σ_1 and as either the arrival probability increases in which case the tree grows faster with more arrivals or the futures volatility increases in which case the futures price jumps higher, the option price would increase to reflect the larger expected total price volatility. Assuming the current intensity level $j(t)$ is observable for parsimonious reasons, then with at least three transactional option prices as the predictor, we can simultaneously back out the values of κ_j, m_j, and σ_1 by using Eq. (9). This set of parameters is sufficient to fully calibrate the futures price process and thus serves as the basis for implementing our market-consensus forward-looking forecasts. As will be illustrated in the next section that on this basis, we can forecast 1) the time-varying transaction arrival probability (m_{t+i}) and the probability of the number of transaction arrivals prior to a hurricane's landfall (M_k), 2) the static probability distribution of the CHI value on the landfall date, and 3) the dynamic evolution of the predicted CHI value over news arrival as a multi-period transaction-time binomial tree.

3. Simulations

We run simulations in two parts. In part 1, we illustrate how to implement our pricing model and calibrate the futures price process by backing out the values of κ_j, m_j, and σ_1 from transactional hurricane derivative prices. Then in part 2 we illustrate how to base our forecasts on this set of implied values.

3.1. Hurricane Futures Option Pricing

We consider a named storm with a traded CHI futures at value $F_t = 8$, and 90 days to expected landfall, or expected maturity at $T = 1/4$. Since Hurricane Katrina, a considerably destructive storm, made landfall with a CHI value of 19.0, while Hurricane Dennis, a mild to medium-sized storm, had a CHI value of 6.9, our named storm is a medium-sized one. Conditional on T, and supposing the number of transactional events that impact the CHI is not more than 30 per quarter, we set $n = 30$, *i.e.* each period is three calendar days. The random arrivals of transactions during $[t, T]$ imply that total number of transactions in this forward period is $k \in [0, 30]$. To implement Eq. (9), we employ N_k time-steps to compute the binomial transaction-time American futures call option prices C_k with maturity k where $k = 1, 2, \ldots, 30$ transactions. We choose $N_k = n$. N_k in principle should be as large as is computationally feasible, and as N_k increases, the binomial trees under transaction maturity should converge to their counterparts under lognormal diffusion. For computational tractability, we demonstrate the methods here using $N_k = n = 30$ for all k. The transaction-time volatility is fixed at $\sigma_1 = 0.2\sqrt{k/N}$ for low volatility and $\sigma_1 = 0.4\sqrt{k/N}$ for high volatility. The strike prices are set at $K = 6$ (in-the-money), $K = 8$ (at-the-money), and $K = 10$ (out-of-the-money). An annual risk-free rate of 2% is assumed. The price results for different sets of parameterizations regarding $j(t)$, κ_j, m_j, and σ_1 are shown in Table 1 below.

TABLE 1

HURRICANE FUTURES OPTION PRICES BASED ON EXPECTED MATURITY OF $T = 0.25$, CURRENT FUTURES PRICE OF $F_t = 8$, DISCRETIZATION SCHEME OF $N = n = 30$, AND DIFFERENT SETS OF PARAMETERIZATIONS SCHEMES REGARDING $j(t)$, κ_j, m_j, AND σ_1.

Prices in CHI value	$\sigma_1 = 0.2$			$\sigma_1 = 0.4$		
Different parameterizations	$K=6$	$K=8$	$K=10$	$K=6$	$K=8$	$K=10$
$m_j = 80,\ \kappa_j = 15,\ j(t) = 100$	3.23	2.53	2.06	4.99	4.58	4.34
$m_j = 80,\ \kappa_j = 30,\ j(t) = 100$	3.21	2.50	2.03	4.96	4.54	4.30
$m_j = 80,\ \kappa_j = 2,\ j(t) = 100$	3.31	2.64	2.19	5.10	4.72	4.51
$m_j = 80,\ \kappa_j = 15,\ j(t) = 60$	3.14	2.41	1.92	4.86	4.41	4.14
Constant intensity $m_j\Delta t = 0.6667$	3.19	2.47	1.99	4.93	4.50	4.25

From Table 1, it is seen that the American-styled hurricane futures option prices increase significantly with increase in transaction-time volatility σ_1, with moneyness, and with decrease in κ_j (since $j(t) - m_j = 20$ here indicates an adjustment downward toward the long-run mean). Comparing with the case of long-run constant intensity by setting $j(t) = m_j = 80$, it is seen that whenever $j(t) > m_j$, the American futures option prices will be higher than in the case of constant intensity.

3.2. Calibration

In Figure 2 below we illustrate a futures call price surface with two underlying parameters. We plot the hurricane futures option price as a function of κ_j taking the range 2 to 30, and of σ_1 under unit transaction time taking the range 0.1 to 0.9. As in before, current futures price is $F_0 = 8$, the strike price is set at $K = 8$, maturity is $T = 1/4$, the risk-free interest rate is assumed to be 2% p.a., and we set $m_j = 80$ and $j(t) = 100$. It is seen that in the case $j(t) > m_j$, the price surface increases in σ_1 and decreases slowly in κ_j.

