Extreme Model of Economic crisis, risks and control interactions countries with different economic development level

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Abstract

For constructed Extreme Economics model and its parameters reducing to the algebraic representations $N_m^p = \sum_{i=1}^m a_{im}^n$ and corresponding with it $N_m^p = \sum a_{im}^n$ and corresponding with it tr 1 and 1 and 1 and 1 and 1 and 1 and corresponding with it tree of numbers is proposed and investigated Polynomial Model in form of curves high degrees **Abstract**

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algebraic representations $N_m^p = \sum_{i=1}^m a_{im}^n$ and corresponding bers is proposed and investigated Polynomial Model in ees
 $P = X^{-(m-1)} + \sum_{i=2}^m A_i X^{-(m-i)}$,
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tions $N_m^p = \sum_{i=1}^m a_{im}^n$ and corresponding with it tree of
nvestigated Polynomial Model in form of curves high
 $\sum_{i=2}^m A_i X^{-(m-i)}$,
 $\sum_{i=2}^m Y_i X^{-(m-i)}$,
 $\sum_{i=1}^m Y_i X^{-($ **Extreme Model of Economic crisis, risks and control interactions countrie

with different economic development level

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Abstract

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For constructed Extreme Economics model and its parameters reducing

the algebraic representations $N_m^u = \sum_{i=1}^m a_{in}^u$ and corresponding with it tree of

abstract propose

$$
N \quad P = X \quad (\text{m} - 1) \quad + \quad \sum_{i=2}^{m} \quad A \, \, \, X \quad (\text{m} - i) \, ,
$$
\n
$$
X = x \quad , \quad Y = y \quad , \quad Z = z \quad , \quad A \, \, \, \, Y = Y \, Z \quad \, i - 2 \, ,
$$

example's which may be considering model economics in the form of

Dushanbe, Tajikistan
\n**Abstract**
\nFor constructed Extreme Economics model and its parameters reducing
\nto the algebraic representations
$$
N_n^* = \sum_{i=0}^n a_{in}^*
$$
 and corresponding with it tree of
\nnumbers is proposed and investigated Polynomial Model in form of curves high
\ndegrees
\n $N^{-p} = X^{-(m-1)} + \sum_{i=2}^m A_i X^{-(m-i)},$
\n $X = x^{n}, Y = y^{n}, Z = z^{p}, A_{i} = YZ^{i-2},$
\nexample's which may be considering model economics in the form of
\n
$$
\frac{dK}{dt} = VAf(K, L), 0 < t \leq t_k, K(0) = K_0, \frac{dL}{dt} = uL, L(0) = L_0,
$$

\n
$$
\frac{dP}{dt} = -(1 - v - MP C)^{-1} \frac{u}{y} \cdot P + \frac{v}{y}, P(0) = P_0,
$$

\n
$$
\frac{d}{dt} = (1 - v - MP C)^{-1} \frac{u}{y} \cdot P + \frac{v}{y}, P(0) = P_0,
$$

\nand on base it introducing necessary and sufficient criteria's of arising
\neconomic crisis problem $Y^i(K, L) \leq Y^i(K^*, L^*) \leq Y^i(K^*, L),$
\n
$$
\frac{1}{t} \int_{0}^{t} K(t) dt \leq K^* \cdot \frac{1}{t} \int_{0}^{t} L(t) dt \geq L, Y^i = \frac{1}{t} \int_{0}^{t} A(t) f(K, L) dt,
$$

\nwhere $K = K$ (t) is size of the capital in time t, L = L (t) is size of a labor
\ndetermined as $L(t) = \int_{0}^{t} \int_{0}^{t} \{(x, a, t)N(x, a, t)dx da, v$ is a share of the national income
\ngong to capital investments, C is size of consumption, A = A (t) is a
\ntechnological level, P = P(t) is a price function, B = B(a) is function of stability of

$$
\frac{1}{t}\int_{0}^{t} K(t) dt \leq K^{*}, \frac{1}{t}\int_{0}^{t} L(t) dt \geq L^{*}, Y^{t} = \frac{1}{t}\int_{0}^{t} A(t) f(K, L) dt,
$$

where $K=K(t)$ is size of the capital in time t, $L=L(t)$ is size of a labor determined as $L(t) = \int_{0}^{a_{max}} \int \{ (x, a, t) N(x, a, t) dx da, v \text{ is a share of the nati}$ 0 going to capital investments, C is size of consumption, A=A (t) is a technological level, $P=P(t)$ is a price function, $B=B(a)$ is function of stability of

a labor determined as $B(a)=B_0(a)e^{-b}$, $B_0(a)$ is function of $\int f_0(\cdot) d\cdot$ $B(a)=B_0(a)e^{-b}$, $B_0(a)$ is function of birth ra $-\int_{0}^{a} f_0(\epsilon) d\epsilon$ $(a) = B_0(a)e^{-\int_0^a 0$ *c* a^a *d n c function of* $0\mu\mu$, $D_0(a)$ ϵ)^{dk}, B₀(a) is function of birth rate of manpower labor, F_0 (a) is function of death rate, N=N (x, a, t) is a solution of population problem with regard to time-age – spaces distributions.

Key words: Polynomial model, capital, labor, function, model economics, economic crisis, national income, catastrophe, risk, model, tree, optimal tree, analyses, numbers tree.

§1. Representation of a complex object by Polynomial model in extreme regime

Theorem. Let any object (or process) operates so that the next time maximizes his condition, then its state is described by a polynomial.

Proof. Let the function $u = u(x, t)$, $x \in E_m, t \ge 0$, , $x = (x_1, x_2, \dots, x_m), x \in G, G \subseteq E^m$ characterizes the state of an object (or process, or a substance) in a point *x* in time t and *L*, L_j $j = \overline{1,m}$ some of the operators realizing the change of state of the object (or process) in general, and direction x_i . Then $L_i u$ is the state of a direction x_i , $L_i u$ a change of state of an object in general. It is natural to assume that the sum is formed, where, $0 < r_i < 1$ and $\sum_{j=1}^{m} r^{\frac{n}{n-s}} = 1$, $n > s$, $s > 0$. We state the following principle, which has important practical interpretation: *Any system (or object) operates so that its status as a whole was extreme (i.e. the best in some sense) in the future*. **EXECUTE:**
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is condition, then its state is described by a polynon

Let the function $u = u(x, t)$, $x \in$
 u, x_m , $x \in G$, $G \subseteq E^m$ characterizes the state

a su **Theorem.** Let any object (or process) operates so that the izes his condition, then its state is described by a polynomial.
 Proof. Let the function $u = u(x, t)$, $x \in E_m$, $t, x_2, ..., x_m$, $x \in G$, $G \subseteq E^m$ characterizes the s Let any object (or process) operates so that the next time
dition, then its state is described by a polynomial.

Let the function $u = u(x, t)$, $x \in E_m, t \ge 0$,
 $\big)$, $x \in G$, $G \subseteq E^m$ characterizes the state of an object (or **Representation of a complex object by Polynomial model in**
 eme regime
 orem. Let any object (or process) operates so that the next time

his condition, then its state is described by a polynomial.
 of. Let the fun Let the function $u = u(x,t)$, $x \in E_m, t \ge 0$,
 x_m , $x \in G$, $G \subseteq E^m$ characterizes the state of an object (or

substance) in a point x in time t and $\frac{L_x L_y}{y} = \frac{1}{1 \cdot m}$ some of the

ting the change of state of the obje **Proof.** Let the function $u = u(x, t)$, $x \in E_m, t \ge$
 Proof. Let the function $u = u(x, t)$, $x \in E_m, t \ge$

s, or a substance) in a point *x* in time t and $L, L_j = \overline{1, m}$ some

ors realizing the change of state of the object (or *his condition, then its state is described by a polynomial.*
 f. Let the function $u = u(x,t)$, $x \in E_m, t \ge 0$,
 \ldots, x_m), $x \in G$, $G \subseteq E^m$ characterizes the state of an object (or

a substance) in a point x in time t an *es his condition, then its state is described by a polynomial.*
 oof. Let the function $u = u(x,t)$, $x \in E_n, t \ge 0$,
 $x_2, ..., x_n$), $x \in G$, $G \subseteq E^*$ characterizes the state of an object (or

or a substance) in a point x in and $\sum_{j=1}^{\infty} r_j \frac{x}{j} = 1, n > s, s > 0$. We state the following princip
important practical interpretation: Any system (or object) operatatus as a whole was extreme (i.e. the best in some sense) in the f
Based on this prin *Lu L* is matural to assume that the sum is formed, where, $\text{d} \sum_{j=1}^n r_j^{\frac{n}{p-2}} = 1$, $n > s$, $s > 0$. We state the following principle, whis mportant practical interpretation: *Any system (or object) operates so tatus* **heorem.** Let any object (or process) operates so that the next time

rest his condition, then its state is described by a polynomial.
 x_5 x_5 \ldots x_n , $x_k \in G$, $G \subseteq E^n$ characterizes the state of an object (or
 x

Based on this principle, we get:

$$
Lu = \max_{\Gamma \in M} \left(\sum_{j=1}^{m} \Gamma_j (L_j u)^s \right)^{1/s}, \quad s > 0,
$$
\n(1)

where $M = \left\{ \Gamma = (\Gamma_1, ..., \Gamma_m): 0 < \Gamma_j < 1, \sum_{j=1}^{n} \Gamma_j \right\}^{n-s} = 1, \sum_{j=1}^{n} \Gamma_j > s > 0$ given nu $\frac{m}{n}$ $\frac{n}{n}$ $\frac{n}{n}$ $\frac{n}{n}$ $\frac{n}{n}$ given numbers.

Lemma 1. Equation (1) describes the best performance of the system (or object) and it equivalent to equations of type $(Lu)^p = \sum_{i=1}^m (L_i u)^{-n}$ $N_m^p = \sum_{i=1}^m a_i^n$ max $\left(\sum_{j=1}^{n} \Gamma_j(L_j u)^s \right)$, $s > 0$,
 $\Gamma = (\Gamma_1, ..., \Gamma_m): 0 < \Gamma_j < 1$, $\sum_{j=1}^{m} \Gamma_j \frac{n}{n-s} = 1$, $n > s > 0$ gi
 na 1. Equation (1) describes the best perform

and it equivalent to equal
 $\sum_{j=1}^{m} (L_j u)^n$ $N_m^p = \sum_{j=1}^{m} a_j$ *Lu* = $\max_{\Gamma \in M} \left(\sum_{j=1}^{m} \Gamma_j(L_j u)^s \right)^{1/s}$, *s* > 0,
 M = { $\Gamma = (\Gamma_1, ..., \Gamma_m): 0 < \Gamma_j < 1$, $\sum_{j=1}^{m} \Gamma_j \frac{n}{n-s} = 1$ }, *n* > *s* > 0 given
 Lemma 1. Equation (1) describes the best performance of and it equivalent to eq $\left(\sum_{j=1}^{m} \Gamma_j(L_j u)^s\right)^{1/s}$, $s > 0$,
 $\Gamma_1,..., \Gamma_m$): $0 < \Gamma_j < 1$, $\sum_{j=1}^{m} \Gamma_j \frac{n}{n-s} = 1$, $n > s$
 \therefore Equation (1) describes the best pe

it equivalent to
 $\left(\sum_{j=1}^{m} a_j \right)^n$ $N_m^p = \sum_{j=1}^{m} a_j^m$ = $\max_{\Gamma \in M} \left(\sum_{j=1}^{m} \Gamma_j(L_j u)^s \right)^{r}$, $s > 0$,
 $\left\{ \Gamma = (\Gamma_1, ..., \Gamma_m): 0 < \Gamma_j < 1, \sum_{j=1}^{m} \Gamma_j \frac{n}{n-s} = 1 \right\}$
 ima 1. Equation (1) describes the bearmoon is and it equivalent to $\sum_{j=1}^{m} (L_j u)^{n}$ $N_m^p = \sum_{j=1}^{m} a_j^n$

$$
\text{or}\ \ Y_k^{\ p}=\sum_{i=1}^k\ Y^n{}_{ik}
$$

$$
(2) \quad
$$

1 $\overline{0}$ $\overline{n-5}$ \overline{l} \overline{u} $\overline{i-1}$ $\overline{2}$ (2)
 $Lu = r^0 \frac{1}{j} h - sL_j u, j = 1, 2, ..., m,$ (2)
= $\Gamma^0_j \frac{1}{n-s} L_j u, j = 1, 2, ..., m,$

 $\left| \cdot \right|$ \int

.

 $\sqrt{2}$

3
\nor
$$
Y_k^p = \sum_{i=1}^k Y^n_{ik}
$$
 (2)
\nmoreover, we have $Lu = \Gamma^0 \frac{1}{j^{n-s}} L_j u, j = 1, 2, ..., m,$
\n
$$
\Gamma^0_j = \left(\frac{(L_{j} u)^n}{\sum_{j=1}^m (L_{j} u)^n} \right)^{\frac{n-s}{n}}
$$
\n
$$
\left(Lu = c, L_j u = c_j, Lu = c, L_j u = c_j, \sum_{j=1}^m c_j n = c^n \right).
$$
\nThe Proof: *The necessity.* We shall introduce a designation $Z = Y_k$,

The Proof: *The necessity*. We shall introduce a designation $Z = Y_k$, $X_j = Y_{jk}$, $j = 1, k$. Let the condition (2) takes place then $(p=n)$

$$
Z^n = \sum_{j=1}^m X_j^n \tag{3}
$$

Let's show, validity (1) i.e.

$$
Z = \max_{\mathbf{S} \in \mathcal{M}} \left(\sum_{j=1}^{k} \tilde{\mathbf{S}}_{j} X_{j}^{s} \right)^{1/s}
$$
(3)

Let $(X_1, X_2, \ldots, X_k, Z)$ is the decision of the equation (3), then having entered a designation $\frac{1}{n}$ and $\frac{1}{n}$ and $\frac{1}{n}$ and $\frac{1}{n}$ $n - s$ $\left| \frac{a}{Z} \right|$, non (5) we have the follow. $\left(X_j^n\right)^{\frac{n}{n}}$ from (3) we have the following $f_j = \left(\frac{X_j^n}{Z_j^n}\right)^{\frac{n-s}{n}}$, from (3) we have the following \int $\left| \begin{array}{ccc} n & & & \end{array} \right|$ $\left(Z^{n}\right)$ $\widetilde{S}_{i} = \left(\frac{X_{i}^{n}}{n}\right)^{n}$, from (3') we have the following system:

$$
\tilde{S}_1 X_1^s + ... + \tilde{S}_k X_k^s - Z^s = 0, \, X_j^s - \Gamma_j^{\frac{s}{n-s}} Z^s = 0 \tag{3'}
$$

As $(X_1, X_2, \ldots, X_k, Z)$ is the decision (3') it is easy to see

 $\frac{1}{2n} = 1$ $\sum_{i=1}^n \tilde{S} \frac{n}{j} = \frac{\sum\limits_{j=1}^n X_{ij}^n}{Z^n} = 1$ as hence determinant of system (3') is equal $\sum_{i=1}^{k} S_i \frac{n}{n-1} = \frac{j-1}{r} = 1$ $\sum_{n=1}^{\infty}$ $\sum_{j=1}^{n}$ $\frac{m}{Z}$ = 1
as hence determinant of system $\frac{k}{\sum}$ *x n* $\sum_{i=1}^{k} \xi_i \frac{n}{n-s} = \frac{\sum_{i=1}^{k} x_i}{n} = 1$ $n = \sum_{i=1}^{n} A_i \hat{j}$ Z^n as hence determinant of system $(3')$ is a X_i^n $\sum_{j=1}^{n} A_j$ $\sum_{j=1}^{n} A_j$ $\sum_{j=1}^{k} \tilde{S} \frac{\overline{n-s}}{j} = \frac{j-1}{Z} \frac{\overline{s}}{n} = 1$ as hence determinant of system (3') is equally , as, hence, determinant of system (3') is equal to zero

 $\sum_{j=1}^{k} \tilde{S} \frac{h}{j} - 1 = 0$. Really, we apply a method of a mathem \equiv 1 J $\sum_{n=1}^{k} \tilde{S} \frac{n}{n-s}$ -1=0. Really, we apply a method *n* $\sum_{j=1}^{k} \tilde{S}_{j}^{\overline{n}-\overline{s}} - 1 = 0$. Really, we apply a method of a mathematical induction,

and as

$$
\Delta_2 = \tilde{S}_1^{\frac{n}{n-s}} - 1, \ \Delta_3 = \tilde{S}_1^{\frac{n}{n-s}} + \tilde{S}_2^{\frac{n}{n-s}} - 1. \text{ It can be assumed, that}
$$

1, $k=2,3,4...$ Let us show it's $1\qquad n$ $\Delta_k = \sum_{j=1}^{k-1} \tilde{S}_j^{\frac{n}{n-s}} - 1$, $k=2,3,4...$ Let us show it's validity at $k+1$, is valid, $=1$ $\sum_{n=1}^{k-1}$ $\sum_{n=1}^{n}$ $k-2$ 3 4 *j* $f_{k} = \sum_{k=1}^{n} \tilde{S}_{j}^{\frac{n}{n}} - 1$, $k=2,3,4...$ Let us show it's validity at $k+1$, is valid, decomposing determinant on $k+1$ elements of a line is received:

$$
\Delta_{k+1} = \begin{vmatrix}\n-1 & \tilde{S}_1 & \tilde{S}_2 & \dots & \tilde{S}_{km-1} & \tilde{S}_m \\
-\tilde{S}_1^{\frac{s}{n-s}} & 1 & 0 & \dots & 0 & 0 \\
-\tilde{S}_2^{\frac{s}{n-s}} & 0 & 1 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
-\tilde{S}_m^{\frac{n}{n-s}} & 0 & 0 & \dots & 1 & 0 \\
-\tilde{S}_m^{\frac{s}{n-s}} & 0 & 0 & \dots & 0 & 1\n\end{vmatrix} =
$$

$$
=(-1)^{k+2} \cdot \left(-\frac{s}{k}\right) \begin{vmatrix} \xi_1 & \xi_2 & \dots & \xi_{k-1} & \xi_k \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{vmatrix} +
$$

$$
+(-1)^{2m+2}\Delta_m = \frac{(-1)^{k+3} \cdot \Gamma_m^{\frac{s}{m-s}} \cdot (-1)^{m+1} \cdot \Gamma_m \begin{vmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 1 \end{vmatrix} +
$$

$$
+\sum_{j=1}^{k-1} \sum_{j=1}^{s} \frac{n}{j} - 1 = (-1)^{2k+4} \cdot \sum_{k}^{n} \frac{n}{n-s} + k\sum_{j=1}^{k} \sum_{j=1}^{n} \frac{n}{j} - 1 = \sum_{j=1}^{k} \sum_{j=1}^{n} \frac{n}{j} - 1
$$

.

