Extreme Model of Economic crisis, risks and control interactions countries with different economic development level

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Abstract

For constructed Extreme Economics model and its parameters reducing to the algebraic representations $N_m^p = \sum_{i=1}^{m} a_{im}^n$ and corresponding with it tree of numbers is proposed and investigated Polynomial Model in form of curves high degrees

$$N^P = X^{(m-1)} + \sum_{i=2}^{m} A_i X^{(m-i)},$$

$$X = x^n, Y = y^n, Z = z^p, A_i = YZ^{i-2},$$

example's which may be considering model economics in the form of

$$\frac{dK}{dt} = £Af(K,L),\ 0 < t \le t_k,\ K(0) = K_0, \frac{dL}{dt} = ¢L, L(0) = L_0,$$

$$\frac{dP}{d²} = -(1 - £ - MPC)^{-1}\frac{u}{y}\cdot P + \frac{\overline{v}}{y}, P(0) = P_0,$$

$$¢ : \int_0^{a_{max}} B(a)e^{-¢a}da = 1,\ ¢ \in (-\infty,\infty), Y = Af(K,L), C = (1-£)Y, \frac{dA}{dt} = a_0A - a_1A^2, A(0) = A_0,$$

and on base it introducing necessary and sufficient criteria's of arising economic crisis problem $Y^{²}(K, L^*) \le Y^{²}(K^*, L^*) \le Y^{²}(K^*, L)$,

$$\frac{1}{2}\int_0^{²} K(t)\,dt \le K^*, \frac{1}{2}\int_0^{²} L(t)\,dt \ge L^*, Y^{²} = \frac{1}{2}\int_0^{²} A(t)\, f(K,L)\,dt,$$

where K=K (t) is size of the capital in time t, L=L (t) is size of a labor determined as $L(t) = \int_0^{a_{max}}\int_G ¨ (x,a,t)N(x,a,t)dx\,da,$ £ is a share of the national income going to capital investments, C is size of consumption, A=A (t) is a technological level, P=P(t) is a price function , B=B(a) is function of stability of

a labor determined as $B(a)=B_0(a)e^{-\int_0^a f_0(\P)d\P}$, $B_0(a)$ is function of birth rate of manpower labor, F_0 (a) is function of death rate, N=N (x, a, t) is a solution of population problem with regard to time-age – spaces distributions.

Key words: Polynomial model, capital, labor, function, model economics, economic crisis, national income, catastrophe, risk, model, tree, optimal tree, analyses, numbers tree.

§1. Representation of a complex object by Polynomial model in extreme regime

Theorem. *Let any object (or process) operates so that the next time maximizes his condition, then its state is described by a polynomial.*

Proof. Let the function $u = u(x,t), \quad x \in E_m, t \geq 0$, $x = (x_1, x_2, ..., x_m)$, $x \in G$, $G \subseteq E^m$ characterizes the state of an object (or process, or a substance) in a point x in time t and $L, L_j \ j=\overline{1,m}$ some of the operators realizing the change of state of the object (or process) in general, and direction x_i. Then $L_j u$ is the state of a direction x_j, $L_j u$ a change of state of an object in general. It is natural to assume that the sum is formed, where, $0 < \sim_j < 1$ and $\sum_{j=1}^{m} \sim_j^{\frac{n}{n-s}} = 1$, $n > s$, $s > 0$. We state the following principle, which has important practical interpretation: *Any system (or object) operates so that its status as a whole was extreme (i.e. the best in some sense) in the future.*

Based on this principle, we get:

$$Lu = \max_{\sim \in M} \left(\sum_{j=1}^{m} \sim_j (L_j u)^s \right)^{1/s}, \quad s > 0, \tag{1}$$

where $M = \left\{ \sim = (\sim_1, ..., \sim_m) \colon 0 < \sim_j < 1, \ \sum_{j=1}^{m} \sim_j^{\frac{n}{n-s}} = 1 \right\}$, $n > s > 0$ given numbers.

Lemma 1. Equation (1) describes the best performance of the system (or object) and it equivalent to equations of type

$$(Lu)^p = \sum_{j=1}^{m} \left(L_j u \right)^n \quad или \quad N_m^p = \sum_{j=1}^{m} a_j^n$$

or $Y_k^{\,p} = \sum_{i=1}^{k} Y^n{}_{ik}$ (2)

moreover, we have $Lu = \tilde{\ }^{0}_{j}{}^{-\frac{1}{n-s}} L_j u,\ j = 1, 2, ..., m,$

$$\tilde{\ }^{0}_{j} = \left(\frac{(L_j u)^n}{\sum_{j=1}^{m} (L_j u)^n} \right)^{\frac{n-s}{n}}$$

$$\left(Lu = c,\ L_j u = c_j\ ,\ Lu = c,\ L_j u = c_j\ ,\ \sum_{j=1}^{m} c_j^{\,n} = c^n \right).$$

The Proof: *The necessity***.** We shall introduce a designation $Z = Y_k$, $X_j = Y_{jk}$, $j = \overline{1,k}$. Let the condition (2) takes place then *(p=n)*

$$Z^n = \sum_{j=1}^{m} X_j^{\,n} \qquad (3)$$

Let's show, validity (1) i.e.

$$Z = \max_{\mu \in M} \left(\sum_{j=1}^{k} \mu_j X_j^{\,s} \right)^{1/s} \qquad (3)$$

Let $(X_1, X_2, ..., X_k, Z)$ is the decision of the equation (3), then having entered a designation $\mu_j = \left(\frac{X_j^{\,n}}{Z^{\,n}} \right)^{\frac{n-s}{n}}$, from (3') we have the following system:

$$\mu_1 X_1^{\,s} + ... + \mu_k X_k^{\,s} - Z^s = 0,\ X_j^{\,s} - \tilde{\ }_j^{\frac{s}{n-s}}\ Z^s = 0\ , \qquad (3')$$

As $(X_1, X_2, ..., X_k, Z)$ is the decision (3') it is easy to see

$\sum_{j=1}^{k} \mu_j^{\frac{n}{n-s}} = \frac{\sum_{j=1}^{k} X_j^n}{Z^n} = 1$, as, hence, determinant of system (3’) is equal to zero

$\sum_{j=1}^{k} \mu_j^{\frac{n}{n-s}} - 1 = 0.$ Really, we apply a method of a mathematical induction, and as

$\Delta_2 = \mu_1^{\frac{n}{n-s}} - 1,\ \Delta_3 = \mu_1^{\frac{n}{n-s}} + \mu_2^{\frac{n}{n-s}} - 1$. It can be assumed, that

$\Delta_k = \sum_{j=1}^{k-1} \mu_j^{\frac{n}{n-s}} - 1$, $k=2,3,4\ldots$. Let us show it’s validity at $k+1$, is valid, decomposing determinant on *k+1* elements of a line is received:

$$\Delta_{k+1} = \begin{vmatrix} -1 & \mu_1 & \mu_2 & \ldots & \mu_{km-1} & \mu_m \\ -\mu_1^{\frac{s}{n-s}} & 1 & 0 & \ldots & 0 & 0 \\ -\mu_2^{\frac{s}{n-s}} & 0 & 1 & \ldots & 0 & 0 \\ \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots & . \\ -\mu_{m-1}^{\frac{n}{n-s}} & 0 & 0 & \ldots & 1 & 0 \\ -\mu_m^{\frac{s}{n-s}} & 0 & 0 & \ldots & 0 & 1 \end{vmatrix} =$$

$$= (-1)^{k+2} \cdot \left(-\sim_k^{\frac{s}{n-s}} \right) \begin{vmatrix} \mu_1 & \mu_2 & \ldots & \mu_{k-1} & \mu_k \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \ldots\ldots\ldots\ldots & \ldots\ldots\ldots & & & \\ 0 & 0 & \ldots. & 1 & 0 \end{vmatrix} +$$

$$+ (-1)^{2m+2} \Delta_m = (-1)^{k+3} \cdot \sim_m^{\frac{s}{n-s}} \cdot (-1)^{m+1} \cdot \sim_m \begin{vmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \ldots\ldots\ldots\ldots\ldots & & & \\ 0 & 0 & \ldots. & 1 \end{vmatrix} +$$

$$+\sum_{j=1}^{k-1}\mu_j^{\frac{s}{n-s}}-1=\left(-1\right)^{2k+4}\cdot\mu_k^{\frac{n}{n-s}}+\sum_{j=1}^{k-1}\mu_j^{\frac{n}{n-s}}-1=\sum_{j=1}^{k}\mu_j^{\frac{n}{n-s}}-1 .$$

As it was shown and as $\mu\in M$, where $\sum_{j=1}^{k}\mu_j^{\frac{n}{n-s}}=1$, i.e. it means $\Delta_{k+1}=0$. From 1st equation (3''') we have:

$$Z^s=\left(\sum_{j=1}^{k}\mu_j X_j^s\right) \quad \text{и} \quad \mu_j=\left(\frac{X_j^n}{Z^n}\right)^{\frac{n-s}{n}} .$$

Hence, as, $\sum_{j=1}^{k}\mu_j X^s\le\left(\sum_{j=1}^{k}\mu_j^0 X_j^s\right)=\sum_{j=1}^{k}\frac{X_j^n}{Z^{n-s}}$ т.е., from here for $Z^s\cdot Z^{n-s}=\sum_{j=1}^{k}X_j^n$ any $\mu\in M$, $Z^n=\sum_{j=1}^{k}X_j^n$, $\mu_j^0=\left(\frac{X_j^n}{\sum_{j=1}^{k}X_j^n}\right)^{\frac{n-s}{n}}$. And in this equality the maximum is reached. Thus we have $Z=\max_{\mu\in M}\left(\sum_{j=1}^{k}\mu_j X^s\right)^{1/s}$.

The sufficiency: The equation (1') let takes place. Let's prove validity (3'). Let's designate $Z=«(\mu)=\left(\sum_{j=1}^{k}\mu_j X_j^s\right)^{1/s}$, $\mu\in M$. It is easy to see, that from a condition $\frac{\partial «}{\partial\mu_j}=0$ the system of the equations follow $X_j^s-\mu_k^{-\frac{s}{n-s}}\cdot\mu_j^{\frac{s}{n-s}}\cdot X_k^s=0,\ j=\overline{1,k}$ and from here $\mu_k^{-\frac{s}{n-s}}\cdot\mu_j^{\frac{s}{n-s}}=\frac{X_j^s}{X_k^s}$ or $\mu_k^{-\frac{n}{n-s}}\cdot\mu_j^{\frac{n}{n-s}}=\frac{X_j^n}{X_k^n}$. To sum last equality on *j* from up *1* to *k*, then we have, $\mu_k^{-\frac{n}{n-s}}=\frac{\sum_{j=1}^{k}X_j^n}{X_k^n}$. and then $\mu_j^{0\ \frac{n}{n-s}}=\frac{X_j^n}{\sum_{j=1}^{k}X_j^n}$ is a point of a maximum of function $«(\mu)$, $\mu \in M$ as $(«''_{\mu\mu} < 0)$. Let's calculate the value of function $«(\mu^0)$. It is easy to see, that

$$Z^s = \sum_{j=1}^{k} \mu_j X_j^s \le \sum_{j=1}^{k} \mu_j^0 X_j^s = \sum_{j=1}^{k} \left(\frac{X_j^n}{Z^n} \right)^{\frac{n-s}{n}} \cdot X_j^s = \sum_{j=1}^{k} \left(\frac{X_j^n X_j^{\frac{sn}{n-s}}}{Z^n} \right)^{\frac{n-s}{n}} =$$

$$= \sum_{j=1}^{k} \left(\frac{X_j^{1+\frac{s}{n-s}}}{Z} \right)^{n-s} = \sum_{j=1}^{k} \frac{X_j^n}{Z^{n-s}} \quad \textit{i.e.} \quad Z^s \cdot Z^{n-s} = \sum_{j=1}^{k} X_j^n$$. And hence

$Z^n = \sum_{j=1}^{k} X_j^n \ ; \forall \mu \in M$. It is easy to see that

$$Z = «\left(\mu_0\right) = \left(\sum_{j=1}^{k} \left(\frac{X_j^n}{\sum_{j=1}^{k} X_j^n} \right)^{\frac{n-s}{n}} \cdot X_j^s \right)^{1/s}$$

$$= \left(\sum_{j=1}^{k} \left(\frac{X_j^n \cdot X_j^{\frac{sn}{n-s}}}{\sum_{j=1}^{k} X_j^n} \right)^{\frac{n-s}{n}} \right)^{1/s} = \left(\sum_{j=1}^{k} \frac{X_j^n}{Z^{n-s}} \right)^{1/s}$$, and $Z^s = «^s\left(\mu_0\right) = \frac{1}{Z^{n-s}} \sum_{j=1}^{k} X_j^n$,

from here $Z = «\left(\mu_0\right) = \left(\sum_{j=1}^{k} X_j^n \right)^{1/n}$, i.e. it takes place (3'). Thus

$$Z = «(\mu_0) = \left(\sum_{j=1}^{k} \left(\frac{X_j^n}{\sum_{j=1}^{k} X_j^n} \right)^{\frac{n-s}{n}} \cdot X^s \right)^{1/s} = \left(\sum_{j=1}^{k} X_j^n \right)^{1/n}$$. The lemma is proved.

Thus, the equation (2), (3) is optimum in sense (1'), and the tree of numbers appropriate by this equation represented on fig. 1 also is an optimum tree.

L__**emma 2**__**.** The tree of numbers to the appropriate equations (2) and (3’) let is given. Then there is transformation *K* which translates the solutions (3’) at $k = m-1$ on the solution (3’) at $k = m$, *i.e.*

$$Y = KX \ , \tag{4}$$

$$K = \begin{pmatrix} x & 0 & \cdots & 0 & 0 & 0 \\ 0 & x & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & x & 0 & 0 \\ 0 & 0 & \cdots & 0 & y & 0 \\ 0 & 0 & \cdots & 0 & 0 & z \end{pmatrix}, \quad x^n + y^n = z^p \ ,$$

$$X = (a_{1,m-1},...,a_{m-1,m}, N_{m-1}, N_{m-1}),\ N_{m-1} = \left(\sum_{j=1}^{m-1} a_{im-1}^n\right)^{\frac{1}{n}},\ Y = (a_{1m}, a_{2m},..., a_{1m}, N_m),\ N_m = \left(\sum_{j=1}^{m} a_{im}^n\right)^{\frac{1}{n}}.$$

__**The proof**__**.** Let be *(x,y,z)* the solution $x^n + y^n = z^p,\ p \ge 2, n \ge 2$.

Transformation (3’) in the form of $a_{im} = xq_{im-1},\ a_{mm} = y^n\sqrt{N_{m-1}^p},\ N_m = zN_{m-1},$ $i = 1,2,...,m-1;\ k = 2,3...$ Let $(a_{im-1},...,a_{m-1m-1}, N_{m-1})$ is the solution (2) at $k = m-1$. As $\sum_{j=1}^{m-i} a_{jm-1}^n = N_{m-1}^p$ multiplying by x^n we have $x^n \sum_{j=1}^{m-1} a_{jm-1}^n = x^n N_{m-1}^p$. Here $\sum_{j=1}^{m-1} (xa_{jm-1})^n = (z^p - y^n) N_{m-1}^p$, and hence $\sum_{j=1}^{m} a_{jm}^n = N_m^p$. Based on this transformation, we have

$$N_m^p = \left(x^{m-1}\right)^n + \sum_{i=2}^{m} \left(yx^{m-i} z^{\frac{p(i-2)}{n}} \right)^n \tag{4}$$

Formula (4) is a formula for a growing tree and can be applied in various fields of natural science. Transformation (3’) with p = 1

$$a_{im} = xa_{im-1},\quad i = \overline{1, m-1}, a_{mm} = y\sqrt[n]{N_{m-1}}, N_m = zN_{m-1}, where\ \ x^n + y^n = z\ ,$$

transforms a solution $\sum_{i=1}^{m-1} a_{im-1}^{n} = N_{m-1}$ to solutions $\sum_{i=1}^{m} a_{im}^{n} = N_m$ and then (4) takes follows:

$$N_m = \left(x^{m-1}\right)^n + \sum_{i=2}^{m} \left(yx^{m-i} z^{\frac{i-2}{n}} \right)^n$$

Thus, for any complex object described by a differential operator of equation (1), or (2) using the vector ***(a₁ … … aₘ)*** $\in E^m$, which is determined from the

$$N_m^p = X^{(m-1)} + \sum_{i=1}^{m} A_i X^{m-i}, X = x^n, A_i = y^n z^{p(i-2)}, (p,n,m) > 1,$$

representation $N_m^p = \sum_{i=1}^{m} a_{im}^n$ is reduced to its description by a finite number of elementary objects $(x,y) \in E^2$ such as a polynomial (4) for x^n, i.e. curves of higher degrees such as $N_m^p = x^{n(m-1)} + \sum_{i=2}^{m} y^n z^{p(i-2)} x^{n(m-i)}$

(Possibly elliptical) used for data protection or form $N_m^p - Y \sum_{i=3}^{m} X^{m-i} Z^{i-2} = X^{(m-1)} + YX^{m-2}$

where Z = X + Y and all the solutions of the equation $x^n + y^n = z^P$ for p = n are represented as $x = zt^{1/n},\ y = z(1-t)^{1/n},\ p = n,\ t \in (0,1)$. Note that $P_m^p(x,y) = N_m^p$ is a polynomial of degree *(m-1)* and x^n are the class of no degenerate curves higher degrees, which is very well used to protect information.

Remark. The basic result of the given work consists in the description of any complex object of Economics type:

$$Lu = \max_{\sim \in M} \left(\sum_{i=1}^{m} \sim_i (L_i\ u)^s\right)^{\frac{n}{ps}}, \quad or$$

$$(Lu)^p = \sum_{j=1}^{m} \left(L_j u\right)^n, \; n > s > o, \; M = \left\{(\sim_1, ..., \sim_m) = \sim; \; \sum_{j=1}^{m} \sim_j^{\frac{n}{n-s}} = 1, \; n > s > 0, \; 0 < \sim_j < 1\right\}$$

by means of a vector *(a_1 … … a_m)* $\in E^m$ which is defined from representation (*)

$$N_m^p = \sum_{i=1}^{m} a_{im}^n, \qquad N = Lu, \; a_{im} = L_i u$$

and it is reduced to its description by means of final number of elementary objects of type $(x, y) \in E^2$ as a polynomial be relative x^n:

$$N_m^p = x^{n(m-1)} + \sum_{i=2}^{m} y^n z^{p(i-2)} x^{n(m-i)},$$

i.e. curve high degrees

$$N^P = X^{(m-1)} + \sum_{i=2}^{m} A_i X^{(m-i)}, \qquad (***)$$

$$X = x^n, \; Y = y^n, \; Z = z^p, \; A_i = YZ^{i-2}$$

§2. The construction of Extreme Economics Model

It is known ([1-13]) that active penetration of scientific methods into practice of modern economy, both in sphere of manufacture of the industry, and in sphere of an agricultural production became prominent feature of our time. It is especially shown by consideration of some questions, in which the decision is connected with creation of the strict, scientifically proven methods in problems of economy and economic development. The decision of these hot questions is impossible without attraction of modern methods of a mathematical science. Creation of the scientific device for research and forecasting a condition of economic resources all over the world is one of the major state problems. Development of methods of qualitative research and, hence, the quantitative forecast of systems of economic development, naturally, demands all-round studying of parameters of a business economics, cities and the countries, at those or other values of parameters anthropogenous and social factors. Thus, experiments with real systems are rather expensive, long and frequently inadmissible, therefore there is a necessity of development of a various sort of mathematical models. By means of mathematical models, there was possible a qualitative and experimental studying of consequences of those or other planned actions touching functioning of economic systems, direct experiments with which are in admissible. The problem of studying of optimum values of resources (size of the capital and a labor) is a key question and to market economy and not having decided it, it is impossible to adjust effective activity of the best economy. Especially, the problem consists in forecasting size of the capital agrees the best manufactures before many countries, and in particular before the countries of the CIS as the condition of economy of many countries of the CIS now is at a low stage of economic development. Huge economic recession should mention a labor market. Forecasting of resources the extensive bibliography is devoted to mathematical questions. Since K. Marks's work, and also Mankyw N.G.'s work, Zanga V.B., Mitina N.A., PetrovA.A., Chernovekov D.S., Starkov N.I., Sh. Cherbakova A.V., Alikariev N.S., and of some other scientists studies various aspects mathematical modeling of economic systems and forecasting of their condition. One of the first mathematical models of Capital size is the model of the capital in view of manufacture Cobb-Douglas and manpower - model Malthus. In particular model of the capital in view of this or that production function not considered questions of optimization of manufacture on parameters of manufacture and it is accepted in model Malthus, that growth rate proportionally number and in it(her) is not taken into account factors of age and space. In view of time, time - age and time - age - spatial distribution works Volterra, Jeffries, Vebb, Alekseev, Svirezhev, Moisseev and many others are devoted to development of models of dynamics of a population. To one of the significant phenomena of a science of last time began the phenomenological theory of growth of the population of the world S.P. Kapitsa in which, with good accuracy the population of the Earth during rather long time it interpreted growth as hyperbolic growth owing to square-law dependence of growth rate on number. In these and other works bases of construction of the device qualitative and quantitative investigations of population's numbers are incorporated. Development of models in view of age and spatial distribution of a population and the problems connected to protection of rare biological kinds, are considered in works. In these works, questions of a correctness of models of biological populations are considered in view of age structure and spatial distribution. Some ideas from the specified works were used for the description of a condition of size of a labor, *i.e.* manpower within the framework of power models of manpower. For a wide class time–age-spaces distributions models described by the integral-differential equations in partial derivatives, questions of modeling of sizes of the capital and manpower in rather general cases are studied. One of achievements of our work is optimization of industrial models and connected with them of economic systems such as «the capital - production ». The purpose of the given book consists in development of models and methods of research of economic systems in view of time age distribution of manpower and research of problem optimization productions, manpower within the framework of models with extreme properties. In it the following questions are considered. The solution of a problem of optimization of productions and economic systems is defined on parameter uses of resources. Construction the best models of manufacture also are connected with them economic systems. Research of the connected problems connected to describing condition of size of the capital and a labor resource and on the basis of simulated functional of size of a labor as integral from a number of labor. A substantiation of the received equation for functional of a labor in cases when number of a human population depends on age and spatial factors is held. A mathematical substantiation initial the mathematical models connected from sizes of the capital and a labor. Mathematical modeling of dynamics of manpower has long enough history. One of the first works in this a direction should count model Malthus about exponential growth of number of the human population which has served by a basic point on creation of mathematical models. It is natural to name the following stage of mathematical models logic model which has formed a basis for a lot of remarkable works Volterra, Kostisin and so on. In these and subsequent works the big attention is given the development of a problem of construction and stability of dot models. Thus, the basic mathematical devices of modeling in these works are the nonlinear equations of dynamics of manpower and a population. Also it is necessary to specify K. Marks's economic works which has carefully studied a condition of economy with the help of Kene diagrams. Models Malthus, the logic model, and some other models have experimental acknowledgement at studying dynamics of a population. At the same time, not enough attention is fair was given modeling of dynamics of a manpower in view of age structure in a class of the differential equations. For economic systems in view of age structure and spatial distributions in our works, there are some ideas on a question of construction of mathematical models in view of time age distribution. In the offered book the general method the decision of the appropriate mathematical problems with the help special entered functional is offered and proved.

We shall propose our model Economics which is considered in our works [14-40].

1. Productions model. It is known, that economy this manufacture and distribution of material benefits. We shall designate through Y quantity made production (or the national income in scale of the country) and it is function of the capital-*N*, labor-*L*, and productivity of technical progress - *A*:

$$Y = A\,f\,(K, L) \qquad (1)$$

where *Y,K,L,f(.)*are non negative function.according to the given law the quantity made production grows with growth of size of the capital, work and a measure of the current level of technical progress and these factors are primary factors of growth of economy. As, Y = εY + (1- ε) Y, $0 < £ < 1$, the part of the received income is designated through I = $£Y$, and refers to in size the investment, and other part is designated, through $C = (1-£)Y$ and refers to as consumption. Besides, the certain part of the aggregate profit of the country which should go on state purchases G. To aggregate profits, it is necessary to add size of pure export - *Nx*. Thus, we have the following equation of balance of distribution of material benefits:

$$y = C+I+G+N_X. \qquad (2)$$

It is necessary as to note, that the size of consumption depends on the available income that is C = C(y-T). Here Tsize of taxes which go on social security payments poor, and payments of social insurance elderly etc. For construction of the equation of mathematical economy we shall take advantage of modeling manufacture (1) and the balance equation (2).