In general with three implied parameters κ_j, m_j, and σ_1 as the independent variables by assuming $j(t)$ is observable, any given futures option price level

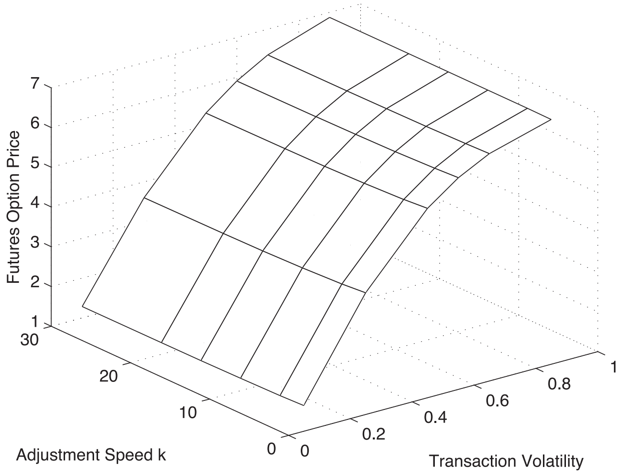


FIGURE 2: Futures Call Price Surface.

The price surface corresponds to $m_j = 80$, $j(t) = 100$, and varying levels of κ_j taking the range 2 to 30, and of σ_1 under unit transaction time taking the range 0.1 to 0.9. Current futures price is $F_0 = 8$, the strike price is $K = 8$, maturity is $T = 1/4$, and risk-free interest rate is 2% p.a.

forms a 3-dimensional surface in the $m_j - \sigma_1 - \kappa_j$ space. Two such surfaces from two derivatives would form an intersection of a curve at points equivalent to the observed market prices of the two derivatives. Three derivatives would be able to provide an intersection equivalent to a point in the $m_j - \sigma_1 - \kappa_j$ space, and hence providing the implied values of $\widehat{m}_j$, $\hat{\sigma}_1$, and $\hat{\kappa}_j$. Once the three parameters are implied at any trading time t before landfall, they can be used to calibrate the transaction-time futures price process to forecast 1) the time-varying transaction arrival probability (m_{t+i}) and the probability of the number of transaction arrivals prior to a hurricane's landfall (M_k), 2) the static probability distribution of the CHI value on the landfall date, and 3) the dynamic evolution of the predicted CHI value over news arrival as a multi-period transaction-time binomial tree.

3.3. Forecasting the Time-Varying Transaction Arrival Probability (m_{t+i}) and the Probability Distribution of the Number of Transaction Arrivals Prior to Landfall (M_k)

We shall first compute, within the same setting, m_{t+I}, the probability of transaction arrivals in the future period $[t + i\Delta t,\ t + (i + 1)\Delta t]$ for $i = 0, 1, 2, \ldots, 29$, using Eq. (5) with values $\kappa_j = 2$, 15, 30, and $m_j = 80$. We also include the case of $\kappa_j = 15$ and $j(t) = 60$ for comparison as the latter is a case of upward adjustment instead. This time-varying transaction-arrival probability forecast under the different scenarios is shown in Figure 3 where each period is 3 days. In practice, the number of intervals may be increased in this discrete framework to improve on the estimates. The limitation of finite discretized intervals

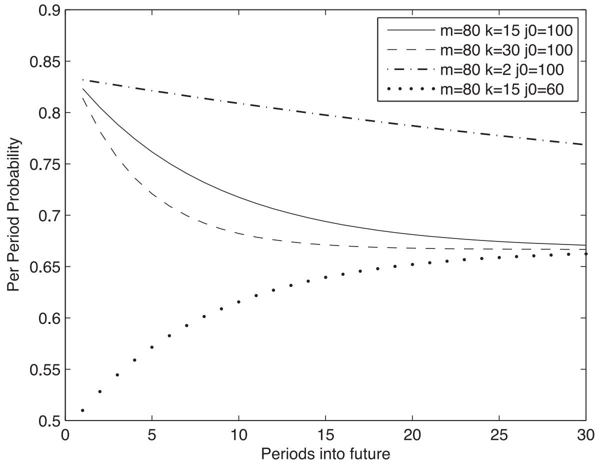


FIGURE 3: News (Transaction) Arrival Probability Forecast.

We assume that the transaction arrival intensity follows a mean-reverting Ornstein-Uhlenbeck process. $T = 1/4$, $n = 30$, $m_j = 80$. Different values of k and initial intensity j_0 are used to forecast the transaction arrival probability m_{t+i} at future period $[t + i\Delta t, t+(i+1)\Delta t]$ for $i = 0, 1, 2, \ldots, 29$. Each period is 3 days.

is that it imposes an arbitrary assumption that the transactions arrive either once or none during these regularly spaced intervals.

Figure 3 shows that m_{t+i} reduces over time if $j(t) > m_j$, but increases over time if $j(t) < m$. The rate of increase or decrease is higher or lower depending directly on the value of κ_j. The sequence of values $\{m_t, m_{t+1}, m_{t+2}, \ldots\}$ for a

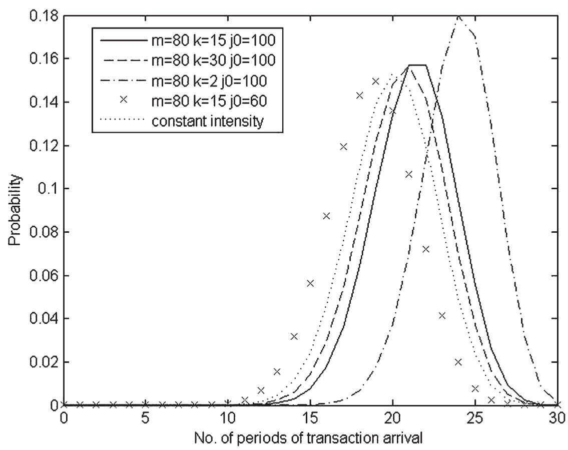


FIGURE 4: Probability of Number of Transaction Arrivals.