As it was shown and as $\mathcal{S} \in M$, where $\sum_{i=1}^{k} \mathcal{S} \frac{n-s}{i} = 1$, i.e. it means $\Delta_{k+1} = 0$. κ +1 $\sum_{i=1}^{n} \tilde{S}_{i}^{n-s} = 1$, i.e. it means $\Delta_{k+1} = 0$. \equiv 1 J $k+1$ $\frac{k}{\Sigma}$ $\leq \frac{n}{n-1}$ is it means Λ = 0 $\bar{j}=1$ J $\overline{n-s}$ = 1, i.e., it means $\Lambda_{s-1}=0$. *n* \overline{S} $\overline{h^{-s}}$ = 1, i.e. it means Δ_{k+1} = 0. From $1st$ equation $(3'')$ we have:

$$
Z^{S} = \left(\sum_{j=1}^{k} \tilde{S}_{j} X_{j}^{S}\right) \qquad \qquad \tilde{S}_{j} = \left(\frac{X_{j}^{n}}{Z^{n}}\right)^{\frac{n-s}{n}} \qquad \qquad \text{Hence,} \qquad \text{as,}
$$

 $\sum_{j=1}^{8} S_j X^s \le \left(\sum_{j=1}^{8} S_j^0 X_j^s \right) = \sum_{j=1}^{8} \frac{1}{Z^{n-s}}$... from here for $Z^{\infty} \cdot Z^{\infty} = \sum_{j=1}^{8} X_j^{\infty}$ any $k \, X_i^n$ from h $\left(\sum_{j=1}^{\infty}$ j^{A} $j\right)^{-}$ $\sum_{j=1}^{\infty}$ \overline{Z}^{n-S} $\left(k \geq 0 \leq s\right)$ k $\left(\begin{array}{c} X_i^n \\ y_i^n \end{array}\right)$ \bar{z} | $j = 1$ $\sqrt{j} = 1$ \sqrt{j} | $j = 1$ $\sqrt{2}$ $\sqrt{n-3}$ $k \propto \frac{X_i^n}{i}$ from here for Z^s . $\frac{n-s}{n-s}$..., from here for Ξ $\frac{k}{\Sigma}$ ζ , ζ ζ $\left(\frac{k}{\Sigma}$ ζ 0 X s $\right) = \frac{k}{\Sigma}$ $\frac{X^n}{j}$..., from here for $Z^s \cdot Z^{n-s} = \frac{k}{\Sigma}$ $j^{\mathbf{\Lambda}}$ \approx $\left(\sum_{j=1}^{n}$ $j^{\mathbf{\Lambda}}$ $j\right)$ \sim $\sum_{j=1}^{n}$ $\overline{Z^{n-s}}$ \cdots , \sim *j* from her $\sum_{j=1}^{j}$ $\sum_{j=1}^{j}$ $\sum_{j=1}^{j}$ $\sum_{j=1}^{j}$ $\sum_{j=1}^{n}$ $\sum_{j=1}^{n}$ $\sum_{j=1}^{j}$ $\sum_{j=1}^{j}$ X_i^n from horo for 7^s 7^{n-s} – $\sum_{k=1}^{k} Y_i^n$ any $X^{s} \leq \left| \sum_{i=1}^{n} S_{i}^{0} X_{i}^{s} \right| = \sum_{i=1}^{n} \frac{1}{n^{s}} \dots$, from here for $Z^{s} \cdot Z^{n-s} = \sum_{i=1}^{n} X_{i}$ $\begin{array}{ccc} 1 & J \\ 1 & 1 \end{array}$ $\begin{array}{ccc} j=1 & J \\ j \end{array}$ $j=1$ \mathbb{Z}^{n} $\sum_{j=1}^{n} X^{s} \leq \left(\sum_{j=1}^{k} \sum_{j=1}^{s} X^{s} \right) = \sum_{j=1}^{k} \frac{X^{n}_{j}}{Z^{n-s}}$..., from here for $Z^{s} \cdot Z^{n-s} = \sum_{j=1}^{k} X^{n}_{j}$ any *j* $S \cdot Z^{n-S} = \sum_{k=1}^{K} X^{n}$ any $\overline{j=1}$ J $Z^{\mathcal{S}} \cdot Z^{n-s} = \sum_{i=1}^{n} X_i^n$ any -1 J any $n - s$ $\overline{}$

 $\frac{1}{n}$ *k n n* and *i* discussed 14 d $j = \lfloor \frac{k}{k} \rfloor$ $\left[\begin{array}{cc} k & x n \\ y n & z 0 \end{array}\right] = \left[\begin{array}{cc} X^n & x n \\ y & x n \end{array}\right]$. And in t j , j $\frac{1}{k}$ $\frac{1}{k}$ $n = \frac{K}{\Sigma} X n \quad \check{\Sigma} 0 = \begin{vmatrix} \Delta & j \end{vmatrix}$. $\sum_{j=1}^{\infty}$ Λ *j* $\Big)$ *j* | . And \overline{y} *X* \overline{y} X_i^n \downarrow And in this equality the M , $Z^n = \sum_{i=1}^{n} X_i^n$, $S_i^0 = \frac{1}{L_i}$ \cdots And in this equally the $\sum_{i=1}^n X_i^n$ $\in M$, $Z^n = \sum_{i=1}^{n} X_i^n$, $S_i^0 = \frac{1}{k}$. And in this equality the contract of the contract of the) and the set of \mathcal{L} and \mathcal{L} and \mathcal{L} $\left\langle n\right\rangle$ $\left\langle n\right\rangle$ $\left(\begin{array}{cc} \sum_{j=1}^{\mathbf{A}} & j \end{array}\right)$ $\left(\begin{array}{cc} & n \end{array} \right)$ \equiv 1 J \equiv 1 J J \mid $\frac{k}{N}$ \mid \mid \mid \mid $1 \quad J$ 1 $J \left| \begin{array}{cc} K & V & N \end{array} \right|$ $\tilde{S} \in M$, $Z^n = \sum_{i=1}^k X_i^n$, $\tilde{S}^0 = \left| \frac{X_i^n}{I} \right|$. And in this equality the

maximum is reached. Thus we have $Z = \max_{s \in \mathbb{R}} \left(\sum_{i=1}^{k} \tilde{S}_i X_s \right)^{1/s}$. $\sum_{j=1}^{\infty}$ *j* Λ $\Big)$ $\Big)$ $\Big)$ X^s \vert \vert \widetilde{M} $\left(\frac{1}{j=1}$ J $\right)$ $Z = \max_{\mathbf{X}} |\sum_{i=1}^{N} \mathbf{S}_i X^s|$ $\frac{1}{s}$ \int_1^1 \int_1 $\sqrt{2}$ $\left(\begin{array}{cc} \frac{\sqrt{2}}{j-1} & j^{A} \end{array}\right)$ $=\max_{S \in M} \left(\sum_{j=1}^k \tilde{S}_j X^s \right)^{7s}.$ $\widetilde{\mathsf{S}}$; X^{s} | . $\sum_{j=1}^{\infty}$ $\sum_{j=1}^{\infty}$ j^{+} .

The sufficiency: The equation (1') let takes place. Let's prove validity (3²). Let's designate $Z = -(\tilde{S}) = \left(\sum_{i=1}^{k} \tilde{S}_i X_i^s\right)^{1/s}, \tilde{S} \in M$. It is easy to see, that $j = \left(\sum_{j=1}^{n} S_j X_j^s\right)$, $S \in M$. It is easy to see, that $\frac{7s}{s}$ $\frac{8}{x}$ It is easy to $\left(\begin{array}{cc} \frac{2}{j-1} & j^{A} & j \end{array}\right)$, \rightarrow \in *M* $y = -(\tilde{S}) = \left(\sum_{j=1}^{k} \tilde{S}_{j} X_{j}^{s}\right)^{7s}, \tilde{S} \in M$. It is easy to see, that $\frac{1}{s}$ $(S) = \left| \sum_{i=1}^{n} S_i X_i^s \right|$, $S \in M$ $\left(1 \quad J \quad J\right)$. It is easy to see, that from a condition $\frac{a}{\sigma} = 0$ the system of the equations S_j ⁻⁰ are system of the equations follows: $t_0 = 0$ the system of the equations follow $j_k^S = 0$, $j = \overline{1,k}$ and from here $\zeta = \frac{s}{n-s}$. $\zeta = \frac{s}{n-s}$. $k = 0, j = 1, \kappa$ and non-neil S_k' $\overline{\overline{n-s}}$ · $X^s = 0$, $i = \overline{1,k}$ and from here ζ *s* $\overline{n-s}$. \overline{S} $\overline{n-s}$. $X_s^s = 0$. $i = \overline{1,k}$ and from $\frac{s}{s}$ $\frac{s}{s}$ $\frac{s}{s}$ $\frac{s}{s}$ $X_j^s - \tilde{S}_k^{\frac{s}{n-s}} \cdot \tilde{S}_j^{\frac{s}{n-s}} \cdot X_k^s = 0$, $j = \overline{1,k}$ and from here $\tilde{S}_k^{\frac{s}{n-s}} \cdot \tilde{S}_j^{\frac{s}{n-s}} = \frac{X_j^s}{X_j^s}$ $-\tilde{S}_k^{\overline{n-s}} \cdot \tilde{S}_j^{\overline{n-s}} \cdot X_k^s = 0$, $j = \overline{1,k}$ and from here $\tilde{S}_k^{\overline{n-s}} \cdot \tilde{S}_j^{\overline{n-s}} = \frac{\tilde{S}_k}{X^s}$ or *k s* $\frac{s}{n-s}$ $\frac{X}{j}$ or $\overline{n-s}$ $\cdot \xi \overline{n-s} = \underline{J}$ or $s \quad s \quad X$; k *X*_{j} $-\frac{X_{k}^{S}}{X_{k}^{S}}$ X_i^s $\oint \frac{s}{n-s} \cdot \oint \frac{s}{n-s} = \frac{X}{Y} \frac{j}{s}$ or $\frac{\partial \mathbf{p}}{\partial n}$. To sum fast equality on *f* from up *f k n* $\frac{n}{n-s}$ $\frac{X^n}{j}$. To sum last equality on *j* free \overline{j} $-\frac{1}{x}$ \overline{n} \cdot 10 sum 1 $\overline{n-s}$ $\cdot \xi \overline{n-s} = \underline{J}$. To sum last equali $\frac{n}{\epsilon}$ $\frac{n}{\epsilon}$ $\frac{X_i^n}{X_i^n}$ k *i X*^{*n*} *X*^{*n*} *X*^{*n}* X_i^n $\tilde{S}_L^{\frac{n}{n-s}} \cdot \tilde{S}_L^{\frac{n}{n-s}} = \frac{X_H^n}{\tilde{S}_L^n}$. To sum last equality on *j* from up *1* to *k*, then we have, $\frac{1}{\zeta}$ and then $\zeta \frac{0}{i}$ $\frac{n}{n-s} = \frac{\zeta}{k}$ is a j *k* $\sum_{i=1}^{k} X_i^n$ *^k ⁿ* \dot{j} and then 0 \dot{a} $\frac{n}{n-s}$ $\frac{2}{n-1}$ *i* and then $\frac{8}{n}$ 0 $\frac{n}{n-s}$ X $k = \frac{X_i^n}{X_i^n}$ $\qquad \qquad \frac{X_i^n}{X_i^n}$ $\qquad \qquad \frac{X_i^n}{X_i^n}$ X^n X^n $j = 1$ and then $\sum_{i=1}^N X_i^n$ $\sum_{n=1}^{\infty} \frac{n}{n-s} = \frac{j-1}{n}$ and then $\sum_{n=1}^{\infty} \frac{n}{n-s} = \frac{X_{ij}^n}{1-x_{ij}}$ is a point of $\sum_{i=1}^n X_i^n$ $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$ is a point of a maximum of \equiv 1 J *k n j* n , n , n , n $\frac{n}{n-s}$ $\frac{X^n}{j}$ is a point of a maximum $j = k$ $j=1$ J X_i^n X_i^n is a noint of a maximum of $1 \quad J$ ζ_i^0 $\frac{n}{n-s} = \frac{X_i^n}{i}$ is a point of a maximum of

function $\sim (\check{S})$, $\check{S} \in M$ as $(\sim \frac{n}{ss} < 0)$. Let's calculate the value of function \sim (ζ^{0}). It is easy to see, that

$$
Z^{s} = \sum_{j=1}^{k} \tilde{S}_{j} X_{j}^{s} \leq \sum_{j=1}^{k} \tilde{S}_{j}^{0} X_{j}^{s} = \sum_{j=1}^{k} \left(\frac{X_{j}^{n}}{Z^{n}} \right)^{\frac{n-s}{n}} \cdot X_{j}^{s} = \sum_{j=1}^{k} \left(\frac{X_{j}^{n} X_{j}^{\frac{sn}{n-s}}}{Z^{n}} \right)^{\frac{n-s}{n}} =
$$

$$
= \sum_{j=1}^{k} \left(\frac{X \, \frac{1 + \, s}{n - s}}{Z} \right)^{n - s} = \sum_{j=1}^{k} \frac{X \, \frac{n}{j}}{Z \, n - s} \quad i.e. \quad Z^s \cdot Z^{n - s} = \sum_{j=1}^{k} X \, \frac{n}{j} \quad \text{And hence}
$$

$$
Z^n = \sum_{j=1}^k X_j^n ; \forall \check{S} \in M \text{ . It is easy to see that}
$$

$$
Z = -\left(\tilde{S}_0\right) = \left(\frac{\sum_{j=1}^{k} \left(\frac{X_j^n}{\sum_{j=1}^{k} X_j^n}\right)^{\frac{n-s}{n}} \cdot X_j^s\right)^{1/s}
$$

$$
= \left(\sum_{j=1}^k \left(\frac{X_j^n \cdot X_j^{\frac{sn}{n-s}}}{\sum_{j=1}^k X_j^n}\right)^{1/s}\right)^{1/s} = \left(\sum_{j=1}^k \frac{X_j^n}{Z^{n-s}}\right)^{1/s}, \text{ and } Z^s = \sim^s \left(\tilde{S}_0\right) = \frac{1}{Z^{n-s}} \sum_{j=1}^k X_j^n,
$$

s

from here $Z = -(\tilde{S}_0) = \left(\frac{k}{\sum X_i^n}\right)^{1/n}$, i.e. it takes place (3'). Thus $0 = \left(\sum_{j=1}^{n} X_j\right)$, i.e. it takes place (5). 11 \int_{Σ}^{k} \mathbf{x} ⁿ i e it takes place (3') Thus $Z = \sim \left(\sum_{j=1}^{6} X_j^n \right)$, i.e. it takes place (3'). The $\begin{pmatrix} 7n & & & & \end{pmatrix}$ $\left(\frac{2}{j=1}^{N} \right)$, i.e. it takes pro- $\hat{\sigma} = -\left(\tilde{\mathcal{S}}_0\right) = \left(\sum_{j=1}^k X_j^n\right)^{7n}$, i.e. it takes place (3'). Thus

 $\left(\frac{k}{N} - \frac{n}{N}\right)^{1/n}$. The lemma is proved. *j s* $\begin{array}{c|c|c|c|c} k & X & n \\ \hline \end{array}$ $\begin{array}{c|c} X & s & \\ \hline \end{array}$ $\begin{array}{c|c} k & x & n \\ \hline \end{array}$ $\begin{array}{c} \n\pi & \pi \end{array}$ including the lemma in X *n* $\frac{n-s}{s}$ $\sqrt{2s}$ $\left[\begin{array}{cc} k \\ y & n \end{array}\right]$ $\left[\begin{array}{cc} \begin{array}{c} n \\ j \end{array}\right]$ *j n* $\overline{j=1}$ $\begin{array}{c} K \\ F \end{array}$ \mathbf{v} n | $\begin{array}{c} \overline{j=1} \end{array}$ $\begin{array}{c} J \end{array}$ $j=1$) $\left| \begin{array}{c} \frac{j}{r} \\ \frac{k}{r} \end{array} \right|$ $= \left| \begin{array}{c} \frac{k}{r} \\ \frac{k}{r} \end{array} \right|^{n}$. The femma is proved. X_i^n $\left| \begin{array}{c} \n\sqrt{2} \end{array} \right|$ X^n \downarrow \qquad $Z = \sim (\tilde{S}_0) = |\sum_{k=1}^{N} |\frac{J}{J_k}| \sim X^{s} | = |\sum_{k=1}^{N} X^{s}|$ $\frac{1}{n}$ The lemma is proved $\frac{1}{s}$ $\left(S_{0}\right) = \left[\sum_{i=1}^{N} \left|\frac{f}{k}\right| \right]$ $\left|X^{s}\right| = \left[\sum_{i=1}^{N} \left|\frac{f}{k}\right| \right]$ $1 \left| \begin{array}{cc} K & V & n \\ V & \end{array} \right|$ $\left| \begin{array}{cc} J & J \\ J & \end{array} \right|$ $1 \quad 1$ \int γ/n . The lemma is proved. $\left(\begin{array}{cc} \angle & \wedge \\ j=1 & \end{array}\right)$ $\left(\begin{array}{cc} k & n \\ k & k \end{array}\right)$ /n. The lemma is \equiv 1 J \mid \int $\sqrt{2}$ $\left(\begin{array}{cc} & \left(\begin{array}{cc} j=1 & \end{array} \right) & \qquad \qquad \end{array} \right)$ $\left(\begin{array}{ccc} & & \frac{n-s}{s} \end{array}\right)$ \int $\left\vert n\right\rangle$ $\left\vert n\right\rangle$ $\begin{pmatrix} 2 & J \\ j = 1 & J \end{pmatrix}$ $\left(\begin{array}{ccc} & & n & \end{array} \right)$ $\sum X_i^n$ | $\sum Y_i^n$ | $\sum_{j=1}^{\infty} \left| \frac{k}{\sum_{j=1}^{k} X_j^n} \right|$ $\cdot X^s$ $= \left(\sum_{j=1}^{\infty} X_j^n \right)$ $\frac{-s}{s}$ \sqrt{s} $= \sim (\mathcal{S}_0) = |\mathcal{S}| - |\mathcal{I}|$ X^s $= |\mathcal{S}| X^n$. The lemma is proved.

Thus, the equation (2) , (3) is optimum in sense (1) , and the tree of numbers appropriate by this equation represented on fig. 1 also is an optimum tree.