Let conditions take place:

1). All economic functions and parameters depend from sets of parameters (t, r, e, a,x, …), where t-time, the r-real rate of interest, e-a rate of an external exchange, a is an age of labor, x-spatial variable, $x = (x_1, x_2 \ldots x_n)$Č*R, the R-sum of regions.*

2). Rates of manufacture are defined by rates of distribution:

$$\frac{dY}{d2} = \frac{dy}{d2} \qquad (1')$$

2. Model of the capital. Let *K*= *K (.)* size of the capital (instruments of production, it is used by workers, monetary resources) at value of parameter t equals *K(t+* Δt*)* at *t= t+* Δt. Then a*K= K(t+* a*t) - K (t)* means a gain of the capital for an interval of parameters ∆t. Hence, a*K= I* a*t* and from here, with the account (1) we receive the equation of the capital

$$\frac{dK}{d²} = £Af(K, L) \quad (3)$$

at what here it is designated

$$\frac{d}{d²} = \frac{\partial}{\partial t} + \frac{\partial}{\partial r}\cdot\frac{dr}{dt} + \sum_i \frac{\partial}{\partial x_{i\cdot}}\cdot\frac{dx_i}{dt} + \frac{\partial}{\partial e}\cdot\frac{de}{dt} - \sum_i \frac{\partial}{\partial xi}(D_i \cdot \frac{\partial}{\partial x_i}),$$

For example, if $² = t,$ that we shall receive the classical equation of the capital: $\frac{dK}{dt} = £Af(K, L),$ and if $² = (t, r),$ that we have the equation of the capital in the following kind:

$$\frac{\partial K}{\partial t} + ¥_0 \frac{\partial K}{\partial r} = £Af(K, L), \qquad ¥_0 = \frac{dr}{dt}.$$

3. **Model of a Labor**. Work of those people who have devoted them to work, those are quantity fulfilled workers of hours. Now in modeling, economy for definition of parameter of work the model is used: $\frac{dL}{dt} = ¢L,$ where ¢ is rate of growth of the population equation. Clearly, that within the framework of the given model many important factors as the erudition, age, a floor, a nationality are not taken into account. In this connection we shall assume, that work is defined as functional manpower:

$$L(t) = \int_{a\min}^{a\max} \int_R ¨(x, a, t) N(x, a, t) dxda, \quad (4)$$

here $¨ = ¨(x, a, t)$ is potential function of working, N=N (x, a, t) - number of working in a point xČR, age and, 0 < a < ∞, at the moment of time t; ~min, ~max - accordingly the minimal and maximal age of the working in sphere of manufacture. As shown in our works, function N=N (x, a, t) is the decision of the following problem:

$$\partial_{tax} N = F(N, a, t), 0 < a < \infty, 0 < t < t_k,$$
$$N_{/t=0} = N_{0,\infty,} X \in R, \quad (5)$$
$$N(x, 0, t) = \int_0^\infty (N, ¶, t) d¶, \; N_{/s=0}.$$

Here $F(\bullet), B(\bullet)$ - according to function of death rate and birth rate of the labor population $\partial_{tax} = \frac{\partial}{\partial t} + \frac{\partial}{\partial \sim} + \sum_i \left[\neg_i \frac{\partial}{\partial X_i} - \frac{\partial}{\partial X_i}(D_i \frac{\partial}{\partial X_i}) \right]$, S is a border of area R; R = $\Sigma R_i, R_i - i - ый$ region. Potential function of working, $¨ = ¨(x,a,t)$ is the decision connected to (5) problems [14] - [40]. In the mentioned works it is shown, that the function of work determined with the help (4) satisfies the equation $\frac{dL}{d²} = ¢L,$ where rates of growth of the population ¢ are the decision of the following so-called equation of survival rate:

$$\int_{-}^{\infty} B(a)e^{-¢ a} da = 1 \qquad (6)$$

Here $B(a) = B_0(a)e^{-\int_0^{\sim} F_0(¶)d¶}$ - is function of survival rate, $B_0(\sim)$ is a factor of birth rate, $F_0(a)$ - a mortality rate coefficient, 0 < a < ∞. The equation (6) has one maximal material root $¢_0 = ¢_{max}$ and accounting number in a complex - connected roots $¢_i = \sim_i \pm i \;\; _i$, i=1,2 …. For the maximal root $¢_0 = ¢_{max}$ takes place

$$¢_{max} = \begin{cases} > 0, & \text{if } h = \sum_0^{\infty} B(a)da > 1, \\ 0, & \text{if } h=1, \\ < 0, & \text{if } h < 1, \end{cases}$$

where h is the potential of a labor. Hence $L(t) = \sum_{i=0}^{\infty} c_i e^{¢ it}$

4. Model of technology productivity level (the Measure of the current technological level). As

$$\frac{dY}{d²} = \frac{dA}{d²} f + A\frac{df}{dk} \cdot \frac{\partial K}{\partial ²} + A\frac{\partial f}{\partial L} \cdot \frac{dL}{d²}, \qquad (7)$$

and

$\frac{dy}{d²} = \frac{dC}{d²} + \frac{dI}{d²} + \frac{dG}{d²} + \frac{dN_x}{d²}$,that with the account $\frac{dC}{d²} = \frac{dC}{dy}(\frac{dy}{d²} - \frac{dr}{d²})$, $\frac{dI}{d²} = £\frac{dy}{d²} + y\frac{d£}{d²}$, from (7) we have: $\frac{dA}{d²} = -\frac{A}{f}\frac{\partial f}{\partial k} \cdot \frac{dk}{d²} - \frac{A}{f}\frac{\partial f}{\partial L} \cdot \frac{dL}{d²} + \frac{A}{y}(1 - £ - MPC)^{-1}$ u,

where u =- $MPC\frac{dT}{d²} + \frac{dG}{dI} + \frac{dN_x}{d²} + y\frac{d£}{d²} = -MPC\,u_0 + u_1 + u_2 + y \cdot u_3.$, MPC = $\frac{dC}{dy}$.

From here, in view of the equation (3), (4) and values of parameters MPK, 1-~ we have:

$$\frac{dA}{d²} = -\sim A^2 + вA, \qquad (8)$$

where $\sim = MPK \cdot £ + ¢ \cdot (1-\sim), в = \frac{1}{y}(1-£-MPC)^{-1} \cdot u$. Thus, the equations for the capital, work, and productivity of technical progress have the following kind:

$$\frac{dK}{d²} = £Af(K, L), K_{/²=0} = K_0,$$

$$\frac{dL}{d²} = ¢L, L_{/²=0} = L_0,$$

I=ε_0 Y, C = (1-ε_0) Y, (9)

$$\frac{dA}{d²} = -\sim A^2 + вA, A_{/²=0} = A_0,$$

$$\frac{dy}{d²} = (1-£-MPC)^{-1}u, y_{/²=0} = y_0, Y = Af(K, L),$$

where ¢ is the decision (6). To the equation (9) it is necessary to add also the equations:

$$\frac{dT}{d²} = u_0, \frac{dG}{d²} = u_1, \frac{dN_x}{d²} = u_2, \frac{d£}{d²} = u_3. \qquad (10)$$

The right parts of the equation (10) are rates of growth of sizes (T, G, N_x,£). They are necessary for defining from a condition of maximization of some economic criterion (or from a condition of minimization cost functional and so on):

***max** y(u0, u1, u2, u3).*

In system (9), (10) the following designation is accepted: $\frac{d}{d²} = \frac{\partial}{\partial t} + ¥_0 \frac{\partial}{\partial r} + \sum_{i=1}^{2} v_i \frac{\partial}{\partial x_i} + ¥_1 \frac{\partial}{\partial e} - \sum_{i=1}^{2} \frac{\partial}{\partial x_i}(D_i \frac{\partial}{\partial x_i})$. In order that if $MPC = 1-£$, at anyone $\frac{dy}{d²} \neq 0.$ It means that $u = (1-£-MPC)\frac{dy}{d²} = 0 \frac{dy}{d²} = 0$, that, if $£ = 1-MPC$, that is $y = £y + (1-£)y = I + C + G + N_x$, and balance of economy $u = -MPC \cdot u_0 + u_1 + u_2 + y \cdot u_3 = 0$. It turns out only due to a choice of rates changes of taxes, rates of the state purchases and pure export, and as rate of a share of the income going on capital investments. If $£ < 1-MPC,$ that occurs increase or reduction of the national income depending on a mark of function

u. Depending on set of values of parameter ² = (t, r, e, x, …) the system of the equation (9) - (10) will be transformed or in system of the ordinary differential equations, or in system of the equations in private(individual) derivatives. For example, if $² = t$, we shall receive system of the ordinary differential equations of 1st order:

$$\frac{dK}{dt} = £Af\,(K, L),\; K(0) = K_0,\; \frac{dL}{dt} = ¢L,\; L(0) = L_0,$$

$$L(t) = \int_{a\,min}^{a\,max} \int_R ¨\,(x,a,t)N(x,a,t)dxda,$$

$$\frac{dA}{dt} = -\sim A^2 + \;\; A,\; A(0) = A_0, \qquad (11)$$

$$\frac{dy}{dt} = (1 - £ - MPC)^{-1}\left[(-MPC\,u_0 + u_1 + u_2) + u_3 y\right],\; y(0) = y_0$$

$$\frac{dT}{dt} = u_0,\; \frac{dG}{dt} = u_1,\; \frac{dN_x}{dt} = u_2,\; \frac{d£}{dt} = u_3.$$

5. Properties of Productions. Under productions we shall understand system of elements (basic and turnaround, information and labor resources) in result of joint functioning of which " the capital and labor " will be transformed to a final product (or national income). The transformation of which carries out this transition refers to as by production function and is designated through *Y=Af(K, L)*, where the K-size of the capital (fixed capital), L–functional of labor resources, which depends from potential of labor resources, educationists, serviceability, floor and age, and also number of the workers, A is technological label. At the moment all over the world distinguish three as modeling of productions.

a). Productions as Cobb-Douglas

$$Y = Y_0 \left(\frac{K}{K_0}\right)^{\sim} \left(\frac{L}{L_0}\right)^{1-\sim},$$

where Y_0 the national income at the appropriate capital K_0 and labor resource L_0. *b). Productions with elastically of replacement CES (Solow):*

$$Y = Y_0\left[\sim \left(\frac{K}{K_0}\right)^{-\circ} + (1-\sim)\left(\frac{L}{L_0}\right)^{-\circ}\right]^{-1/\circ},\; 0 < \sim < 1,\; \circ > 0.$$

c). *The constant proportion productions CP (Leontev)*

$$Y = Y_0 \min\{K/K_0, L/L_0\}$$

d).μ-production , « **- (m yu):** $Y = Aä(K, L)$,

$$f(K,L)=f_0\left[\sim\left(\frac{K}{K_0}\right)^{-p}+(1-\sim^{n/_{n-s}})^{(n-s/n}\left(\frac{L}{L_0}\right)^{-p}\right]^{-1/_p}$$, n>s>0, ρ=ρ0s,

0<ρ0<∞or introducing $«(\sim)=\left[\frac{f(K,L)}{f_0}\right]^{-°_0}$, $X=\left[\frac{K}{K_0}\right]^{-°_0}$, $Y=\left[\frac{L}{L_0}\right]^{-°_0}$ we have

$$«(\sim)=\left(\sim X^s+\left(1-\sim^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}}Y^s\right)^{\frac{1}{s}}$$, 0<~ <1,

It is shown in works [1,2] from CES productions at ° ì 0 follows Cobb-Douglas productions and at ° ì ã the constant proportion productions CP and from μ-production at s ì 0 or n ì ãall productions.

It is shall out, that all types of production, i.e. the production functions should satisfy to following conditions:

1). $f(K,L)\in C^2[K_0,K_{\max}]\times[L_0,L_{\max}]$, i.e. entrance variables smoothly are chaining and results of activity of manufacture - the national income rather smoothly varies at changes quantity of used resources. It is natural at forecasting large systems, for example, economy of country.

2). F (0, L) =0, f (K, 0) =0, i.e. at absence though of one industrial resource of manufacture it is impossible. $\frac{\partial f(K,L)}{\partial K}>0$, $\frac{\partial f(K,L)}{\partial L}>0$.

3). At K> 0 and L> 0, it means, that growth of used quantity of fixed capital and growth of number of the workers results in growth of the national income.

4) $\frac{\partial^2 f}{\partial K^2}\leq 0,\ \frac{\partial^2 f}{\partial L^2}\leq 0,$ I.e. in conditions of pure economic growth of manufacture (without technical progress) grows of expenses only of one industrial resource results in decrease(reduction) of efficiency it using.

5). $f(ªK,ªL)>f(K,L)$ at ª >1 or $f(ªK,ªL)>ª^m f(K,L)$ at ª >1, and $m\geq 1$.

6). This condition provides that at proportional growth the quantity of used resources occurs proportional growth made products or national income. We shall notice, that all above mentioned functions submit to these conditions.

6. **The basic parameters of production**. The basic parameters of manufacture are: K=K(t) is the size of the capital at the moment of time t, i.e. K=K (t); L=L (t) is functional of a labor resource, ¥= *dK/dL* is limiting

norm of replacement, $\pm = \frac{d(K/L)}{d¥} \cdot \frac{¥}{K/L}$ is elasticity replacement of resources, $E_k = \frac{K}{f}\frac{\partial f}{\partial K}$, $E_L = \frac{L}{f}\frac{\partial f}{\partial L}$ are coefficients of elasticity of issue on resources, $¤_k = \frac{f}{K}$, $¤_L = \frac{f}{L}$ are medium capital productivity. We shall notice, that for example shows how many fixed capital an expense of labor per unit of and on the contrary for the preservation of the national income at a former level f (K, L) =fc can be released at increase. Parameter defines speeds of change of limiting norm of replacement of resources. Factors of elasticity of issue on resources E_K, E_L show on how many of percent will change manufactures of the national income at change of expenses of the appropriate resource of manufacture on one percent. It is necessary to note, that if to enter concept of value k=K/ L in productions, we have: y=F (), where $Y=Y/Y_0$, F (.) =f (,1), and F'() > 0, F"() < 0.

7. **Model of the basic resources**. Following our work we shall write general economic model for definition of sizes of the basic industrial resources K=K (t), L=L (t) and consumption C=C (t):

$$\begin{cases} \frac{dK}{dt} = £\ f(K,L), K(O) = K_0, C(t) = (1-£)\ f(K,L). \\ L(t) = \int_{a_{\min}}^{a_{\max}} ¤(a,t)N(a,t), 0 \le t \le t_k, \end{cases} \qquad (1")$$

Here the share of the national income *Y=Af (K, L)* going on process of manufacture, a_{min}, a_{max} are minimum and maximum age of the workers, (a, t) - potential function of the workers is defined as the decision of the following problem

$$\begin{cases} \frac{\partial ¤}{\partial t} + \frac{\partial ¤}{\partial a} = A(a,t)¤(a,t) + B(a,t)¤(o,t) + f(t), 0 \le t < t_k, \\ ¤\big|_{t_k} = 0, ¤\big|_{a=\infty} = 0, \end{cases}$$

A(.), B(.), f (.) are given functions, N=N (a, t) is the number of the workers of age at the moment of time t:

$$\begin{cases} \dfrac{\partial N}{\partial t} + \dfrac{\partial N}{\partial a} = F(N, a, t), 0 < a < \infty, 0 < t \le \infty \\ N(a, 0) = N^0(a), 0 \le a < \infty, \\ N(0, t) = \int\limits_0^\infty B(N(a, t), a, t)\, da \end{cases},$$

$F(\ ), N^0(\ ), B(\ )$ are given functions of the arguments, and F (.) - means the death-function of the workers, and B (.) function of their birth rate, N_0 (a) - initial number of the workers. Solving a problem we find functions φ= φ(a, t) and *N=N (a, t),* and then we shall define functional of labor resources *L=L (t).* At a known kind of manufacture *Y=Af (K, L)* from a problem (1) we shall define dynamics of the size of fixed capital, i.e. a size of the capital *K=K (t), 0< t<*t_k, and a size consumption C=C (t) in any moment of time.

Definition. Economic system connected with manufacture Y=A f (K, L), we shall name system consisting from the following elements: (K (t), L (t), C (t),I(t)), where C=C (t), K=K (t), L=L (t) are the decision of system (1”).

8. **General model productions: *μ-production***. As marked above from CES productions, Cobb-Douglas productions follow in particular at ρ→ 0 and a constant proportion productions at ρ→∞. In our work by us it was offered following modeling productions(A=1, f=Y):

$$Y = Y_0 \left[\sim \left(\frac{K}{K_0} \right)^{-\circ} + \left(1 - \sim^{\frac{n}{n-s}} \right)^{\frac{n-s}{n}} \left(\frac{L}{L_0} \right)^{-\circ} \right]^{-1/\circ}, n > s, s > 0. \qquad (12)$$

We shall notice that is higher resulted modeling productions special cases of the given manufacture are. From production function (12) at n→∞ follows function CES which as have noted is more general than Cobb-Douglas functions and a constant proportion functions. Thus from production function of (12) are follow all known production functions. We shall calculate parameters of manufacture. Easily to see, that

$$¥ = -\frac{(1 - \sim^{\frac{n}{n-1}})^{\frac{n-1}{n}}}{\sim} \left(\frac{K_0}{L_0} \right)^{\circ}, \left(\frac{K}{L} \right)^{1+\circ}$$

, i.e. the norm of replacement is function of asserts, and therefore

$$g=\frac{K}{L}=\left[-\frac{\sim}{(1-\sim^{\frac{n}{n-1}})^{\frac{n-1}{n}}}\left(\frac{K_0}{L_0}\right)\text{¥}\right]^{\frac{1}{1+\circ}}.$$

The meanings of elasticity replacement are defined as follows: $\pm=\frac{1}{1+\circ}$.

Coefficients of elasticity of issue on resources are defined accordingly under the formulas:

$$E_K=\frac{\sim\left[\frac{K}{K_0}\right]^{-\circ}}{\left[\sim\left(\frac{K}{K_0}\right)^{-\circ}+\left(1-\sim^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}}\left(\frac{L}{L_0}\right)^{-\circ}\right]}$$

$$E_L=\left(1-\sim^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}}\cdot\left[\frac{L}{L_0}\right]^{-\circ}\Bigg/\left[\sim\left(\frac{K}{K_0}\right)^{-\circ}+\left(1-\sim^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}}\cdot\left(\frac{L}{L_0}\right)^{-\circ}\right]$$

and $E_K=\sim\frac{(f/f_0)^{\circ}}{(K/K_0)^{\circ}}, E_L=(1-\sim^{\frac{n}{n-1}})^{\frac{n-1}{n}}\frac{(f/f_0)^{\circ}}{(L/L_0)^{\circ}}, E_K+E_L=1$.

We shall consider offered manufactures (12) at $\rho\to 0$. As *F (K) =f (K, 1),* it is necessary to find a limit $\lim Y_0\left[\sim(\frac{K}{K_0})^{-\circ}+(1-\sim^{\frac{n}{n-s}})^{\frac{n-s}{n}}\right]^{-\frac{1}{\circ}}=A$,Easily to see, that $\ln A=\ln Y_0-\frac{1}{\circ}\ln\left[\sim(\frac{K}{K_0})^{-\circ}+(1-\sim^{\frac{n}{n-s}})^{\frac{n-s}{n}}\right]$,and therefore $\lim_{\circ\to 0}A=\sim\ln\frac{K}{K_0}$ i.e. $f=Y_0F(\frac{K}{K_0})^{\sim}$. As F (K) unequivocally defines function f (K, L) we have received the statement that the function Cobb-Douglas at $\rho\to 0$ (and $n\to\infty$) is a special case our function. Similarly, at ($\rho\to\infty$ and $n\to\infty$) we have

$$\lim_{^\circ\to\infty} A = \begin{cases} Y_0 & \text{at} \quad K \ge K_0 \\ Y_0 \dfrac{K}{K_0} & \text{, at} \quad K < K_0 \end{cases}$$, or $$F(K) = Y_0 \min\{\frac{K}{K_0}, 1\}$$, and therefore

$$f(K,L) = LY_0 \min\{\frac{K}{K_0}, 1\} = L\frac{Y_0}{L_0} \min\left\{\frac{K}{L}\cdot\frac{L_0}{K_0}, 1\right\} = Y_0 \min\left\{\frac{K}{L}\cdot\frac{L_0}{K_0}\right\}$$, i.e. have received function with a constant proportion. We shall define limiting meaning of economic parameters (s=1)

$$\text{¥} = -\frac{(1-\sim^{\frac{n}{n-1}})^{\frac{n-1}{n}}}{\sim}\frac{K}{L}, \pm = 1$$,

$$E_K = \frac{\sim}{\sim + (1-\sim^{\frac{n}{n-1}})^{\frac{n-1}{n}}}, \; E_L = \frac{(1-\sim^{\frac{n}{n-1}})^{\frac{n-1}{n}}}{\sim + (1-\sim^{\frac{n}{n-1}})^{\frac{n-1}{n}}}.$$

$$\text{¥} = \begin{cases} -\infty & (K > L),\, at \pm = 0 \\ 0 & \text{at} \; K < L \end{cases}$$,

$$E_K = \begin{cases} 1 \quad \text{at} \; \text{K} > \text{L} \\ 0 \quad \text{at} \; \text{K} < \text{L} \\ \sim / [\sim + (1-\sim^{\frac{n}{n-1}})^{\frac{n-1}{n}}] \; \text{at} \; K = L \end{cases}$$

$$E_L = \begin{cases} 0 \quad \text{at} \; \text{K} > \text{L} \\ 1 \quad \text{at} \; \text{K} < \text{L} \\ (1-\sim^{\frac{n}{n-1}})^{\frac{n-1}{n}} \big/ [\sim + (1-\sim^{\frac{n}{n-1}})^{\frac{n-1}{n}}] \; \text{at} \quad \text{K} = L \end{cases}$$

Optimal model of productions. Easily to see, that ranked production functions Cobb-Douglas, CES, from a constant proportion but one parameter is not optimized, i.e. the condition of the appropriate productions cannot be improved. The μ- function offered us on parameter is optimized. Easily to see, that

$$\frac{dY}{d\sim} = 0$$, at $$\sim * = \left[\frac{(\frac{K}{K_0})^{-^\circ n}}{(\frac{K}{K_0})^{-^\circ n} + (\frac{L}{L_0})^{-^\circ n}}\right]^{\frac{n-1}{n}}$$ and $$\left.\frac{d^2Y}{d\sim^2}\right|_{\sim=\sim^*} < 0$$ i.e. takes place

$$Y^* = \max_{0<\sim<1} Y(\sim),$$

Substituting α=α * in the formula (4) we shall receive:

$$Y^{*} = Y_0 \left[\left(\frac{K}{K_0}\right)^{-\circ n} + \left(\frac{L}{L_0}\right)^{-\circ n} \right]^{-1/\circ n} \qquad (13)$$

Model production as (5) we shall name best model production and appropriate economic system (K, L, C) connected with manufacture (13) best economic system. From (13) we have

$$\left(\frac{Y^*}{Y_0}\right)^{-\circ_0 n} = \left(\frac{K}{K_0}\right)^{-\circ_0 n} + \left(\frac{L}{L_0}\right)^{-\circ_0 n} \quad or \quad z^n = x^n + y^n$$

and for m resource we have

$$z^n = x_1^n + x^n{}_2 + \ldots + x_m^n \qquad (@)$$

The optimal economic systems. *A triad (K * (t), L * (t), C * (t))*K *y =y *, where K=K* (t),L=L*(t) is the decision (1) with production function f (K*, L*) = y * and consumption C * (t) = (1-* £ *) y * we shall name as the optimal economic system.*

We shall notice, that if to enter a designation

$$Z = \left(\frac{Y^*}{Y_0}\right)^{-\circ}, \quad X = \left(\frac{K}{K_0}\right)^{-\circ} \quad Y = \left(\frac{L}{L_0}\right)^{-\circ}$$, we shall receive the equation

of type $X^n + Y^n = Z^n$, which has not the solution in the whole positive

numbers at n> 2: $\sim^* = \left(\frac{(\frac{K}{K_0})^{-\circ n}}{\left(\frac{K}{K_0}\right)^{-\circ n} + \left(\frac{L}{L^0}\right)^{-\circ n}} \right)^{\frac{n-1}{n}}, 1 - \sim^{*\frac{n}{n-1}} = \left(\frac{L}{L_0}\right)^{-\circ n} / \left[\left(\frac{K}{K_0}\right)^{-\circ n} + \left(\frac{L}{L_0}\right)^{-\circ n} \right].$

The appropriate economic parameters for optimum manufacture (13) are represented as:

$$¥ = -\left(\frac{K_0}{L_0}\right)^{-\circ n} \left[\frac{K}{L}\right]^{1+\circ n}, \qquad \frac{K}{L} = \left[-\left(\frac{K_0}{L_0}\right)^{\circ n} ¥ \right]^{1/(1+\circ n)}, \quad ± = \frac{1}{1+\circ n},$$

$E_K = \left(\frac{K}{K_0}\right)^{-\circ n} \cdot \left(\frac{y^*}{y_0}\right)^{\circ n}$, $E_L = \left(\frac{L}{L_0}\right)^{-\circ n} \cdot \left(\frac{y^*}{y_0}\right)^{pn}$, $E_K + E_L = 1$. We find limiting meanings of parameters: At $° \to 0$: $¥ = -\frac{K}{L}$, $± = 1$, E_K=1, $E_K = \begin{cases} 0, \overline{K<K_0} \\ \infty, \overline{K>K_0}, \ \overline{K} = \frac{y^*}{K}, \\ 1, \overline{K=K_0} \end{cases}$

$\overline{K_0} = \frac{y_0}{K_0} \overline{K_0}$, E_L=1. At ρ→∞, $E_L = \begin{cases} 0, \overline{L<L_0} \\ \infty, \overline{L>L_0}, \ \overline{L}_0 = \frac{y^*}{L}, \ \overline{L_0} = \frac{y_0}{L_0}, ¥ = \begin{cases} -\infty under K>L \\ 0 under K<L, ± = 0 \end{cases}, \\ 1, \overline{L=L_0} \end{cases}$

$\lim \sim^* = \left(\frac{1}{2}\right)^{\frac{n-1}{n}}$. From received results follows, that for functions Cobb-Douglas, CES, from constant proportions there are no best condition and necessary introduced some regularization.