We assume that the transaction arrival intensity follows a mean-reverting Ornstein-Uhlenbeck process. $T = 1/4$, $n = 30$, $m_j = 80$. Different values of k and initial intensity j_0 are used to forecast the transaction arrival probability m_{t+i} at future period $[t + i\Delta t, t + (i+1)\Delta t]$ for $i = 0, 1, 2, \ldots, 29$. The sequence of values $\{m_t, m_{t+1}, m_{t+2}, \ldots\}$ for a particular parameterization $\{\kappa_j, m_j\}$ is then employed to find the probability of number of transactions M_k shown below. The case of constant intensity m is derived using $m_j\Delta t = 80 \times 0.0083 = 0.6667$ as per period probability.

particular parameterization $\{\kappa_j, m_j\}$ is then employed to find the probability of number of transactions M_k shown in Figure 4.

This probability is different from existing models that assume constant intensity in that we accommodate a stochastic intensity specification as in (1). We vary κ_j to examine the probability distribution M_k and find that for a given m_j and $j(t) > m_j$, as κ_j increases, the mode of the distribution tends to decrease and likewise its probability. This is because under downward adjustment since $j(t) > m_j$, increasing κ_j implies reduction in future probability m_{t+i} of transaction arrival, and hence lower probabilities for total number of arrivals. The figure also shows that for $j(t) < m_j$, the lower intensities typically produces a probability distribution that is lower in number of arrivals and its attendant probabilities. A comparison with a constant intensity specification $m_j \Delta t = 80 \times 0.0083 = 0.6667$ as in the dotted curve shows that for an upward adjustment $j(t) > m_j$, the probability distribution dominates that from using an averaged constant intensity. The situation is converse for the case of downward adjustment where $j(t) < m_j$.

3.3. Static Forecast of the Final CHI value on the Landfall Date

Next we forecast the probability distribution of the CHI values at the expected landfall or maturity time. This is done using the random variable $\tilde{F}_T = u_k^i d_k^{N-i} F_t$ for different $k = 1$ to n, and for each k, $i = 1$ to N, where as seen earlier, $u_k = e^{\sigma_1 \sqrt{k/N}}$ and $d_k = \frac{1}{u_k}$. We employ all the binomial trees for each k to construct the implied risk-neutral distribution of $\tilde{F}_T$. For each k, we have N nodal values of F_T at T, and thus N probability values. Conditional on k, these probability values sum to one. Since the probability of observing k transactions in T is M_k, we have the unconditional probability of nodal value $\tilde{F}_T = u_k^i d_k^{N-i} F_t$ as

$$\text{(10)} \qquad \Pr\left(\tilde{F}_T\right) = M_k \times P_k(i) = M_k \times \frac{N!}{i!(N-i)!} \, p_k^i (1 - p_k)^{N-i}.$$

Next suppose three traded futures options with strikes at $K = 6$, $K = 8$, and $K = 10$, are priced at 2.73, 1.92, and 1.38 in terms of CHI units respectively in the market. Using the above theoretical model, we can imply out the parameters $\widehat{m}_j$, $\widehat{\sigma}_1$, and $\widehat{\kappa}_j$ to be 80, 0.15, and 20 respectively. Then the risk-neutral distribution of CHI values at expected landfall is shown in Figure 5. This implied probability distribution provides a forecast of how destructive it will be when it makes landfall. As the implied probability distribution is tracked over time, it also provides information on how its expected destructive power will behave over time from inception to landfall. Figure 5 shows that the mean and also mode of the distribution is 8, the current future value, with a probability of about 30%. The distribution is skewed to the right.

The distribution also provides a way of measuring the risk or probability of hurricane devastation when the CHI value is expected to exceed certain thresholds. Hurricane Katrina for example made landfall with a CHI value of

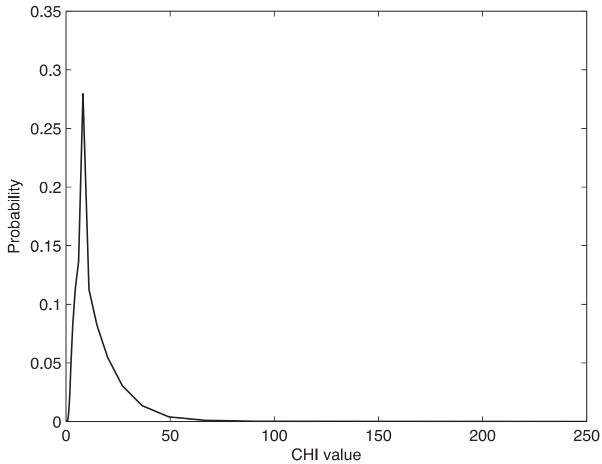


FIGURE 5: Discrete Risk-Neutral Probability of CHI at Maturity.