Lemma 2. The tree of numbers to the appropriate equations (2) and (3') let is given. Then there is transformation *K* which translates the solutions (3[']) at $k = m - 1$ on the solution (3[']) at $k = m$, *i.e.*

7
\n**Lemma 2.** The tree of numbers to the appropriate equations (2) and
\n(3') at
$$
k = m - 1
$$
 on the solution (3') at $k = m$, *i.e.*
\n $Y = KX$, (4)
\n
$$
K = \begin{pmatrix} x & 0 & \cdots & 0 & 0 & 0 \\ 0 & x & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & x & 0 & 0 \\ 0 & 0 & \cdots & 0 & y & 0 \\ 0 & 0 & \cdots & 0 & 0 & z \end{pmatrix}
$$
\n
$$
X = (a_{1,m-1}, \ldots, a_{m-1,m}, N_{m-1}, N_{m-1}), N_{m-1} = \left(\sum_{j=1}^{m-1} a_{im}^n\right)^{\frac{1}{n}}, Y = (a_{1,m}, a_{2,m}, \ldots, a_{1m}, N_m), N_m = \left(\sum_{j=1}^{m} a_{jm}^n\right)^{\frac{1}{n}}.
$$
\n**The proof.** Let be (x, y, z) the solution $x^n + y^n = z^n$, $p \ge 2, n \ge 2$.
\nTransformation (3') in the form of $a_{im} = x a_{im+1}, a_{mm} = y^n \sqrt{N_{m+1}^p}, N_m = z N_{m+1}$,
\n $i = 1, 2, ..., m-1; k = 2, 3...$ Let $(a_{m-1}, ..., a_{m-1,m-1}, N_{m-1})$ is the solution (2) at $k = m - 1$. As

$$
X=(a_{1,m-1},...,a_{m-1,m},N_{m-1},N_{m-1}),\;N_{m-1}=\left(\sum_{j=1}^{m-1}a_{im-1}^n\right)^{\frac{1}{n}},\;Y=(a_{1m},a_{2m},...,a_{1m},N_m),\;N_m=\left(\sum_{j=1}^{m}a_{im}^n\right)^{\frac{1}{n}}.
$$

The proof. Let be (x, y, z) the solution $x^n + y^n = z^p$, $p \ge 2, n \ge 2$.

Transformation (3) in the form of $a_{im} = x a_{mn}$, $a_{mn} = y^n \sqrt{N_{m-1}^p}$, $N_m = z N_{m-1}$, $i = 1, 2, ..., m-1$; $k = 2, 3...$ Let $(a_{im-1}, ..., a_{m-1}, N_{m-1})$ is the solution (2) at $k = m-1$. As $\sum_{j=1}^{m-i} a_{jm-1}^n = N_{m-1}^p$ multiplying by x^n we have $x^n \sum_{j=1}^{m-1} a_{jm-1}^n = x^n N$ $1 - \mu_{m-1}$ muniprying by x we $\sum_{j=1}^{m-i} a_{jm-1}^n = N_{m-1}^p$ multiplying by x^n we have $x^n \sum_{j=1}^{m-1} a_{jm-1}^n = x^n N_{m-1}^p$. Here $x^n\sum^{m-1}a^n_{jm-1}=x^nN^{\,p}_{m-1}$. Here 1 $1 - \lambda$ $1\mathbf{v}_{m-1}$. **IICIC** $\sum_{j=1}^{m-1} a_{jm-1}^n = x^n N_{m-1}^p$. Here $\sum_{m=1}^{m-1} (xa_{jm-1})^n = (z^p - y^n)N_{m-1}^p$, and hence $\sum_{m=1}^{m} a_{jm}^n = N_m^p$. Based on this $=1$ $\sum_{n=1}^{n}$)ⁿ = (z^p - yⁿ) N_{m-1}^p , and hence $\sum a_{jm}^n = N_m^p$. Ba 1 1 $\sum^{m-1}(xa_{jm-1})^n=(z^p-y^n)N^p_{m-1}$, and hence $\sum^m a^n_{jm}=1$ *j* $\int_{a}^{m} (x^{p} - y^{n}) N_{m-1}^{p}$, and hence $\sum_{j=1}^{m} a_{jm}^{n} = N_{m}^{p}$. Based on this . Based on this on (3') in the form of $d_{lm} = x d_{lm1}$, $d_{mm} = y \sqrt{N_{m-1}}$, $N_m = z N_{m-1}$,
 $k = 2,3...$ Let $(a_{lm-1},...,a_{m-lm-1}, N_{m-1})$ is the solution (2) at $k = m - 1$. As
 L_{-1} multiplying by x^n we have $x^n \sum_{j=1}^{m-1} a_{jm-1}^n = x^n N_{m-1}^p$. H *im imaging in the form of* $a_{lm} = x a_{lm}$, $a_{mm} = y^{\alpha} \sqrt{N_{m-1}^{p}}$, $N_{m} = z N_{m}$;
 nsformation (3') in the form of $a_{lm} = x a_{lm}$, $a_{mm} = y^{\alpha} \sqrt{N_{m-1}^{p}}$, $N_{m} = z N_{m}$;
 i.2..... $m-1$; $k = 2,3...$ Let $(a_{mn-1}, a_{m-1}, N_{m-$

transformation, we have

$$
N_{m}^{p} = \left(x^{m-1}\right)^{n} + \sum_{i=2}^{m} \left(yx^{m-i}z^{\frac{p(i-2)}{n}}\right)^{n}
$$
(4)

Formula (4) is a formula for a growing tree and can be applied in various fields of natural science. Transformation $(3')$ with $p = 1$

$$
a_{im} = xa_{im-1}
$$
, $i = \overline{1,m-1}, a_{mn} = y\sqrt[n]{N_{m-1}}, N_m = zN_{m-1}$, where $x^n + y^n = z$,

transforms a solution $\sum a_{im-1} = N_{m-1}$ to solutions $\sum a_{im}$ 1 1 1^{-1} \mathbf{v}_{m-1} to solutions $\sum a_{ii}$ -1 $\sum_{i=1}^{m-1} a_{im-1}^n = N_{m-1}$ to solutions $\sum_{i=1}^{m} a_{im}^n = N_m$ and then (4) $\sum_{i=1}^{m-1} a_{im-1}^n = N_{m-1}$ to solutions $\sum_{i=1}^{m} a_{im}^n = N_m$ and then (4) and then (4) takes follows:

$$
N_{m} = (x^{m-1})^{n} + \sum_{i=2}^{m} \left(yx^{m-i} z^{\frac{i-2}{n}} \right)^{n}
$$

Thus, for any complex object described by a differential operator of equation (1), or (2) using the vector $(a_1 \ldots \ldots a_m) \in E^m$, which is determined from the

$$
N_m^p = X^{(m-1)} + \sum_{i=1}^m A_i X^{m-i}, X = x^n, A_i = y^n z^{p(i-2)}, (p, n, m) > 1,
$$

representation N $_{m}^{p} = \sum_{i=1}^{m} a_{i m}^{n}$ is reduced to its description \sum_{m} \sum_{m} *a* $\sum_{i=1}^{n}$ *a* is reduced to its descript 1 is reduced to its description by a finite number of elementary objects $(x, y) \in E^2$ such as a polynomial (4) for x^n , i.e. curves of higher degrees such as $N_{m}^{p} = x^{n(m-1)} + \sum_{i=2}^{m} y^{n} z^{p(i-2)} x^{n(m-i)}$ $i=2$ $N_{m}^{p} = x^{n(m-1)} + \sum y^{n} z^{p(i-2)} x^{n(m-i)}$ 2

(Possibly elliptical) used for data protection or form $-Y \sum_{i=3}^{m} X^{m-i} Z^{i-2} = X^{(m-1)} + YX^{m-2}$ $i=3$ $N \frac{p}{m} - Y \sum X^{m-i} Z^{i-2} = X^{(m-1)} + Y X^{m-2}$

the properties such as
 $N_{m}^{P} = x^{n(m-1)} + \sum_{i=2}^{m} y^{n} z^{p(i-2)} x^{n(m-i)}$

Possibly elliptical) used for data protection or form
 $N_{m}^{P} - Y \sum_{i=3}^{m} X^{m-i} z^{i-2} = X^{(m-1)} + YX^{m-2}$

where $Z = X + Y$ and all the solutions of the eq where $Z = X + Y$ and all the solutions of the equation $X + Y = Y$ $x^n + y^n = z^p$ for $p - n$ are for $p = n$ are represented as $x = zt^{1/n}$, $y = z(1-t)^{1/n}$, $p = n$, $t \in (0,1)$. Note that $P_m^p(x, y) = N_m^p$ is a polynomial of degree $(m-1)$ and x^n are the class of no degenerate curves higher degrees, which is very well used to protect information. egree $(m-1)$ and x^n are the class very well used to protect information The basic result of the given ect of Economics type:
 $\sum_{i=1}^{m} \Gamma_i (L_i u)^s)^{\frac{n}{p}s}$, or $X + Y$ and all the solutions of the solutions of the solutions of the solutions of the solutions x^n are the consider the solution is very well used to protect in **ark.** The basic result of the give αx object of Econom

Remark. The basic result of the given work consists in the description of any complex object of Economics type:

$$
L u = \max_{r \in M} \left(\sum_{i=1}^{m} r_i (L_i u)^s \right)^{\frac{n}{p} s}, \quad or
$$

$$
(Lu)^{P} = \sum_{j=1}^{m} \left(L_{j} u \right)^{n}, n > s > o, M = \left\{ (r_{1}, ..., r_{m}) = r; \sum_{j=1}^{m} r^{\frac{n}{n-s}} = 1, n > s > 0, 0 < r_{j} < 1 \right\}
$$

by means of a vector $(a_{1} ..., a_{m}) \in E^{m}$ which is defined from representation (*)

$$
N \Big|_{m}^{p} = \sum_{i=1}^{m} a \Big|_{m}^{n}, \qquad N = L u, a_{im} = L_{i} u
$$

and it is reduced to its description by means of final number of elementary
objects of type $(x, y) \in E^{2}$ as a polynomial be relative x^{n} :

by means of a vector $(a_1 \ldots \ldots a_m) \in E^m$ which is defined from representation (*)

$$
N_{m}^{p} = \sum_{i=1}^{m} a_{im}^{n}, \qquad N = L u, a_{im} = L_{i} u
$$

and it is reduced to its description by means of final number of elementary objects of type $(x, y) \in E^2$ as a polynomial be relative x^n :

,

$$
N_{m}^{p} = x^{n(m-1)} + \sum_{i=2}^{m} y^{n} z^{p(i-2)} x^{n(m-i)},
$$

i.e. curve high degrees

$$
U_{\mu}P = \sum_{j=1}^{m} \left(L_{j} u \right)^{n}, n > s > o, M = \left\{ (r_{1},...,r_{m}) = r; \sum_{j=1}^{m} \sum_{j=1}^{n} \frac{1}{r_{j}} = 1, n > s > 0, 0 < r_{j} < 1 \right\}
$$

by means of a vector $(a_{1},..., a_{m}) \in E^{m}$ which is defined from representation (*)

$$
N \Big|_{m}^{p} = \sum_{i=1}^{m} a_{i,m}^{n}, \qquad N = L u, a_{im} = L_{i} u
$$

and it is reduced to its description by means of final number of elementary
objects of type $(x, y) \in E^{2}$ as a polynomial be relative x^{n} :

$$
N \Big|_{m}^{p} = x^{n(m-1)} + \sum_{i=2}^{m} y^{n} z^{p(i-2)} x^{n(m-i)}
$$

i.e. curve high degrees

$$
N \Big|_{n}^{p} = X^{-(m-1)} + \sum_{i=2}^{m} A_{i} X^{-(m-i)}, \qquad (* * *)
$$

$$
X = x^{n}, Y = y^{n}, Z = z^{p}, A_{i} = Y Z^{i-2}
$$

§2. The construction of Extreme Economics Model
It is known (I1-13) that active penetration of scientific methods into

§2. The construction of Extreme Economics Model

It is known ([1-13]) that active penetration of scientific methods into practice of modern economy, both in sphere of manufacture of the industry, and in sphere of an agricultural production became prominent feature of our time. It is especially shown by consideration of some questions, in which the decision is connected with creation of the strict, scientifically proven methods in problems of economy and economic development. The decision of these hot questions is impossible without attraction of modern methods of a mathematical science. Creation of the scientific device for research and forecasting a condition of economic resources all over the world is one of the major state problems. Development of methods of qualitative research and, hence, the quantitative forecast of systems of economic development, naturally, demands all-round studying of parameters of a business economics, cities and the countries, at those or other values of parameters anthropogenous and social factors. Thus, experiments with real systems are rather expensive, long and frequently inadmissible, therefore there is a necessity of development of a various sort of mathematical models. By means of mathematical models, there was possible a qualitative and experimental studying of consequences of those or other planned actions touching functioning of economic systems, direct experiments with which are in admissible. The problem of studying of optimum values of resources (size of the capital and a labor) is a key question and to market economy and not having decided it, it is impossible to adjust effective activity of the best economy. Especially, the problem consists in forecasting size of the capital agrees the best manufactures before many countries, and in particular before the countries of the CIS as the condition of economy of many countries of the CIS now is at a low stage of economic development. Huge economic recession should mention a labor market. Forecasting of resources the extensive bibliography is devoted to mathematical questions. Since K. Marks's work, and also Mankyw N.G.'s work, Zanga V.B., Mitina N.A., PetrovA.A., Chernovekov D.S., Starkov N.I., Sh. Cherbakova A.V., Alikariev N.S., and of some other scientists studies various aspects mathematical modeling of economic systems and forecasting of their condition. One of the first mathematical models of Capital size is the model of the capital in view of manufacture Cobb-Douglas and manpower - model Malthus. In particular model of the capital in view of this or that production function not considered questions of optimization of manufacture on parameters of manufacture and it is accepted in model Malthus, that growth rate proportionally number and in it(her) is not taken into account factors of age and space. In view of time, time - age and time - age - spatial distribution works Volterra, Jeffries, Vebb, Alekseev, Svirezhev, Moisseev and many others are devoted to development of models of dynamics of a population. To one of the significant phenomena of a science of last time began the phenomenological theory of growth of the population of the world S.P. Kapitsa in which, with good accuracy the population of the Earth during rather long time it interpreted growth as hyperbolic growth owing to square-law dependence of growth rate on number. In these and other works bases of construction of the device qualitative and quantitative investigations of population's numbers are incorporated. Development of models in view of age and spatial distribution of a population and the problems connected to protection of rare biological kinds, are considered in works. In these works, questions of a correctness of models of biological populations are considered in view of age structure and spatial distribution. Some ideas from the specified works were used for the description of a condition of size of a labor, *i.e.* manpower within the framework of power models of manpower. For a wide class time–age-spaces distributions models described by the integral-differential equations in partial derivatives, questions

of modeling of sizes of the capital and manpower in rather general cases are studied. One of achievements of our work is optimization of industrial models and connected with them of economic systems such as «the capital - production ». The purpose of the given book consists in development of models and methods of research of economic systems in view of time age distribution of manpower and research of problem optimization productions, manpower within the framework of models with extreme properties. In it the following questions are considered. The solution of a problem of optimization of productions and economic systems is defined on parameter uses of resources. Construction the best models of manufacture also are connected with them economic systems. Research of the connected problems connected to describing condition of size of the capital and a labor resource and on the basis of simulated functional of size of a labor as integral from a number of labor. A substantiation of the received equation for functional of a labor in cases when number of a human population depends on age and spatial factors is held. A mathematical substantiation initial the mathematical models connected from sizes of the capital and a labor. Mathematical modeling of dynamics of manpower has long enough history. One of the first works in this a direction should count model Malthus about exponential growth of number of the human population which has served by a basic point on creation of mathematical models. It is natural to name the following stage of mathematical models logic model which has formed a basis for a lot of remarkable works Volterra, Kostisin and so on. In these and subsequent works the big attention is given the development of a problem of construction and stability of dot models. Thus, the basic mathematical devices of modeling in these works are the nonlinear equations of dynamics of manpower and a population. Also it is necessary to specify K. Marks's economic works which has carefully studied a condition of economy with the help of Kene diagrams. Models Malthus, the logic model, and some other models have experimental acknowledgement at studying dynamics of a population. At the same time, not enough attention is fair was given modeling of dynamics of a manpower in view of age structure in a class of the differential equations. For economic systems in view of age structure and spatial distributions in our works, there are some ideas on a question of construction of mathematical models in view of time age distribution. In the offered book the general method the decision of the appropriate mathematical problems with the help special entered functional is offered and proved.

We shall propose our model Economics which is considered in our works [14- 40].

1. Productions model. It is known, that economy this manufacture and

distribution of material benefits. We shall designate through Y quantity made

production (or the national income in scale of the country) and it is function of

the capital-*N*, labor-*L*, and productivity of technical progress - *A*:

$$
Y = A f(K, L) \tag{1}
$$

where *Y,K,L,f(.)* are non negative function.according to the given law the quantity made production grows with growth of size of the capital, work and a measure of the current level of technical progress and these factors are primary factors of growth of economy. As, $Y = \varepsilon Y + (1 - \varepsilon) Y$, $0 \le y \le 1$, the part of the received income is designated through $I = VY$, and refers to in size the investment, and other part is designated, through $C = (1 - v)Y$ and refers to as consumption. Besides, the certain part of the aggregate profit of the country which should go on state purchases G. To aggregate profits, it is necessary to add size of pure export \cdot *N*. Thus, we have the following equation of balance of distribution of material benefits:

$$
= +I+G+N_X. \t\t(2)
$$

It is necessary as to note, that the size of consumption depends on the available income that is $C = C(y-T)$. Here size of taxes which go on social security payments poor, and payments of social insurance elderly etc. For construction of the equation of mathematical economy we shall take advantage of modeling manufacture (1) and the balance equation (2).