§3. The optimal model productions in a class of regularization productions of Cobb-Douglas

Occurrence of the theory of production functions is accepted for carrying by 1927 when article of the American scientists of the economist of item has appeared. Douglas (P. Douglas) and D. Cobb's mathematics (D. Cobb) « the Theory of manufacture ». In this article, attempt was undertaken, in the empirical way to define influence of the spent capital and work on output in a manufacturing industry of USA. As is known, modeling manufacture Cobb-Douglas looks like: $Y = A\ f(K, L)$, where $f(K, L) = f_0 \left(\frac{K}{K_0}\right)^{\sim 1} \left(\frac{L}{L_0}\right)^{\sim 2}$,

where the and A is theological level, K is a size of the capital, L is a size labor resources, f_0, K_0, L_0 positive constants, $f_0 = f(K, L)$ at $K = K_0$, $L = L_0$, $\sim_1 + \sim_2 = 1$, $0 \le \sim_j \le 1$, $j = 1,2$. Here parameters $\sim_j$ characterized degrees use of resources during manufacture. It is necessary to note, that modeling manufacture (1) is "rigid" manufacture and does not accept the maximal condition on one parameter. In this connection there is a question on change of area of change of entrance parameters, functions (1). For example, areas of change of a degree of use of resources (the capital and a labor) during manufacture the set of straight lines лини in an individual square is $M = \left\{\sim_j : \sim_1 + \sim_2 = 1, 0 \le \sim_j \le 1\right\}$. As M we take set curvilinear lines on sphere $M_n^s = \left\{\sim = (\sim_1 ... \sim_m) : \sum_{j=1}^{m} \sim_j^{\frac{n}{n-s}} = 1,\ 0 \le \sim_j \le 1\right\}$. For function (1) m=2 also we shall take n=2, s=1 also a problem of maximization function (1) on set of M is reduced to the following problem:

$$Z = \max_{\sim\, \in M} \left\{ A\, f_0 \left(\frac{K}{K_0} \right)^{\sim_1} \left(\frac{L}{L_0} \right)^{\sim_2} \right\}, \text{ where } M = \left\{ \sim : \sim_1^2 + \sim_2^2 = 1, 0 \le \sim_j \le 1 \right\}.$$

Having entered a designation, $\text{«}(\sim) = A\, f_0 \left(\frac{K}{K_0} \right)^{\sim_1} \left(\frac{L}{L_0} \right)^{\sim}$ we shall receive

$Z = \max_{\sim\, \in M} \text{«}(\sim)$. Thus, the initial problem consists in a presence of parameter $\sim_1$ and $\sim_2$ a degree of use of resources during manufacture and the maximal condition of modeling manufacture Z.

<u>The statement 1.</u> Takes place $\sim\; = \frac{x}{\sqrt{x^2 + y^2}}, \quad z = z_0 e^{\sqrt{x^2 + y^2}},$

where $x = \ln \frac{K}{K_0}$, $y = \ln \frac{L}{L_0}$, $Z_0 = A f_0$, $x^2 + y^2 = z^2$

The proof. As

$$\frac{d\text{«}}{d\sim} = A f_0 \left[\left(\frac{K}{K_0} \right)^{\sim} \ln \frac{K}{K_0} \left(\frac{L}{L_0} \right)^{\sqrt{1 - \sim^2}} + \left(\frac{L}{L_0} \right)^{\sqrt{1 - \sim^2}} \cdot \ln \frac{L}{L_0} \cdot \left(\frac{K}{K_0} \right)^{\sim} \cdot \left(\frac{2\sim}{2\sqrt{1 - \sim^2}} \right) \right] =$$

$$= A f_0 \left(\frac{K}{K_0} \right)^{\sim} \cdot \left(\frac{L}{L_0} \right)^{\sqrt{1 - \sim^2}} \left[\ln \frac{K}{K_0} - \frac{\sim}{\sqrt{1 - \sim^2}} \cdot \ln \frac{L}{L_0} \right]$$

That from a condition $\frac{d\text{«}}{d\sim} = 0$ we have $\frac{\sim^2}{1 - \sim^2} = \left(\frac{\ln \frac{K}{K_0}}{\ln \frac{L}{L_0}} \right)^2$ i.e.

$$\sim^2 = \frac{\left(\frac{\ln \frac{K}{K_0}}{\ln \frac{L}{L_0}}\right)^2}{1+\left(\frac{\ln\left(\frac{K}{K_0}\right)}{\ln\left(\frac{L}{L_0}\right)}\right)^2} = \frac{\left(\ln \frac{K}{K_0}\right)}{\left(\ln \frac{L}{L_0}\right)^2 + \left(\ln \frac{K}{K_0}\right)^2} \text{ and} 1-\sim^2 = \frac{\left(\ln \frac{L}{L_0}\right)^2}{\left(\ln \frac{L}{L_0}\right)^2 + \left(\ln \frac{K}{K_0}\right)^2}$$

Let’s transform function $«(\sim)$. It is easy to see, that

$$Z = \ln(Af_0) + \sim_1 \ln\left(\frac{K}{K_0}\right) + \sim_2 \ln\left(\frac{L}{L_0}\right) = \ln(Af_0) + \frac{\left(\ln \frac{K}{K_0}\right)^2}{\sqrt{\left(\ln \frac{L}{L_0}\right)^2 + \left(\ln \frac{K}{K_0}\right)^2}} \cdot \ln \frac{K}{K_0} +$$

$$+ \frac{\left(\ln \frac{L}{L_0}\right)}{\sqrt{\left(\ln \frac{L}{L_0}\right)^2 + \left(\ln \frac{K}{K_0}\right)^2}} \cdot \ln \frac{L}{L_0}$$ From here taking into account a designation

$x = \ln \frac{K}{K_0}$, $y = \ln \frac{L}{L_0}$, we have $Z = Z_0 e^{\sqrt{x^2 + y^2}}$, $\sim^* = \frac{x}{\sqrt{x^2+y^2}}$, $\sqrt{1-\sim^{*2}} = \frac{y}{\sqrt{x^2+y^2}}$,

that it was required to proved. We shall enter $z = \ln \frac{Z}{Z_0}$ then from (3) we have the equations $x^2 + y^2 = z^2$ or $x^n + y^n = z^n$

and in generally for m resource we have

$$x_1^n + x_2^n + \ldots + x_m^n = z^n \quad (@)_1$$

As is known [3-6], the equation (4) we have accounting number of decisions. Including the whole decisions of a kind from here knowing x, y, z definition size K, L, Y for optimal modeling manufacture Собб - Douglas is represented in the form

$K = K_0 e^x$, $L = L_0 e^y, Z = Z_0 e^z$.

Optimization of size of the capital under law Cobb – Douglass .*The given paragraph it is devoted to optimization of process of formation of the capital according to law Cobb – Douglass and formation of a labor on экспоненциальному to the law.* As is known sizes of the capital it is formed according to the law: $\begin{cases} \frac{dK}{dt} = \pounds A f_0 \left(\frac{K}{K_0}\right)^{\sim} \left(\frac{L}{L_0}\right)^{1-\sim}, 0 < t \le t_k \\ K(0) = K_0 \end{cases}$,where $\pounds, A, f_0, K_0, L_0, \sim$ the given positive numbers $0 < \pounds < 1,\ 0 < \sim < 1$ characterize economic parameters. For example, the size $\sim$ means degrees of use of resources. We shall assume, that $0 < \sim < 1$, $\sum \sim_i^2 = 1$, where $\sim_1 = \sim,\ \sim_2 \equiv 1 - \sim$. Size L is characterized a manpower and within the framework of the given work we shall assume, that $L = L_0 \ell^{¢t}$ where ¢ mean rate of growth of manpower, t is a time.

<u>**The statement 2**</u>**.** This Model is optimized on $\sim$; $0 < \sim < 1$, $\sum_{i=1}^{2} \sim_i^2 = 1$, $\sim_1 = \sim,\ \sim_2 = 1 - \sim$ also the following assumes

$$\begin{cases} \frac{dk}{dt} = \pounds A f_0\, e^{\sqrt{\left(\ell n \frac{K}{K_0}\right)^2 + \left(\ell n \frac{L}{L_0}\right)^2}} \\ K(0) = K_0 \end{cases} \qquad (1)$$

Really, that as in (1) $\frac{dK}{dt} > 0$, that taking the logarithm both parts (1) we shall receive $\ell n \left(\frac{dK}{dt}\right) = \ell n \left(\pounds A f_0\right) + \sim_1 \ell n \left(\frac{K}{K_0}\right) + \sim_2 \ell n \left(\frac{L}{L_0}\right) =$

$\ell n \left(\pounds A f_0\right) + \sim \ell n \left(\frac{K}{K_0}\right) + \left(1 - \sim^2\right)^{-\frac{1}{2}} \ell n \left(\frac{L}{L_0}\right)$. We shall find extreme of functions $Z = \ell n\left(\frac{dK}{dt}\right)$ on $\sim$ i.e.

$\frac{dz}{dt} = \ell n\frac{K}{K_0} - \left(1-\sim^2\right)^{-\frac{1}{2}}\sim \quad \ell n\frac{L}{L_0} = 0$ and from here

$$\sim^2 = \frac{\left(\ell n \frac{K}{K_0}\right)^2}{\left(\ell n\frac{K}{K_0}\right)^2 + \left(\ell n\frac{L}{L_0}\right)^2}, \quad \frac{d^2 z}{d\sim^2} < 0 \text{ ,therefore}$$

$$\ell n\frac{dK}{dt} = \ell n\left(\pounds A f_0\right) + \frac{\ell n\frac{K}{K_0}}{\sqrt{\left(\ell n\frac{K}{K_0}\right)^2 + \left(\ell n\frac{L}{L_0}\right)^2}} \cdot \ell n\frac{K}{K_0} + \frac{\ell n\frac{L}{L_0}}{\sqrt{\left(\ell n\frac{K}{K_0}\right)^2 + \left(\ell n\frac{L}{L_0}\right)^2}} \cdot \ell n\frac{L}{L_0} =$$

$$= \ell n\left(\pounds A f_0\right) + \sqrt{\left(\ell n\frac{K}{K_0}\right)^2 + \left(\ell n\frac{L}{L_0}\right)^2}\text{. Thus,} \quad \frac{dK}{dt} = \pounds A f_0 \exp\sqrt{\left(\ell n\frac{K}{K_0}\right)^2 + \left(\ell n\frac{L}{L_0}\right)^2}$$

that was required factors. Using $L = L_0 e^{\text{¢}t}$ model we shall copy as

$$\begin{cases} \frac{dk}{dt} = \pounds A f_0 \ \ell^{\sqrt{\left(\ell n\frac{K}{K0}\right)^2 + \text{¢}^2 t^2}} \\ K(0) = K_0 \end{cases}, \quad 0 \le t \le t_k$$

.This Model characterized formations of the capital it agrees the law of optimization by Cobb-Douglas and for media manpower under law Malthus. As (3) is nonlinear model for definition size of capital according to model (3), we use a method of broken lines Eelier, i.e.

$$K_{i+1} = K_i + h\pounds\ A\ f_0 \exp\sqrt{\left(\ell n\frac{K_i}{K_0}\right)^2 + (\text{¢}ih)^2} \quad i = 0,1,2.......n$$

The optimal model productions in a class of manufactures of Cobb-Douglas in a case m resources. In the given paragraph the model optimization of size of the capital in a class of manufactures Cobb - Douglas in a case *m* resource is offered. As is known the size of the capital in a case *m* resources is formed according to the law:

$\begin{cases} \frac{dK_1}{dt} = £Af \; \mathop{\Pi}\limits_{i=1}^{m} \left(\frac{K_i}{K^i{}_0} \right)^{~_i}, \quad 0 < t \le t_k \\ K_1(0) = K_{10} \end{cases}$, where $£, A, f_0, K_0, L_0, ~_i$ the given positive numbers $0 < £ < 1$, $0 < ~_i < 1$ characterize economic parameters, K_i the size *i*-a resource, $~_i$ means degrees of use *i*-a resource. We shall assume, that $0 < ~_i < 1$, $\sum ~_i^{\frac{n}{n-s}} = 1$, where *n > s > 0.*

The statement 4. Model is optimized on~ : $0 < ~_i < 1$, $\sum ~_i^{\frac{n}{n-s}} = 1$ also the following assumes $\begin{cases} \frac{dK_1}{dt} = £Af_0 \, e^{\sqrt[n]{\sum_{i=1}^{m} \left(\frac{K_i}{K^i{}_0} \right)^n}} \\ K_1(0) = K_{10} \end{cases}$. Really, $\frac{dK_1}{dt} > 0$, that taking the logarithm both parts we shall receive $\ell n \left(\frac{dK}{dt} \right) = \ell n \left(£Af_0 \right) + \sum_{j=1}^{m} ~_j \ell n \left(\frac{K_j}{K^j{}_0} \right)$. We shall enter designations $z = \ell n \left(\frac{1}{£Af_0} \frac{dK_1}{dt} \right)$, $x_j = \ell n \left(\frac{K_j}{K_0} \right)$ and we shall receive $z = \max_{~ \in A} «(~)$, $«(~) = \sum_{j=1}^{m} ~_j x_j$, where $~ \in M, M = \left\{ \{ ~ = (~_1, ..., ~_m) : 0 < ~_j < 1, \sum_{j=1}^{m} ~_j^{\frac{n}{n-s}} = 1 \right\}$.. We maximize function $«(~)$ on set *A* in view of the entered designations we shall receive the proof of the given statement and we have

$$z^n = \sum_{j=1}^{m} x_j^n \qquad\qquad (@\ )_2$$

§4. On the energetic theory of population growth

The new model of growth of the population, so-called energetic model of growth of the population is offered. It is shown; that power model it is possible to receive from lines of groups of initial models in view of age structure and spatial distributions.

To questions of modeling and forecasting of growth of the population numerous works (for example, [14-40]), since known Malthus model $\frac{dn}{dt} = ¢n$ are devoted, to logistical model, model in view of immigration and to emigration, model in view of age structures and spatial distributions, and also S.P. Kapitsa’s to model in last time. In many models or assume proportionality of growth rate to number to the population (Malthus model), or to a square of their number (model Kapitsa). These assumptions impose also on some age and пространственных models of growth of number of a population. In our work, proceeding from some initial groups of the models, describing growth of a population in view of age structure and spatial distributions the new model so-called by us power will be received. Initial group of models of growth of the population we shall write in the following kind:

$$\begin{cases} \left(\dfrac{\partial}{\partial t}+\dfrac{\partial}{\partial a}\right)N=-F_0(a)N, \ 0<a<\infty, \ 0<t<t_r \\ N(a,0)=N_0(a), \quad 0\le a<\infty, \\ N(0,t)=\sqrt[p]{\displaystyle\int_0^\infty B_0(a)N^p(a,t)\,da}, \end{cases}$$

$$\begin{cases} \left(\dfrac{\partial}{\partial t}+\dfrac{\partial}{\partial a}+r\dfrac{\partial}{\partial x}\right)N=-F_0(a)N, \ 0<a<\infty, 0<t<t_k \\ N(x,a,0)=N_0(x,a), \quad 0\le a<\infty, \ 0<x<L, \\ N(x,0,t)=\sqrt[p]{\displaystyle\int_0^\infty B_0(a)N^p(x,a,t)\,da}, \ 0<x<L, \\ N\big|_{x=0}=0=N\big|_{x=0}, \end{cases} \qquad (1)$$

Here N is a number population, $N=N(a,t)$ – for a case a) and $N=N(x,a,t)$ – for a cases a)-b), $t\in[0,t_k]$ is time, $a\in[0,\infty)$ is an age, $x\in G=[0,L]$, $F_0(a)\ge 0$, $B_0(a)\ge 0$ are factors death rates and birth rate, p is parameter, $0<p<\infty$.

a). Model in view of age distribution. Let’s enter function

$$L(t)=\left(\int_0^\infty \text{¨}\,(a)\,N^p(a,t)\,da\right)^{1/p}, \ 0<\text{p}<\infty, \qquad (2)$$

where $\text{¨}\,(a)$ is some non-negative function with a condition $\int_0^\infty \text{¨}\,(a)\,da=1$. The basic result of the given work we shall formulate as the following theorem.

<u>The theorem.</u> Let function $N = N(a,t)$ is a solution of a problem (1) in case a). Then there will be a function $¨\,(a) \ge 0$, $\int_0^\infty ¨\,(a)\,da = 1$ for which function $L(t)$ is a solution of the equation:

$$\frac{dL}{dt} = ¢L, \; ¢ : \int_0^\infty B(a) e^{-¢a} da = 1 \qquad (3)$$

where $B(a) = B_0(a) e^{-p\int_0^a F_0(¶)d¶}$ is function of survival rate of the population.

<u>The proof</u> of the theorem. We consider the case a) from (1). The first equation a) we shall increase on - N^{p-1} then we shall receive identity $-\frac{1}{p}(\frac{\partial}{\partial t} + \frac{\partial}{\partial a})N^p = F_0(a)N^p$. The given equation we shall increase on function $¨ = ¨\,(a) \ge 0$, and result we shall integrate on $a \in [0,\infty)$. Further, проводя simple transformations we shall receive:

$-\frac{1}{p} ¨\; N^p \int_0^\infty + \frac{1}{p}\int_0^\infty \frac{d¨}{da} N^p da - \frac{1}{p}\frac{d}{dt}\int_0^\infty ¨\,(a) N^p da + \int_0^\infty ¨\; F_0(a) N^p da = 0$. Taking into account a boundary condition at $a = 0$, adding and subtracting a member $\frac{¢}{p}\int_0^\infty ¨\,(a) N^p da$ in the left part of last identity, and also choosing function $¨ = ¨\,(a)$ as the solution of a problem $\begin{cases} \frac{d¨}{da} = (¢ + pF_0(a))¨\,(a) - B_0(a)¨\,(0) \\ ¨\,(\infty) = 0 \end{cases}$, i.e.

$¨\,(a) = ¨\,(0)\int_a^\infty B_0(s) e^{-¢(s-a) - p\int_a^s F_0(¶)d¶} ds$, we have the equation of type equation Malthus $\frac{dL^p}{dt} = ¢L^p$. From (xxx) at $a = 0$ we shall receive $\int_0^\infty B_0(a') e^{-¢a' - p\int_0^{a'} F_0(¶)d¶} da' = 1$. Having entered replacement $B(a) = B_0(a) e^{-p\int_0^a F_0(¶)d¶}$ we receive the proof of the theorem.

Definition. *Model (1), (2) we shall name power model, and the problem appropriate to this model power (p=2). Function (3) we shall name potential of the population.*

b). Polynomials model. The decision of a power problem is represented as

$L(t)^p = \sum_{j=0}^{\infty} c_j^{\ p}\ e^{\sim_j t} \cos(\ \ _j t)$, i.e. $L(t) = \left(\sum_{j=0}^{\infty} c_j^p e^{\sim_j t} \cos\left(\ \ _j t\right) \right)^{1/p}$, where factors c_j are defined from a condition $L(t)^p = \sum_{j=0}^{\infty} c_j^{\ p}$, at p=*2* we have ***number tree model in the form of:*** $L^2(0) = \sum_{j=0}^{\infty} c_j^2$ or $x_1^{\ n} + x_2^{\ n} + \ldots + x_m^n = z^n$ (@)3

and sizes $\sim_j$ u $\ \ _j$ - are decisions of system $\begin{cases} \int_0^{\infty} B(a) e^{-\sim_j a} \cos\ \ _j a\ da = 1 \\ \int_0^{\infty} B(a) e^{-\sim_j a} \sin\ \ _j a\ da = 0 \end{cases}$

The remark. For definition of a population in view of age structure we shall receive the following formula $N(a,t) = \frac{\ddot{\ }(a)}{\sqrt[p]{\int_0^{\infty} \ddot{\ }^{2}(a)\, da}} L(t)$ where $\ddot{\ }(a)$ it is defined under the formula and $\ddot{\ }(0)$ from a condition $\int_0^{\infty} \ddot{\ }(a)\, da = 1$.

c). Model in view of age and spatial distribution. As is known, the concept of potential of the population was entered in works [1-6] in case of time, age distribution. Distinctive feature of the present work is that during change of manpower spatial parameters are taken into account. We shall enter function

$$L(t) = \left(\int_0^{\infty} \int_0^{L} \ddot{\ } (x,a,t) N^p (x,a,t) dx da \right)^{1/p}, \qquad (4)$$

where function ¨(.) characterizes the function describing serviceability, erudition of the population and ¨ $= ¨(x,a,t) \geq 0$ $\int_0^\infty ¨(a)\,da = 1$, $N = N(x,a,t)$ is a population of age *a* in a point $x \in [0, L]$ at the moment time *t* also satisfies (1), б).

$$\begin{cases} \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a} + r\frac{\partial}{\partial x}\right)N = -F_0(a)N, \; 0 < a < \infty, 0 < t < t_k \\ N(x,a,0) = N_0(x,a), \quad 0 \leq a < \infty, \; 0 < x < L, \\ N(x,0,t) = \sqrt[p]{\int_0^\infty B_0(a)N^p(x,a,t)\,da}, \; 0 < x < L, \\ N\big|_{x=0} = 0 = N\big|_{x=0}, \end{cases}$$

where t is time, *a*-age, x is spatial coordinate, *r=r (x)* is the given function describing speeds of change of number on a direction *x*, $F_0(a)$ is a mortality rate coefficient $B_0(a)$ is factor of birth rate. Using our the theorem for definition of function ¨(.) from (.) we have next equation

$$¨(x,a,t) = \int_a^\infty B_o(¶)e^{-p\int_a^{¶} F_o(¦)d¦ + ¢(a-¶)} ¨(x + r(s - a),0,t - a + ¶)d¶ \qquad (5)$$

In (4) we shall put a =0 then having taken, $¨(x,0,t) = ce^{-t+\ a+¥x}$ we have the equation of type $\frac{dL}{dt} = ¢L,\; ¢ : \int_0^\infty B(a)e^{-¢a}da = 1$, where ª $= ¢ + \quad + ¥\, r$.

d). **Model of the population for** n **countries in view of age distribution.**

Let's assume, that the population n of the countries satisfies to the equation

$$\begin{cases} \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a} + r\frac{\partial}{\partial x}\right)N = -F_0 N, \; 0 < a < \infty, 0 < t < t_k, x \in G = \bigcup_1^n G_i, G_i \in E^m, \\ N(x,a,0) = N_0(x,a), \quad 0 \leq a < \infty, \; x \in \bar{G} = G + S, \\ N(x,0,t) = \int_0^\infty B_0(a)N(x,a,t)\,da, x \in \bar{G}, \end{cases}$$

where $B_0(a)$, F_0 are the given matrixes n of the order with elements

$$F_0 = \begin{pmatrix} f_{11} & f_{12}\dots & f_{1n} \\ f_{21} & f_{22} & f_{2n} \\ \dots & & \\ f_{n1} & f_{n2} & f_{nn} \end{pmatrix} \quad B_0 = \begin{pmatrix} b_{11} & b_{12}\dots & b_{1n} \\ b_{21} & b_{22} & b_{2n} \\ \dots & & \\ b_{n1} & b_{n2} & b_{nn} \end{pmatrix}$$