We suppose 3 traded futures options with strikes at $K = 6$, $K = 8$, and $K = 10$, are priced at 2.73, 1.92, and 1.38 in terms of CHI units respectively in the market. Current futures price is $F_0 = 8$, maturity is $T = 1/4$, and $j(t) = 100$. Risk-free interest rate is 2% p.a. Using these prices, we employ our theoretical model to imply out the parameters $\hat{m}_j$, $\hat{\sigma}_1$, and $\hat{\kappa}_j$ as 80, 0.15, and 20 respectively. These values are used to find the probability of occurrences of number of transactions over T based on the mean-reverting Ornstein-Uhlenbeck process. The unconditional risk-neutral distribution of CHI values at expected landfall can be obtained via the binomial trees. The histogram is smoothed as follows.

19.0, a considerably destructive storm. In contrast, Florida's Hurricane Dennis had only a CHI value of 6.9, a mild to medium-sized storm. From the distribution, we can infer that the probability of exceeding CHI value of 20 is about 4.95% or close to 5%. Hence there is a 5% chance of a serious hurricane hit within 90 days in this example.

3.4. Dynamic Forecast of the Evolution of the CHI value over News Arrival

Finally, using implied parameters $m = 80$, $k = 20$, $\sigma_1 = 0.15$; $u = 1.1618$, $d = 0.8607$, $p = 0.4625$, and assuming an expected number of transactional arrival of 30 over 90 days, in Table 2 below, we implement a dynamic market-consensus forward-looking forecasting model as to show how the expected destructive power of a hurricane would evolve from news arrival to news arrival as a multi-period transaction-time binomial tree, As shown in the Table, the value in the cell of each node of the binomial tree denotes the expected destructive power in that period with an initial CHI value of 8.00. This initial power would evolve in the following fashion: there is a probability of 0.82 that news will arrive in the next time period and upon news arrival there is a probability of 0.4625 that the CHI value will jump up to 9.29 but a probability of 0.5375 that it will jump down to 6.89. After 30 news arrivals, the final CHI value will range from a low of 0.09 with probability of 0.15529 to a high of 720.14 with probability of 0.08982. These probability values are computed as

TABLE 2

The market-consensus forward-looking forecast as to how the expected destructive power of a hurricane would evolve from news arrival to news arrival is displaced as a multi-period transaction-time binomial tree using implied parameters $m = 80$, $k = 20$, $\sigma = 0.15$; $u = 1.1618$, $d = 0.8607$, $p = 0.4625$. Parameters are conditioned on 30 transaction arrivals over 90 days, and each period in the binomial tree denotes 3 days. The term "prob" denotes the probability news will arrive in the next 3 days, and the term "period" labels the transaction count with a total of 30 expected transaction arrivals in 90 days. The initial CHI value is 8.00 but after 30 arrivals the value will range from a low of 0.09 with probability of 0.15529 to a high of 720.14 with probability of 0.08982.

																75.9
															65.3	56.2
														56.2	48.4	41.7
													48.4	41.7	35.9	30.9
												41.7	35.9	30.9	26.6	22.9
											35.9	30.9	26.6	22.9	19.7	16.9
										30.9	26.6	22.9	19.7	16.9	14.6	12.6
									26.6	22.9	19.7	16.9	14.6	12.6	10.8	9.29
								22.9	19.7	16.9	14.6	12.6	10.8	9.29	8	6.89
							19.7	16.9	14.6	12.6	10.8	9.29	8	6.89	5.93	5.1
						16.9	14.6	12.6	10.8	9.29	8	6.89	5.93	5.1	4.39	3.78
					14.6	12.6	10.8	9.29	8	6.89	5.93	5.1	4.39	3.78	3.25	2.8
				12.6	10.8	9.29	8	6.89	5.93	5.1	4.39	3.78	3.25	2.8	2.41	2.07
			10.8	9.29	8	6.89	5.93	5.1	4.39	3.78	3.25	2.8	2.41	2.07	1.79	1.54
		9.29	8	6.89	5.93	5.1	4.39	3.78	3.25	2.8	2.41	2.07	1.79	1.54	1.32	1.14
	8	6.89	5.93	5.1	4.39	3.78	3.25	2.8	2.41	2.07	1.79	1.54	1.32	1.14	0.98	0.84
prob	0.82	0.8	0.78	0.76	0.75	0.73	0.72	0.71	0.71	0.7	0.7	0.69	0.69	0.68	0.68	0.68
period	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