Let conditions take place:

1). All economic functions and parameters depend from sets of parameters (t, r, e, a,x, …), where t-time, the r-real rate of interest, e-a rate of an external exchange, a is an age of labor, x-spatial variable, $x = (x_1, x_2, ...$ x_n Θ *R, the R-sum of regions. d* and the mathematical economic of mathematical economic (1) and the balance equalities that the balance equality.
 differentions take place:
 differentio differentio eters (*t, r, e, a,x, ...), where*
 dre R-s

2). Rates of manufacture are defined by rates of distribution: $\frac{dY}{dt} = \frac{dy}{dt}$ $(1')$

2. Model of the capital. Let $K = K(1)$ size of the capital (instruments of production, it is used by workers, monetary resources) at value of parameter t

equals $K(t+\Delta t)$ at $t=t+\Delta t$. Then $\bigcup K=K(t+\bigcup t)$ - K (t) means a gain of the capital for an interval of parameters Δt . Hence, $\bigcup K = I \bigcup t$ and from here, with the account (1) we receive the equation of the capital

$$
\frac{dK}{dt} = VAf(K, L)
$$
\n(3)

at what here it is designated

$$
\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{\partial}{\partial r} \cdot \frac{dr}{dt} + \sum_{i} \frac{\partial}{\partial x_{i}} \cdot \frac{dx_{i}}{dt} + \frac{\partial}{\partial e} \cdot \frac{de}{dt} - \sum_{i} \frac{\partial}{\partial x_{i}} (D_{i} \cdot \frac{\partial}{\partial x_{i}})
$$

For example, if $\ddagger = t$, that we shall receive the classical equation of the capital: $\frac{dK}{dt}$ = vAf(K, L), and if ^{\uparrow} = (*t*, *r*), that we have the equation of the capital in the following kind:

$$
\frac{\partial \mathbf{K}}{\partial t} + \mathbf{x}_0 \frac{\partial \mathbf{K}}{\partial r} = \mathbf{V} \mathbf{A} f(\mathbf{K}, L), \qquad \mathbf{x}_0 = \frac{dr}{dt}.
$$

3. **Model of a Labor**. Work of those people who have devoted them to work, those are quantity fulfilled workers of hours. Now in modeling, economy for definition of parameter of work the model is used: $\frac{dL}{dt} = uL$, where U is rate of growth of the population equation. Clearly, that within the framework of the given model many important factors as the erudition, age, a floor, a nationality are not taken into account. In this connection we shall assume, that work is defined as functional manpower:

$$
L(t) = \int_{a \min}^{a \max} \int_{R} \left\{ (x, a, t) N(x, a, t) dx da, \right\} \tag{4}
$$

here $\{ = \{ (x, a, t) \}$ is potential function of working, N=N (x, a, t) - number of working in a point xèR, age and, $0 < a < \infty$, at the moment of time t; $\int_{0}^{\infty} \sin \theta \, d\theta$ - accordingly the minimal and maximal age of the working in sphere of manufacture. As shown in our works, function $N=N(x, a, t)$ is the decision of the following problem:

$$
\partial_{tax} N = F(N, a, t), 0 < a < \infty, 0 < t < t_k,
$$

\n
$$
N_{t=0} = N_{0,\infty,} X \in R,
$$

\n
$$
N(x, 0, t) = \int_0^\infty (N, \langle , t \rangle) d\langle , N_{t=0}.
$$
\n(5)

Here $F(\bullet)$, $B(\bullet)$ according to function of death rate and birth rate of the labor population $\int_{0}^{\delta_{\text{max}}} = \frac{1}{\delta t} + \frac{1}{\delta t} + \sum_{i=1}^{\infty} \left[\frac{\epsilon_i}{\delta x_i} - \frac{\epsilon_i}{\delta x_i} \right]_{0}^{\infty}$ (*D_i* $\frac{\delta_{\text{max}}}{\delta x_i}$) S is a border the contract of the contract of the contract of $\left[\begin{array}{cc} \epsilon_i & \frac{\partial X_i}{\partial X_i} \end{array} - \frac{\partial X_i}{\partial X_i} (D_i \frac{\partial X_i}{\partial X_i}) \right]$, S $\begin{bmatrix} 0 & \partial & \partial & \mathbb{R} \end{bmatrix}$ ∂X_i $\left| \right|$ S is a horder ∂ , \Box $\frac{\partial}{\partial t} + \frac{\partial}{\partial \Gamma} + \sum_{i} \left[\epsilon_i \frac{\partial}{\partial X_i} - \frac{\partial}{\partial X_i} (D_i \frac{\partial}{\partial X_i}) \right]$ S is a border $\partial_{t} u x = \frac{\partial}{\partial t} + \frac{\partial}{\partial r} + \sum_{i} \left[\epsilon_i \frac{\partial}{\partial x_i} - \frac{\partial}{\partial x_i} (D_i \frac{\partial}{\partial x_i}) \right]$, S is a border

of area R; $R = \sum R_i, R_i - i -$ region. Potential function of working, $\{\}$ = { (*x, a,t*) is the decision connected to (5) problems [14] - [40]. In the mentioned works it is shown, that the function of work determined with the

help (4) satisfies the equation $\frac{dE}{dt} = uL$, where rates of growth of the population U are the decision of the following so-called equation of survival rate: *dL* $\frac{L}{t}$ = uL, where rates of growth of the

$$
\int_{0}^{\infty} B(a)e^{-u} da = 1
$$
 (6)

Here $B(a) = B_0(a)e^{-0}$ - is function of survival rate, $B_0(r)$ is a factor of birth rate, $F_0(a)$ - a mortality rate coefficient, $0 < a < \infty$. The equation (6) has one maximal material root $u_0 = u_{\text{max}}$ and accounting number in a complex - connected roots $u_i = r_i \pm i s_i$, $i=1,2$ For the maximal root $u_0 = u_{\text{max}}$ takes place $-\int F_0(\cdot) d\cdot$ $(a) = B_0(a)e^{-0}$ - is function of $F_0(\epsilon) d\epsilon$

$$
0, \quad \text{if} \quad h = 0
$$

\n
$$
0, \quad \text{if} \quad h = 1,
$$

\n
$$
< 0, \quad \text{if} \quad h < 1,
$$

where h is the potential of a labor. Hence $\sum c_i e^{au}$ $=\sum_{i=0}^{\infty} c_i e^{u_i}$ $(t) = \sum_{i} c_i e^{u_i t}$ $L(t) = \sum_{i=0}^{6} c_i e^{i t}$ *it* u it

4. Model of technology productivity level (the Measure of the current technological level). As

$$
\frac{dY}{dt} = \frac{dA}{dt} f + A \frac{df}{dk} \cdot \frac{\partial K}{\partial t} + A \frac{\partial f}{\partial L} \cdot \frac{dL}{dt},
$$
\n(7)

and

$$
\frac{dy}{dt} = \frac{dC}{dt} + \frac{dI}{dt} + \frac{dG}{dt} + \frac{dN_x}{dt}
$$
, that with the account
$$
\frac{dC}{dt} = \frac{dC}{dy}(\frac{dy}{dt} - \frac{dr}{dt}),
$$

$$
\frac{dI}{dt} = \frac{V}{dt} \frac{dy}{dt} + y \frac{dV}{dt}
$$
, from (7) we have:
$$
\frac{dA}{dt} = -\frac{A}{f} \frac{\partial f}{\partial k} \cdot \frac{dk}{dt} - \frac{A}{f} \frac{\partial f}{\partial L} \cdot \frac{dL}{dt} + \frac{A}{y} (1 - v - MPG^{-1})
$$

$$
u,
$$

where
$$
u = -\frac{MPC}{dt} \frac{dT}{dt} + \frac{dG}{dt} \frac{dN_x}{dt} + y \frac{dv}{dt} = -MPCu_0 + u_1 + u_2 + y \cdot u_3
$$
, $MPC = \frac{dC}{dy}$.

$$
\frac{dA}{dt} = -rA^2 + A,
$$
\n(8)
\n
$$
r = \cdot v + u \cdot (1 - r), = \frac{1}{y}(1 - v - MPC)^{-1} \cdot u
$$
\nwhere\n(8)
\n
$$
r = \cdot v + u \cdot (1 - r), = \frac{1}{y}(1 - v - MPC)^{-1} \cdot u
$$
\nThus, the equations for

the capital, work, and productivity of technical progress have the following kind:

$$
\frac{dK}{d\ddagger} = VAf(K, L), K_{/ \ddagger = 0} = K_0,
$$
\n
$$
\frac{dL}{d\ddagger} = uL, L_{/ \ddagger = 0} = L_0,
$$
\nI=ε₀ Y, C = (1-ε₀) Y,
\n
$$
\frac{dA}{dt} = -\Gamma A^2 + A, A_{/ \ddagger = 0} = A_0,
$$
\n
$$
\frac{dy}{dt} = (1 - V - MPC)^{-1}u, y_{/ \ddagger = 0} = y_0, Y = Af(K, L),
$$
\n(9)

where μ is the decision (6). To the equation (9) it is necessary to add also the equations:

$$
\frac{dT}{dt} = u_0, \frac{dG}{dt} = u_1, \frac{dN_x}{dt} = u_2, \frac{dV}{dt} = u_3
$$
\n(10)

The right parts of the equation (10) are rates of growth of sizes (T, G, N_x, V) . They are necessary for defining from a condition of maximization of some economic criterion (or from a condition of minimization cost functional and so on): V).

max y(u0, u1, u2, u3).

In system (9), (10) the following designation is accepted: . In order that if $MPC=1-\nu$, at anyone $\frac{dy}{dt} \neq 0$. It means that $u = (1 - v - MPC)\frac{dy}{dt} = 0\frac{dy}{dt} = 0$, that, if $v = 1 - MPC$, that is $y = Vy + (1 - V)y = I + C + G + N_x$, and balance of economy $u = -MPC \cdot u_0 + u_1 + u_2 + y \cdot u_3 = 0$. It turns out only due to a choice of rates changes of taxes, rates of the state purchases and pure export, and as rate of a share of the income going on capital investments. If $V < 1 - MPC$, that occurs increase or reduction of the national income depending on a mark of function $\sum_{i=1}^{n} v_i \frac{1}{\partial x_i} + x_1 \frac{1}{\partial e} - \sum_{i=1}^{n} \frac{1}{\partial x_i} (D_i \frac{1}{\partial x_i})$ In order that if $MPC=1-\nu$, at ∂ $\partial x_i \stackrel{(D_i)}{\sim} \partial x_i'$ In order that if $MPC=1-\sqrt{ }$ $-\frac{2}{2}\frac{\partial}{\partial}(D_i\frac{\partial}{\partial})$ ∂e_{i} $\stackrel{\sim}{\sim} \partial x_i$ $\stackrel{\sim}{\sim}$ ∂x_i In order that if MP ∂ 2 ∂ ∂ $+X_1 - \sum_{i=1}^{N} -\sum_{i=1}^{N} (D_i - \sum_{i=1}^{N})$ ∂x_i $\bigcap_{i=1}^n \partial e_i \bigcap_{i=1}^{\infty} \partial x_i$ ∂x_i In order that ∂ ∂ ∂ ∂ ∂ $+\Sigma v_i \frac{U}{2} + x_1 \frac{U}{2} - \Sigma \frac{U}{2} (D_i \frac{U}{2})$ $\partial r \stackrel{\simeq}{=} \frac{1}{2} i \partial x_i$ $\stackrel{\simeq}{=} \frac{1}{2} \partial x_i$ $\stackrel{\simeq}{=} \frac{1}{2} \partial x_i$ $\stackrel{\simeq}{=} \frac{1}{2} \partial x_i$ ∂ ∂ ∂ ∂ ∂ ∂ ∂ $+X_0 - \frac{C}{2} + \bar{\Sigma}v_i - \frac{C}{2} + X_1 - \bar{\Sigma} - \bar{\Sigma} - (D_i - C)$ $\partial t \stackrel{\cdots}{\cdots} \partial r \stackrel{\cdots}{_{i=1}^{i}} \partial x_i \stackrel{\cdots}{\cdots} \partial e \stackrel{\cdots}{_{i=1}^{i}} \partial x_i \stackrel{\cdots}{\cdots} \partial x_i$ $=\frac{\partial}{\partial}+X_0\frac{\partial}{\partial}+\frac{2}{\Sigma}y_i\frac{\partial}{\partial}+X_1\frac{\partial}{\partial}-\frac{2}{\Sigma}\frac{\partial}{\partial}(D_i\frac{\partial}{\partial})$ 1° αx_i° $\alpha e_{i=1}\alpha x_i^{\circ}$ αx_i° 2 ∂ ∂ ∂ $0 \frac{\partial}{\partial r} + \bar{\Sigma} v_i \frac{\partial}{\partial x_i} + X_1 \frac{\partial}{\partial e} - \bar{\Sigma} \frac{\partial}{\partial x_i} (D_i \frac{\partial}{\partial x_i})$ In order that if MPC=1 *i*=1 α *i* α *i* α *i*=1 α *i* α *i i* α *i i i n* order that $\overline{i} \overline{\lambda}$ $\frac{\partial}{\partial x_i} + x_1 \frac{\partial}{\partial e} - \sum_{i=1}^{\infty} \frac{\partial}{\partial x_i} (D_i \frac{\partial}{\partial x_i})$. In order that if $MPC=1-\nu$, at $\frac{\partial}{\partial t} = \frac{\partial}{\partial t} + X_0 \frac{\partial}{\partial r} + \frac{\Sigma}{i} v_i \frac{\partial}{\partial x_i} + X_1 \frac{\partial}{\partial e} - \frac{\Sigma}{i} \frac{\partial}{\partial x_i} (D_i \frac{\partial}{\partial x_i})$ In order that if MPC=1 *d* $\frac{d}{dt} = \frac{\partial}{\partial t} + X_0 \frac{\partial}{\partial r} + \sum_{i=1}^{N} v_i \frac{\partial}{\partial x_i} + X_1 \frac{\partial}{\partial e} - \sum_{i=1}^{N} \frac{\partial}{\partial x_i} (D_i \frac{\partial}{\partial x_i})$ In order that if $MPC=1-\nu$, at $\frac{dy}{dt} \neq 0.$ $u = (1 - v - MPC)\frac{dy}{dt} = 0$ $\frac{dy}{dt} = 0$ \ddagger d dt , that, if $V = 1 - MPC$. $V - MPC \frac{dy}{dx} = 0 \frac{dy}{dx} = 0$ $d\ddagger$, that, if $V = 1 - MPC$, *dy* $d\ddagger$ $d\ddagger$ that, if $V = 1 - MPC$, $u = (1 - v - MPC)\frac{dy}{dt} = 0 \frac{dy}{dt} = 0$ $V = 1 - MPC$

u. Depending on set of values of parameter $\bar{f} = (t, r, e, x, ...)$ the system of the equation (9) - (10) will be transformed or in system of the ordinary differential equations, or in system of the equations in private(individual) derivatives. For example, if $\ddagger = t$, we shall receive system of the ordinary differential equations of $1st$ order:

$$
\frac{dK}{dt} = VAf(K, L), K(0) = K_0, \frac{dL}{dt} = uL, L(0) = L_0
$$
\n
$$
L(t) = \int_{a \text{ min}}^{a \text{ max}} \int_R \{ (x, a, t)N(x, a, t)dxda,
$$
\n
$$
\frac{dA}{dt} = -r A^2 + sA, A(0) = A_0
$$
\n
$$
\frac{dy}{dt} = (1 - V - MPC)^{-1} [(-MPCu_0 + u_1 + u_2) + u_3y]y(0) = y_0
$$
\n
$$
\frac{dT}{dt} = u_0, \frac{dG}{dt} = u_1, \frac{dN}{dt} = u_2, \frac{dV}{dt} = u_3
$$
\n(11)

5. Properties of Productions. Under productions we shall understand system of elements (basic and turnaround, information and labor resources) in result of joint functioning of which " the capital and labor " will be transformed to a final product (or national income). The transformation of which carries out this transition refers to as by production function and is designated through *Y=Af(K, L),* where the K-size of the capital (fixed capital), L–functional of labor resources, which depends from potential of labor resources, educationists, serviceability, floor and age, and also number of the workers, A is technological label. At the moment all over the world distinguish three as modeling of productions.

a). Productions as Cobb-Douglas

$$
Y = Y_0 \left(\frac{K}{K_0} \right)^{r} \left(\frac{L}{L_0} \right)^{1-r},
$$

where Y_0 the national income at the appropriate capital K_0 and labor resource L0. *b). Productions with elastically of replacement CES (Solow):*

.

$$
Y = Y_0 \left[\Gamma \left(\frac{K}{K_0} \right)^{-m} + (1 - \Gamma) \left(\frac{L}{L_0} \right)^{-m} \right]^{-1} , 0 < \Gamma < 1, \dots > 0
$$

c). *The constant proportion productions CP (Leontev)*

$$
Y = Y_0 \min\{K/K_0, L/L_0\}
$$

d), *u*-production, \sim - (**m** y**u**): $Y = A\lambda(K, L)$,

$$
f(K,L) = f_0 \left[\Gamma \left(\frac{K}{K_0} \right)^{-p} + (1 - \Gamma^{\frac{n}{n-s}})^{(n-s/n)} \left(\frac{L}{L_0} \right)^{-p} \right]^{-\frac{1}{p}} , \quad n > s > 0, \quad \rho = \rho_0 s,
$$

0 $\langle \rho_0 \langle \infty \text{or introducing } \sim (r) \rangle = \left[\frac{f(K,L)}{f_0} \right]^{-\frac{1}{\rho_0}}, \quad X = \left[\frac{K}{K_0} \right]^{-\frac{1}{\rho_0}}, \quad Y = \left[\frac{L}{L_0} \right]^{-\frac{1}{\rho_0}} \text{ we have}$

$$
\sim (r) = \left[rX^s + \left(1 - r^{\frac{n}{n-s}} \right)^{\frac{n-s}{n}} Y^s \right], \quad 0 < r < 1,
$$

It is shown in works [1,2] from CES productions at ... È 0 follows Cobb-*Douglas productions and at the constant proportion productions CP and from µ-production at s* $\hat{\mathsf{E}}$ 0 or $n\hat{\mathsf{E}}$ *all productions.*

It is shall out, that all types of production, i.e. the production functions should satisfy to following conditions:

1). $f(K, L) \in C^2[K_0, K_{\text{max}}] \times [L_0, L_{\text{max}}]$, i.e. entrance var. $f(K,L) \in C^2[K_0, K_{\text{max}}] \times [L_0, L_{\text{max}}]$, i.e. entrance variables smoothly are chaining and results of activity of manufacture - the national income rather smoothly varies at changes quantity of used resources. It is natural at forecasting large systems, for example, economy of country.

2). F $(0, L) = 0$, f $(K, 0) = 0$, i.e. at absence though of one industrial resource of manufacture it is impossible. $\frac{\partial f(K,L)}{\partial K} > 0$, $\frac{\partial f(K,L)}{\partial K} > 0$. ∂K ∂L ∂L $\partial f(K,L)$ $\partial f(K,L)$ *K* $\frac{f(K,L)}{g(K,L)} > 0$, $\frac{\partial f(K,L)}{\partial K} > 0$. ∂L and ∂L and ∂L and ∂L $\partial f(K,L)$ *L* $\frac{f(K,L)}{g(K)} > 0$.

3). At > 0 and L > 0 , it means, that growth of used quantity of fixed capital and growth of number of the workers results in growth of the national income.

4) $\frac{U}{2V^2} \le 0$, $\frac{U}{2I^2} \le 0$, I.e. $2f \qquad \qquad \lambda^2 f$ $\leq 0, \frac{0}{\sqrt{2}} \leq 0,$ I.e. in conditions of ∂K^2 - 3, ∂L^2 - 3, 1.0. In community $\partial^2 f$ (0 $\partial^2 f$ (0 I, in sent) $\frac{\partial^2 f}{\partial K^2} \le 0$, $\frac{\partial^2 f}{\partial L^2} \le 0$, I.e. in conditions of p ≤ 0 , I.e. in conditions of pure ∂L^2 - 3, 1.0. In conditions on $\partial^2 f$ (0 I, in conditions of L^2 , we have the continuous of \mathbf{r} $\frac{f}{2} \leq 0$, I.e. in conditions of pure economic growth of manufacture (without technical progress) grows of expenses only of one

industrial resource results in decrease(reduction) of efficiency it using.