, $N_0(x,a)$ is an initial number of populations. Let's enter function $L(t) = \int_0^\infty \int_G ¨(x,a,t)N(x,a,t)dxda,$ and $\frac{dL}{dt} = ¢L,$ where $¨(.)$, n - a measured vector of function, $¨ = ¨(x,a,t) \geq 0$,

$\int_0^\infty \int_G ¨(x,a,t)N(x,a,t)dxda = 1,$ $¨(x,a,t) = \int_a^\infty e^{-\int_a^{¶} F_o^*(¦)d¦ + ¢(a-¶)} B_0^*(s)¨(x+r(s-a),0,t-a+¶)d¶$, where B_0^*, F_0^* are conjugates to B_0, F_0 matrices. Hence we have next equation

$$¢ : \det\left(\int_0^\infty B(a)e^{-ªa}da - I\right) = 0, \text{ and } L(t) = \sum_{j=0}^{\infty} c_j \; e^{~_j t} \cos(\;_j t), \text{where } ª = ¢ + \; + ¥\, r \,.$$

e). <u>Computer experiments</u>. For the first series of computer experiments initial functions we take in the following kind:

$$B_0(a) = \begin{cases} 0, & a < a\min \\ b, & a\min \leq a \leq a\max \\ 0, & a \geq a\max \end{cases} . \quad F_0(a) \equiv F_0,\; a\min = 0,\; a\max = 90 .$$

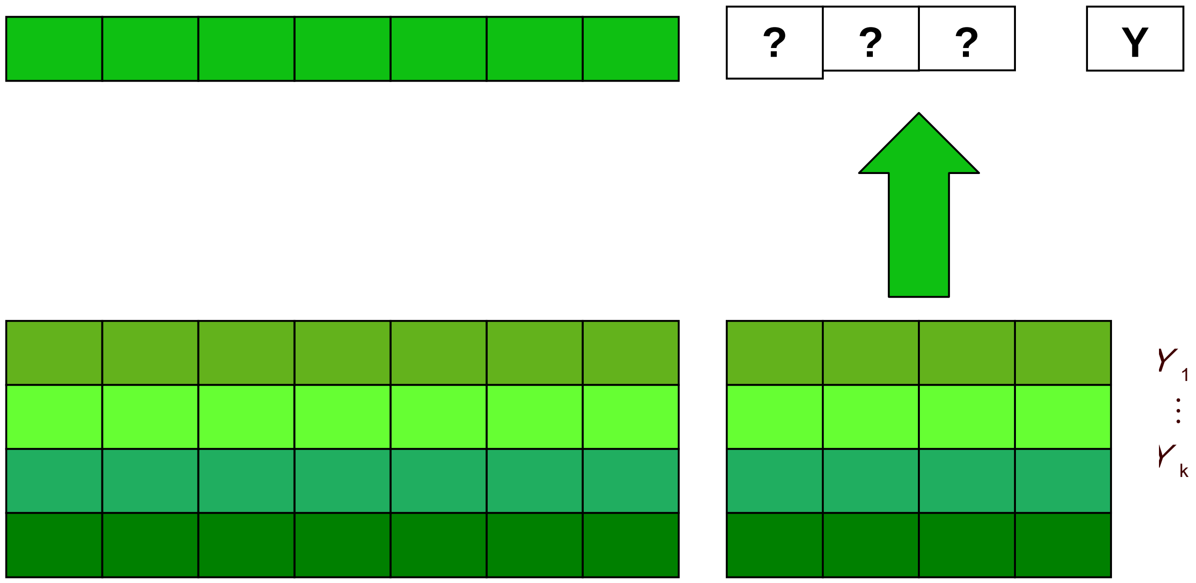
For definition of factors of decomposition (@) we shall write as

$$L_m^2(0) = \sum_{j=1}^{m} c_{jm}^2$$

where *m=1,2,3 …*, and it is solved this equation on the basis of initial the equation $L_2^2(0) = c_{12}^2 + c_{22}^2$ with the help of transformation offered by us. Then on the basis of the decision $L_2^2(0) = c_{12}^2 + c_{22}^2$ we shall receive the decision the equation $L_3^2(0) = C_{13}^2 + C_{23}^2 + C_{33}^2$ etc. up to the decision of the required equation $L_m^2(0) = \sum_{j=1}^{m} C_j^2, \dots,\; m = 2,3\dots$. The written algorithm was programmed in language Borland Delphi and a series of computing experiments is carried out.

§5. Polynomials Models of Development of Losses in the Worst Condition by Kinds with Long Settlement - a modification method of the nearest neighbor

Now we consider questions of construction and investigating a new method of calculation a size of losses by kinds with long settlement: a modification method of the nearest neighbor. The proposition method is to define a size of losses in the worst condition of system and is based on using of so-called model of numbers tree. By using this method for a model, the data showed and carried out some computational experiments. It is noticed that the propose method basis on numbers tree model, is a simple and universal method for definition of value of Losses in the Worst Condition by Kinds with Long Settlement and it is easy programmed on all computer languages. The receiving formulas are controlled our calculations. As the methodology actuaries calculations uses the probability theory, given and the long-term statistical data, financial calculations that on faculty are in full read to a demography rates connected with System mathematical and the statistical regularities establishing mutual relation between the insurer and the insurant. They reflect as mathematical formulas the mechanism of formation (education) and an expenditure of insurance fund in long-term insurance operations. To them also carry calculations of tariffs on any kind of insurance: life's pensions, from accidents, property, work capacity. The methodology actuary's calculations use the probability theory, given to demography and the long-term statistical data, financial calculations. By means of the last in tariffs the income which is received by the insurer from use as credit resources of the accumulated payments of insurance is taken into account. Except for that rates on actuaries are read to calculations which is connected to one of the widespread problems(tasks) of such -statistical calculations connected to definition of norms and conditions of insurance, is, that the sum of insurance payments minus relying payments guaranteed reception by insurance firm (or the state organization) expected results. Making "tree" of decisions, it is necessary to draw "trunk" and the "branches" displaying structure of a problem. "Trees" from left to right settle down."Branches" designate possible alternative decisions which can be accepted, and the possible outcomes arising as a result of these decisions. On the circuit we use two kinds of "branches": The first - the dashed lines connecting squares possible decision, the second - the continuous lines connecting circles of possible outcomes. Square "units" designate places where it is made a decision, round "units" - occurrence of outcomes. As accepting the decision cannot influence occurrence of outcomes, it needs to calculate probability of their occurrence only. When all decisions and their outcomes are specified on "tree", each of variants is counted, and in the end its monetary income is put down. All charges caused by the decision, are put down appropriate "branch". As is known, the forecast of the future losses, using observably average development of the nearest neighbors, usually define under the following circuit:



On the basis of given to the circuit the model of the nearest neighbor assumes the following (***ICA 2010, CA, Cape Town***): $Y = \sum_{i=1}^{k} \mu_i Y_i$,where $\mu_i \geq 0, i = 1,...,k$; $\sum_{i=1}^{k} \mu_i = 1$ .We shall consider more general model, than this model: $Y_k = \left(\sum_{i=1}^{k} \mu_i Y_{ik}{}^{s} \right)^{1/s}$,where $\sum_{i=1}^{k} \mu_i^{\frac{n}{n-s}} = 1$, $n > s > 0$, $\mu_i \geq 0$, $i = 1,...,k$; $k=2,3,$. Having entered a designation $X_{ik} = \mu_i^{1/s} Y_{ik}$ from this model we have

$$Y_k = \left(\sum_{i=1}^{k} X^{s}{}_{ik} \right)^{1/s}$$ or

$$Y_k^{s} = \sum_{i=1}^{k} X^{s}{}_{ik} \qquad (@)$$ [1]

and $\sum_{i=1}^{k}\left(\frac{X_{ik}}{Y_{ik}}\right)^{\frac{ns}{n-s}}=1,$ *n>s>0, k=2,3,4…….*The first equation is the equation of a degree *s* with *k+1* unknown and has infinite number of decisions. For allocation of the necessary decisions it is necessary that we use echo the equation (@).

***Model of the worst development of losses*:** For everything $\sum_{i=1}^{k}\mu_i^{\frac{n}{n-s}}=1$, *n > s > 0* $\mu_i \geq 0,\ i=1,...,k$ right part of the equation has the maximal value that there corresponds the worst condition of system, i.e. $Y_k=\max_{\mu\in M}\left(\sum_{i=1}^{k}\mu_i Y_{ik}^{\ s}\right)^{1/s}$

where $M=\left\{\mu : 0\leq\mu_j\leq 1,\ \sum_{j=1}^{k}\mu_j^{\frac{n}{n-s}}=1,\ n>s,\ s>0,\ k>1\right\}$.

The equation (4) we shall call *"Model of the worst development of losses"*

$$z_k^{\ s}=\sum_{i=1}^{k}x_{ik}^{\ s} \qquad (@)_5$$

§6. Extreme Economics and Economic Crisis

1. Model of an economic crisis. We shall consider following modeling Economy offered in works:

$$\begin{cases}\dfrac{dK}{dt}=£Af(K,L),\ 0<t\leq t_k,\quad K(0)=K_0,\\ \dfrac{dL}{dt}=¢L,\qquad L(0)=L_0,\\ ¢:\displaystyle\int_0^{a_{max}}B(a)e^{-¢a}da=1,\quad ¢\in(-\infty,\infty),\\ Y=Af(K,L),\qquad C=(1-£)Y,\\ \dfrac{dA}{dt}=a_0A-a_1A^2,\qquad A(0)=A_0,\end{cases} \qquad (1)$$

where *K=K (t)* - size of the capital at the moment of time *t, L=L (t)* - size of a manpower, £ a share of the national income - *Y* going on capital investments, C- size of consumption, *A=A (t) is* a technological level, *B-B (a) function* of stability of a manpower determined as $B(a)=B_0(a)e^{-\int_0^a f_0(¶)d¶}$, B_0 (a) function of birth rate of manpower resources, F0 (a) - function of death rate. It is necessary to note, that function *L=L (t)* is represented as

$$L(t)=\int_0^{a_{\max}} ¨(a,t)N(a,t)da, \qquad (2)$$

where $N=N(a,t)$ is the decision of a problem:

$$\begin{cases} \left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\right)N=-F_0(a)N, \quad 0<t\le t_0, \quad 0\le a\le a_{\max} \\ N(a,0)=N_0(a) \\ N(0,t)=\int_0^{a_{\max}} B_0(¶)N(¶,t)d¶ \end{cases}$$

Here $¨=¨(a,t)$ is the decision of the connected problem and refers to potential to function of manpower resources.

Definition 1. Let entrance functions of a problem (1) are given in ranges of definition of the parameters, and also known entrance parameters $K_0,\ L_0,\ A_0,\ £\ \ f(0)$ some production function then a vector function (K,L,Y,C,A) determined as the decision of a problem(task) (1) we name modeling economy the appropriate production function f (K, L). Thus production function $A\cdot f(K,L)$ we shall name modeling to manufactures.

Definition 2. We shall tell, that the modeling economy (1) is in a condition of crisis if there are such constant positive numbers K *, L* and ¢ for which have places of an inequality

$$Y^{²}(K,L^{*})\le Y^{²}(K^{*},L^{*})\le Y^{²}(K^{*},L), \qquad (3)$$

what decision (1) satisfying conditions would not be

$$\frac{1}{²}\int_0^{²} K(t)\,dt\le K^{*}, \qquad \frac{1}{²}\int_0^{²} L(t)\,dt\ge L^{*}, \qquad (4)$$

where $Y^{²}=\frac{1}{²}\int_0^{²} A(t)f(K,L)dt$. For example, if we shall consider manufactures Cobb-Douglas $f(K,L)=f_0\left(\frac{K}{K_0}\right)^{\sim}\left(\frac{L}{L_0}\right)^{1-\sim}$ where~ a degree of use of a manpower during manufacture, $0 < \sim < 1$. It is easy to see, that if there is a pair $\left(K^*, L^*\right)$ satisfy (3) $\frac{1}{2}\int_0^2 K(t)d \le K^*$, $\frac{1}{2}\int_0^2 L(t)dt \ge L^*$, i.e. has places of an inequality (4).

The statement 1. For anyone modeling manufacture of type $Y = Af(K, L)$ the condition (3) is necessary and sufficient. Really, by definition of production $\frac{\partial f}{\partial x} > 0$, $\frac{\partial f}{\partial l} > 0$, and $0 < A_{\min} < A(t) < A_{\max}$, we have

$$f^2\left(k, L^*\right) - f^2\left(k^*, L^*\right) = \frac{1}{2}\int_0^2 A(t)\left[f\left(K, L^*\right) - f\left(k^*, L^*\right)\right]dt =$$

$$= \frac{1}{\tau}\int_0^{\tau} A(t)\frac{\partial f}{\partial K}\Big|_{x,L}\left(K - K^*\right)dt \qquad (5)$$

From here, if are executed condition (3), $f^2\left(K, L^*\right) - f^2\left(K^*, L^*\right) \le 0$ and $\frac{1}{2}\int_0^2 K(t)dt \le K^*$. Similarly, using a condition $f^2\left(K^*, L^*\right) - f^2\left(K^*, L\right) \le 0$, we shall receive $\frac{1}{2}\int_0^2 L(t)dt \ge L^*$. The proof of the given inequality also follows from an inequality (5). Thus, necessary and sufficient conditions of crisis of modeling economy are inequalities (4). It is necessary to note, that numbers $\left(K^*, L^*\right)$ are defined from the decision of the differential equation. For a presence of the decision of the equation $\frac{dL}{dt} = ¢ L$, we should solve the equations $\int_0^{a_{\max}} B(a)e^{-¢a}da = 1$ all over again. Generally, if $B(a) \ge 0$, this equation has one maximal material root $¢ = ¢_{\max}$ and accounting numbers of in a complex connected roots of type $¢_j = \sim_j \pm i \;\; _j$ so we have: $L_k(t) = c_k e^{¢_k t}$, $k = 0,1,2...$, and therefore $L(t) = L_0 e^{¢_{\max} t} + \sum_{i=1}^{\infty} c_j e^{\sim_j t}\cos \;\; _j t$, $0 \le t \le t_k$, where C_j are Fourier coefficients of the given decomposition. For legality of the given decomposition, we should consider a class of function $B(a) \ge 0$ for which system of function $\{\cos \;\; , t\}$ is ортонормированной. Numbers $\sim_j$ also $\;_j$ we shall define from system

$$\begin{cases} \int_0^{a_{\max}} B(a)e^{-\sim_j a}\cos \;\; _j ada = 1 \\ \int_0^{a_{\max}} B(a)e^{\sim_j a}\sin \;\; _j ada = 0 \end{cases}$$

From here, it is easy to see, that $\sin(\bar{}_j a) = 0$, i.e. $_j = \frac{2® j}{a_{max}}$, $j = 0,1,2,....$ and roots $\sim_j, ¢_{max}$ удовлетворяемой to conditions $|\sim_j| \le |¢_{max}|$, $\sim_j = \frac{1}{\bar{a}}\ln(B_j)$, $\sim_j = 0$ *at* $B_j = 1$,

$\sim_j < 0$ *at* $B_j < 1$, $\sim_j > 0$ *and* $B_j > 1$, *where* $B_j = \int_0^{a_{max}} B(a)\cos(\frac{2®j}{a_{max}}a)\,da.$

The statement 2. For the uniform law of distribution

$$B(a) = \begin{cases} 0, & a \le 0 \\ \frac{1}{a_{max}}, & 0 < a \le a_{max} \\ 0, & a > a_{max} \end{cases}$$

we have $\sim_j < 0$ *at* $B_j = 0$, $L(t) = \sum_{j=1} c_j e^{\sim_j t}\cos(\frac{2® j}{a_{max}}t)$, $L^* = \frac{1}{2}\int_0^2 L(t)dt$

The statement 3. Let B (a) function of distribution of the normal law is defined as the normal law of distribution i.e. $B(a) = \frac{1}{\sqrt{2®}\,±} e^{-\frac{(a-a_0)}{2±2}}$ then $¢_j$ are defined from the decision of the equation $F\left(\frac{a_0}{±} - ±¢\right) = e^{¢a_0 - \frac{±^2¢^2}{2}}$ where a_0 ,± parameters of $F(a)$.

The proof. We shall enter replacement $t = ±¢ + \frac{a - a_0}{±}$, т.е $a = a_0 - ±^2¢ + ±\,t$, and

$$dx = ±\,dt, -¢a_0 + ±^2¢^2 - ±¢t - \frac{(¢t - ±¢)^2}{2} = -±a_0 + ±^2¢^2 - ¢±t - \frac{t^2}{2} + \quad t\,±¢ - \frac{±^2¢^2}{2} = -¢a_0 + \frac{1}{2}±^2¢^2 - \frac{t^2}{2}$$

Then we have $\frac{1}{\sqrt{2®}}\int_{-\frac{a_0}{±}+±¢}^{\infty} e^{-\frac{t^2}{2}}dt \cdot e^{-¢a_0 + \frac{±^2¢^2}{2}} = 1$, $\frac{1}{\sqrt{2®}}\int_{-\infty}^{±¢-\frac{a_0}{±}} e^{-\frac{t^2}{2}}dt = e^{¢a_0 - \frac{±^2¢^2}{2}}$,as was to be shown we shall notice, that if to enter replacement $x = \frac{a_0}{±} - ¢±$, that we shall receive $f(x) = e^{-\frac{1}{2}\left(x^2 - \frac{a_0^2}{±^2}\right)}$, $0 \le f(x) \le 1$, $x^2 - \frac{a_0^2}{±^2} \ge 0$, $|x| \ge \frac{a_0}{±}$,

and therefore $x = \frac{a_0}{±} - ¢± \le -\frac{a_0}{±}$, $¢ \ge \frac{a_0}{±^2}$, $¢ \prec \infty$. Similarly, we shall receive

$x = \frac{a_0}{±} - ¢± \ge \frac{a_0}{±}$, $¢ \le 0$, $¢ \succ -\infty$. Thus $¢ \ge ¢_{\min} = \frac{a_0}{±^2}$, is unique material a root of the equation $\int_0^{a_{\max}} B(a)e^{-¢a}da = 1$ in the first and $¢ \le ¢_{\max} = 0$, in the second cases. These values are used at definition of crisis values.

2. Polynomials model of economic crisis. We shall consider

$$y^{2} = y^{2}\left(©, l^*\right) = \frac{1}{2}\int_0^{2} A(t) f\left(©, l^*\right)dt$$

Case 1.As $y^{2} = \frac{1}{2}\sum_{i=0}^{m}\int_{t_i}^{t_{i+1}} A(t)f\left(©, l^*\right)dt = \frac{1}{2}\sum_{i=0}^{m} \sim_i \int_{t_i}^{t_{i+1}} f\left(©, l^*\right)dt$,

where $\sim_i = A(¶)$, $t_i \le ¶ \le t_{i+1}$, we shall take $\sim_i \in M = \left\{\sim_i : \sum_{i=1}^{m} \sim_i^{\frac{n}{n-1}} = A_0^{\frac{n}{n-1}}\right\}$, and designating $f_i = \frac{1}{2}\int_{t_i}^{t_{i+1}} f\left(©, l^*\right)dt$, we shall receive $y^{2} = \sum_{i=1}^{m} \sim_i f_i$. Now we shall consider a problem of maximization on set of M, i.e.

$y^{2} = \max_{\sim \in M} \sum_{i=1}^{m} \sim_i f_i$. Using conditions of a maximum of function m of variables from here we have

$$\left[¥^{2}\right]^{n} = A_0 \sum_{i=1}^{m} f_i^{n},$$

$$\sim_i^{\frac{n}{n-1}} = A_0^{\frac{n}{n-1}} \frac{f_i}{f_1^n + f_2^n + \dots f_n^n}, \quad i = \overline{1, m}$$

Thus maximization occurs on parameter of technology. How to exhaust an alignment, we represent the basic equation of model a tree of numbers. Each member of the sum of the right part (or some of them) are decomposed as the sum less composed in the same are sedate, and members of the right parts of last sums in turn are again decomposed as the sum less composed in the same are sedate.

Let y^{2}, f_i are the decision the equation a tree of numbers then we shall define sizes © and l from a condition $\int_{t_{i+1}}^{t_{i+1}} f\left(©, l^*\right)dt = f_i$, $\frac{1}{2}\int_0^{2} f\left(©^*, l\right)dt = ¥^{2}$.

Case 2. In this case we maximize sizes $y^{²}$ on size of the capital т.e

$y^{²} = \frac{1}{2}\sum f_i \int_{t_i}^{t_{i+1}} \mathrm{A}(t)dt$, where $f_i = f(©(¶), l^*)$, $t_i \le ¶ \le t_{i+1}$ .Let's enter $\mathrm{A}_i = \frac{1}{2}\int_t^{tin} \mathrm{A}(t)\, dt$

and let the set M is defined as $\mathrm{M} = \left\{ f_i : \sum_{i=1}^{m} f_i^{\frac{n}{n-1}} = f_0^{\frac{n}{n-1}} \right\}$, then considering a

problem $y^{²} = \max_{f_i \in \mathrm{M}} \sum_{i=1}^{m} f_i \mathrm{A}_i$. Let's receive the equation

$$y^{²\,n} = f_0 \sum_{i=1}^{m} \mathrm{A}_i^n , \qquad (@)_6$$

$$f_i^{\frac{n}{n-1}} = f_0^{\frac{n}{n-1}} \frac{\mathrm{A}_i^n}{\mathrm{A}_1^n + \mathrm{A}_2^n + ...\mathrm{A}_m^n}, \quad i = 1, m$$

This equation such is the basic equation a tree of numbers. Solving it we find $¥^{²}, \mathrm{A}_i$ and then sizes f_i , т.e the sizes capitals and general forces.

§7. Model of monetary circulation

Monetary circulation, as is known, according to the quantitative theory of money for cumulative demand (dependence between quantity of made production on which purchasing demand is showed and general a price level) takes place:

$$\mathrm{M\,V = P\,Y} \qquad (1)$$

Here *M* is the offer of money, *V* is a speed of the manipulation of money, a *P* is a price level, and Y_{ia} the quantity of the made goods and services. This equation approves, that the offer of money defines volume of manufacture in nominal expression which in turn, depends from price levels and quantity of made production: M=$к_0$Py, $к_0 = \frac{1}{V}$. From here P =к0 $\frac{1}{y}$, $K = \frac{M}{V}$, and therefore between a price level and to volumes of manufacture there is an inverse relationship. As the volume of manufacture is defined by various kinds made production *Y = (У1, У2, … .*$У_n$*)* and the vector of price level P is connected to it *P= (P1, P2 …* P_n*)* the basic equation will be defined(determined) in the following kind: *(Y, P) =MV,* where *(p, y) =* $\sum_{i=1}^{u} P_i y_i$. Besides we shall assume, that price levels and volume of manufacture P, at are functions of some parameter $² = (t, r, e, x),$ where t-time, r-the real rate of interest, e-the exchange rate, x-the spatial factor. Then basically the equation (1) scalar product (P, y) is defined as: $(p,y)=\sum_{i=1}^{n}\int_{e\min}^{r\max}\int_{e\min}^{e\max}\int_{G}P_i(t,r,e,x)y_i(t,r,e,x)drdedx$

If will designate through $P_{\min}(t)$ and $P_{\max}(t)$ - accordingly minimal and maximum levels of the prices at the moment of time t from the basic equation (1) we shall receive an inequality: $P_{\min}(t)y(t)\le M(t)v\le P_{\max}(t)y(t), 0\le t\le t_k$, where $y(t)=\sum_{i}\int_{0}^{r\max}\int_{e\min}^{e\max}\int_{R}y_i(t,r,e,x)drdedx$ is total amount of manufacture. Naturally, minimal and to maximum levels of the prices the minimal and maximal offers of money accordingly answer. Then $P_{\min}(t)=\frac{VM_{\min}(t)}{y(t)}$, $P_{\max}(t)=\frac{VM_{\max}(t)}{y}$. From here $\frac{M_{\min}(t)}{P_{\min}(t)}=\frac{M_{\max}(t)}{P_{\max}(t)}$. The attitude of the minimal offer of money on a minimum level of the prices equally to the attitude of the maximal offer of money on maximal price levels. This attitude refers to as a stock of money. Thus, at a constancy of volume of manufacture on parameters (r, e, x) stocks of money does not change. Using the theorem of average for average value price levels on (r, e, x) we have: $P_{cp}(t)=\frac{VM}{y(t)}, P_{\min}(t)\le P_{cp}(t)$, and therefore, $M_{\min}(t)\le M_{cp}(t)\le M_{\max}(t)$ .The received results are fair for average values P, M, at on time in the considered time interval of supervision. *i.e* $\overline{P}_{\min}\le \overline{P}_{cp}\le \overline{P}_{\max}$, $\overline{M}_{\min}\le \overline{M}_{cp}\le \overline{M}_{\max}$, where feature above in sizes means averaging on time of values of these sizes, for example, $\overline{P}=\frac{1}{t_k}\int_{0}^{tk}P(t)dt$.