														720
													620	533
												533	459	395
											459	395	340	293
										395	340	293	252	217
									340	293	252	217	187	161
								293	252	217	187	161	138	119
							252	217	187	161	138	119	102	88.2
						217	187	161	138	119	102	88.2	75.9	65.3
					187	161	138	119	102	88.2	75.9	65.3	56.2	48.4
				161	138	119	102	88.2	75.9	65.3	56.2	48.4	41.7	35.9
			138	119	102	88.2	75.9	65.3	56.2	48.4	41.7	35.9	30.9	26.6
		119	102	88.2	75.9	65.3	56.2	48.4	41.7	35.9	30.9	26.6	22.9	19.7
	102	88.2	75.9	65.3	56.2	48.4	41.7	35.9	30.9	26.6	22.9	19.7	16.9	14.6
88.2	75.9	65.3	56.2	48.4	41.7	35.9	30.9	26.6	22.9	19.7	16.9	14.6	12.6	10.8
65.3	56.2	48.4	41.7	35.9	30.9	26.6	22.9	19.7	16.9	14.6	12.6	10.8	9.29	8
48.4	41.7	35.9	30.9	26.6	22.9	19.7	16.9	14.6	12.6	10.8	9.29	8	6.89	5.93
35.9	30.9	26.6	22.9	19.7	16.9	14.6	12.6	10.8	9.29	8	6.89	5.93	5.1	4.39
26.6	22.9	19.7	16.9	14.6	12.6	10.8	9.29	8	6.89	5.93	5.1	4.39	3.78	3.25
19.7	16.9	14.6	12.6	10.8	9.29	8	6.89	5.93	5.1	4.39	3.78	3.25	2.8	2.41
14.6	12.6	10.8	9.29	8	6.89	5.93	5.1	4.39	3.78	3.25	2.8	2.41	2.07	1.79
10.8	9.29	8	6.89	5.93	5.1	4.39	3.78	3.25	2.8	2.41	2.07	1.79	1.54	1.32
8	6.89	5.93	5.1	4.39	3.78	3.25	2.8	2.41	2.07	1.79	1.54	1.32	1.14	0.98
5.93	5.1	4.39	3.78	3.25	2.8	2.41	2.07	1.79	1.54	1.32	1.14	0.98	0.84	0.73
4.39	3.78	3.25	2.8	2.41	2.07	1.79	1.54	1.32	1.14	0.98	0.84	0.73	0.62	0.54
3.25	2.8	2.41	2.07	1.79	1.54	1.32	1.14	0.98	0.84	0.73	0.62	0.54	0.46	0.4
2.41	2.07	1.79	1.54	1.32	1.14	0.98	0.84	0.73	0.62	0.54	0.46	0.4	0.34	0.3
1.79	1.54	1.32	1.14	0.98	0.84	0.73	0.62	0.54	0.46	0.4	0.34	0.3	0.25	0.22
1.32	1.14	0.98	0.84	0.73	0.62	0.54	0.46	0.4	0.34	0.3	0.25	0.22	0.19	0.16
0.98	0.84	0.73	0.62	0.54	0.46	0.4	0.34	0.3	0.25	0.22	0.19	0.16	0.14	0.12
0.73	0.62	0.54	0.46	0.4	0.34	0.3	0.25	0.22	0.19	0.16	0.14	0.12	0.1	0.09
0.68	0.68	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30

shown before using $\Pr\left(\tilde{F}_T\right) = M_k \times P_k(i) = M_k \times \frac{N!}{i!(N-i)!}\, p_k^i (1-p_k)^{N-i}$. The probability values at the bottom rows denoted as "prob" represents the probability news will arrive in the next 3 days, and the term "period" labels the transaction count with a total of 30 expected transaction arrivals in 90 days.

4. A Practical Example – Hurricane Irene

The recent tropical cyclone activity in the Atlantic during 2011 was heavy, chalking up 19 storms including 7 hurricanes. A major event was Hurricane Irene which was actively tracked by the National Hurricane Center (NHC) during the period 20th to 28th August when it made landfall on eastern U.S. first at Cape Look, North Carolina, on 27th August about 7:30 am, and then at Little Egg inlet, New Jersey, the next morning. As the hurricane increased wind speed to 120 mph on 24th August, it was upgraded to a Category 3 hurricane on the Saffir-Simpson Hurricane Scale through August 25th. Towards the close of market trading day on August 25th, there was widespread fear that Irene would turn out to be deadly with extensive to extreme damages as reported by most media. At that time then it would be comparable to Katrina, Rita, or Wilma in the past with landfall CHI values of above 10 or even close to 20 and above. This assessment was borne out by the NHC final advisory report as follows: "However, official intensity errors for Irene were higher than the mean official errors for the previous 5-yr period at all times. This was the result of a consistent high bias during the U.S. watch/warning period. The main reason for the high bias in the official forecast was that Irene was anticipated to maintain category 3 intensity through landfall in North Carolina, given that the hurricane was forecast to remain in an environment of relatively light wind shear while moving over a warm ocean. However, Irene surprisingly did not maintain or increase its strength while moving between the Bahamas and North Carolina. Rather, it weakened to a category 1 hurricane (two categories below what was originally anticipated) by the time it made landfall near Cape Lookout… It is important to note that NHC does not have reliable tools to anticipate these structural changes. Developing improved intensity forecast guidance is a top priority of NOAA Hurricane Forecast Improvement Project now in its early stages".