5). $f(\frac{1}{K}, \frac{1}{L}) > f(K, L)$ at $\} > 1$ or $f(\frac{1}{K}, \frac{1}{L}) > \frac{1}{K} f(K, L)$ at $\} > 1$, and $m \ge 1$.

6). This condition provides that at proportional growth the quantity of used resources occurs proportional growth made products or national income. We shall notice, that all above mentioned functions submit to these conditions.

6. The basic parameters of production. The basic parameters of manufacture are: $K=K(t)$ is the size of the capital at the moment of time t, i.e. K=K (t); L=L (t) is functional of a labor resource, $x = dK/dL$ is limiting

norm of replacement, $\tau = \frac{d(K/L)}{dx} \cdot \frac{x}{K/L}$ is elasticity replacement of $/L$ (K/L) x \vdots electricity replease $X \times K/L$ $t = \frac{u(N/L)}{l} \cdot \frac{\lambda}{N/L}$ is elasticity replacement of resources, $E_k = \frac{K}{f} \frac{\partial f}{\partial K}$, $E_k = \frac{L}{f} \frac{\partial f}{\partial L}$ are coefficients of elasticity of $f \partial K$ ^{, - L} $f \partial L$ $E_k = \frac{K}{f} \frac{\partial f}{\partial K}$, $E_L = \frac{L}{f} \frac{\partial f}{\partial L}$ are coefficients of elasticity of is $E_L = \frac{L}{f} \frac{\partial f}{\partial L}$ are coefficients of elasticity of issue on resources, $W_k = \frac{f}{K}$, $W_L = \frac{f}{L}$ are medium capital productivity. We shall notice, that for example shows how many fixed capital an expense of labor per unit of and on the contrary for the preservation of the national income at a former level f $(K, L) = f$ can be released at increase. Parameter defines speeds of change of limiting norm of replacement of resources. Factors of elasticity of issue on resources E_K , E_L show on how many of percent will change manufactures of the national income at change of expenses of the appropriate resource of manufacture on one percent. It is necessary to note, that if to enter concept of value $k=K/L$ in productions, we have: y=F (), where Y=Y/Y₀, F (.) =f (,1), and F'() > 0, F''() < 0. resources, $w_k = \frac{y}{K}$, $w_k = \frac{y}{L}$ are medium capital productivity. We shall no
that for example shows how many fixed capital an expense of labor
unit of and on the contrary for the preservation of the national income
 and to four more more than the preservation of the national income at a
function data on the contrary for the preservation of the national income at a
former level f (K, L) =f can be released at increase. Parameter define *a d* resources, $w_k = \frac{1}{K}$
that for example
unit of and on the
former level f (*H*
speeds of change
elasticity of issue
change manufact
appropriate resources, that if to e
y=F (), where Y=
7. **Model of the ba**
economic model that for example shows how many fixed capital and the contrary for the preservation of former level $f(K, L) = f$ can be released at increspeeds of change of limiting norm of replacement elasticity of issue on resources E_K , sources, $w_k = \frac{f}{K}$, $w_L = \frac{f}{L}$ are medium capital productivity. We shall not
to for example shows how many fixed capital an expense of labor
it of and on the contrary for the preservation of the national income
mer l resources, $w_i = \frac{f}{K}$, $w_i = \frac{f}{L}$ are medium capital productivity. We shall notice,
that for example shows how many fixed capital an expense of labor per
unit of and on the contrary for the preservation of the nationa that for example shows how many fixed capital an expense of labor per
unit of and on the contrary for the preservation of the national income at a
former level $f(K, L) = f$ can be released at increase. Parameter defines
spee

7. Model of the basic resources. Following our work we shall write general economic model for definition of sizes of the basic industrial resources

change manufacturers of the national income at change of expenses of the appropriate resource of manufacture on one percent. It is necessary to note, that if to enter concept of value k=K/L in productions, we have:
$$
y=F(0)
$$
, where $Y=Y/Y_0$, $F(.)=f(.1)$, and $F'(0) > 0$, $F''(0) < 0$.
\n7. Model of the basic resources. Following our work we shall write general economic model for definition of sizes of the basic industrial resources $K=K(t), L=L(t)$ and consumption C=C(t):
\n
$$
\begin{cases}\n\frac{dK}{dt} = v f(K, L), K(0) = K_0, C(t) = (1-v) f(K, L).\n\end{cases}
$$
\n(1")
\nHere the share of the national income $Y=Af(K, L)$ going on process of manufacture, a_{\min} , a_{\max} are minimum and maximum age of the workers, (a, t)-potential function of the workers is defined as the decision of the following problem
\n
$$
\begin{cases}\n\frac{\partial W}{\partial t} + \frac{\partial W}{\partial a} = A(a, t)W(a, t) + B(a, t)W(o, t) + f(t), 0 \leq t < t_k,\n\end{cases}
$$
\n
$$
W|_{I_k} = 0, W|_{a=\infty} = 0,
$$

Here the share of the national income $Y=At(K, L)$ going on process of manufacture, a_{min} , a_{max} are minimum and maximum age of the workers, (a, t) potential function of the workers is defined as the decision of the following problem

$$
\begin{cases}\n\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{W}}{\partial a} = A(a, t) \mathbf{W}(a, t) + B(a, t) \mathbf{W}(o, t) + f(t), 0 \le t < t_k, \\
\mathbf{W}\Big|_{t_k} = 0, \mathbf{W}\Big|_{a = \infty} = 0,\n\end{cases}
$$

A(.), B(.), f(.) are given functions, N=N (a, t) is the number of the workers of age at the moment of time t: $0(a)$ $0 < a < \infty$ 0 19

are given functions, N=N (a, t) is the number of the w

the moment of time
 $(N, a, t), 0 < a < \infty, 0 < t \le \infty$
 $a), 0 \le a < \infty,$
 $N(a, t), a, t) da$ 19

, B(.), f (.) are given functions, N=N (a, t)

at the moment
 $\frac{N}{t} + \frac{\partial N}{\partial a} = F(N, a, t), 0 < a < \infty, 0 < t \le \infty$

(a, 0) = N^o(a), 0 \le a $< \infty$,

(0, t) = $\int_{0}^{\infty} B(N(a, t), a, t) da$

, N^o(), B() are given functions of (0 , 1) I9

(0 , 1) F(.) are given functions, N=N (a, t) is

at the moment
 $\frac{N}{t} + \frac{\partial N}{\partial a} = F(N, a, t), 0 < a < \infty, 0 < t \le \infty$

(a, 0) = $N^0(a), 0 \le a < \infty$,

(0, t) = $\int_0^{\infty} B(N(a, t), a, t) da$

(N) F() are given functions of the *N* $B(.)$, $f(.)$ are given functions, $N=N$ (a, t) is the numb at the moment of $\frac{N}{2t} + \frac{\partial N}{\partial a} = F(N, a, t), 0 < a < \infty, 0 < t \le \infty$
 $(a, 0) = N^0(a), 0 \le a < \infty,$, $(0, t) = \int_{0}^{\infty} B(N(a, t), a, t) da$ *t*, B(.), f (.) are given functions, N=N (

at the moment
 $\frac{N}{t} + \frac{\partial N}{\partial a} = F(N, a, t), 0 < a < \infty, 0 < t \le$
 $(a, 0) = N^0(a), 0 \le a < \infty,$
 $(0, t) = \int_0^{\infty} B(N(a, t), a, t) da$
 $M^0(\), B(\)$ are given functions of the arg *(.), B(.), f (.) are given functions, N=N (a, t) is the nun*
 ne at the moment of
 $\frac{\partial N}{\partial t} + \frac{\partial N}{\partial a} = F(N, a, t), 0 < a < \infty, 0 < t \le \infty$
 N (a, 0) = $N^0(a), 0 \le a < \infty$,
 N (0,t) = $\int_0^{\infty} B(N(a, t), a, t) da$

(*),N*⁰(*),B*(¹⁹

(b), B(b), f (c) are given functions, N=N (a, t) is the number of
 $\frac{\partial N}{\partial t} + \frac{\partial N}{\partial a} = F(N, a, t), 0 < a < \infty, 0 < t \le \infty$
 $N(a, 0) = N^0(a), 0 \le a < \infty,$
 $N(0, t) = \int_0^{\infty} B(N(a, t), a, t) da$

(e), $N^0(0, t) = \int_0^{\infty} B(N(a, t), a, t) da$

(e A(.), B(.), f (.) are given functions, N=N (a, t) is the number of the workers of
age at the moment of time t:
 $\begin{cases} \frac{\partial N}{\partial t} + \frac{\partial N}{\partial a} = F(N, a, t), 0 < a < \infty, 0 < t \le \infty \\ N(a, 0) = N^0(a), 0 \le a < \infty, \\ N(0, t) = \int_0^{\infty} B(N(a, t), a, t) da \end{cases}$.), B(.), f (.) are given functions, N=N (a, t) is the number of the worker

e at the moment of time
 $\frac{\partial N}{\partial t} + \frac{\partial N}{\partial a} = F(N, a, t), 0 < a < \infty, 0 < t \le \infty$
 $V(a, 0) = N^0(a), 0 \le a < \infty,$
 $V(0,t) = \int_0^{\infty} B(N(a,t), a, t) da$ 19

A(.), B(.), f (.) are given functions, N=N (a, t) is the number of the workers of

age at the moment of time t:
 $\left(\frac{\partial N}{\partial t} + \frac{\partial N}{\partial a} = F(N, a, t), 0 < a < \infty, 0 < t \le \infty\right)$
 $N(a, 0) = N^0(a), 0 \le a < \infty$,
 $N(0, t) = \int_0^{\infty} B(N(a, t$ ∞ and ∞ $= | B(N(a,t),a,t)da$ $N(0,t) = \int_{0} B(N(a,t),a,t) da$,

 $F($), $N^0($), $B($) are given functions of the arguments, and F(.) - means the deathfunction of the workers, and B (.) function of their birth rate, N_0 (a) - initial number of the workers. Solving a problem we find functions $\varphi = \varphi(a, t)$ and $N=N$ *(a, t),* and then we shall define functional of labor resources $L=L(t)$. At a known kind of manufacture $Y=Af(K, L)$ from a problem (1) we shall define dynamics of the size of fixed capital, i.e. a size of the capital $K=K(t)$, $0 < t < t_k$, and a size consumption $C=C(t)$ in any moment of time. B (.) function of their birth rate, N₀ (a) - initial
g a problem we find functions $\varphi = \varphi(a, t)$ and $N=N$
functional of labor resources $L=L(t)$. At a known
L) from a problem (1) we shall define dynamics
a size of the cap *n* of the workers, and **B** (*s*) functions of the arguments, and **P** (*s*) - means the decann-
of the workers, solving a problem we find functions φ_0 (a) - initial
or of the workers. Solving a problem we find functi *Kers, and B (.) function of their birth rate, N₀ (a) - initial

ers. Solving a problem we find functions* $\varphi = \varphi(a, t)$ *and* $N = N$ *

hall define functional of labor resources* $L = L(t)$ *. At a known
* $E = Af(K, L)$ *from a problem xere given functions of the arguments, and F (.) - means the de workers, and B (.) function of their birth rate, N₀ (a) - in workers. Solving a problem we find functions* $\varphi = \varphi(a, t)$ *and* Λ *we shall define functiona* are given functions of the arguments, and F (.) - means the death-
e workers, and B (.) function of their birth rate, N₀ (a) - initial
workers. Solving a problem we find functions $\varphi = \varphi(a, t)$ and $N = N$
we shall define), *B*() are given functions of the arguments, and F (.) - means the death-
of the workers, and B (.) function of their birth rate, N₀ (a) - initial
of the workers. Solving a problem we find functions $\varphi = \varphi(a, t)$ and

Definition. *Economic system connected with manufacture Y=A f (K, L), we shall name system consisting from the following elements: (K (t), L (t), C* (t), $I(t)$, where $C=C(t)$, $K=K(t)$, $L=L(t)$ are the decision of system (1").

8. General model productions: *µ-production*. As marked above from CES productions, Cobb-Douglas productions follow in particular at $\rho \rightarrow 0$ and a constant proportion productions at $\rho \rightarrow \infty$. In our work by us it was offered following modeling productions $(A=1, f=Y)$: consisting from the following elements: (*K*, t), $K=K(t)$, $L=L(t)$ are the decision of system
 l productions: μ -production. As marked abobb-Douglas productions follow in particular

trion productions at $\rho \rightarrow \infty$. In

$$
Y = Y_0 \left[\Gamma \left(\frac{K}{K_0} \right)^{-\pi} + \left(1 - \Gamma^{\frac{n}{n-s}} \right)^{\frac{n-s}{n}} \left(\frac{L}{L_0} \right)^{-\pi} \right]^{-\frac{1}{2}} , n > s, s > 0. \quad (12)
$$

We shall notice that is higher resulted modeling productions special cases of the given manufacture are. From production function (12) at $n \rightarrow \infty$ follows function CES which as have noted is more general than Cobb-Douglas functions and a constant proportion functions. Thus from production function of (12) are follow all known production functions. We shall calculate parameters of manufacture. Easily to see, that

$$
x = -\frac{(1 - r^{\frac{n}{n-1}})^{\frac{n-1}{n}}}{r} \left(\frac{K_0}{L_0}\right)^{\frac{n}{n}}, \left(\frac{K}{L}\right)^{1+\dots}
$$
, i.e. the norm of replacement is function of

asserts, and therefore

$$
\mathcal{L} = \frac{K}{L} = \left[-\frac{r}{(1 - r^{\frac{n}{n-1}})^{\frac{n-1}{n}}} \left(\frac{K_0}{L_0} \right) \mathbf{k} \right]^{1 + \dots}
$$

The meanings of elasticity replacement are defined as follows: $+$... $1 + \dots$

Coefficients of elasticity of issue on resources are defined accordingly under the formulas:

The meanings of elasticity replacement are defined as follows:
$$
1 + ...
$$

\nCoefficients of elasticity of issue on resources are defined accordingly under the
\nformulas:
\n
$$
E_K = \frac{r\left[\frac{K}{K_0}\right]^{-...}}{r\left[\frac{K}{K_0}\right]^{-...} + \left(1 - r^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}} \left(\frac{L}{L_0}\right)^{-...}}\right]
$$
\n
$$
E_L = \left(1 - r^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}} \cdot \left[\frac{L}{L_0}\right]^{-...} \left[r\left(\frac{K}{K_0}\right)^{-...} + \left(1 - r^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}} \cdot \left(\frac{L}{L_0}\right)^{-...}\right]
$$
\nand $E_K = r \frac{(f/f_0)^{-}}{(K/K_0)^{-}}, E_L = (1 - r^{\frac{n}{n-1}})^{\frac{n+1}{n}} \frac{(f/f_0)^{-}}{(L/L_0)^{-}}, E_K + E_L = 1.$
\nWe shall consider offered manufacturers (12) at $\rho \rightarrow 0$. As $F(K) = f(K, 1)$, it is necessary to find a limit $\lim_{k \to \infty} Y_0 \left[r\left(\frac{K}{K_0}\right)^{-1} + (1 - r^{\frac{n}{n-1}})^{\frac{n-1}{n}}\right]^{\frac{1}{n}} = A$, Easily to see, that

and
$$
E_K = r \frac{(f/f_0)^m}{(K/K_0)^m}
$$
, $E_L = (1 - r^{\frac{n}{n-1}})^{\frac{n-1}{n}} \frac{(f/f_0)^m}{(L/L_0)^m}$, $E_K + E_L = 1$.

necessary to find a limit $\lim_{\epsilon \to 0} Y_0 \left[\frac{K}{K} \right]^{-1} + (1 - \epsilon^{\frac{1}{n-s}})^{\frac{1}{n}} = A$, Easily to see, that K_0 $\qquad \qquad$ \qquad $\qquad \qquad$ \qquad $\$ Y_0 $\left| \Gamma\left(\frac{K}{K}\right)^{-1} + (1 - \Gamma^{\frac{n}{n-s}})^{\frac{n-s}{n}} \right|^{-1} = A$, Easily to see, that \mathbf{r} $\begin{bmatrix} \overline{a} & \overline{b} & \overline{c} \\ \overline{c} & \overline{d} & \overline{c} & \overline{c} \end{bmatrix}$ $\left| \Gamma(\frac{1}{K})^{-1} + (1 - \Gamma^{\frac{n-s}{n}})^n \right| =$ $\begin{bmatrix} & K_0 & \cdots & \cdots & \cdots \end{bmatrix}$ K \overline{K} $+(1-r^{\frac{1}{n-s}})^{\frac{1}{n}}$ = A, Easily to see, that $-\frac{1}{2}$ -1 + $(1 - r^{\frac{n}{n-s}})^{\frac{n-s}{n}}$ = -4 Fasily $\left| \Gamma\left(\frac{K}{\sigma}\right)^{-n} + (1 - \Gamma^{\frac{n}{n-3}})^{\frac{n-3}{n}} \right|^{-n} = A$, Easily to see, that $\mathbf{1}$ and $\mathbf{1}$ and $\mathbf{1}$ and $\mathbf{1}$ $\lim_{n \to \infty} Y_0 \mid r \left(\frac{R}{\sigma} \right)^{-1} + (1 - r^{\frac{1}{n-s}})^{-n} \mid = A$, Easily to see, that $\begin{array}{ccc} 0 & & \end{array}$ $\int_0^{\infty} \left| \int_0^{\infty} \left| \int_0^{\infty} e^{-x} \right|^{x} dx \right|^{x} dx = A$, Easily to see, that

$$
E_L = \left(1 - r^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}} \cdot \left[\frac{L}{L_0}\right]^{-\frac{n}{n}} \sqrt{r\left(\frac{K}{K_0}\right)^{-\frac{n}{n}} + \left(1 - r^{\frac{n}{n-1}}\right)^{\frac{n-1}{n}} \cdot \left(\frac{L}{L_0}\right)^{-\frac{n}{n}}}
$$
\nand $E_K = r \frac{(f/f_0)^{-\frac{n}{n}}}{(K/K_0)^{-\frac{n}{n}}}, E_L = (1 - r^{\frac{n}{n-1}})^{\frac{n-1}{n}} \frac{(f/f_0)^{-\frac{n}{n}}}{(L/f_0)^{-\frac{n}{n}}}, E_K + E_L = 1.$ \nWe shall consider offered manufacturers (12) at $\rho \to 0$. As $F(K) = f(K, 1)$, it is necessary to find a limit $\lim_{k \to \infty} \left[r\left(\frac{K}{K_0}\right)^{-\frac{n}{n}} + (1 - r^{\frac{n}{n-1}})^{\frac{n}{n}}\right]^{-\frac{1}{n}} = A$, Easily to see, that $\ln A = \ln Y_0 - \frac{1}{\ln} \ln \left(r\left(\frac{K}{K_0}\right)^{-\frac{n}{n}} + (1 - r^{\frac{n}{n-1}})^{\frac{n-1}{n}}\right)$, and therefore $\frac{\lim_{k \to \infty} A = r \ln \frac{K}{K_0}}{\frac{1}{n}}$ i.e. $f = Y_0 F(\frac{1}{\sqrt{n}})^{-\frac{n}{n}}$. As $F(K)$ unequivocally defines function $f(K, L)$ we have received the statement that the function Cobb-Douglas at $\rho \to 0$ (and $n \to \infty$) is a special case our function. Similarly, at $(\rho \to \infty$ and $n \to \infty$) we have

.