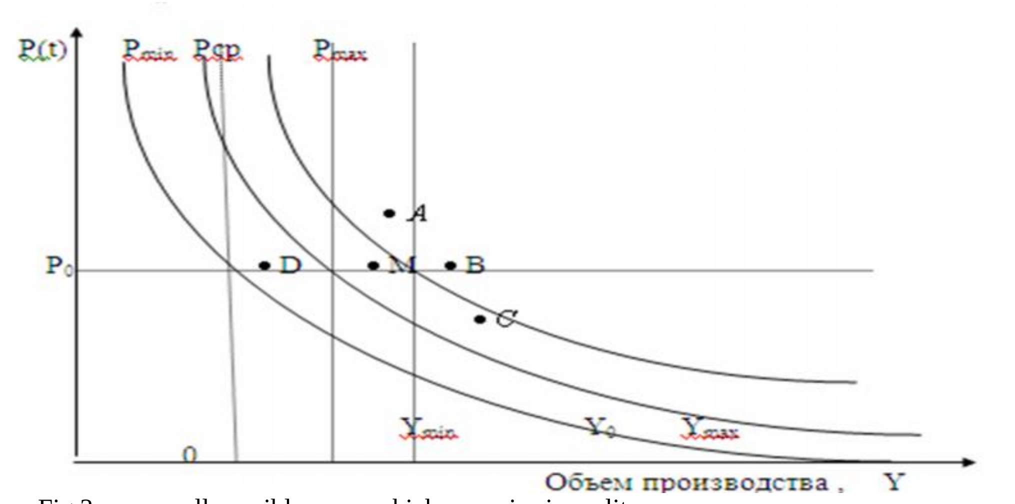


Fig 3.covers, all possible cases which can arise in reality

Depending upon that in which part of figure to be point $M_0 = M$ ($У_0$, P_0), those who offers money, it is necessary for a society, conduct appropriate a policy of change of value of volume of manufacture and price levels. For example, for a constant level of volume of manufacture $У_0$ of the price can vary from a minimum level up to maximal. Similarly, we can hold price levels on some favorably all level P0, and volume of manufacture to reduce or increase (from $У_{min}$ up to $У_{max}$). In result, the reasonable policy under the attitude the offer of money is defined. At any price level, the offer will result increase in increase of a stock of money and reduction will result the offer of money in its reduction. In the first case when the volume of manufacture is increased, and in the second is decreases. If the economy in the beginning of supervision is in condition M_0 that at decrease of cumulative demand connected with reduction from the offer of money there is a transition from point M0 to point D in which the volume of manufacture of below real level, and then in process of reduction of prices occurs growth of economy up to level У0. In the same figure other picture is observed also. At volume of manufacture equal У0, all over again a price level it is increased up to P_{max}, that is up to a point And, and then it is smoothly reduced up to a point In. In result, there is a jump in economy, which is manufactures to become maximal. As $MV=Py, \frac{dP}{d²}y+\frac{dy}{d²}\text{P}=\frac{d\text{M}}{d²}V+\frac{dV}{d²}\text{M}$, also we shall enter designations $v_0 = \frac{dM}{d²}, v_1 = \frac{dV}{d²}, \overline{v} = v_0 \cdot V + v_1 \cdot M$, we have $\frac{d\text{P}}{d²} = -\frac{1}{y}\frac{dy}{d²}\text{P} + \frac{1}{y}\overline{v}$ and from here, we shall receive

$\frac{d\text{P}}{d²} = -\frac{1}{y}\frac{dA}{d²}f - \frac{A}{y}\frac{\partial f}{\partial \text{K}}\cdot\frac{d\text{K}}{d²} - \frac{A}{y}\frac{\partial f}{\partial L}\frac{dL}{d²} + \frac{1}{y}\overline{v}$. Considering values $\frac{dA}{d²}$, we receive

the **price equation**:

$$\frac{d\text{P}}{d²} = -(1 - £ - MPC)^{-1}\frac{u}{y}\cdot\text{P} + \frac{\overline{v}}{y}, P(0) = P_0,$$

where $\frac{d\text{P}}{d²} = \frac{\partial \text{P}}{\partial t} + ¥_0\frac{\partial \text{P}}{\partial r} + \frac{\partial \text{P}}{\partial e}¥_1 + \sum v_i\frac{\partial \text{P}}{\partial x_i}$.

This equation is the basic equation of the quantitative theory of money. As, into a designation for $\overline{v}$ enter $v_0 = \frac{d\text{M}}{d²}$ also $v_1 = \frac{dV}{d²}$, that in our arrangement, there is

$$\text{P}(t) = P_0 e^{-\frac{£_{0u}}{y}t} + \frac{\overline{v}}{£_0 u}(-e^{\frac{-£_0 u}{y}t} + 1), \quad \text{P}(t) = P_0 e^{-\int_0^t (1-£-MPC)^{-1}\frac{u}{y}d¶} + \int_0^t \frac{\overline{v}}{y} e^{-\int_¶^t (1-£-MPC)^{-1}\frac{u}{y}d¶} d¶$$

a choice of their change, that is change of rates of the offer of money and speed of the reference of money. These rates are allowable by management, and they are defined from the decision of some of a typical problem of optimum control. At $^{2} = t,$ from receiving equation we shall receive the formulas:

From which at a constancy u, $\bar{v}, A, y$ we have: $£_0 = (1 - £ - MPC)^{-1} > 0$

This formula characterizes time change price levels at a constancy of other parameters (see fig. 4).

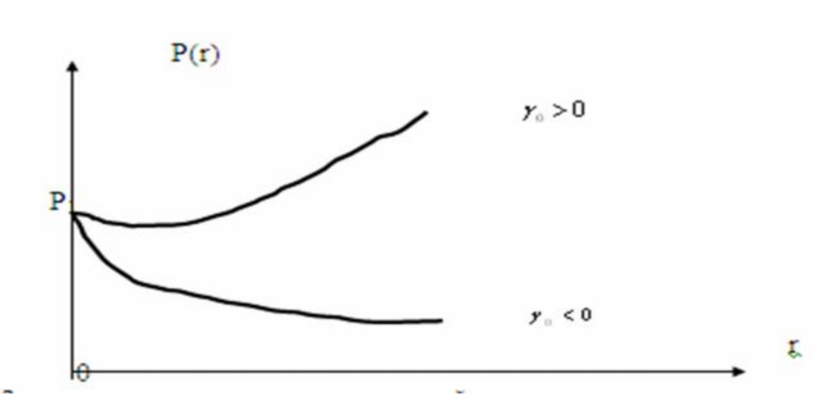


Fig4. Dependence price levels from the real rate of interest at constant parameters $(P_1 = \frac{\bar{v}}{£_0 ¥ u}, ¥_0 = \frac{dr}{dt} = const$ *)*

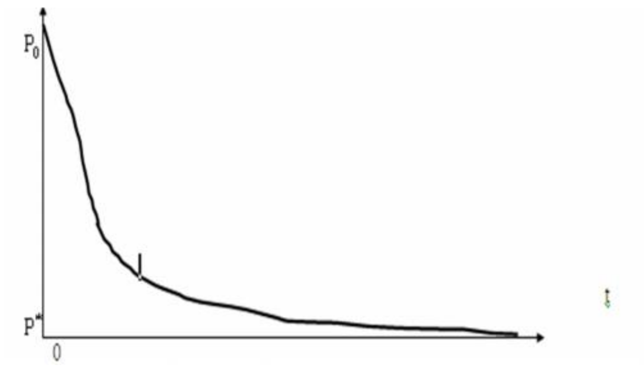


Fig 5. Dependence price levels from the real rate of interest at constant parameters

If ² $= (t, r)$, that we receive the equations in private derivatives of 1-st order of next type:

$$\frac{\partial \mathrm{P}}{\partial t} + ¥_0 \frac{\partial \mathrm{P}}{\partial r} = \frac{£_0 u}{y}\mathrm{P} + \frac{\overline{v}}{y}.$$

At $t \to \infty$, the solution of this equation (with a condition $\mathrm{P}_{/r=0} = \mathrm{P}_1$) is submitted on Fig 3. For solution of this equation we shall set also initial conditions $\mathrm{P}_{/t=0} = \mathrm{P}_0(r)$ and boundary conditions such as formation (education) price levels depending on parameter r, that is

$$\mathrm{P}_{/r=r\max} = \int_0^{r\max} ¨(r)\mathrm{P}(r,t)dr. \text{Here } ¨(r) \geq 0, \int_0^{r\max} ¨(¶)d¶ = 1, ¥_0 = \frac{dr}{dt} = const.$$

The received problem represents an example of problems with functional entry conditions which are entered and investigated in works of the author [14]. It is easy to see that the solution of equation in this case represents as:

$$\mathrm{P}(r,t) = \mathrm{P}(0, t - \frac{r}{¥_0})e^{£_0 \int_r^{r\max} \frac{u}{y}d¶} - \int_r^{r\max} \frac{\overline{v}}{y} e^{\int_r^{¶} £_0 \frac{u}{Y}du} d¶$$

Function $«(t) = P(0,t)$ we shall define from a boundary condition of formation (education) of the prices, that is

$$«(t) = \int_0^{r\max} ¨(r)e^{£_0 \int_r^{r\max} \frac{u}{Y}d¶} \cdot «(t - \frac{£}{¥_0})d¶ + f_0(t), \text{where } f_0(t) = -\int_0^{r\max}\int_r^{r\max} \frac{\overline{v}}{y} e^{£_0 \int_r^{r\max} \frac{u}{Y}du} d¶dr.$$

The equations restoration represents non-uniform integrated the equations of type. At $t \to \infty$ we shall receive:

$$\mathrm{P}(r) = \mathrm{P}(r_{\max})e^{£_0 ¥_0^{-1} \int_r^{r\max} \frac{u}{Y}d¶} - \frac{1}{¥_0}\int_r^{r\max} \frac{\overline{v}}{y} e^{\frac{£_0}{¥_0}\int_r^{¶} \frac{u}{Y}du} d¶ \quad ,$$

where

$$\mathrm{P}(r_{\max}) = \frac{-\frac{1}{¥_0}\int_0^{r\max}\int_r^{r\max} \frac{\overline{u}}{Y} \ell^{\frac{£_0}{¥_0}\int_r^{¶} \frac{u}{Y}du}}{1 - \int_0^{r\max} ¨(r)\ell^{\frac{£_0}{¥_0}\int_r^{r\max} \frac{u}{Y}dud¶}} \geq 0 \cdot$$

From this formula, at constants Y, u$\frac{1}{v}$, we have

$P(r) = \frac{\bar{v}}{£_0 u} + \left[P(r_{\max}) - \frac{\bar{v}}{£_0 u} \right] \ell^{\frac{£_0 u}{¥_0 Y}(r_{\max} - r)}$. The received formula is interpreted as the following figure.

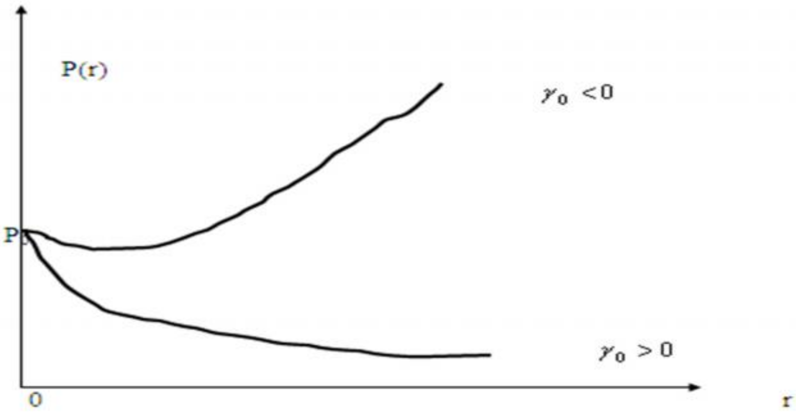


Fig.6.*Dependence price levels from the real rate of interest at constant parameters* $\left(P_0 = P(r_{\max}), ¥_0 = \frac{dr}{dt} = const \right)$.

Let's notice, that figures 1,2 are identical, though is present different boundary conditions. Similarly, we consider the case, when $^2 = \ell, ^2 = x$ *and* $^2 = (t, r, e, x)$.

§8. Model of interaction of countries with different economic level of development

It is known, that in epoch of stable and steady growth of economics the method on the basis of statistical processing various economic parameters yielded normal results. But the situation has sharply changed with the beginning of an economic crisis. The forecast on the basis of the analysis of long-term tendencies became unusable. Burst world financial and economic crisis, which approach became for many unexpectedness, has sharply raised the question about opportunities of economic forecasting, about ability of a science adequately to describe complex social processes and to predict their development. And the decision in such cases is with the help of other methods.

Methods of nonlinear dynamics (changes) concern to these methods, economic synergetrics, aimed on the description of none equilibrium processes, on the analysis laws destructions old and formations of new social and economic structures. Thus, we consider the use of nonlinear dynamics for research of economic and sociopolitical processes in scale the separate country and the world as a whole, the basic attention having given the analysis: laws of formation of steady social and economic structures; laws of transients, crises, and phase transitions from one structure to another. For development of economy of the countries, there are countries with HP and the countries with LP with following the circuit of development:

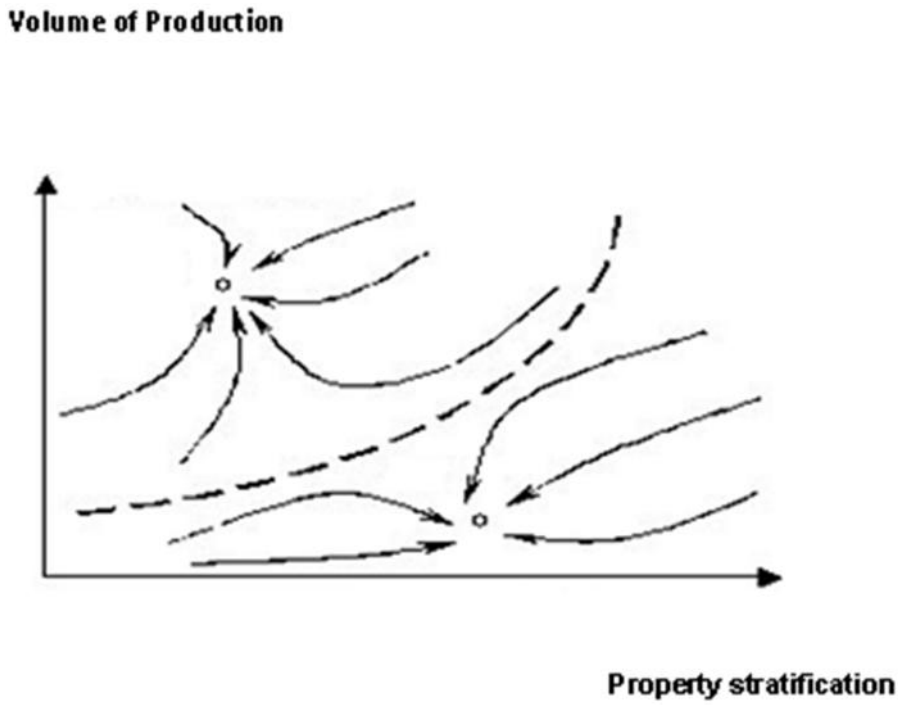


Fig 7.The countries with high production (HP) and low production (LP).

and mathematical model of type

$$\frac{dx_i}{dt} = a_i x_i^2 - \sum_{j=1}^{n} (a_{ij} x_i - \sim_j b_{ij} y_j), \quad \frac{dy_i}{dt} = a_i y_i^2 - \sum_{j=1}^{n} (b_{ij} y_i - \;_j a_{ij} x_j),$$

$i = 1...,n;\ 0 \le t \le t_k, \sum_j \sim_j = 1, \sum_j \;_j = 1,$ where $x_i, y_i, i = 1,...,n;$ - gross national product per capita cooperating countries with high production (*HP*) and low production (*LP*), a_{ij} - the factor describing intensity of interaction, x_0, y_0 initial conditions ($x_{i0} > y_{i0}$, that is y_i - countries - leaders, x_i - catching up countries).

Example. We shall consider a case when we have system “one catching up country with two leaders” then the written model is higher looks like

$$\frac{dx_i}{dt} = ax^2 - (d_1 x - \quad {}_1 d_2 y - ¥_1 d_3 z),$$

$$\frac{dy_i}{dt} = by^2 + (\sim_1 d_1 x - d_2 y + ¥_2 d_3 z),$$

$$\frac{dz_i}{dt} = cz^2 + (\sim_2 d_1 x + \quad {}_2 d_2 y - d_3 z),$$

where ***x(0)=x*$_0$*< y(0)=y*$_0$*, x*$_0$*< z(0)=z*$_0$**, ***α$_j$,β$_j$,γ$_j$,a,b,c,d$_j$*** are parameters of the equation. It is easy to see, that in the field of values

(x,y,z) ≥0:
$\beta_1 d_2 y+\gamma_1 d_3 z \le d_1^2/4a,$
$\alpha_1 d_1 y+\gamma_2 d_3 z \le d_2^2/4b,$
$\alpha_2 d_1 x+\beta_2 d_2 y \le d_3^2/4c$

the given system has two equilibrium conditions of type fig. 1.
Using differential system of this example it is possible to show, that all of its solutions satisfy to a condition for total national income

$$x+y+z=\int_0^t (ax^2+by^2+cz^2)dt+C_0,$$

where C_0 - means the left part (2) at $t=0$, i.e. the Gross National Product (GNP) countries at the initial moment of time. It is necessary to note, that the left part (2) represents GNP all countries at the moment of time t, and the right part (integral) is energy of system (« energy of economy »). For the decision of the received problem we shall take advantage system Borland DELPI c modeling given on

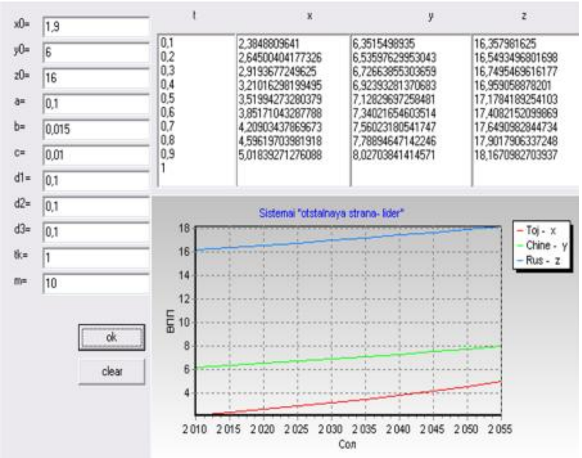


Fig.8. Results of computer experiments at a =0,1.

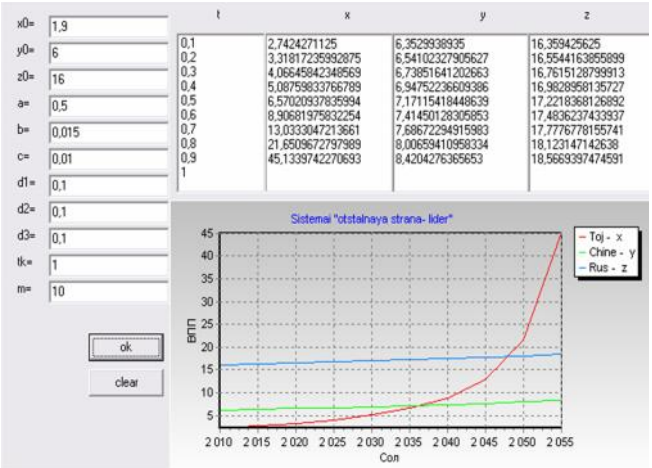


Fig. 9. Results of computer experiments at a =0,5.

fig. 8 then we shall receive results were catching up country very strongly lags behind the countries of leaders. Let's operate now the given system with the help of parameter, ***a*** from an interval **(0,1).** We shall **t**ake for example parameter ***a*** **=0.5** then we have *(fig 9).*Thus, as a result of successful management with the help of parameter ***a*,** we shall receive transitions of the country from crisis state approximately in 2047 on a highway of advanced production.

§9. MODEL OF POPULATION TURBULENCE AND CATASTROPHE IN EXTREME ECONOMICS

This item deals with a study of turbulence models of population with the time-space distribution of age-related changes in the parameters (diffusion coefficient) in a defined area of nonlinear equations. Consider a model population with the time-space-age distributions :

$$\begin{cases} \dfrac{\partial N}{\partial t}+\dfrac{\partial N}{\partial a}+\sum\limits_j g_j\dfrac{\partial N}{\partial x_j}=F_0(a)N+\sum\limits_j D_j\dfrac{\partial^2 N}{\partial x_j^2},\ 0<x_j<L_j,\ 0<a\le a_{\max},\quad 0<t\le t_k, \\ N(x,a,0)=N_0(x,a),\quad 0\le x\le L_j,\ 0\le a\le a_{\max}, \\ N(x,0,t)=\int\limits_0^{a_{\max}} B_0(\P)N(x,\P,t)d\P,\quad 0\le x_j\le L_j,\ 0\le t\le t_k, \\ \dfrac{\partial N}{\partial x}-\sim_j N\Big|_{x_j=(0,L_j)}, \end{cases} \qquad (1)$$

where $N=N(x,a,t)$ is the size of the population at the point x, age a, at time t, $F_0=F_0(a)$ is death rate, $B_0=B(a)$ is birth rate, $N_0=N_0(x,a)$ is the population size at the initial time. On the base of our replacement $a^{'} = a,\ t^{'} = a + ²,\ ¨(x,a,²) = N(x,a,a+²),$

$$U(x,a,²) = ¨(x,a,²)exp\left(\int_0^a F_0(¶)d¶ + \sum_j g_j \frac{x_j}{2D_j} - \sum_j \frac{g_j^2 a}{4D_j}\right),$$

instead of (1) we obtain the problem:

$$\begin{cases} \dfrac{\partial u}{\partial a} = \sum_j D_j \dfrac{\partial^2 u}{\partial x_j^2},\ 0 \le x_j \le L_j,\ 0 < a \le a_{max},\ 0 < t \le t_k \\ U(x,0,²) = \int_0^{a_{max}} B_0(¶)U(x,¶,t)d¶, \\ \left.\dfrac{\partial u}{\partial x_j}\right|_{\substack{x_j=0 \\ x_j=L_j}} = 0. \end{cases} \qquad (2)$$

Suppose that. $D_j = D\sim_j,\ \sim_j \in M$, where $M = \left\{\sim : \sim_0 = (\sim_1,...,\sim_m),\ \sum_j \sim_j^{\frac{n}{n-s}} = 1\right\}$

Using the idea of the method of separation of variables for the problems (2) formulate two classes of possible solutions:

1). Class of simple solutions $D_j \dfrac{\partial^2 u}{\partial x_j^2} = C_j,\ \dfrac{\partial u}{\partial a} = C.$

2). Class of exponential solutions $D_j \dfrac{\partial^2 u}{\partial_j^2} = C_j u,\ \dfrac{\partial u}{\partial a} = Cu,$ which D defines the representation $D_j = D\sim_j,\ \ j = 1,2,\ \ 0 \le \sim_j \le 1.$

The definition. A population "turbulence" in the framework of the model (2) (or (1)), we call this state population, which at a certain value of the vector $\sim,\ \sim = (\sim_1,...,\sim_m) \in M$, the value

$$\left(\sum_{j=1}^m \sim_j \left(\frac{\partial^2 u}{\partial x_j^2}\right)^s\right)^{1/s}, \qquad s > 0, \qquad (3)$$

takes its maximum value, i.e.

$$\frac{\partial u}{\partial a} = \max_{\sim \in M} \sum_{j=1}^m \sim_j \frac{\partial^2 u}{\partial x_j^2}. \qquad (4)$$

The following theorem:

Theorem 1. The Equation $Z = \max_{\sim \in M} (\sim, X^s)^{1/s}$ and the equation $Z^n = \sum_{j=1}^{m} X_j^n$ are equivalent. Now we consider the equation

$$\frac{\partial u}{\partial a} = \max_{\sim \in M} \left(\sim \left(\frac{\partial^2 u}{\partial x^2} \right)^s \right)^{1/s} \quad (5)$$

Theorem 2. Let conditions 1.2 then there is a value of M for which the equation (5) and the equation

$$\left(\frac{\partial u}{\partial a} \right)^n = \sum_{j=1}^{m} \left(D \frac{\partial^2 u}{\partial x_j^2} \right)^n \quad (6)$$

equivalent. Indeed, under the conditions of 1.2 (5) is transformed into the equation of the type that corresponds to the maximization of the functional (3). Thus, equation (6) $Z^n = \sum x_j^n$, is the equation of the "turbulence", i.e addressed by the functional (3) takes the maximum value. Solve the equation (6) in the class of possible simple solutions: $\frac{\partial u}{\partial a} = C,\ \frac{\partial^2 u}{\partial x_j} = C_j,\ \sum_{j=1}^{m} C_j = C.$

Theorem 3. The solution of equation (6) in the class of possible simple solutions can be represented as

$$U(x,a,^2) = U(0,a,^2) + \sum_{i=1}^{m} \frac{\partial u}{\partial x_j}\bigg|_{x=0} x_j + \sum_{j \neq j} \frac{\partial^2 u}{\partial x_i \partial x_j}\bigg|_{x=0} x_i x_j + \sum_{j=i}^{m} C_j \frac{x^2}{2},\ ^2 = t - a. \quad (7)$$

Proof. For simplicity, the proof proceeds in the case where m = 2 and D = 1.