The U.S. National Oceanic and Atmospheric Administration (NOAA) in conjunction with NHC make hurricane forecasts based on a large range of climatology equipment and meteorological methods including satellite images of gathering cyclones, airborne, sea, as well as land-based surveillance. As cyclones become hurricanes of critical wind speed with landfall imminence, regular surveillance, analyses, and public announcements are made up to within the hour or more usually, reporting at 3-hourly intervals. Surveillance measurements include aircraft-based microwave remote sensing for Doppler radar images to establish near-range estimates of cyclonic intensity and movements of the cyclonic air and vapor masses, air pressure gauges and wind speeds at different heights, as well as visual pictures of the cyclonic movements. Storm surges and rainfalls in nearby areas are also monitored to lend collaborating evidence to the scale of the intensities and path directions of the evolving hurricane. Besides the key measurements of maximum wind velocities and sea level pressure dips drawing in the cyclonic air masses, the various other measurements are used to analyze physical characteristics including wind-shear ratios, dynamics of the cyclonic circular eye-walls, and thus yield invaluable physical information on likely changes in the intensities and momentum possibilities of the evolving Hurricane.

As mentioned in our introduction, the CHI value is an alternative measure besides the NHC Saffir-Simpson Hurricane scale to enable consideration of both maximum wind speed as well as the impact area size of the hurricane, and thus

$$CHI = \left(\frac{V}{V_0}\right)^3 + \frac{3}{2}\left(\frac{R}{R_0}\right)\left(\frac{V}{V_0}\right)^2,$$

where R is the radius of the extent of hurricane force winds from the center of the hurricane, V is the maximum sustained wind speed, $R_0 = 60$ miles, and $V_0 = 74$ miles per hour. CME and Eurex started using this CHI index as the underlying for trading hurricane futures options about 2007. Besides these exchanges, many large financial trading firms with a specialization in weather derivatives also provide brokerage as well as market-making for speculative and hedging trades on hurricane futures options. Their clients include insurance and re-insurance firms that may buy the options to hedge against large payouts when hurricanes cause extensive damages to businesses and properties. The way the futures option works is that at landfall, the higher the CHI value or the more destructive the hurricane's wind speed and its area of devastation, the more a hurricane futures call option would pay out to the buyer. On the other hand, hedge funds speculating a smaller than expected hurricane would sell such options.

Although there was no reported open interest for hurricane options on CME during the end August 2011 period, a couple of examples of indicative prices from brokerages include for example, 0.5 for a futures call of strike or trigger at first landfall of 10.0, and 4.5 for a call with trigger at 4.0. Call option prices are stated in units of CHI values just as settlement profit is $\max(F_T - K, 0)$ where F_T is futures price in CHI value at settlement date within several days of landfall. The actual dollar value per contract is typically \$1,000 times the index value. Such prices are also reflected during trading sessions on CME Globex for example, if that is where the brokerage clears the trade. The indicative price examples were consistent with the market expectation at about August 25th afternoon for a 35% probability of a massive landfall CHI value such as 10.0 and above. The report was sourced at Business Insider, a trade journal with reporting on weather derivatives. On August 25th afternoon, the NHC had reported hurricane maximum speeds of 115 mph and a radius of about 80 miles, yielding a computed CHI value of 8.6. This is the underlying price at the market forecast time on August 25th late afternoon. The risk-free rate is taken at 1% p.a. Market forecast is conditioned on the NHC report then which indicated estimated landfall within a couple of days. Thus, for market-based valuation purpose and for implying out the market model parameters for market-based forecast, the horizon as at August 25th was kept short at about 2 days or about 17 sub-periods of 3-hour intervals coinciding with the updates by NHC every 3 hours at that time. During the updates, transactions events in the form of changes or news in the hurricane movements vide different wind speeds or different radii may or may not occur. The frequencies of such transaction event occurrences were, however, public information, and their historical data up to the point of forecast could be utilized by the market for calibrating the model parameters as well as the utilization of market prices of the options.

It is perhaps instructive to re-emphasize the advantage of the market-based approach relative to the mechanical meteorological approach for hurricane impact forecasting. There is really no contradiction in the market-based approach with the meteorological approach as the whole idea of using market information in the market-based approach is that it incorporates all information to-date including publicly available meteorological reporting by NHC and NOAA. Whereas the NHC and NOAA analyses and forecasts are purely based on mechanically measuring and assessing the physical hurricane system at that point in time per se, other probable and relevant *additional* information may become available to a trader. Possible additional information could include non-NOAA news and weather related observations by just about any feasible sources including eyewitness accounts of the brewing storms, pattern recognitions of past storms that may or may not relate to the current hurricane, and meteorological analytical methods that are superior but are not yet used by NOAA and NHC. It has become a well-known fact that large financial trading houses on weather derivatives these days typically have meteorologists on their payroll to provide analyses and advice to the weather derivative traders of the firms. The key upshot of these is that market firms and traders put their money where their mouth is — it is plausible, rather than just betting instincts, that a particular market-based trade and price indeed had incorporated some inside information over and above the publicly available NHC information, and that the market price would therefore yield more information than the publicly available NHC broadcasts and reports. This is the critical argument for a market-based model approach in hurricane forecasting. Another way of viewing it is that it is less likely for the market to be wrong than for NHC to be wrong about a particular issued forecast.