 $=\frac{1}{\sqrt{2}}$

.

1

$$
\lim_{\Delta t \to \infty} A = \begin{cases} Y_0 & \text{at } K \ge K_0 \\ Y_0 \frac{K}{K_0} & , \text{at } K < K_0 \end{cases} \quad \text{or} \quad \begin{cases} Z_1 & \text{at } K \le K_1 \\ Y_0 & , \text{at } K < K_0 \end{cases} \quad \text{or} \quad \begin{cases} K < L_0 \\ \frac{K}{L_0} & , \text{at } K \le K_0 \end{cases} \quad \text{or} \quad \begin{cases} K < L_0 \\ \frac{K}{L} \cdot \frac{L_0}{K_0} & , \text{at } K \le K_0 \end{cases} \quad \text{and} \quad \begin{cases} \frac{K}{L} \cdot \frac{L_0}{K_0} \cdot \frac{K_0}{K_0} & , \text{at } K \le K_0 \end{cases}
$$

$$
f(K, L) = LY_0 \min\{\frac{K}{K_0}, 1\} = L\frac{Y_0}{L_0} \min\left\{\frac{K}{L} \cdot \frac{L_0}{K_0}, 1\right\} = Y_0 \min\left\{\frac{K}{L} \cdot \frac{L_0}{K_0}\right\}, \text{ i.e. have}
$$

received function with a constant proportion. We shall define limiting meaning of economic parameters (s=1)

,

$$
\lim_{n \to \infty} A = \begin{cases}\nY_0 & \text{at } K \ge K_0 \\
Y_0 \frac{K}{K_0}, & \text{at } K < K_0\n\end{cases}, \text{ or } F(K) = Y_0 \min\{\frac{K}{K_0}, 1\}
$$
\nand therefore\n
$$
f(K, L) = LY_0 \min\{\frac{K}{K_0}, 1\} = L\frac{Y_0}{L_0} \min\{\frac{K}{L} \cdot \frac{L_0}{K_0}, 1\} = Y_0 \min\{\frac{K}{L} \cdot \frac{L_0}{K_0}\}\
$$
\nreceived function with a constant proportion. We shall define limiting meaning\nof economic parameters (s=1)\n
$$
X = -\frac{(1 - r^{\frac{n}{n-1}})^{\frac{n-1}{n}} K}{r} = \frac{1}{r} = \frac{(1 - r^{\frac{n}{n-1}})^{\frac{n-1}{n}}}{r} = \frac{1}{r} = \frac{(1 - r^{\frac{n}{n-1}})^{\frac{n-1}{n}}}{r} = \frac{1}{r} =
$$

Optimal model of productions. Easily to see, that ranked production functions Cobb-Douglas, CES, from a constant proportion but one parameter is not optimized, i.e. the condition of the appropriate productions cannot be improved. The µ- function offered us on parameter is optimized. Easily to see, that

$$
\frac{dY}{d\,\Gamma} = 0 \quad , \quad \text{at} \quad \Gamma * = \left[\frac{(\frac{K}{K_0})^{-1}}{(\frac{K}{K_0})^{-1}}\right]^{\frac{n-1}{n}} \quad \text{and} \quad \frac{d^2Y}{d\Gamma^2}\Big|_{\Gamma = \Gamma^*} < 0 \quad \text{i.e.} \quad \text{takes} \quad \text{place}
$$
\n
$$
Y^* = \max_{0 \le \Gamma \le 1} Y(\Gamma),
$$

 -1 and -1 and -1 and -1

Substituting $\alpha = \alpha *$ in the formula (4) we shall receive:

$$
Y^* = Y_0 \left[\left(\frac{K}{K_0} \right)^{-n} + \left(\frac{L}{L_0} \right)^{-n} \right]^{-\frac{1}{2-n}} \qquad (13)
$$

\nModel production as (5) we shall name best model production and appropriate
\neconomic system (K, L, C) connected with manufacture (13) best economic
\nsystem. From (13) we have
\n
$$
\left(\frac{Y^*}{Y_0} \right)^{-n} = \left(\frac{K}{K_0} \right)^{-n} + \left(\frac{L}{L_0} \right)^{-n} \qquad or \qquad z^n = x^n + y^n
$$
\nand for m resource we have
\n
$$
z^n = x_1^n + x^n z + ... + x_m^n \qquad (\textcircled{0})
$$

 $(\frac{K}{K_0})^{-1/n} + (\frac{L}{L_0})^{-1/n} \Big]^{-\frac{1}{2/n}}$ (13)
ction as (5) we shall name best model production and appropriate
stem (K, L, C) connected with manufacture (13) best economic
(13) we have ²²
= $Y_0 \left[\left(\frac{K}{K_0} \right)^{-1} \right]^{1/n} + \left(\frac{L}{L_0} \right)^{-1/n} \right]^{-1/n}$ (13)
el production as (5) we shall name best model production and appropriate
omic system (K, L, C) connected with manufacture (13) best economic
m. From Model production as (5) we shall name best model production and appropriate economic system (K, L, C) connected with manufacture (13) best economic system. From (13) we have + $(\frac{L}{L_0})^{-m}$
we shall name best model pro
C) connected with manufact
e
 $^{0^n}$ + $(\frac{L}{L_0})^{-m}$ or zⁿ ²²
 $^* = Y_0 \left[\left(\frac{K}{K_0} \right)^{-n} + \left(\frac{L}{L_0} \right)^{-n} \right]^{-\frac{1}{\sqrt{n}}}$ (13)

odel production as (5) we shall name best model production and appropriate

onomic system (K, L, C) connected with manufacture (13) best economic

s ²²

²

^o ^{*n*} ^{*n*} *k* (*L*₀^{*n*} ^{*n*} ^{*n*} ^{*x*_{*x*}}
 o *n* as (5) we shall name best model production and appropriate
 n (*K*, *L*, *C*) connected with manufacture (13) best economic
 S) we have
 K ²²

ⁿ + $(\frac{L}{L_0})^{-1}$ $\Big]^{-\frac{1}{2-n}}$ (13)

5) we shall name best model production and appropriat

L, C) connected with manufacture (13) best economiave
 $\Bigg|^{-1}$ $\Bigg|^{-1}$ $\Bigg|^{-1}$ $\Bigg|^{1-n}$ or $z^n = x^n + y^n$ ²²
 *x*₀ $\left[(\frac{K}{K_0})^{-1} + (\frac{L}{L_0})^{-1} \right]^{-\frac{1}{\sqrt{n}}}$ (13)

production as (5) we shall name best model production and appropriate

mic system (K, L, C) connected with manufacture (13) best economic
 *x*₀ From (13) we ²²
 $\left(\frac{K}{K_0}\right)^{-n} + \left(\frac{L}{L_0}\right)^{-n}$
 $\left(\frac{K}{K_0}\right)^{-n} + \left(\frac{L}{L_0}\right)^{-n}$ (13)

(13)

(action as (5) we shall name best model production and appropriate

system (K, L, C) connected with manufacture (13) best economic

$$
\left(\frac{Y^*}{Y_0}\right)^{-\infty} = \left(\frac{K}{K_0}\right)^{-\infty} + \left(\frac{L}{L_0}\right)^{-\infty} \circ n \quad or \quad z^n = x^n + y^n
$$

and for m resource we have

$$
z^{n} = x_{1}^{n} + x_{2}^{n} + ... + x_{m}^{n}
$$
 (@)

²²
 $= Y_0 \left[\left(\frac{K}{K_0} \right)^{-n} + \left(\frac{L}{L_0} \right)^{-n} \right]^{-\frac{1}{2-n}}$ (13)

1 production as (5) we shall name best model production and appropriate

mic system (K, L, C) connected with manufacture (13) best economic

n. From (*z x x x* **The optimal economic systems**. A triad $(K^*(t), L^*(t), C^*(t))$ $y = y^*$, where $K=K^*$ (t), $L=L^*(t)$ is the decision (1) with production function $f(K^*, L^*) = y^*$ *and consumption C* $*(t) = (1 - V)y * we shall name as the optimal economic$ *system.*

We shall notice, that if to enter a designation

$$
Z = \left(\frac{Y^*}{Y_0}\right)^{-1}
$$
, $X = \left(\frac{K}{K_0}\right)^{-1}$ $Y = \left(\frac{L}{L_0}\right)^{-1}$, we shall receive the equation

of type $X^n + Y^n = Z^n$, which has not the solution in the whole positive

numbers at n > 2:
$$
r^* = \left(\frac{\frac{K}{K_0}}{\left(\frac{K}{K_0}\right)^{-1}} + \left(\frac{L}{L_0}\right)^{-1}}\right)^{\frac{n-1}{n}}, 1 - r^*\frac{n}{n-1} = \left(\frac{L}{L_0}\right)^{-1} \left(\frac{K}{K_0}\right)^{-1} + \left(\frac{L}{L_0}\right)^{-1}.
$$

The appropriate economic parameters for optimum manufacture (13) are represented as:

$$
\mathbf{x} = -\left(\frac{K_0}{L_0}\right)^{-m} \left[\frac{K}{L}\right]^{1+...n}, \qquad \frac{K}{L} = \left[-\left(\frac{K_0}{L_0}\right)^{-n} \mathbf{x}\right]^{1+...n}, \quad \mathbf{t} = \frac{1}{1+...n},
$$

 $, E_L = \frac{\cdot}{\cdot}$ $\cdot \frac{\cdot}{\cdot}$ $0/$ ($-0/$ $\sqrt{2}$ * λ ⁻ⁿ $\{L\}$ λ λ 0) $\sqrt{v_0}$ $\sqrt{v_0}$ $n \left(1 + \frac{1}{2} \right)$ $\ldots n \left(1 + \frac{1}{2} \right)$ $K = \begin{pmatrix} K_0 \end{pmatrix} \begin{pmatrix} y_0 \end{pmatrix}$, $\begin{pmatrix} \Sigma_L \\ \end{pmatrix} \begin{pmatrix} L_0 \end{pmatrix} \begin{pmatrix} y_0 \end{pmatrix}$, Λ y^* $\begin{bmatrix} F & -\frac{L}{L} \end{bmatrix}$ $\begin{bmatrix} y \\ y \end{bmatrix}$ $\begin{bmatrix} F_{xx} + F_{yy} \end{bmatrix}$ K_0 $\left(y_0\right)$ $\left(y_0\right)$ $\left(y_0\right)$ $E_{K} = \left(\frac{K}{K}\right)^{-1} \left(\frac{y^{*}}{K}\right)^{-1}, E_{L} = \left(\frac{L}{K}\right)^{-1} \left(\frac{y^{*}}{K}\right)^{-1}, E_{K} + E_{L} = 1.$ \mathbb{R}^n (\mathbb{R}^n) \mathbb{R}^n (\mathbb{R}^n) \mathbb{R}^n (\mathbb{R}^n) \mathbb{R}^n \int , $E_L - \left(\overline{L_0}\right)$ $\left(\overline{y_0}\right)$, E_K $\begin{bmatrix} \begin{array}{c} \begin{array}{c} \end{array} \\ \end{bmatrix} \begin{array}{c} \begin{array}{c} \\ \end{array} \\ \end{array} \\ \begin{bmatrix} \end{array} \end{bmatrix} \begin{array}{c} \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} \begin{array}{c} \end{array} \\ \end{array} \begin{array}{c} \end{array} \end{bmatrix} \begin{array}{c} \begin{array}{c} \end{array} \\ \end{array} \begin{array}{c} \begin{array}{c} \end{array} \\ \end{array} \begin{array}{c} \end{array} \begin{array}{c} \end{array} \$ $\left(\frac{V_0}{y_0}\right)$, $L_L - \left(\frac{V_0}{L_0}\right)$ $\left(\frac{V_0}{y_0}\right)$ $\sum_{i=1}^{n} \binom{n}{i}$ $\sum_{i=1}^{n} \binom{n}{i}$ $\sum_{i=1}^{n} \binom{n}{i}$ $\left(\overline{K_0}\right)$ $\left(\overline{y_0}\right)$ $, E_L - \left(\overline{L_0}\right)$ $=\left(\frac{K}{K}\right)^{-n} \cdot \left(\frac{y^*}{K}\right)^{-n}$, $E_L = \left(\frac{L}{K}\right)^{-n} \cdot \left(\frac{y^*}{K}\right)^{pn}$, $E_K + E_L = 1$. We $\overline{0}$) * $\sum_{n=1}^{\infty}$ 0) $\sqrt{y_0}$ $n \times p$ *n* $L = \left(\frac{L}{L_0}\right) \left(\frac{V_0}{V_0}\right)$, $K = L$ if $W = 1$ y^* $\begin{bmatrix} F & F & F & -1 \end{bmatrix}$ We find lim $E_L = \left(\frac{L}{L_0}\right)^{-1} \cdot \left(\frac{y^*}{y_0}\right)^{-1}$, $E_K + E_L = 1$. We find lim $\Big)^{pn}$ $F + F = 1$ We find 1 $\left(\frac{L}{y_0}\right)$, $E_K + E_L = 1$. We in $\begin{pmatrix} v^* & v^* \end{pmatrix}^{pn}$ $E + E = 1$ W $\left(\overline{L_0}\right)$ $\left(\overline{y_0}\right)$ $, E_K + E_L - 1$ $E = \left(\frac{L}{I}\right)^{-1} \cdot \left(\frac{y^*}{I}\right)^{pn}$, $E_K + E_L = 1$. We find limiting meanings

of parameters: At
$$
\dots \to 0
$$
: $x = -\frac{K}{L}$, $t = 1$, $E_K = 1$, $E_K = \begin{cases} \infty, & \text{if } K > K_0, \\ K > K_0, & \text{if } K = \frac{y^*}{K}, \end{cases}$

$$
\overline{K_0} = \frac{y_0}{K_0} \overline{K_0}, \quad E_L = 1. \quad \text{At} \quad \rho \to \infty, E_L = \left\{ \infty, \sum_{1, L = L_0}^{\infty} \frac{1}{L_0}, \overline{L_0} = \frac{y^*}{L}, \quad \overline{L_0} = \frac{y_0}{L_0}, \quad \overline{K} = \left\{ \begin{array}{l} \text{--cylinder K
$$

 $n \rightarrow$ $n-1$ * $2)$ $\lim_{n \to \infty} \left(\frac{1}{n} \right)^n$. From received results follows, that for -1 \vert . From received result \int $\left(\frac{1}{2}\right)^{\frac{n}{n}}$. From received result (2) $r^* = \left(\frac{1}{2}\right)^n$. From received results follows, that for functions Cobb-Douglas,

S, from constant proportions there are no best condition and necessary introduced some regularization.

§3. The optimal model productions in a class of regularization productions of Cobb-Douglas

Occurrence of the theory of production functions is accepted for carrying by 1927 when article of the American scientists of the economist of item has appeared. Douglas (P. Douglas) and D. Cobb's mathematics (D. Cobb) « the Theory of manufacture ». In this article, attempt was undertaken, in the empirical way to define influence of the spent capital and work on output in a manufacturing industry of USA. As is known, modeling manufacture Cobb-

Douglas looks like:
$$
Y = A f(K, L)
$$
, where $f(K, L) = f_0 \left(\frac{K}{K_0}\right)^{r_1} \left(\frac{L}{L_0}\right)^{r_2}$,

where the and A is theological level, K is a size of the capital, L is a size labor resources, f_0, K_0, L_0 positive constants, $f_0 = f(K, L)$ at $f_{\Omega} = f(K, L)$ at $K = K_0$, $L = L_0$, $r_1 + r_2 = 1$, $0 \le r_j \le 1$, $j = 1, 2$. Here parameters r_j characterized degrees use of resources during manufacture. It is necessary to note, that modeling manufacture (1) is "rigid" manufacture and does not accept the maximal condition on one parameter. In this connection there is a question on change of area of change of entrance parameters, functions (1). For example, areas of change of a degree of use of resources (the capital and a labor) during manufacture the set of straight lines in an individual square is $\}$. As M we take set curvilinear $M = \left\{ r_j : r_1 + r_2 = 1, 0 \le r_j \le 1 \right\}$. As M we take set curvilinear lines on sphere \int $\overline{}$ Eq. function (1) $m-2$ else w For function (1) m=2 also w $\mathcal{L} = \{ \mathcal{L} \mid \mathcal{L} \in \mathcal{L} \}$ $j = 1$ j^{n} $\begin{array}{|c|c|c|c|c|c|c|c|}\n\hline\nm & m & m & n\n\end{array}$ $\{\Gamma = (\Gamma_1...\Gamma_m): \sum \Gamma_i \overline{n-s} = 1, 0 \le$ $\begin{array}{ccc} \n\mu & n\n\end{array}$ $\sum_{i} r_i = 1, 0 \le r_i \le 1$. For function (1) m=2 also we $=1$ $\left\lfloor \frac{1}{2} \right\rfloor$ $= \{r = (r_1...r_m): \sum r \overline{r_{n-s}} = 1, 0 \le r \le 1\}$. For function (1) m=2 also v $(r_1...r_m)$: $\sum_{j=1}^r r_j \overline{n-s} = 1, 0 \le r_j \le 1$. For function *n* $jn - s$ $-1, 0 \leq t \leq t$ m n $\sqrt{2}$ $\sqrt{2}$ $\sqrt{2}$ $\sqrt{2}$ m' , $\frac{2}{j}$ $\frac{1}{j}$ $jn - s$ -1 , $0 \leq 1$ $j \leq 1$ $\left(1, 0, 1 \text{ and } 1 \right)$ $s = \frac{1}{r} - (r - r) \cdot \frac{m}{r} - \frac{r}{r}$ $M_n^s = \left\{ r = (r_1...r_m): \sum_{i=1}^n r_i = 1, 0 \le r_j \le 1 \right\}.$ For function (1) m=2 also we

shall take $n=2$, $s=1$ also a problem of maximization function (1) on set of M is reduced to the following problem:

$$
Z = \max_{\Gamma \in M} \left\{ A f_0 \left(\frac{K}{K_0} \right)^{\Gamma} 1 \left(\frac{L}{L_0} \right)^{\Gamma} 2 \right\}, \text{ where } M = \left\{ \Gamma : \Gamma_1^2 + \Gamma_2^2 = 1, 0 \le \Gamma_j \le 1 \right\}.
$$