Consider the class of simple solutions $\frac{\partial u}{\partial a} = C,\quad \frac{\partial^2 u}{\partial x_1^2} = 0,\ \frac{\partial^2 u}{\partial x_2^2} = 0.$

It is easy to see that it follows $\frac{\partial u}{\partial a} = C,\quad \frac{\partial^2 u}{\partial x_1^2} = 0,\ \frac{\partial^2 u}{\partial x_2^2} = 0$. Delivering this into condition, $\frac{\partial^2 u}{\partial x_2^2} = 0$, we obtain (7). Formula (7) corresponds to the maximization problem (3) that sum of the second derivatives, which corresponds to the "population of turbulence." Now consider the case where the population in the process of turbulence at which point the field reached the maximum number of populations. From $\max_{(x_1,x_2)} U(x_1,x_2,a,²)$, we have:

$$\begin{cases} \dfrac{\partial u}{\partial x_1} = \left.\dfrac{\partial u}{\partial x_1}\right|_{(x_1,x_2)=0} + \left.\dfrac{\partial^2 u}{\partial x_1 \partial x_2}\right|_{(x_1,x_2)=0} x_2 + C_1 x_1 = 0 \\ \dfrac{\partial u}{\partial x_2} = \left.\dfrac{\partial u}{\partial x_2}\right|_{(x_1,x_2)=0} + \left.\dfrac{\partial^2 u}{\partial x_1 \partial x_2}\right|_{(x_1,x_2)=0} x_1 + C_2 x_2 = 0 \end{cases}$$

Hence

$$x_1^0 = \frac{\left(\left.\dfrac{\partial^2 u}{\partial x_1 \partial x_2}\right|_0\right) - C_2 \left.\dfrac{\partial u}{\partial x_1}\right|_0}{C_1 C_2 - \left.\left(\dfrac{\partial^2 u}{\partial x_1 \partial x_2}\right)^2\right|_0}, \qquad x_2^0 = \frac{\left.\dfrac{\partial^2 u}{\partial x_1 \partial x_2}\right|_0 \cdot \left.\dfrac{\partial u}{\partial x_1}\right|_0 - \left.\dfrac{\partial u}{\partial x_2}\right|_0 \cdot C_1}{C_1 C_2 - \left(\left.\dfrac{\partial^2 u}{\partial x_1 \partial x_2}\right|_0\right)^2}.$$

Compute the matrix of second derivatives

$$\frac{\partial^2 u}{\partial x_1^2} = C_1, \quad \frac{\partial^2 u}{\partial x_2 \partial x_1} = \left.\frac{\partial^2 u}{\partial x_1 \partial x_2}\right|_0, \quad \frac{\partial^2 u}{\partial x_1 \partial x_2} = \left.\frac{\partial^2 u}{\partial x_1 \partial x_2}\right|_0, \quad \frac{\partial^2 u}{\partial x_2^2} = C_2.$$

Thus, if $(C_1;C_2) > 0$, and $C_1 C_2 > \left.\left(\dfrac{\partial^2 u}{\partial x_1 \partial x_2}\right)^2\right|_0$, $(C_1) > 0$, then at the point $\left(x_1^0,\, x_2^0\right)$ function $u = u(x_1, x_2, a, ²)$ defined by (7) reaches its maximum.

Remark. In (7) all the coefficients $\left.\dfrac{\partial u}{\partial x_j}\right|_0$, $\left.\dfrac{\partial^2 u}{\partial x_1 x_2}\right|_0$, $j = \overline{1,2}$ depend on $(a, t-a)$, i.e, in fact, they are functions and they are determined by the initial conditions $\left(u|_{t=0},\ u|_{a=0}\right)$. Representation (7) can be written as the following polynomial:

$$u = u_0 + u_1 x_1 + u_2 x_2 + u_3 x_1 x_2 + \frac{C_1}{2} x_1^2 + \frac{C_2}{2} x_2^2, \qquad (7')$$

where $u_0 = u(0,a,²)$, $u_1 = \left.\dfrac{\partial u}{\partial x_1}\right|_0$, $u_1 = \left.\dfrac{\partial u}{\partial x_1}\right|_0$, $u_2 = \left.\dfrac{\partial u}{\partial x_2}\right|_0$, $u_3 = \left.\dfrac{\partial^2 u}{\partial x_1 \partial x_2}\right|_0$, $C_1^n + C_2^n = C^n$, $n > 1$.

From this polynomial, it follows that in the case of turbulence population behavior "forgotten" the initial conditions.

Theorem 4. Let the representation (7 ’) u_j, $j = 0,1,2,3$ is a known quantity and C_1, C_2 is a solution $C_1^n + C_2^n = C^n$ for some $n > 1$ *and* $C \geq 4u_3^{2n}$, then every process of population turbulence described function (7’) makes an emergency landing at only two points (C_1^+, C_2^+) *and* (C_1^-, C_2^-), ,

$$C_1^+ = \frac{u_3^2}{C_2^+}, C_1^+ = \left(\frac{C^n + \sqrt{C^n - 4u_3^{2n}}}{2}\right)^{1/n}, C_1^- = \frac{u_3^2}{C_1^-}, C_2^- = \left(\frac{C^n - \sqrt{C^n - 4u^{2n}}}{2}\right)^{1/n}, U_3 = \left.\frac{\partial^2 u}{\partial x_1 \partial x_2}\right|_0 = \left.\frac{\partial^2 u}{\partial x_2 \partial x_1}\right|_0.$$

where the proof follows from the definition of disasters: $\frac{\partial u}{\partial x_i} = 0, \quad i = 1,2, \quad \det K = 0,$

$$\text{where } K = \begin{pmatrix} C_1 & \left.\frac{\partial^2 u}{\partial x_1 \partial x_2}\right|_0 \\ \left.\frac{\partial^2 u}{\partial x_2 \partial x_1}\right|_0 & C_2 \end{pmatrix}.$$

§10. Computational experiments

Crisis. In next figure we show results of computational experiments with economical crisis parameters and we defined the structure of the crisis.

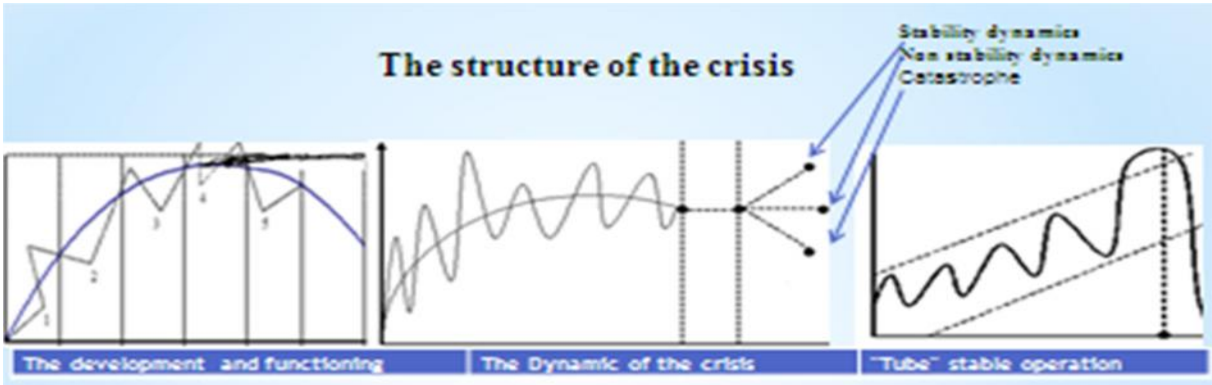
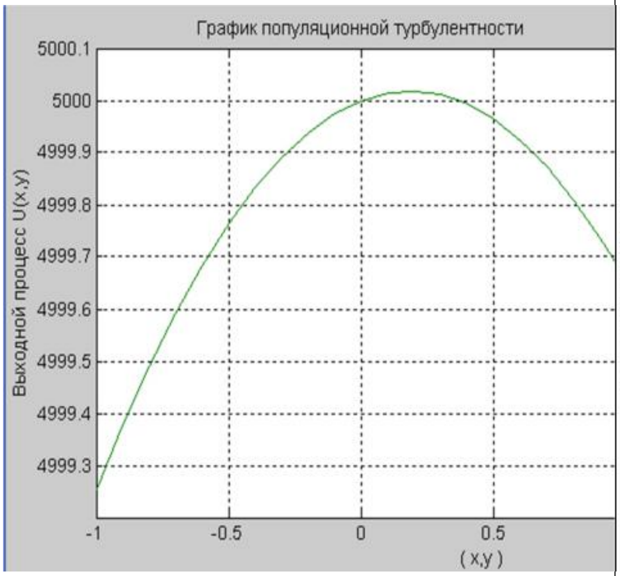


Fig. 10.The structure of the crisis

Population turbulence. Now, here are some computer simulations with the following model data: u0 = 5000; u1 = 0.1; u2 = 0.1; u3 = -0.1; n = 2; c = 1; x = -1: .1: 1; y = -1: .1: 1; c1 = ((c ^ n-(c ^ (2 * n) -4 * u3 ^ 2) ^ (1/2)) / 2) ^ (1 / n); c2 * c2 = u3 ^ 2; c1 ^ n + c2 ^ n = c ^ n. For these data, the following are some of the results of numerical experiments.

u3=-0.1



u3=0.1

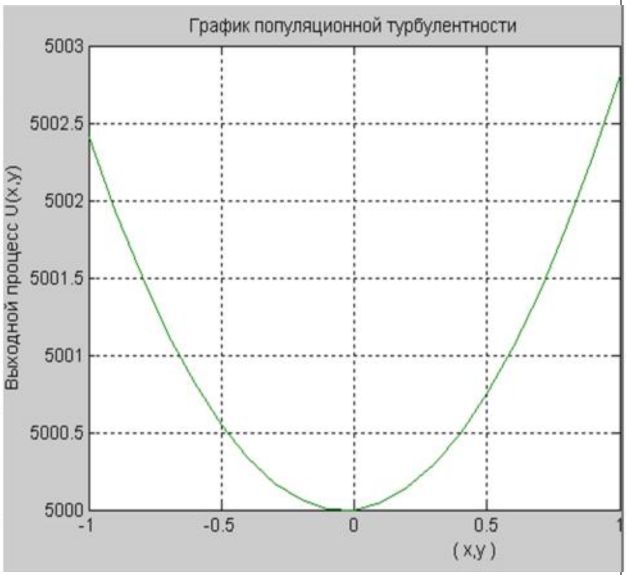
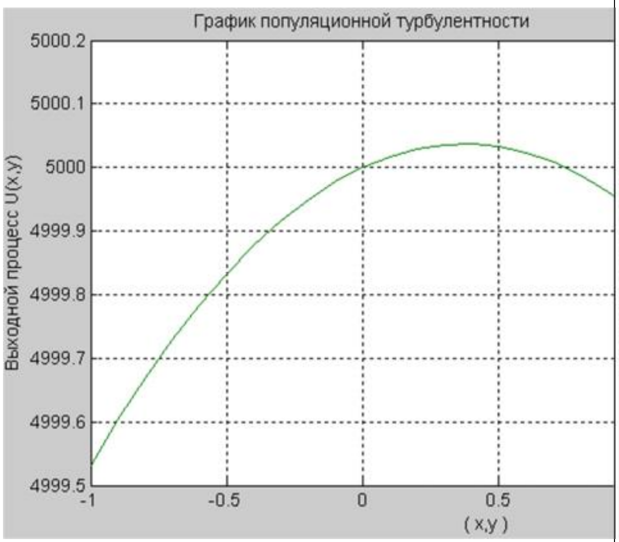


Fig. 10. Dependence of the population turbulence size for different value parameters

u3=-0.000000004



u3=0.000000004

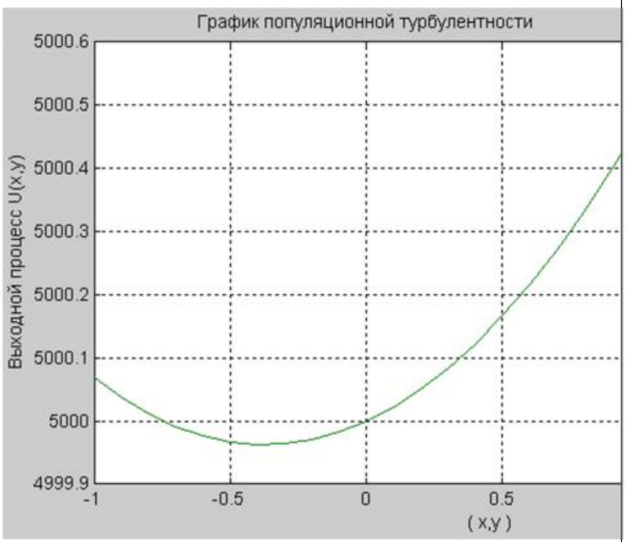
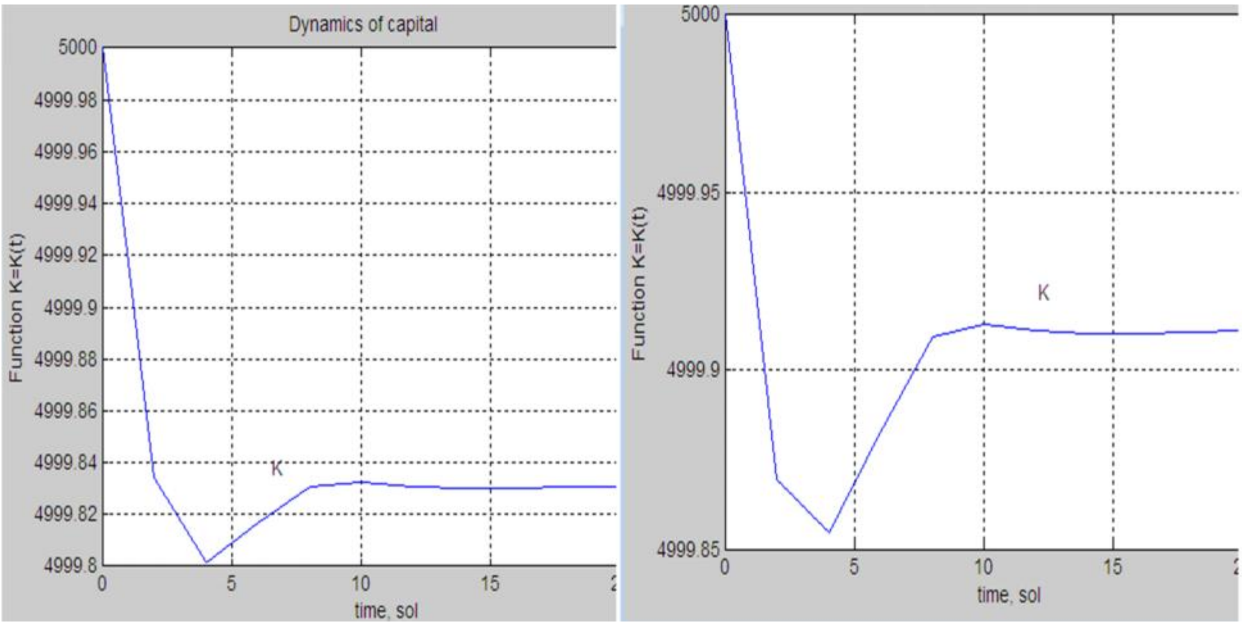


Fig. 11. Dependence of the population turbulence size for different value parameters

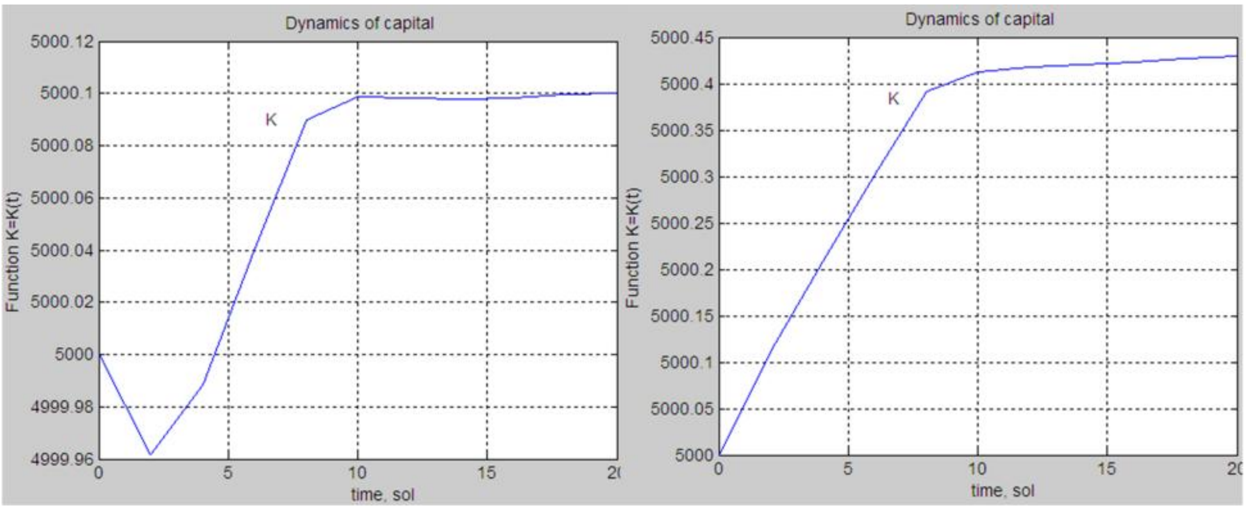
From the given results it is visible, that at change parameters there is "turbulence" population number.

Experiments with capital size in different case. *Now we consider some results of computational experiments with capital size in time extreme regime at next model data* ε*=0.7,a0=1,a1=0.000001,a2=0.00000001,*

f0=1,de=-0.5,be=10, l0=70 ***al=0.1****al=0.2*



al=0.1 **a)** al=0.2 b)



al=0.3c) al=0.4 d)

Fig. 12.Dependence of the capital size on time for different value parameters (case of a)-d))

The Conclusion

Any type of Economics in different periods may by pass through the development crises. In a broad sense, it is a process that threatens the existence of the Extreme Economics in some organizations. Crises often come to the organization's management suddenly. However, in practice, the emergence of the crisis shows many symptoms: loss of income from the sale of goods (works, services), reduction of other indicators of financial and economic activities of the organization. Many companies are not able to overcome such crises develop and leave the market. The very possibility of a crisis is defined risky development of any organization that is always there. Therefore, it is advisable to consider the typology of crises in the development of the organization, the causes and patterns of their occurrence, the criteria specific to the crisis. Any complex objects (country, organization) in the different periods may by-pass through the development crises. In a broad sense, it is a process that threatens the existence of the organization. Crises often come to the organization's management suddenly. However, in practice, the emergence of the crisis shows many symptoms: loss of income from the sale of goods (works, services), reduction of other indicators of financial and economic activities of the organization. Many companies are not able to overcome such crises develop and leave the market. The maximum possibility of a crisis is defined risky development of any organization that is always there. Therefore, it is advisable to consider the typology of crises in the development of the organization, the causes and patterns of their occurrence, the criteria specific to the crisis.

Appendix 1. **Polynomial model information security**

It is possible it used at protection of the information or a kind

$$N_m^p - Y\sum_{i=3}^{m} X^{m-i} Z^{i-2} = X^{(m-1)} + YX^{m-2}$$

where $Z=X+Y$ and all decisions of the equation $x^n + y^n = z^P$ at $p=n$ are represented as

$$x = zt^{1/n}, \quad y = z(1-t)^{1/n}, \quad p = n, \ t \in (0,1).$$

$P_m^p(x,y) = N_m^p$ is a polynomial of a degree *(m-1)* on x_n and it is a class curve higher degrees which are very well used at protection of the information. For example, we shall write a class of such curves in special cases with the help of displays:

$N_m = z^{m-1}$, $a_{1m} = x^{m-1}$, $a_{2m} = yx^{m-2}$, $a_{im} = yz^{\frac{i-2}{n}p} x^{m-i}$, $i = 1, ..., m;$ I.e. curves

$P^p{}_2(x,y) = X + Y$, $P_3^p(x,y) = X^2 + YX + YZ$ and $P_4^p(x,y) = X^3 + YX^2 + YZX + YZ^2$, ….,

and also type $N_4^2 = X^3 + YX^2 + YZX + Z^2Y$, $Z=X+Y$, which belongs to a class elliptic curves. Pairs (x, y) is usual to name "point" which can "be put" about other similar point of an elliptic curve. « The sum » two points, in turn, too "lays" on an elliptic curve. Except for the points laying on an elliptic curve, « the zero point » is considered also. The sum of two points A with coordinates (XA is considered, that, YA) and B with coordinates (XB, YB) is equal O, if XA =

XB, YA = - YB (mod p). The zero point does not lay on an elliptic curve, but, nevertheless, participates in calculations; it can be considered as indefinitely removed from a curve. Set of points of an elliptic curve together with a zero point and with the entered operation of addition, we shall name them "group". For each elliptic curve the number of points in group certainly, but is great enough. The important role in algorithms of the signature with use of elliptic curves is played with "multiple" points. Point Q refers to as a point of frequency rate k if for some point P k time it is executed equality: P = Q + Q + Q +. + Q = kQ. If for some point P there is such number k, that kP = 0, this number is named the order of point P. Multiple points of an elliptic curve are analogue of degrees of numbers in a simple field. The problem of calculation of frequency rate of a point is equivalent to a problem of calculation of the discrete logarithm. Follows noticed that reliability of the digital signature is based on complexity of calculation of "frequency rate" of a point of an elliptic curve and. Though equivalence of a problem discrete logarithm and problems of calculation of frequency rate also is proved, the second has the big complexity. For this reason at construction of algorithms of the signature in group of points of an elliptic curve appeared possible to do without shorter keys in comparison with a simple field at maintenance of the greater stability. A confidential key, consider some random number. The open key considers coordinates of some point on elliptic curve P which is defined as P = xQ where Q - special image the chosen point of an elliptic curve named « a base point. Coordinates of point Q together with factors of the equation specifying a curve, are parameters of the circuit of the signature and they should be known to all participants of an exchange messages. From here follows, that anyone "modern" cryptosystem can "be shifted" on elliptic curves.

Appendix 2. Algebraic representation of tree numbers of some differential models

The Model of Tree Numbers

Definition of numbers tree and its representation. *Let N is some natural number. We shall tell, that the number N forms a tree of numbers if there will be natural numbers $n, m \geq 2$ and integers $a_1, a_2, ..., a_m$ for which*

$$N^n = a_1^n + a_2^n + ... + a_m^n \ , \qquad (1)$$

and in turn some a_j (or all) represented as

$$a_j^n = a_{1j}^n + a_{2j}^n + \ldots + a_{m_1 j}^n\,,\; m_1 \le m \qquad (2)$$

and some a_{ij} *of (2) also can be submitted as*

$$a_{ij}^n = a_{1ij}^n + \ldots + a_{m_2 ij}^n\,,\;\; m_2 \le m_1, \ldots,$$

and at last decomposition takes place

$$a_{ijj_1 \ldots j_{m_k}}^n = a_{1ijj_1 \ldots j_{m_k}}^n + a_{2ijj_1 \ldots j_k}^n\,, \qquad (3)$$

in which members of the right part (3) cannot beat are submitted as the final sum composed n-th degrees of some integers so-called by a basis (basis) of a tree.