Using the option data above, we employ our analytical method described in section 2 for market-based forward-looking forecasting. In particular, Eq. (9) links each option value to $j(t)$, k_j, m_j, and σ_1. As in the simulation illustration in section 3, we suppose at forecasting date, historical information including initial intensity at start date $j(0)$ can be estimated from the reporting history by NHC. Instead of a third option price, the average intensity over past histories of similar cyclones may be used as a proxy for the long-run m_j. For Hurricane Irene on August 25th afternoon about 5pm, $j(0)$ and $\widehat{m}_j$ are estimated at 3.60 and 3.33 respectively. For example, $\widehat{m}_j = 3.33$ represents a historical long-term average of about 10 news or transactions arrivals for every 3 days. Our calibration here is on a per day basis, rather than per annum basis, and may be more convenient here due to the short-horizon forecasting at imminence of landfall. The other two prices allow implied values for $\hat{\kappa}_j$ and $\hat{\sigma}_1$ using Eq. (9). Solving two price equations in two unknowns gives $\hat{\kappa}_j = 28.5$ and $\hat{\sigma}_1 = 0.1078$ or 10.78% per 3 hour period volatility of CHI rate of change when there is arrival of transaction or news.

From the above set of implied parameters $\widehat{m}_j$, $\hat{\sigma}_1$, and $\hat{\kappa}_j$ and estimated $j(0)$, the sequence of values $\{m_t, m_{t+1}, m_{t+2}, \ldots\}$ are computed from Eq. (3), (4), and (5) based on the mean-reverting Ornstein-Uhlenbeck intensity process. The sequence is then used to find the probability of number of transactions M_k as we had illustrated in the previous section and had shown an example of such a probability distribution in Figure 4. The dynamic evolution of the predicted CHI value over news arrival as a multi-period transaction-time binomial tree can then be derived based on the similar Eq. (10) shown in subsections 3.3 and 3.4 for a horizon at T. Specifically, Eq. (10) solves for the probability of a forward value of CHI at time horizon T. The forward value is the i^{th} node from the top of the binomial tree based on N_k nodes at T (using the notation in subsection 3.3). If we use the same number of N periods to partition a binomial tree regardless of k number of transaction or news arrivals, then all $N_k = N$. We adhere to the latter so all lattice implementations have fixed lattice structures regardless of k. This does not pose any theoretical or numerical problem as long as N is sufficiently large. Then conditioned on k transaction arrivals, the horizon T probability distribution of CHI values was shown in Figure 5 and the binomial tree structure with the forward evolution of the CHI values and their attendant probabilities were shown in Table 2. In those cases we use $k = 30$ or a fully active transaction arrival rate as illustration.

For the forward CHI value forecast of Hurricane Irene on August 25th late afternoon, we also employ a 30-period binomial lattice partition. However, we consider all possibilities of arrivals or the complete set of values of k. We provide two forecasts for the real-case demonstration here. The first is for horizon just beyond a day or $T = 1$ day. This is equivalent to number of transaction periods or maximum number of transaction arrivals $n = 9$. In other words, over a 1-day ahead horizon, there would at most be 9 transaction arrivals since NHC released news only at about 3-hour intervals (we ignore a couple of patched broadcasts at odd hourly gaps). The second is for horizon just beyond 2 days or $T = 2$. This is assumed to be the maximal horizon when landfall would have occurred if at all. In this case, maximum $n = 17$. In the first case, for $k = 0, 1, 2,$ and so on to 9, we compute the complete set of paired values $\{\tilde{F}_T, \Pr(\tilde{F}_T)\}$ for every k. As in Eq. (10), $\Pr(\tilde{F}_T) = M_k \times P_k(i) = M_k \times \frac{N!}{i!(N-i)!} p_k^i (1-p_k)^{N-i}$. for a particular k, and also $\tilde{F}_T = u_k^i d_k^{N-i} F_0$ for similar values of k, where $F_0 = 8.6$ is the CHI value on August 25th about 5 pm. The complete set of values form

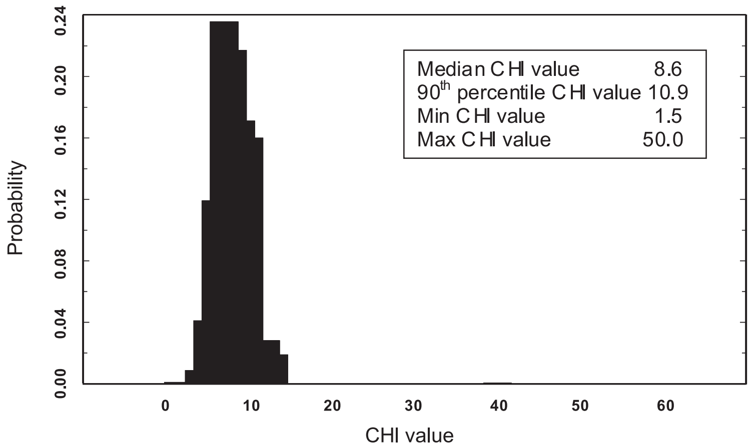


FIGURE 6A: Histogram of 1-day Ahead Forward Discrete Risk-Neutral Probability.

The histogram shows the forward risk-neutral probabilities of CHI values of Hurricane Irene 1 day ahead of August 25th 2011 if it hit landfall, computed using the market-based model and implied parameters $\widehat{m}_j$, $\hat{\sigma}_1$, and $\hat{\kappa}_j$.