Having entered a designation, ~(r Γ_{1} Γ $\gamma(r) = A f_0 \left| \frac{R}{K_s} \right| = \left| \frac{E}{L_s} \right|$ we shall receive) and the state \overline{a} $\left\{ \frac{1}{2} \right\}$ $\left| \overline{L} \right|$ we shall receive $\binom{L}{0}$ $\begin{pmatrix} 1 \end{pmatrix}$ $\left| \right| \left| \overline{L_{\odot}} \right|$ we shall receive $\left(\begin{array}{c} L_0 \end{array} \right)$ $\left(\begin{array}{c} 1 \\ 1 \end{array} \right)$ $\left| \overline{K_{\circ}} \right|$ $\left| \overline{L_{\circ}} \right|$ we shall for $\binom{n}{0}$ $\binom{n}{0}$ $\begin{pmatrix} k \\ 1 \end{pmatrix}$ $= A f_0 \left| \frac{R}{m} \right|$ $\left| \frac{E}{m} \right|$ we shall receive $\left(0\right)$ $1\binom{1}{L}$ $0)$ ($\binom{0}{0}$) $F(r) = A f_0 \left(\frac{K}{K_0} \right)^{-1} \left(\frac{L}{L_0} \right)$ we shall receive K_0 \vert \vert L_0 \vert $A f_0 \left| \frac{K}{K} \right|^{-1} \left| \frac{L}{K} \right|$ we shall receive

max \sim (Γ). Thus, the initial problem of $\Gamma \in M$ $Z = \max$ \sim (r). Thus, the initial proble = max \sim (r). Thus, the initial problem consists in a presence of parameter r_1
 $r \in M$ and $r₂$ a degree of use of resources during manufacture and the maximal condition of modeling manufacture Z. ake n=2, s=1 also a problem of maximization function (1) on set of M is

d to the following problem:
 $\int_{\text{max}}^{\text{max}} \left\{ A f_0 \left(\frac{K}{K_0} \right)^{\Gamma_1} \left(\frac{L}{L_0} \right)^{\Gamma_2} \right\}$, where $M = \left\{ r : r_1^2 + r_2^2 = 1, 0 \le r_j \le 1 \right\}$.

g mization function (1) on set of M is
 $\left\{ r : r_1^2 + r_2^2 = 1, 0 \le r_j \le 1 \right\}.$
 $\left\{ \frac{\kappa}{r_0} \right\}^r \left\{ \frac{L}{L_0} \right\}^r$ we shall receive

usists in a presence of parameter r_1

ing manufacture and the maximal
 $\frac{x}{2+y^2}$ ization function (1) on set of M is
 $\left\{r : r_1^2 + r_2^2 = 1, 0 \le r_j \le 1\right\}.$
 $\left[\int_0^r \left(\frac{L}{L_0}\right)^r$ we shall receive

sists in a presence of parameter r_1

ing manufacture and the maximal
 $\frac{x}{x+y^2}$, $z = z_0 e^{\sqrt{x^2 + y^$ function (1) on set of M is
 $r_2^2 = 1, 0 \le r_j \le 1$.
 \int_0^r we shall receive

presence of parameter r_1

ifacture and the maximal
 $z = z_0 e^{\sqrt{x^2 + y^2}}$. imization function (1) on set of M
 $=\left\{r : r_1^2 + r_2^2 = 1, 0 \le r_j \le 1\right\}.$
 $\left(\frac{K}{K_0}\right)^{r_1} \left(\frac{L}{L_0}\right)^{r}$ we shall receive

onsists in a presence of parameter r

irring manufacture and the maxin
 $\frac{x}{x^2 + y^2}$, $z =$ 24

maximization function (1) on set of M is
 $M = \left\{ r : r_1^2 + r_2^2 = 1, 0 \le r_j \le 1 \right\}.$
 $f_0 \left(\frac{K}{K_0} \right)^{r_1} \left(\frac{L}{L_0} \right)^{r}$ we shall receive

n consists in a presence of parameter r_1

during manufacture and the x ~(r). Thus, the initial pro
 M

a degree of use of resou

on of modeling manufacture

The statement 1. Takes place
 $x = \ln \frac{K}{K_0}$, $y = \ln \frac{L}{L_0}$, $Z_0 = Af_0$,
 $z_{2} = y^2 = z^2$
 oof. As $\left(A f_0 \left(\frac{K}{K_0}\right)^{r_1} \left(\frac{L}{L_0}\right)^{r_2}\right), \text{ where } M = \left\{r : r_1^2 + r_2^2 = 1, 0 \leq x \right\}$
tered a designation, $-(r) = A f_0 \left(\frac{K}{K_0}\right)^{r_1} \left(\frac{L}{L_0}\right)^{r} \text{ we sh}$
 $-(r)$. Thus, the initial problem consists in a presence
degree o $\frac{x}{\sqrt{1 + x^2}} \left[\frac{K}{L_0} \right]^r 1 \left(\frac{L}{L_0} \right)^r 2$, where $M = \left\{ r : r_1^2 + r_2^2 = 0 \right\}$

d a designation, $-(r) = A f_0 \left(\frac{K}{K_0} \right)^r 1 \left(\frac{L}{L_0} \right)^r$

d. Thus, the initial problem consists in a prespective of resources d $\max_{x \in M} \left\{ A f_0 \left(\frac{K}{K_0} \right)^{-1} \left(\frac{L}{L_0} \right)^{-2} \right\}$, where $M =$
g entered a designation, $-(r) = A f_0 \left(\frac{K}{K} \right)$
 $\max_{x \in M}$ $-(r)$. Thus, the initial problem con
 $\sum_{x \in M}$ a degree of use of resources duri
ion of m $\int_{M} \left\{ A f_{0} \left(\frac{K}{K_{0}} \right)^{r} 1 \left(\frac{L}{L_{0}} \right)^{r} 2 \right\}$, where $M = \left\{ r : r_{1}^{2} + r_{2}^{2} = 1.0 \right\}$

entered a designation, $-(r) = A f_{0} \left(\frac{K}{K_{0}} \right)^{r} 1 \left(\frac{L}{L_{0}} \right)^{r}$ we s
 $\int_{M} \frac{dx}{\sqrt{1 - r}}$. Thus, the initial to the following problem:
 $\int_{\tau} \left\{ A f_0 \left(\frac{K}{K_0} \right)^{\tau} \left[\frac{L}{L_0} \right]^{\tau} 2 \right\}$, where $M = \left\{ r : r_1^2 + r_2^2 = 1, 0 \le r_j \le 1 \right\}$.
 τ
 $\int_{\tau}^{2} (1 + r_0)^{\tau} \left(\frac{L}{K_0} \right)^{\tau} \left(\frac{L}{K_0} \right)^{\tau} \left(\frac{L}{L_0} \right)^{\tau}$ we s $\int_{\alpha} \left\{ A f_0 \left(\frac{K}{K_0} \right)^{\Gamma_1} \left(\frac{L}{L_0} \right)^{\Gamma_2} \right\}$, where $M = \left\{ r : r_1^2 + r_2^2 = 1, 0 \le r_j \le 1 \right\}$.

Intered a designation, $-(r) = A f_0 \left(\frac{K}{K_0} \right)^{\Gamma_1} \left(\frac{L}{L_0} \right)^{\Gamma}$ we shall receive
 $\int_{\alpha} \left(r \right)$. Thus

 $r = \frac{x}{\sqrt{2\pi}}$, $z = z_0 e^{\sqrt{x^2 + y^2}}$, $+y^2$

where K $\mathfrak{0}$,

The proof. As

Having entered a designation,
$$
-(r) = A f_0 \left(\frac{K}{K_0}\right)^{r_0} \left(\frac{L}{L_0}\right)^{r_0}
$$
 we shall receive
\n $Z = \max_{r \in M} -(r)$. Thus, the initial problem consists in a presence of parameter r_1
\nand r_2 a degree of use of resources during manufacture and the maximal
\ncondition of modeling manufacturer Z.
\n**The statement 1.** Takes place $r = \frac{x}{\sqrt{x^2 + y^2}}$, $z = z_0 e^{\sqrt{x^2 + y^2}}$,
\nwhere $x = \ln \frac{K}{K_0}$, $y = \ln \frac{L}{L_0}$, $Z_0 = Af_0$.
\n $x^2 + y^2 = z^2$
\n**The proof.** As
\n $\frac{dz}{dr} = Af_0 \left[\left(\frac{K}{K_0}\right)^{r_0} \ln \frac{K}{K_0} \left(\frac{L}{L_0}\right)^{\sqrt{1-r^2}} + \left(\frac{L}{L_0}\right)^{\sqrt{1-r^2}} \cdot \ln \frac{L}{L_0} \left(\frac{K}{K_0}\right)^{r_0} \cdot \left(\frac{2r}{2\sqrt{1-r^2}}\right)\right] =$
\n $= Af_0 \left(\frac{K}{K_0}\right)^{r_0} \cdot \left(\frac{L}{L_0}\right)^{\sqrt{1-r^2}} \left[\ln \frac{K}{K_0} - \frac{r}{\sqrt{1-r^2}} \cdot \ln \frac{L}{L_0}\right]$
\nThat from a condition $\frac{dz}{dr} = 0$ we have $\frac{r^2}{1-r^2} = \left(\frac{\ln \frac{K}{K_0}}{\ln \frac{L}{L_0}}\right)^2$ i.e.

 $\begin{pmatrix} 0 \end{pmatrix}$

 \int

$$
r^{2} = \frac{\left(\frac{\ln \frac{K}{K_{0}}}{\ln \frac{L}{L_{0}}}\right)^{2}}{\left(\frac{\ln \left(\frac{K}{K_{0}}\right)}{\ln \left(\frac{L}{L_{0}}\right)}\right)^{2}} = \frac{\left(\ln \frac{K}{K_{0}}\right)}{\left(\ln \frac{L}{L_{0}}\right)^{2} + \left(\ln \frac{K}{K_{0}}\right)^{2}} \text{ and } 1 - r^{2} = \frac{\left(\ln \frac{L}{L_{0}}\right)^{2}}{\left(\ln \frac{L}{L_{0}}\right)^{2} + \left(\ln \frac{K}{K_{0}}\right)^{2}}
$$

Let's transform function $-(r)$. It is easy to see, that

$$
Z = \ln(Af_0) + r_1 \ln\left(\frac{K}{K_0}\right) + r_2 \ln\left(\frac{L}{L_0}\right) = \ln(Af_0) + \frac{\left(\ln\frac{K}{K_0}\right)^2}{\sqrt{\left(\ln\frac{L}{L_0}\right)^2 + \left(\ln\frac{K}{K_0}\right)^2}} \cdot \ln\frac{K}{K_0} + \frac{\left(\ln\frac{K}{K_0}\right)^2}{\sqrt{\left(\ln\frac{K}{L_0}\right)^2 + \left(\ln\frac{K}{K_0}\right)^2}} \cdot \ln\frac{K}{K_0} + \frac{\left(\ln\frac{K}{K_0}\right)^2}{\sqrt{\left(\ln\frac{K}{K_0}\right)^2 + \left(\ln\frac{K}{K_0}\right)^2}} \cdot \ln\frac{K}{K_0} + \frac{\left(\ln\frac{K}{K_0}\right)^2}{\sqrt{\left(\ln\frac{K}{K_0}\right)^2 + \left(\ln\frac{K}{K_0}\right)^2}} \cdot \ln\frac{K}{K_0}
$$

Let's transform function
$$
-(t^2)
$$
. It is easy to see, that
\n
$$
Z = \ln(Af_0) + r_1 \ln\left(\frac{K}{K_0}\right) + r_2 \ln\left(\frac{L}{L_0}\right) = \ln(Af_0) + \frac{\left(\ln\frac{K}{K_0}\right)^2}{\sqrt{\left(\ln\frac{L}{L_0}\right)^2 + \left(\ln\frac{K}{K_0}\right)^2}} \cdot \ln\frac{K}{K_0} + \frac{\left(\ln\frac{L}{L_0}\right)^2 + \left(\ln\frac{K}{K_0}\right)^2}{\sqrt{\left(\ln\frac{L}{L_0}\right)^2 + \left(\ln\frac{K}{K_0}\right)^2}} \cdot \ln\frac{L}{L_0}
$$
 From here taking into account a designation
\n
$$
x = \ln\frac{K}{K_0}, y = \ln\frac{L}{L_0}, \text{ we have } Z = Z_0 e^{\sqrt{x^2 + y^2}}, \quad r^* = \frac{x}{\sqrt{x^2 + y^2}}, \sqrt{1 - r^2} = \frac{y}{\sqrt{x^2 + y^2}},
$$
\nthat it was required to proved. We shall enter $z = \ln\frac{Z}{Z_0}$ then from (3) we have
\nthe equations $x^2 + y^2 = z^2$ or $x^n + y^n = z^n$
\nand in generally for m resource we have
\n $x_1^n + x_2^n + \dots + x_m^n = z^n$ (ω)₁
\nAs is known [3-6], the equation (4) we have accounting number of decisions.
\nInduding the whole decisions of a kind from here knowing x, y, z definition size
\nK, L, Y for optimal modeling manufacture C - Douglas is represented in the form

that it was required to proved. We shall enter $z = \ln \frac{z}{z}$ 0 $\ln \frac{E}{\sigma}$ then from (3) we have Z_0 $z = \ln \frac{Z}{Z}$ then from (3) we have the equations $x^2 + y^2 = z^2$ or $x^n + y^n = z^n$

and in generally for m resource we have

$$
x_1^{n} + x_2^{n} + \ldots + x_m^{n} = z^{n} \quad (\text{C})_1
$$

As is known [3-6], the equation (4) we have accounting number of decisions. Including the whole decisions of a kind from here knowing x, y, z definition size K, L, Y for optimal modeling manufacture $C -$ Douglas is represented in the form

$$
K = K_0 e^x, \ L = L_0 e^y, Z = Z_0 e^z.
$$

Optimization of size of the capital under law Cobb – Douglass .*The given paragraph it is devoted to optimization of process of formation of the capital according to law Cobb – Douglass and formation of a labor on to the law.* As is known sizes of the capital it is formed

according to the law:
$$
\begin{cases} \frac{dK}{dt} = vAf_0 \left(\frac{K}{K_0}\right)^r \left(\frac{L}{L_0}\right)^{1-r}, 0 < t \le t_k, \text{where } v, A, f_0, K_0, L_0, r \le t_k \end{cases}
$$

the given positive numbers $0 < v < 1$, $0 < r < 1$ characterize economic parameters. For example, the size Γ means degrees of use of resources. We shall assume, that $0 < r < 1$, $\sum r_i^2 = 1$, where $r_1 = r$, $r_2 = 1-r$. Size *L* is characterized a manpower and within the framework of the given work we shall assume, that $L = L_0 e^{iUt}$ where U mean rate of growth of manpower, t is a time.

The statement 2. This Model is optimized on r ; $0 < r < 1$, $\sum_{n=1}^{n} r_i^2 = 1$, 1^l 1 $\sum_{i=1}^{n} r_i^2 = 1,$ $=1,$ $r_1 = r$, $r_2 = 1 - r$ also the following assumes

$$
\begin{cases}\n\frac{dk}{dt} = v A f_0 e \sqrt{\left(\ln \frac{K}{K_0}\right)^2 + \left(\ln \frac{L}{L_0}\right)^2} \\
K(0) = K_0\n\end{cases}
$$
\n(1)

Really, that as in (1) $\frac{dK}{dt} > 0$, that taking the logarithm both parts (1) we shall receive $ln\left(\frac{dR}{dt}\right) = ln\left(VAf_0\right) + \Gamma_1ln\left(\frac{R}{K_0}\right) + \Gamma_2ln\left(\frac{L}{L_0}\right) =$ \int $\bigg)$ $\left| \frac{L}{L} \right|$ = $\left(\begin{array}{c} L_{0} \end{array} \right)$ $\begin{pmatrix} 1 \end{pmatrix}$ $\left| + \Gamma_2 \ell n \right| \frac{E}{L_2}$ = $\left(\begin{array}{cc} L_0 \end{array} \right)$ $\left(\begin{array}{cc} I \end{array} \right)$ $\left| \frac{K}{K} \right|$ $\left| \frac{1}{2}$ $\frac{1}{2}$ $\left| \frac{1}{L_{\infty}} \right|$ $\begin{pmatrix} R_0 & 0 \end{pmatrix}$ $\begin{pmatrix} L_0 & 0 \end{pmatrix}$ $\begin{pmatrix} k \end{pmatrix}$ $\begin{pmatrix} 1 \end{pmatrix}$ $\vert = \ell n \vert \vee Af_{\Omega} + \Gamma_1 \ell n \vert \frac{K}{K} + \Gamma_2 \ell n \vert \frac{L}{K} \vert =$ $\left| \begin{array}{c} \hline \end{array} \right|$ $\left| \frac{u}{u} \right| = \ln \left| \sqrt{A} f \right|$ $\begin{pmatrix} dt \end{pmatrix}$ $\begin{pmatrix} 0 \end{pmatrix}$ $\begin{pmatrix} dK \end{pmatrix}$ $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $\begin{array}{c} 0 \end{array}$ 2^{n} L_{\circ} 0^{f+1} 1^{h} $\left(K_{0}\right)^{h+2}$ 2^{h} $\left(L_{0}\right)^{-1}$ $\left| \frac{L}{L} \right| =$ K_{Ω} $2 \left[L_{\Omega} \right]$ $n\left(\forall Af\right)$ + Γ , $\ell n\left|\frac{K}{\tau}\right|$ + Γ , $\ell n\left|\frac{L}{\tau}\right|$ = dt) $\qquad \qquad$ 0 / $\ell n \left(\frac{dK}{dL} \right) = \ell n \left(\sqrt{A} f \right) + r \left(\ell n \right) \left| \frac{K}{K} \right| + r \left(\sqrt{n} \right) \left| \frac{L}{K} \right| =$

$$
\ell n \left(v A f_0\right) + r \ell n \left(\frac{K}{K_0}\right) + \left(1 - r^2\right)^{-\frac{1}{2}} \ell n \left(\frac{L}{L_0}\right)
$$
. We shall find
extreme of functions $Z = \ell n \left(\frac{dK}{L_0}\right)$ on r i.e.