Conceptual Model of Numbers Tree in general case is given in fig. 1.

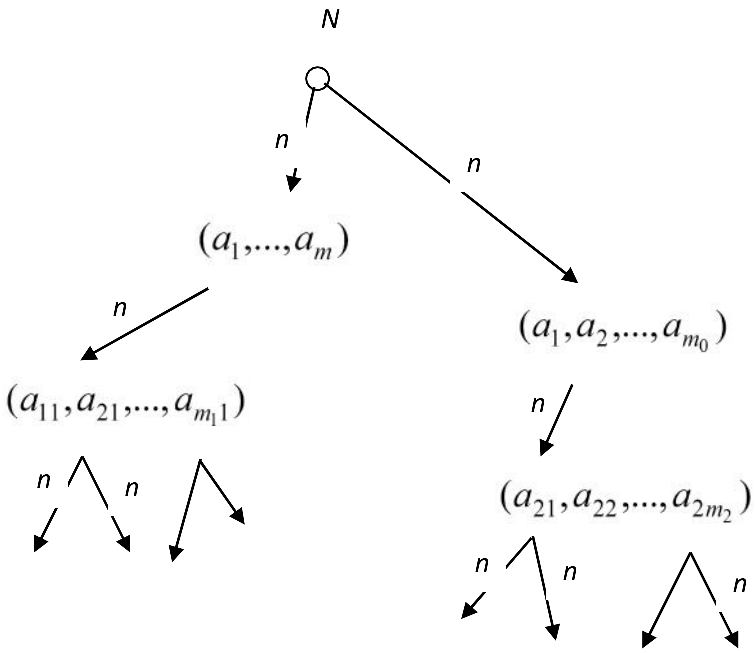


Fig.1. Conceptual Model of Numbers Tree in general case

So last a level the tree consists of the sum such as (3).base elements can enter into each level a tree, therefore from each level we take only those elements,

which not представимы as (2), (3). Then in result the number N *is uniquely represented* as

$$N^n = \sum_{j_q} k_{j_q} a_{ijj_1 \ldots j_q}^n\,, \qquad (4)$$

where k_{j_q} - number of occurrence of a basic element $a_{ijj_1 \ldots j_q}$ in a tree of numbers.

We shall consider a problem for number N and a vector $a=(a_1,...a_k),\ k\geq 2$ from the equation

$$N = \max_{\sim \in M} (\sim, a), \text{ where } M=\left\{(\sim_1,...,\sim_k)=\sim;\ \sum_{j=1}^{k} \sim_j^{\frac{n}{n-s}}=1,\ n>s>0,\ 0<\sim_j<1\right\}.$$

The set M represents a measured curvilinear spheroid and at $s=1,\ n=2$ turns in usual m-a measured spheroid. Using theorems 1 from work [12] we shall receive:

$$\begin{cases} N^n = a_1^n + ... + a_k^n, \\ N_k^n = a_{1k}^n + ... + a_{kk}^n, \quad k \geq 2 \end{cases} \tag{4'}$$

Thus, this equation is optimum in sense (4), and the tree of numbers appropriate by this equation represented on rice. 1 also is an optimum tree.

Now, we consider questions of the best representation some character of any objects (elements) with the help of some properties (elements) given object and its applications in some cases where distributions processes of growth processes, heats and waves processes, diffusion processes, at some parameters values take place in the maximal regimes. Extreme regimes of such physical processes are arising in the case when the values of their parameters are chaining in some given set. They can be an accumulation of the warmth, particles, wave energy in some areas where the considered physical processes are arising. Series computational experiments also carried out with models data for some considering heat transfer processes under complex conditions and in extreme regime. Let$H$and L - some normalized spaces and the set of M is given .Let's assume, that for any objects (elements) $z\in H$ and any properties of objects

$x_j \in L,\ j=\overline{1,m}$ norms $P=\|z\|_H,\ h_j=\|x_j\|_L,\ j=\overline{1,m},\ m>1$ and also ways of display of properties of the considered objects are determined. For example, we can define their norms in the appropriate spaces, i.e.

$$P = «(\sim) = \left(\int_T \left(\sum_{j=1}^{m} \sim_j \left| x_j \right|^s \right)^{\frac{n}{s}} dt \right)^{1/n}, \quad \|x_j\|_{L_m^n(T)} = \left(\int_T \sum_{j=1}^{m} \left| x_j \right|^n dt \right)^{1/n} < \infty \cdot$$

Definition. We shall tell, that any object (element) $z \in H$ is in the best way submitted with the help of some properties (elements) $x_j \in L,\ j = \overline{1,m}$,

if for some element $\sim^0 \in M$ fairly a parity $P = \max\limits_{\sim \in M} (\sim, h^s)^{1/s} = \left(\sim_1^0 h_1^s + ... + \sim_m^0 h_m^s \right)^{1/s}$ For anyone $s > 0$ and $s < n < \infty.$.

The Theorem 1. That the object (element) $z \in H$ the best is submitted (maximal) image through properties $x_j \in L,\ j = \overline{1,m}$, it was necessary and enough that had places about parity:

$$\sim_j \frac{n}{n-s} = \frac{h_j^n}{\sum\limits_{j=1}^{m} h_j^n},\ j = \overline{1,m} \qquad ,\ n > s > 0,\ P^n = \sum_{j=1}^{m} h_j^n$$

Example 1. *Distribution problem of resources*. As an example it is possible to result a problem of representation of a monetary stream, or a word at coding etc. as the sum m numbers:

$\left(\sum\limits_{j=1}^{m} \sim_j h_j^s \right)^{\frac{1}{s}}$ then $P = \max\left(\sim, h^s \right)^{1/s}$ we shall receive the equation $P^n = \sum\limits_{j=1}^{m} h_j^n$.

Last equation for anyone m > 2, n > 1 has accounting number of integer decisions $h_1, h_2, ...h_m$ and P .

Example 2. *Growth of plant*. Let us consider model of growth of plants with *m, m≥2* parts (for example, roots, trunk, leaves, buds). Let first part makes means of production biomass x_1 , which can be spent for development of all other parts of plant. Let $x_j(t),\ \ j = \overline{2,m}$ are biomass of part now of time t. Development of plant we shall set to the following model:

$$\dot{x}_1 = \sim_1 f(x_1, l),\ \dot{x}_j = \sim_j x_j,\ x_1(0) = x_1^0,\ x_j(0) = x_j^0,\ j = \overline{2,m}\ ,$$

$$I(\sim) = \ddot{} \, (x(t_{t_k}), t_k) \quad - \max \quad , \quad \sim = (\sim_1, ..., \sim_m) \in M, \quad 0 \le t \le t_k ,$$

where $x_j^0 \ge 0$, $j = 1, ..., m$ are given numbers, $f = f(.)$ the law of formation of a new biomass, l - size of photosynthesis parameters. On the base of our theorem Bellman function

correspondents to optimal control problem satisfy next equation

$$-\frac{\partial «}{\partial t} = \max_{\sim \in M} \left\{ \left(\sim , x_1 \frac{\partial «}{\partial x} \right) \right\} \text{ i.e.} \left(\frac{\partial «}{\partial t} \right)^n = \sum_{j=1}^{m} \left(x_1 \frac{\partial «}{\partial x_j} \right)^n$$

or $$« (x_1, x_2, ..., x_m, t) = «_0 + Ct + \ln \left| \left(\frac{x_1}{x_1^0} \right)^{C_1} \left(\frac{x_2}{x_2^0} \right)^{C_1} ..... \left(\frac{x_m}{x_m^0} \right)^{C_m} \right| , \quad C_1^n + C_2^n + ... + C_m^n = C^n$$

<u>Example 3</u>. *Differential equations with extreme properties*. Many processes (distributions processes of heats and waves, diffusion processes) belong to so-called model equations with extreme properties. In the general case such equations may be represented in the form of:

$$Lu = \max_{\sim \in M} \left\{ \sum_{j=1}^{m} \sim_j \left(L_j u \right)^s \right\}^{1/s}, \text{ or } (Lu)^n = \sum_{j=1}^{m} \left(L_j u \right)^n, \quad n > s > o,$$ here L, L_j are given operators, which characterize the considered physical processes.

Let it is given area $G \subseteq E^m$ with border Γ: $\Gamma = \cup \Gamma_j$, and then it is known, that the temperature mode inside area G and wave process satisfy to next equations[2]:

$$\frac{\partial u}{\partial t} = \sum_{j=1}^{m} \frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right), \quad x \in G, \quad o <\le t \le t_k ,$$

$$\frac{\partial^2 u}{\partial t^2} = \sum_{j=1}^{m} \frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right), \quad x \in G, \quad o < t \le t_k$$. Here

$k_j = k_j(\cdot)$, $j = \overline{1, m}$ are coefficients of heat conductivity (or wave velocity) characterizing a version of environment. We shall make the following assumption. Let the environment of heat conductivity is those, that

$$\frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right) = \sim_j \frac{\partial}{\partial x_j} \left(k \frac{\partial u}{\partial x_j} \right), \quad j = \overline{1, m}, \quad \text{where } \sim_j(\cdot) \ge o, \quad \sum_{j=1}^{m} \sim_j^{\frac{n}{n-s}} = 1, \; n > s > o, \; m > 1,$$

$\sim = (\sim_1 \ldots \sim_m) \in M$. The equation of heat conductivity (1) corresponds (meets) a case when s=1, i.e. heat exchange occurs under the usual law of Newton. Therefore at a choice of a set of parameters $\sim = (\sim_1 \ldots \sim_m)$ from M at $s=1$. Then we have the equations with extreme properties

$$\frac{\partial u}{\partial t} = \max_{\sim \in M|_{s=1}} \sum_{j=1}^{m} \sim_j \frac{\partial}{\partial x_j}\left(K \cdot \frac{\partial u}{\partial x_j}\right), \quad \text{and} \quad \frac{\partial^2 u}{\partial t^2} = \max_{\sim \in M|_{s=1}} \sum_{j=1}^{m} \sim_j \frac{\partial}{\partial x_j}\left(K \frac{\partial u}{\partial x_j}\right), \; x \in G,\; o < t \le t_k \qquad (2)$$

And therefore, on the basis of the basic theorem we shall receive the nonlinear equations of type

$$\left(\frac{\partial u}{\partial t}\right)^n = \sum_{j=1}^{m}\left[\frac{\partial}{\partial x_j}\left(K \frac{\partial u}{\partial x_j}\right)\right]^n, \; \begin{matrix} x \in G \\ o < t \le t_k \end{matrix} \quad \left(\frac{\partial^2 u}{\partial t^2}\right)^n = \sum_{j=1}^{m}\left[\frac{\partial}{\partial x_j}\left(K \frac{\partial u}{\partial x_j}\right)\right]^n, \; x \in G,\; o \le t \le t_k \qquad (3)$$

The right parts of last equations corresponds to a maximum quantity of heat and a wave in area G, formed as a result of maximization of the previous equations (2) on a set $\sim \in M|_{s=1}$. For the solutions of last equations (for example, the first equation (3)) is necessary to set a class of possible solutions or simple type,

$\frac{\partial u}{\partial t} = c$, $\frac{\partial}{\partial x_j}\left(K \frac{\partial u}{\partial x_j}\right) = c_j$, $j = \overline{1,m}$, or exponent type $\frac{\partial u}{\partial t} = cu$, $\frac{\partial}{\partial x_j}\left(K \frac{\partial u}{\partial x_j}\right) = c_j u$, $j = \overline{1,m}$,

where c_j, c are solutions of the coordination equation: $\sum_{j=1}^{m} c_j^n = c^n$.

The Lemma 1. Let area G is a rectangular with the sides: l_1, l_2 , And in the first equation (3) is given (similarly for the second), and also are given initial and boundary conditions: $u|_{t=0} = u_0(x_1, x_2)$, $(x_1, x_2) \in \bar{G}$, $u|_{x_j=0} = 0 = u|_{x_j = l_j}$, $j = \overline{1,2}$. Then the solution is represented in the following kind: $u(x_1, x_2, t) = \frac{2}{\sqrt{l_1 l_2}} \sum_{n_1, n_2 = 1} D_{n_1 n_1} e^{c_{n_1 n_2} t} \sin\frac{® n_1}{l_1} x_1 \sin\frac{® n_2}{l_2} x_2$, where $D_{n_1 n_2}$ are coefficients Fourier of function $u_0(x_1, x_2)$, and parameters $c_{n_1 n_2} = c$ are the solutions of the coordination equation $c_1^n + c_2^n = c^n$, $c_{n_1 n_2} = n\sqrt[n]{\left(\frac{® n_1}{l_1}\right)^n + \left(\frac{® n_2}{l_2}\right)^n}$, $n_j = 2,3,\ldots, j = \overline{1,2}$.

The Lemma 2. Let $G \subset E^2$ is a rectangular with the sides l_1, l_2 and next initial and boundary conditions are given $u\Big|_{t=0} = u_0(x_1, x_2)$, $u_t\Big|_{t=0} = u_2(x_1, x_2)$, $u\Big|_{x_i = 0} = 0$, Then the solution of the second equation (3) is represented as

$$u(x_1, x_2, t) = \sum_{n_1 n_2 = 1} \left(A_{n_1 n_2} \cos\sqrt{c_{n_1 n_2}}\, t + B_{n_1 n_2} \sin\sqrt{c_{n_1 n_2}}\, t \right) \sin\frac{(2n_1 + 1)®x_1}{2l_1} \sin\frac{(2n_2 + 1)®x_2}{2l_2},$$

where $c_{n_1 n_2} = \sqrt[n]{\left(\frac{(2n_1 + 1)®x_1}{2l_1}\right)^n + \left(\frac{(2n_2 + 1)®x_2}{2l_2}\right)^n}$, $A_{n_1 n_2}$, $B_{n_1 n_2}$ are defined from initial conditions.

Example 4. *The function* $\cdot$. Let there is a bunch of particles with the certain pulse and some energy. It is known, that such bunch of particles is described by wave function $\cdot$, i.e. amplitude of probability. This wave falls on the screen in a crack, and is further from these cracks leaves as a spherical wave and on the following screen, these waves are unreferenced. Summing wave from the top and bottom cracks of the screen is organized as a wave $\cdot = \sum_{j=1}^{k} \sim_{j,k} \cdot_{jk}$,

where $k = 2$ for one particle and $k = 2^m$ for m particles, $\sim_{jk}$ accordingly shares of waves leaving cracks in general summing waves. We shall assume, that $\sum_{j=1}^{k} \sim_{jk}^{\frac{n}{n-s}} = 1$, where $\sim^n_{jk}$ the appropriate probabilities, n, s parameters of the environment of distribution (for example $n = 2,\ s = 1$). Then considering integrated approach wave function, i.e. $\cdot = P + iP^*$, $\cdot_{jk} = h_{jk} + ih_{jk}$ on the basis of a principle of optimum representation we have: $extrP_k^n = \sum_{j=1}^{k} h_{jk}^n$, $extrP_k^{*s} \cdot extrP_k^{n-s} = \sum_{j=1}^{k} h_{jk}^{*s} h_{jk}^{n-s}$,where $extr$=(max at n>s, and min at n<s). The first formula is characterized probability of particles detection, and the second is interference picture. Using « - transformation $h'_{ik+1} = x\, h_{ik}$, $h'_{k+1k+1} = y\, extrP_k$, $P'_{k+1} = z\, extr\, P_k$ we are easily established connections by conditions for various particles. Here *(x, y, z)* is some solutions of the equation $x^n + y^n = z^n$.

Example 5. *Some properties of* « $-$ *function and it application*. Let's consider so-called « – function:

$$\text{«}(\sim)=\left(\sim x^{s}+\left(1-\sim^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}} y^{s}\right)^{1/s}, \quad s>0,$$

where x, y - positive numbers, n, s - natural numbers, $0<\sim<1$.

<u>*Properties 1.*</u> Function $\text{«}(\sim), 0<\sim<1$ in a point $\sim^{*}=\left[\frac{x^{n}}{x^{n}+y^{n}}\right]^{\frac{n-s}{n}}$ at $s<n$ has maximal, and at $s>n$ minimal values equal $z=\left(x^{n}+y^{n}\right)^{1/n}$,

i.e. $x^{n}+y^{n}=z^{n}$, where $z=\max\limits_{0<\sim<1} \text{«}(x)$ at $s<n$ and $z=\min\limits_{0<\sim<1} \text{«}(x)$ at $s>n$.

Really, as

$$\frac{d\text{«}}{d\sim}=\frac{1}{s}\left[\sim x^{s}+\left(1-\sim^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}} y^{s}\right]^{\frac{1-s}{s}} \cdot\left[x^{s}-\sim^{\frac{s}{n-s}}\left(1-\sim^{\frac{n}{n-s}}\right)^{-\frac{s}{n}} \cdot y^{s}\right]$$

and

$$\frac{d^{2}\text{«}}{d\sim^{2}}=\frac{1}{s}\left(\frac{1}{s}-1\right)\left[\sim x^{s}+\left(1-\sim^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}} \cdot y^{s}\right]^{\frac{1}{s}-2}\left[x^{s}-\sim^{\frac{s}{n-s}}\left(1-\sim^{\frac{n}{n-s}}\right)^{-\frac{s}{n}} \cdot y^{s}\right]^{2}+$$

$$-\frac{1}{n-s}\left[\sim^{\frac{s}{n-s}-1}\left(1-\sim^{\frac{n}{n-s}}\right)^{-\frac{s}{n}}+\left(1-\sim^{\frac{n}{n-s}}\right)^{-\frac{s}{n}-1} \cdot \sim^{\frac{2s}{n-s}}\right] \cdot y^{s}$$

$$\left[\sim x^{s} + \left(1 - \sim^{\frac{n}{n-s}} \right)^{\frac{n-s}{n}} \cdot y^{s} \right]^{\frac{1-s}{s}},$$

that from a condition $\left. \frac{d«}{d\sim} \right|_{\sim *} = 0$ we have $\sim * = \left[\frac{x^{n}}{x^{n} + y^{n}} \right]^{\frac{n-s}{n}}, 0 < \sim^{*} < 1$. It is easy to see, that $\left. \frac{d^{2}«}{d\sim^{2}} \right|_{\sim *} < 0,$ at $s < n$ and $\left. \frac{d^{2}«}{d\sim^{2}} \right|_{\sim *} > 0,$ at $s > n$. Hence, the point $\sim *, 0 < \sim * < 1$ at $s < n$ is unique

point of a maximum of function « : $z = \max\limits_{0 < \sim < 1} «(x) = «(\sim *)$ and at $s < n$ is a unique point of a minimum of function « : $z = \min\limits_{0 < \sim < 1} «(x) = «(\sim *)$. Besides $z = \left(x^{n} + y^{n} \right)^{1/n}$ and $x^{n} + y^{n} = z^{n}.$

<u>*Properties 2.*</u> If *(x, y, z)* is the decision of the equation $x^{n} + y^{n} = z^{n}$ at some n the point $\sim = \left(\frac{x^{n}}{x^{n} + y^{n}} \right)^{\frac{n-s}{n}}$ is a unique point extreme of the functions $«(\sim), 0 < \sim < 1.$

Really, let (*x, y, z)* is the decision of the equation $x^{n} + y^{n} = z^{n}$. As $x^{n-s} \cdot x^{s} + y^{n-s} y^{s} - z^{n-s} z^{s}$, having entered designations

$\sim = \left(\frac{x}{z} \right)^{n-s}, \quad = \left(\frac{y}{z} \right)^{n-s}$, we shall receive system of the equation be relative (x, y, z):

$$\begin{cases} \sim x^{s} + \quad y^{s} = z^{s} \quad , \sim + \quad > 1 , \sim + \quad < 2 \\ x^{s} - \sim^{\frac{s}{n-s}} z^{s} = 0 \\ y^{s} - \quad^{\frac{s}{n-s}} z^{s} = 0 \end{cases}$$

Last system is relative (x^s, y^s, z^s) has the unique decision (it is a positive) as the determinant of system is equal to zero $\det = \sim^{\frac{n}{n-s}} + {}^{\frac{n}{n-s}} - 1 = 0.$. From here $= \left(1 - \sim^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}}$ and from 1-st equation of system we shall receive value $z = \left(\sim x^s + \left(1 - \sim^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}} y^s\right)^{1/s}$ and function $« = \left(\sim x^s + \left(1 - \sim^{\frac{n-s}{n}}\right)^{\frac{n-s}{n}} y^s\right)^{1/s}$.

<u>*Properties 3*</u>. All decisions of the equation $x^n + y^n = z^n$ are represented in the following parametrical formulas: $x = z\, t^{\frac{1}{n}},\ y = z(1-t)^{\frac{1}{n}},\ 0 < t < 1$, (*)

where *t* and z any positive numbers, and $t = \sim^{\frac{s}{n-s}}$, $0 < \sim < 1$, $z = k^{\frac{1}{n}}$. Really, using presentations $\sim^{\frac{n}{n-s}} = \frac{x^n}{x^n + y^n}$, ${}^{\frac{n}{n-s}} = \frac{y^n}{x^n + y^n}$ we have homogeneous system of the algebraic equations be relative (x^n, y^n):

$$\begin{cases} \left(1 - \sim^{\frac{n}{n-s}}\right) x^n + \sim^{\frac{n}{n-s}} y^n = 0 \\ -{}^{\frac{n}{n-s}} x^n + \left(1 - {}^{\frac{n}{n-s}}\right) y^n = 0 \end{cases} \quad \text{or} \quad \begin{cases} \left(1 - \sim^{\frac{n}{n-s}}\right) x^n + \sim^{\frac{n}{n-s}} y^n = 0 \\ -\left(1 - \sim^{\frac{n}{n-s}}\right) x^n + \sim^{\frac{n}{n-s}} y^n = 0 \end{cases}$$

As the determinant of system is equal to zero it has the not trivial decision $x^n = \sim^{\frac{n}{n-s}}$, $y^n = \left(1 - \sim^{\frac{n}{n-s}}\right)$, $0 < \sim < 1$. By virtue of uniformity of system, its decision are represented as $x^n = k \sim^{\frac{n}{n-s}}$, $y^n = k\left(1 - \sim^{\frac{n}{n-s}}\right)$, $k = const.$. From here $x = k^{1/n} \sim^{\frac{1}{n-s}}$, $y = k^{1/n}\left(1 - \sim^{\frac{n}{n-s}}\right)^{1/n}$, $z = k^{1/n}$ and having entered designations

$t=\sim^{\frac{s}{n-s}}$ all decisions of the equation $x^n + y^n = z^n$ we shall copy as (*).

The remark. *For any decisions (*) are fair estimations*

1). $2\ z^s < x^s + y^s < 2^{1-\frac{s}{n}} z^s,\quad s < n,$ *2).* $2^{1-\frac{s}{n}} z^s < x^s + y^s < z^s,\quad n > s,$ *and also for them takes place formulas:*

$$\int_0^1 \frac{x^s}{y^s} dt = \frac{\left(\frac{s}{n}\right)\text{®}}{\sin\left(\frac{s}{n}\right)\text{®}},\qquad \int_0^1 \frac{y^s}{x^s} dt = \frac{\left(\frac{s}{n}\right)\text{®}}{\sin\left(\frac{s}{n}\right)\text{®}}.$$

Corollary. *At n > 2 decisions (*) are not the whole positive numbers. It is necessary to note, that at n=2 decisions (*) can be integers and no integers. For example, all decisions of type (*) at* $t=\left(\frac{2j}{j^2+1}\right)^2$ *are integer's numbers and they are represented as:* $x = j\ ,\ y = \frac{j^2-1}{2}\ ,\ z = \frac{j^2+1}{2}$ *at odd j and* $x = 2j\ \ y = \left(j^2-1\right),\ z = \left(j^2+1\right)$ *at even j, j=1,2,3,4, ….*

2. Some examples of Numbers Tree

Now we shall consider now examples of Numbers Tree for different numbers.