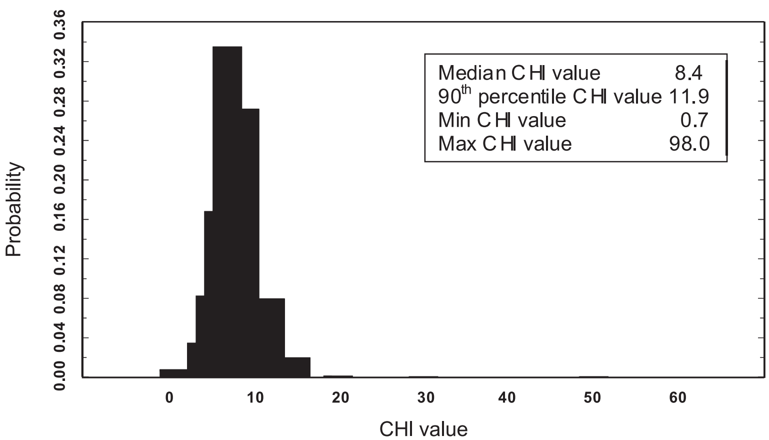


FIGURE 6B: Histogram of 2-day Ahead Forward Discrete Risk-Neutral Probability.

The histogram shows the forward risk-neutral probabilities of CHI values of Hurricane Irene 2 days ahead of August 25th 2011 if it hit landfall, computed using the market-based model and implied parameters $\widehat{m}_j$, $\hat{\sigma}_1$, and $\hat{\kappa}_j$.

the unconditional market-based forward-looking probability distribution at horizon T of CHI values subject to the intensity process Eq. (1), volatility process conditional on transaction arrival – Eq. (6), and market risk-neutral option valuation Eq. (9) and (10). This distribution based on the estimated market parameters: $j(0) = 3.60$, $\widehat{m}_j = 3.33$, $\hat{\kappa}_j = 28.5$, and $\hat{\sigma}_1 = 0.1078$, is computed and is shown as a histogram in Figure 6a.

The 1-day ahead forward discrete risk-neutral probability forecast in Figure 6a shows that by about August 26th early evening, Hurricane Irene's median CHI value was expected to be 8.6 with a 10% chance that the hurricane would increase in ferocity to over 10.9 in CHI value in Category 3. Based on similar information available as on August 25th afternoon, the 2-day ahead forward discrete risk-neutral probability forecast in Figure 6b shows that Hurricane Irene's median CHI value would decrease a bit to 8.4 with a 10% chance that the hurricane would increase in ferocity to over 11.9 in CHI value in Category 3. The maximum-minimum spread of possible (though with trivially small probabilities of about 1×10^{-16}) CHI values widened, as expected. This is due to increasing uncertainty on the fringes as in a longer horizon. Overall, however, Figures 6a and 6b indicate that past one day, the mean-reverting intensity could bring about some central tendencies for the hurricane CHI values (hence also physical wind speeds and radii of damage) to cluster toward a lower median. The means are about the same as the medians in both cases.

Using the market-based approach, market option prices are supposed to contain and reflect as a large subset all NHC information that were publicly available. The output forecasts shown in Figures 6a and 6b showed that over 1 to 2 day horizons till expected landfall, the market expected the hurricane intensity to remain about the same at 8.6 after 24 hours and to drop a little to 8.4 within 48 hours. This is not significantly different from the HNC reports up to August 25th afternoon that anticipated Hurricane Irene to maintain in Category 3 intensity through landfall in North Carolina. However, our market-based approach does evidence a slight ability to recognize some slow reversion toward a lower intensity. This was indeed the case with Hurricane Irene after August 25th. By August 26th evening, it was clearly at a less threatening Category 2 level, and by actual landfall in August 27th morning, it was only at Category 1 with a first landfall CHI value of 5.1. The second landfall CHI value another day later was 4.2. As it turned out 2 days after August 25th, Hurricane Irene brought much wind and rain that caused widespread power outages along the coast north of NC. Most of the $7 billion or more losses were due to the huge wind-swept floods that caused extensive property and business damages to Maryland, Pennsylvania, New Jersey, New York City, and Long Island areas. Hurricane Irene, though much publicized and feared during the two weeks of end August, did not become as damaging as Katrina, Rita, or Wilma.

5. Concluding Remarks and Futures Research directions

By using transactional price changes of traded hurricane derivative contracts as the predictor, we have developed a novel dynamic aggregate market-consensus forward-looking hurricane forecasting model and demonstrated its application using simulation. Our model forecasts how news regarding a hurricane will arrive, how will the expected destructive power of a hurricane changes upon news arrival, and how this power will evolve over news arrival from inception to landing. Since prevailing meteorological dynamic simulations and statistical models neither forecast well hurricane intensity nor produce clear-cut consensus, our novel market-consensus forward-looking forecast could provide a functional alternative. While the NHC blends 50 plus individual forecasting results for its consensus model forecasts using a subjective approach, our aggregate is market-based. A real case analysis of Hurricane Irene in 2011 using our methodology vis-à-vis the NHC approach highlighted our methodology's efficacy. Believing their proprietary forecasts are sufficiently different from our market-based forecasts, traders could also examine the discrepancy for a potential trading opportunity using hurricane derivatives.

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Carolyn W. Chang
Department of Finance
California State University
Fullerton

Jack S.K. Chang (Corresponding author)
Department of Finance & Law
California State University
Los Angeles
19150 Hastings Street
Rowland Heights, CA 91748
jskchang@roadrunner.com

Kian Guan Lim
Singapore Management University
Singapore