1). Let $N = 25,\ \ n = 2$, then we have

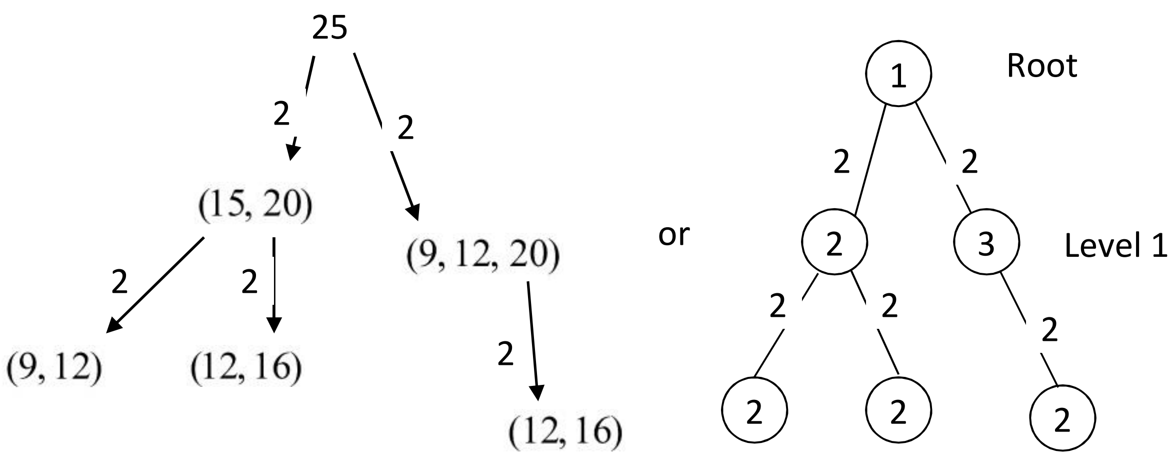


Fig.2. Model of Numbers Tree for $N = 25,\ \ n = 2$

$\textcircled{k}$ - means quantity (amount) of an element of the given top of a tree, and number on edges paw decomposition. From here follows, that representation (4) takes the following kind:

$$25^2 = 9^2 + 2 \cdot 12^2 + 16^2 .$$

2). Now we shall consider number $N = 50$.

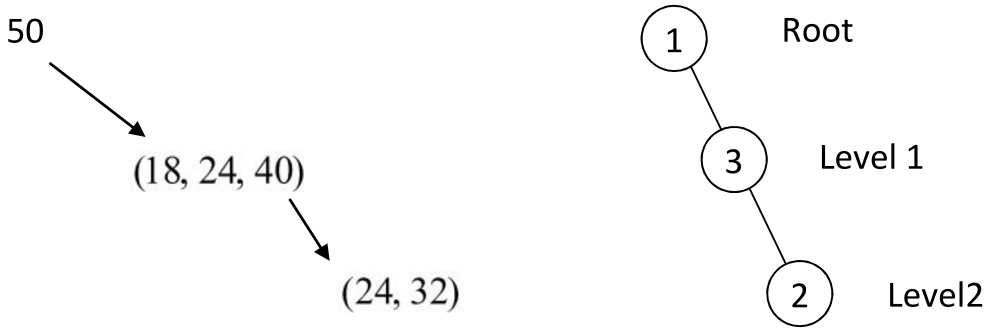


Fig.3. Model of Numbers Tree for $N = 50\,(50^2 = 18^2 + 2 \cdot 24^2 + 32^2)$

3). $N = 75$.

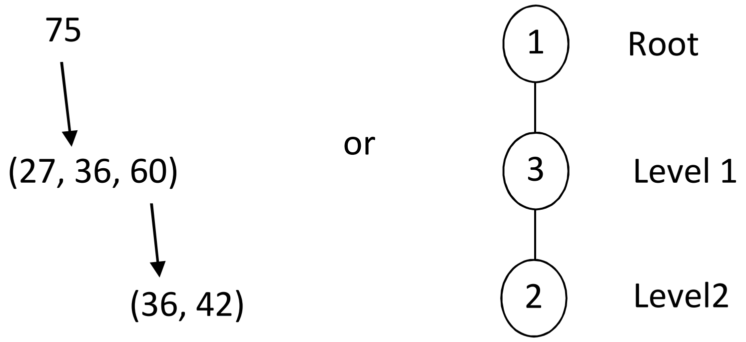


Fig.4. Model of Numbers Tree for $N = 75$ $(75^2 = 27^2 + 2 \cdot 36^2 + 42^2)$

4). $N = 100$.

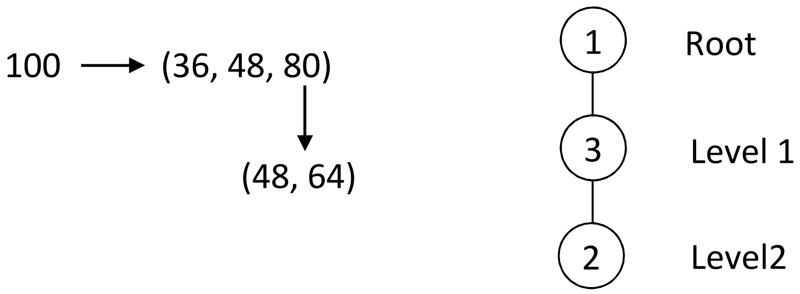


Fig.5. Model of Numbers Tree for $N = 100$ and therefore $100^2 = 36^2 + 2 \cdot 48^2 + 64^2$.

5). $N = 125$.

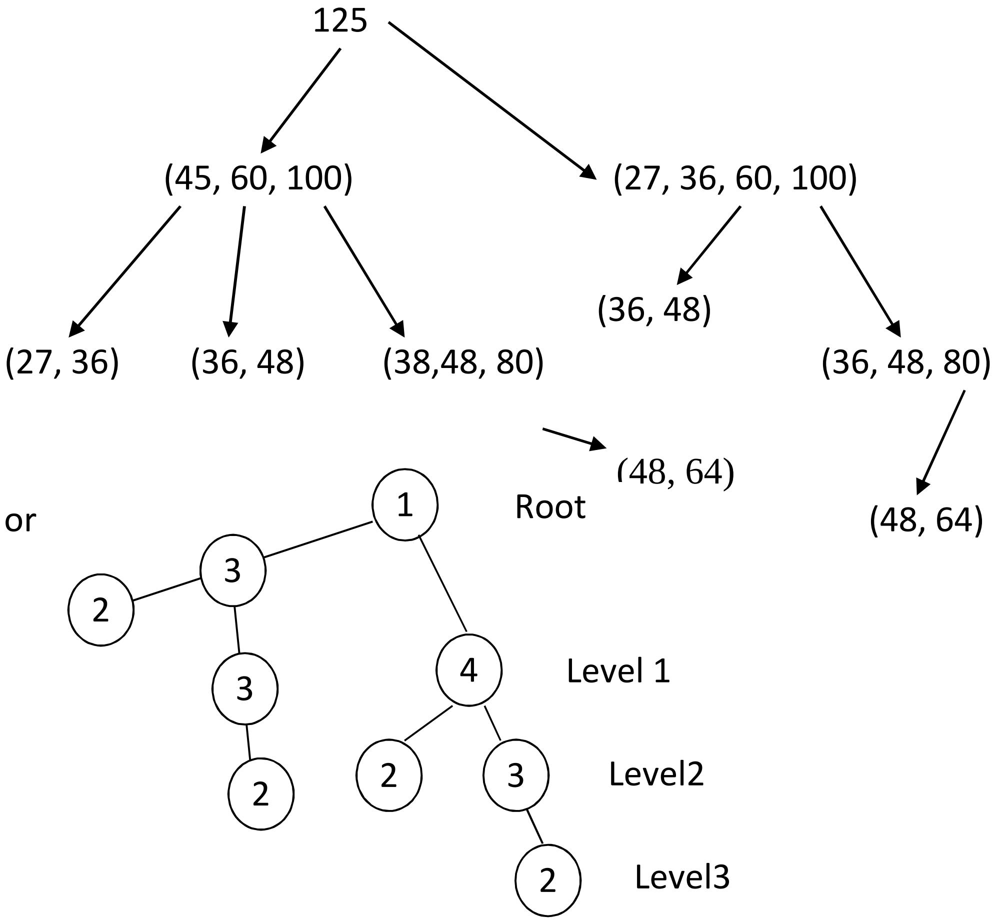
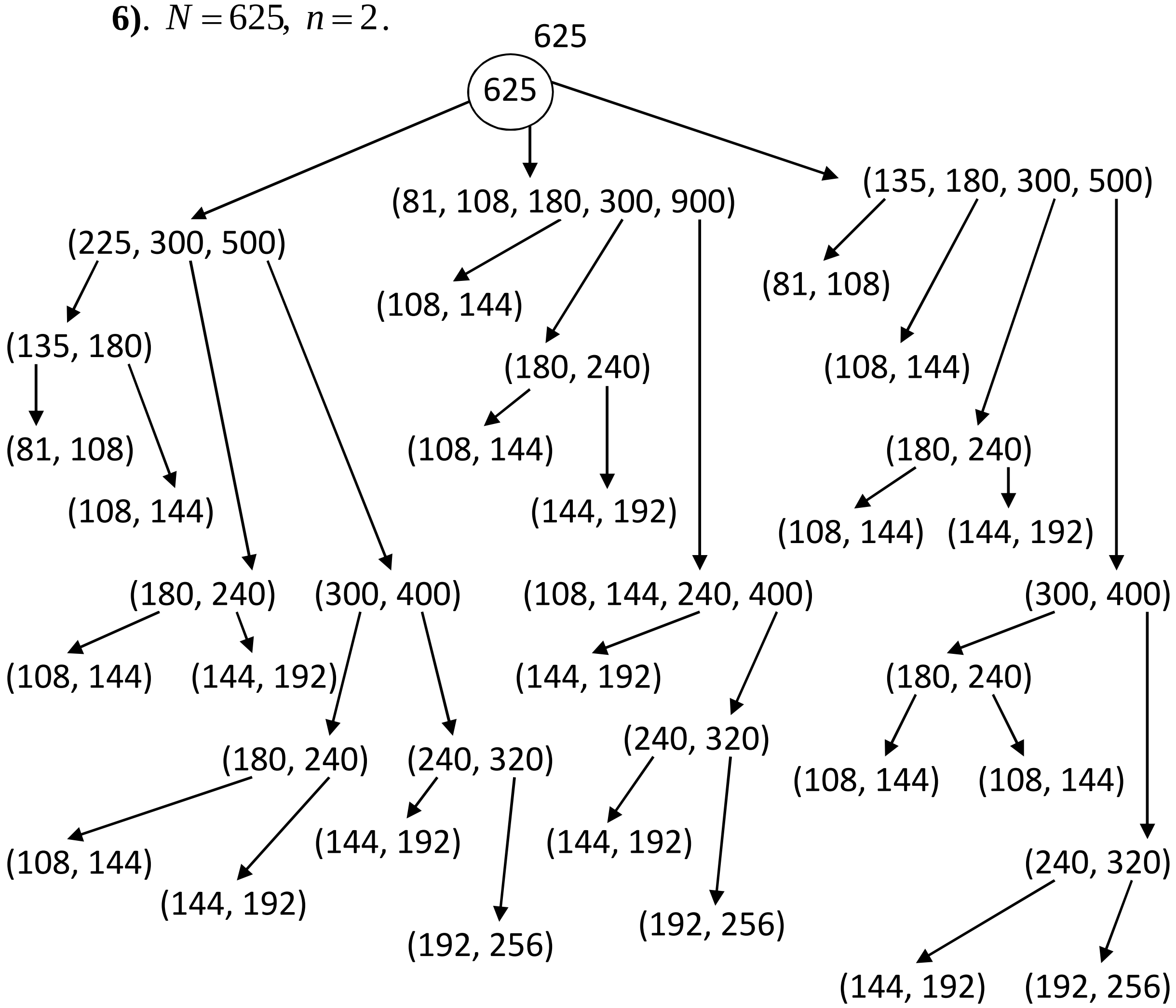


Fig.6. Model of Numbers Tree for $N = 125$, $125^2 = 27^2 + 3 \cdot 36^2 + 3 \cdot 48^2 + 64^2$

6). $N = 625,\ n = 2$.



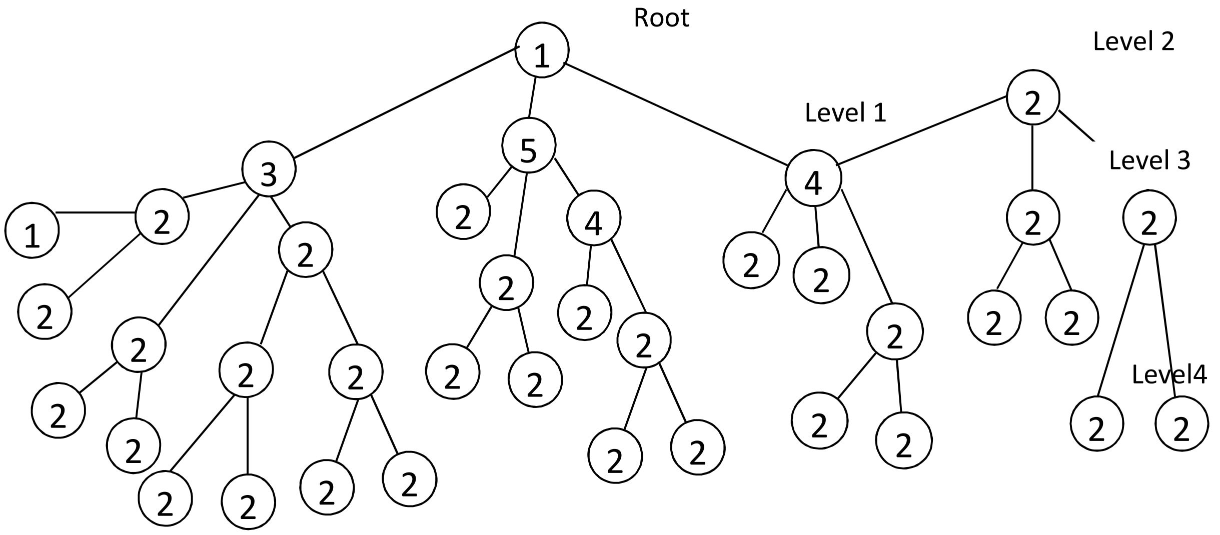
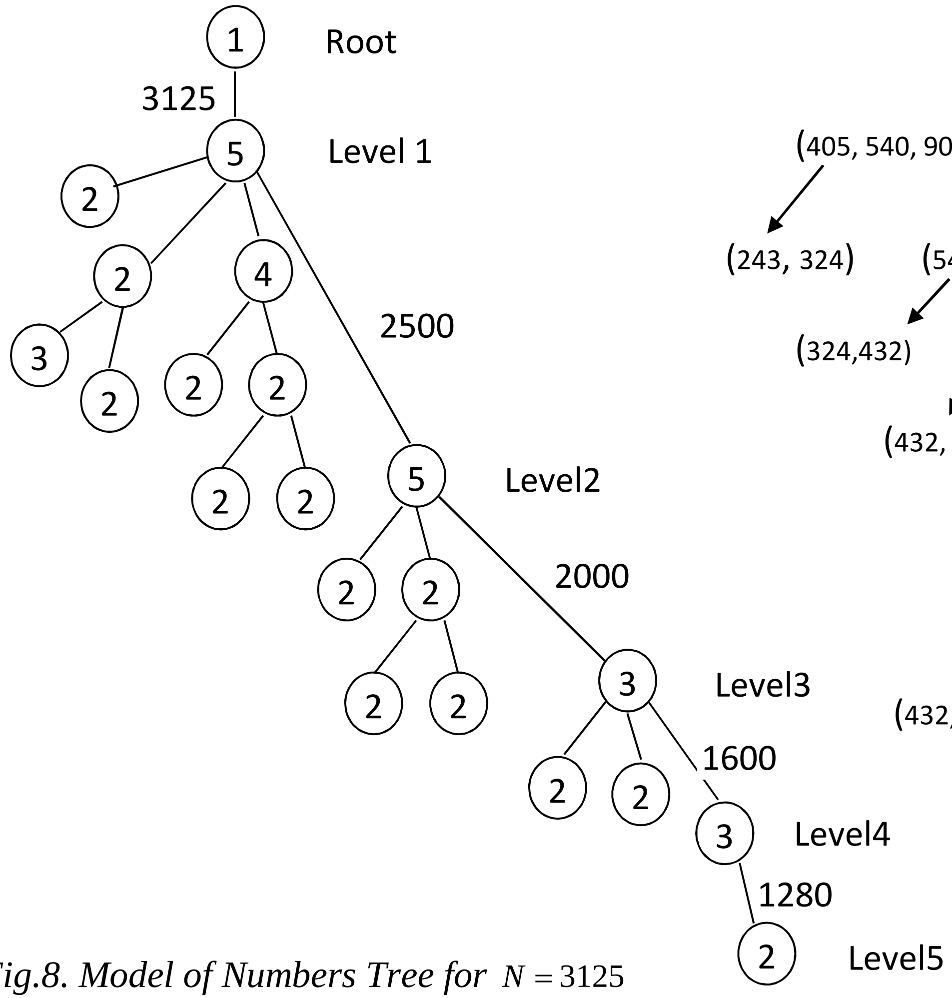


Fig.7. Model of Numbers Tree for $N = 625,\ n = 2$

$(625^2 = 81^2 + 4 \cdot 108^2 + 6 \cdot 144^2 + 4 \cdot 192^2 + 256^2)$.

7). Similarly, for $N = 3125$ we shall receive representation



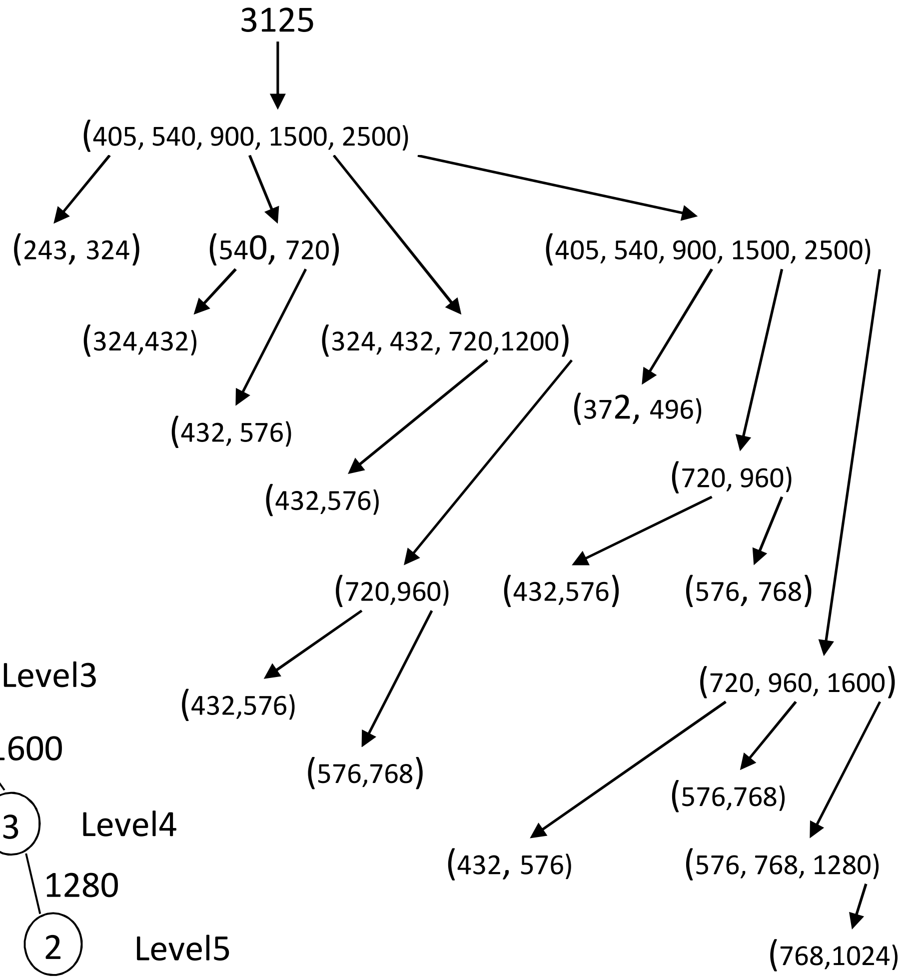


Fig.8. Model of Numbers Tree for $N = 3125$

$$3125^2 = 243^2 + 4 \cdot 324^2 + 372^2 + 8 \cdot 432^2 + 496^2 + 9 \cdot 576^2 + 5 \cdot 768^2 + 1024^2 .$$

For cases when $n > 2$ not always it is possible to construct graceful examples. We shall result one example. Let $N = 35$, $n = 3$, then

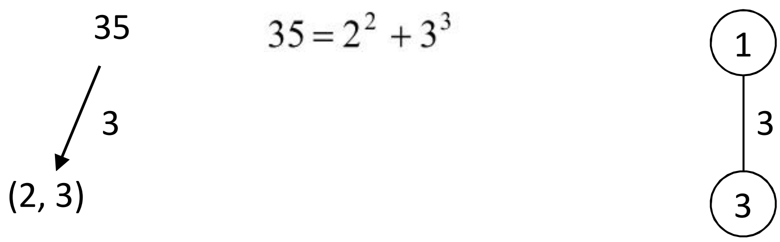


Fig.9

From the given example follows, that we have not received a tree, but only it(him) ветку. We shall name such "trees" not growing trees. For growth of such tree it is necessary to make "cuttings" from another (for example) $n = 2$ a tree and to insert them into not growing trees. For example

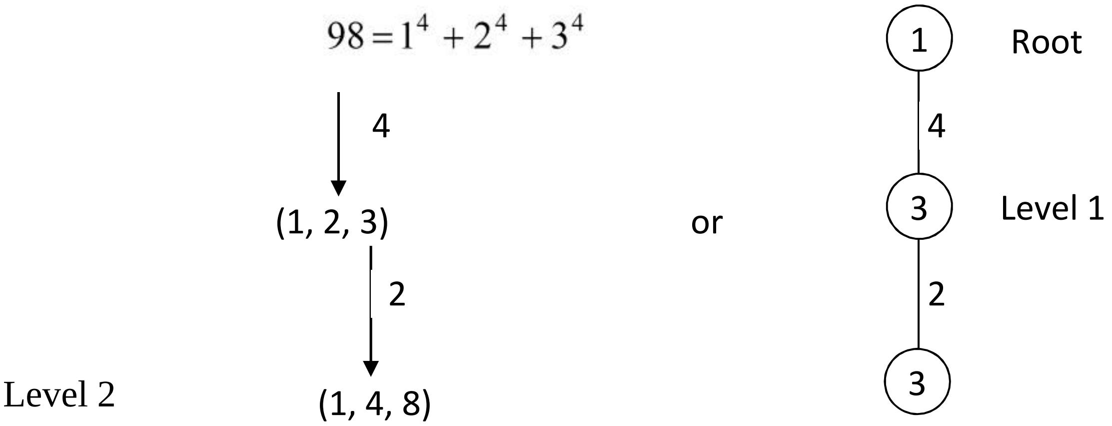


Fig.10. $98 = 1^4 + 2^4 + 1^2 + 4^2 + 8^2$.

4. Economical interpretation, structure of a tree of solutions. The tree of decisions represents one of ways of splitting of set of the data on classes or categories. The root of a tree implicitly contains all classified data, and leaves - the certain classes after performance of classification. Intermediate units of a tree represent items (points) of decision making on a choice or performance of testing procedures with attributes of elements of the data which serve for the further division of the data in this unit. Usually] the tree of decisions is determined as structure which consists from Units - leaves, each of which represents the certain class; Units of acceptance of decisions, специфицирующих the certain test procedures which should be executed in relation to one of values of attributes; the unit of acceptance of decisions is left with branches which quantity corresponds to quantity (amount) of possible(probable) outcomes of testing procedure. It is possible to consider a tree of decisions and from other point of view: intermediate units of a tree correspond to attributes of classified objects, and arches - to possible alternative values of these attributes. The example of a tree is submitted on figure. On this tree intermediate units represent attributes supervision, humidity, is windy. Leaves of a tree are marked by one of two classes P or H. It is possible to count, that P corresponds to a class of positive copies consultation, and H - to a class negative. For example, P can represent a class " to leave on walk ", and H- the class " to sit at home ". Though it is obvious, that the tree of decisions is the way of representation which is distinct from inducing rules, the tree can compare the certain rule of classification which gives for each object having the appropriate set of attributes it is submitted by set of intermediate units of a tree), the decision to what from classes to attribute (relate) this object (a set of classes is submitted by set of values of leaves of a tree). In the given example the rule will carry objects to class P or H.

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