 \int

 $\left(\begin{array}{c}dt\end{array}\right)$

dt

$$
\frac{dz}{dt} = \ln \frac{K}{K_0} - \left(1 - r^2\right)^{-1/2} r \quad \ln \frac{L}{L_0} = 0 \quad \text{and} \quad \text{from} \quad \text{here}
$$

$$
r^{2} = \frac{\left(\ell n \frac{K}{K_{0}}\right)^{2}}{\left(\ell n \frac{K}{K_{0}}\right)^{2} + \left(\ell n \frac{L}{L_{0}}\right)^{2}}, \frac{d^{2} z}{d r^{2}} < 0, \text{therefore}
$$

$$
\ell n \frac{dK}{dt} = \ell n \left(\nu A f_0 \right) + \frac{\ell n \frac{K}{K_0}}{\sqrt{\left(\ell n \frac{K}{K_0} \right)^2 + \left(\ell n \frac{L}{L_0} \right)^2}} \cdot \ell n \frac{K}{K_0} + \frac{\ell n \frac{L}{L_0}}{\sqrt{\left(\ell n \frac{K}{K_0} \right)^2 + \left(\ell n \frac{L}{L_0} \right)^2}} \cdot \ell n \frac{L}{L_0} =
$$

$$
= \ln\left(vAf_0\right) + \sqrt{\left(\ln\frac{K}{K_0}\right)^2 + \left(\ln\frac{L}{L_0}\right)^2}.
$$
 Thus,
$$
\frac{dK}{dt} = vAf_0 \exp\sqrt{\left(\ln\frac{K}{K_0}\right)^2 + \left(\ln\frac{L}{L_0}\right)^2}
$$

that was required factors. Using $L = L_0 e^{Ut}$ model we shall copy as

 $t \leq t_k$. I fils iviouel characterized t^2 $K0$ $n \frac{K}{I}$ + u² + u² + 2 $K(0) = K_0$ $\frac{dk}{dt} = vAf_0 e^{\int_0^{\ln t} \frac{dE}{dV}}$ $\leq t \leq t_{\text{r}}$. I fils Model characterized form $+u^2t^2$ $K(0) = K_{0}$ $\sqrt{\frac{dt}{dt}} = \nabla A f_0 \, \ell \, V \qquad \text{if } 0$ $\left\lceil dk \right\rceil_{\mathcal{U}}$ $\left\lceil \frac{dk}{\mathcal{U}} \right\rceil$ $\left\lceil \frac{m}{K0} \right\rceil$ $\left\lceil \frac{m}{K0} \right\rceil$ $= K_{\Omega}$ $= \mathsf{V} \mathsf{A} \mathsf{f}_{\Omega} \ell \mathsf{V}(\mathsf{K} \mathsf{0})$ $(K0)$ $(K)^2$ 0.0 $0 \leq t \leq t_{\text{L}}$. I fils induct characterize 2^{2} + 11 $2t^{2}$ $0)$ and \Box $(0) = K_{0}$ 0 $0 \le t \le t$. u^2t^2 $V A f_{\alpha} \ell V^{\alpha}$ $\ln \frac{H}{I(0)}$ + U $^{2}t^{2}$ ℓ ^V Λ ^O) $0 \le t \le t$. This Model characterized formations

of the capital it agrees the law of optimization by Cobb-Douglas and for media manpower under law Malthus. As (3) is nonlinear model for definition size of capital according to model (3), we use a method of broken lines Eelier, i.e.

$$
K_{i+1} = K_i + h \vee A f_0 \exp \sqrt{\left(\ln \frac{K_i}{K_0}\right)^2 + (uih)^2} \quad i = 0, 1, 2, \dots, n
$$

The optimal model productions in a class of manufactures of Cobb-Douglas in a case m resources. In the given paragraph the model optimization of size of the capital in a class of manufactures Cobb - Douglas in a case *m* resource is offered. As is known the size of the capital in a case *m* resources is formed according to the law:

$$
\begin{cases}\n\frac{dK_1}{dt} = vAf & \text{if } \left(\frac{K_i}{K^i_0}\right)^{r_i}, \ 0 < t \le t_k \\
K_1(0) = K_{10}\n\end{cases}
$$
, where v, A, f_0, K_0, L_0, r_i the given positive

numbers $0 < v < 1$, $0 < r_i < 1$ characterize economic parameters, K_i the size *i*-a resource, r_i means degrees of use *i*-a resource. We shall assume, that $0 < r_i < 1$, $\frac{n}{n-s}$ = 1, where $n > s > 0$.

$$
\sum r \frac{n-s}{i} = 1
$$
, where $n > s > 0$.

The statement 4.Model is optimized on Γ : $0 < \Gamma$ _i < 1, $\sum \Gamma$ ^{n-s} = 1 also the *n* $\int_{i}^{n-s} = 1$ also the

following assumes $\left\{\frac{dK_1}{dt} = v A f_0 e^{\int_0^t \left| \sum_{i=1}^l \left(\frac{1}{K^i 0} \right) \right|} \right\}$. Really $m \left(K \right)$ ⁿ $\sum_{i=1}$ $\left(K^i\right)$ *i* $K^i(0)$ and $\overline{K^i(0)}$ K_i ⁿ $K_1(0) = K_{10}$ at the set of the s $A f_0 e^{\int i = 1 \Delta_0}$ A *dt* $\frac{dK_1}{dK_2} = \sqrt{4f} \cdot e^{\pi i \left(\frac{\mathbf{A}}{K}i\right)} \left(\frac{\mathbf{A}}{K}i\right)$ $\bigcap_{n=1}^n$ $\left(\overline{K^i_0}\right)$ dk $\left(K_i\right)^n$ $K_1(0) = K_{10}$ $\int \frac{1}{dt} = V A f_0 e V^{i=1} (M U)$ $\begin{cases} dt & 0 \end{cases}$. Really, $\left\| dK_1 \right\|_{\mathcal{M}\left(0,1\right)} \leq \left\| \frac{1}{n^2} \right\| \overline{K^i \Omega}$ $= K_{10}$ at $= \mathsf{V} \mathsf{A} \mathsf{f}_{\Omega} e \sqrt{\mathsf{I} \mathsf{I}(\mathsf{A} \mathsf{I})}$ $\mathsf{I} \mathsf{I} \mathsf{I}$ $1^{(0)} - n_{10}$ $\frac{1}{\epsilon}$ = $\sqrt{4f}$ $e^{\sqrt{\frac{1}{i-1}}(K^{\prime}0)}$ X_{10} at a set of X_{10} $V A f_0 e^{\sqrt{i-1} (K'0)}$ Really $\frac{dK_1}{dx} > 0$. Really, $\frac{dA_1}{d} > 0$, that taking the *dt* $\frac{dK_1}{dt} > 0$, that taking the *z* r_i < 1 characterize economic parameters of use *i*-a resource. We shall a
 z s > 0.
 4. Model is optimized on Γ : 0 < r_i < 1
 4. Model is optimized on Γ : 0 < r_i < 1
 $\frac{dK_1}{dt} = vAf_0 e^{\sqrt{\sum_{i=1}^{m} \left$ characterize economic parar
 Af Q A Af Q ^{*A*} *Af Q* ^{*A*} *Af Q Af Q Af* $0 < t \le t_k$, where v, A, f_0 , K_0 , L_0 , r_i the given positive
 r_i <1 characterize economic parameters, K_i the size *i*-a

grees of use *i*-a resource. We shall assume, that $0 < r_i < 1$,
 $s > 0$.

4. Model is opti 1 characterize economic parameters, K_i the size *i*-a
 x so f use *i*-a resource. We shall assume, that $0 < r_i < 1$,
 x 0.
 x 0.
 x 0.
 x 1 c $\sqrt{\sum_{i=1}^{n} \left(\frac{K_i}{K'0}\right)^n}$
 x 2 x 1 c $\sqrt{\sum_{i=1}^{n} \left(\frac{K_i}{K'0}\$, A, f_0 , K_0 , L_0 , r_i the given positive

conomic parameters, K_i the size i -a

ource. We shall assume, that $0 < r_i < 1$,

d on $r : 0 < r_i < 1$, $\sum r_i^{\frac{n}{n} - s} = 1$ also the
 $\int_{\frac{r}{n}}^{\infty}$

Really, $\frac{dK_1}{dt} >$ omic parameters, K_i the size *i*-a

e. We shall assume, that $0 < r_i < 1$,
 n^{Γ} : $0 < r_i < 1$, $\sum r_i^{\frac{n}{n-s}} = 1$ also the

Really, $\frac{dK_1}{dt} > 0$, that taking the
 $= ln\left(\frac{\mu A f_0}{m}\right) + \sum_{j=1}^{m} r_j ln\left(\frac{K_j}{K'0}\right)$. We shal resource, Γ_i means degrees of use *i*-a resource. We sh
 $\sum \Gamma_i^{\frac{n}{n-s}} = 1$, where $n > s > 0$.
 The statement 4. Model is optimized on $\Gamma : 0 < r$
 The statement 4. Model is optimized on $\Gamma : 0 < r$

following assumes $\$ *j n* $\frac{n}{\sqrt{n-s}} = 1$, where *n* > *s* > 0.
 i $r \frac{n}{\sqrt{n-s}} = 1$, where *n* > *s* > 0.
 i $\frac{n}{\sqrt{n-s}} = 1$, where *n* > *s* > 0.
 ii $\frac{n}{\sqrt{n-s}} = 1$, where *n* > *s* > 0.
 ii f $\frac{dK_1}{dt} = vAf_0 e^{\frac{t}{2} \left(\frac{K_1}{k \cdot$ $(0 < r_i < 1, \sum r_i^{\frac{n}{n-s}} = 1 \text{ also the}$
 $\left(\frac{dK_1}{dt} > 0, \text{ that taking the} \right)$
 $\left(\frac{dK_1}{dt} > 0, \text{ that taking the} \right)$
 $\left(\frac{dM_0}{m}\right) + \sum_{j=1}^{m} r_j \ln\left(\frac{K_j}{K^j0}\right).$ We shall
 $\left(\frac{1}{m_1}, \dots, n_m\right): 0 < r_j < 1, \sum_{j=1}^{m} r_j^{\frac{n}{n-s}} = 1$. We entered desi $\frac{dK_1}{dt} > 0$, that taking the
 $+\sum_{j=1}^m \Gamma_j \ln\left(\frac{K_j}{K^j 0}\right)$. We shall

nd we shall receive
 $m_H: 0 < \Gamma_j < 1$, $\sum_{j=1}^m \Gamma_j \frac{n}{K^j} = 1$. We

red designations we shall Economic parameters, K_i die size $i-a$

source. We shall assume, that $0 < r_i < 1$,
 $\frac{n}{\sqrt{n}}$
 $\left(\frac{1}{n}\right)^n$
 $\left(\frac{dK}{dt}\right) = \ln(Mf_0) + \sum_{j=1}^m r_j \ln\left(\frac{K_j}{K^j_0}\right)$. We shall
 $\left(\frac{dK}{dt}\right) = \ln(Mf_0) + \sum_{j=1}^m r_j \ln\left(\frac{K_j}{K^j$ momic parameters, K_i the size i -a
rce. We shall assume, that $0 < r_i < 1$,
on $\Gamma : 0 < r_i < 1$, $\sum r_i^{\frac{n}{n}-s} = 1$ also the
. Really, $\frac{dK_1}{dt} > 0$, that taking the
 $= \ln \left(\frac{k_1}{K_0}\right) + \sum_{j=1}^{m} r_j \ln \left(\frac{K_j}{K_0}\right)$. We sha but
only parameters, K_i are size $t-a$
urce. We shall assume, that $0 < r_i < 1$,
 $1 \text{ on } r : 0 < r_i < 1$, $\sum r_i^{\frac{n}{n-s}} = 1$ also the
 $\frac{\pi}{n}$.
Really, $\frac{dK_1}{dt} > 0$, that taking the
 $\sum_{i=1}^{K} \sum_{j=1}^{K} f_j \ln \left(\frac{K_j}{K^j 0} \right)$ commeters, K_i the size *i*-a

all assume, that $0 < r_i < 1$,
 $\sum_{i=1}^{n} \sum_{j=1}^{n} r_j = 1$ also the
 $\frac{dK_i}{dt} > 0$, that taking the
 $+\sum_{j=1}^{m} r_j \ln\left(\frac{K_j}{K^j0}\right)$. We shall

d we shall receive
 $\sum_{j=1}^{m} \sum_{j=1}^{n} r_j \ln\$

logarithm both parts we shall receive $ln\left(\frac{dK}{dt}\right) = ln\left(Af_0\right) + \sum_{j=1} r_j ln\left(\frac{K_j}{K^j 0}\right)$. We shall $\overline{1}$ $\overline{1}$ $\overline{1}$ $\overline{1}$ $\overline{1}$ $\left(\overline{K^j_0}\right)$. We shall (K_i) W $(1, 1)$ $= ln[VAf_{\Omega}] + \sum r$, $ln \left| \frac{f(t)}{f(t)} \right|$. We shall) $\sqrt{0}$ $\frac{1}{j=1}$ $\sqrt{K^2}$ $\left(\frac{dK}{dt}\right) = ln\left(xA_{f}\right) + \sum_{i=1}^{m} \left(r\right) \left(\frac{K_{i}}{K_{i}}\right).$ $\left(\begin{array}{ccc} dt \end{array}\right)$ $\left(\begin{array}{ccc} 0 & \stackrel{\frown}{\longrightarrow} 0 \end{array}\right)$ $\left(K\right)$ (dK) expect $\sum_{i=1}^{m}$ K_i $\sum_{i=1}^{m}$ K_j $\sum_{i=1}^{m}$ $\sum_{j=1}^{\infty}$ *j*^{on} $\left(K^{j}0\right)$ *i* $\sum_{j=1}^{\infty}$ *j* | W_{α} ab $K^{j}(0)$ K_i **K** $n\left(\frac{dK}{dt}\right) = ln\left(x\frac{d}{dt}\right) + \sum_{i=1}^{m} \left(r\frac{d}{dt}\right)^{i}$ *Ne* shall $\ln\left(\frac{dK}{dt}\right) = \ln\left(\frac{k}{4f_0}\right) + \sum_{j=1}^r \int f\left(n\right) \frac{f\left(n\right)}{K^j\left(0\right)}$. We shall

enter designations $z = ln \left| \frac{1}{z} \frac{dK_1}{dt} \right|$, $x_i = ln \left| \frac{K_i}{K} \right|$ and w $0 \left(\begin{array}{c} 0 \\ 0 \end{array} \right)$ $\ell n \left| \frac{1}{\mu} \frac{dK_1}{dK_1} \right|$, $x_i = \ell n \left| \frac{K_i}{K_i} \right|$ and we shall receive $0₀$ $j \mid$ and j ^{- \mathcal{L} \mathcal{L} | \mathcal{L} |} K_i and K_{i-1} K_0 $\ln \left| \frac{R_i}{K} \right|$ and we shall receive

 $\sum_{j=1}^{\lfloor n/2 \rfloor} j^{i}$, where $\sum_{j=1}^{\lfloor n/2 \rfloor} j^{i}$, where $\sum_{j=1}^{\lfloor n/2 \rfloor} j^{i}$ $\binom{n-1}{j}$, $\sum_{j=1}^{\lfloor n/2 \rfloor} j^{i}$ $\sum_{j=1}^{\lfloor n/2 \rfloor} j^{i}$. We

maximize function (\cap) on set A in view of the entered designations we shall receive the proof of the given statement and we have

$$
z^{n} = \sum_{j=1}^{m} x^{n} \qquad (\omega)_{2}
$$

§4. On the energetic theory of population growth

The new model of growth of the population, so-called energetic model of growth of the population is offered. It is shown; that power model it is possible to receive from lines of groups of initial models in view of age structure and spatial distributions.

To questions of modeling and forecasting of growth of the population numerous works (for example, [14-40]), since known Malthus model $\frac{dn}{l} =$ un are devoted, *dt* $\frac{dn}{dt}$ = un are devoted, to logistical model, model in view of immigration and to emigration, model in view of age structures and spatial distributions, and also S.P. Kapitsa's to model in last time. In many models or assume proportionality of growth rate to number to the population (Malthus model), or to a square of their number (model Kapitsa). These assumptions impose also on some age and

models of growth of number of a population. In our work, proceeding from some initial groups of the models, describing growth of a population in view of age structure and spatial distributions the new model so-called by us power will be received. Initial group of models of growth of the population we shall write in the following kind: migration and to emigration, model in
tions, and also S.P. Kapitsa's to model
roportionality of growth rate to number
to a square of their number (model
o on some age and
ulation. In our work, proceeding from
bing growth

to logistical model, model in view of immigration and to emigration, model in view of age structures and spatial distributions, and also S.P. Kapitsa's to model in last time. In many models or assume proportionality of growth rate to number to the population (Malthus model), or to a square of their number (model Kapitsa). These assumptions impose also on some age and models of growth of number of a population. In our work, proceeding from some initial groups of the models, describing growth of a population in view of age structure and spatial distributions the new model so-called by us power will be received. Initial group of models of growth of the population we shall write in the following kind:\n

$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right)N = -F_0(a)N, 0 < a < \infty, 0 < t < t_c$
$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right)N = -F_0(a)N, 0 < a < \infty, 0 < t < t_c$
$N(a,0) = N_0(a), 0 \le a < \infty$
$N(0,t) = \sqrt{\int_0^t B_0(a)N''(a,t)da}$
Here <i>N</i> is a number population, $N = N(a,t)$ - for a case <i>)</i> and $N = N(x,a,t)$ - for a cases a) -b), $t \in [0,t_k]$ is time, $a \in [0,\infty)$ is an age, $x \in G = [0,L]$, $F_0(a) \ge 0, B_0(a) \ge 0$ are factors death rates and birth rate <i>n</i> is parameter $0 < n < \infty$

Here *N* is a number population, $N = N(a,t)$ –for a case) and $N = N(x,a,t)$ –for a cases a)-b), $t \in [0, t_k]$ is time, $a \in [0, \infty)$ is an age, $x \in G = [0, L]$, $F_0(a) \ge 0$, $B_0(a) \ge 0$ are factors death rates and birth rate, *p* is parameter, $0 < p < \infty$.

a). Model in view of age distribution. Let's enter function

$$
L(t) = \left(\int_{0}^{\infty} \{ (a) N^{p}(a,t) da \right)^{1/p}, 0 < p < \infty,
$$
 (2)

where { () is some non-negative function with a condition \int_{a}^{∞} { (*a*) *da* = 1. The basic 0 result of the given work we shall formulate as the following theorem.

The theorem. Let function $N = N(a,t)$ is a solution of a problem (1) in case). Then there will be a function { () \ge 0, \int_{0}^{∞} { (a) da = 1 for which function $\{(a) da = 1 \text{ for which function}$ $L(t)$ is a solution of the equation:

$$
\frac{dL}{dt} = \mathsf{u}L, \ \mathsf{u} : \int_{0}^{\infty} B(a)e^{-\mathsf{u}a}da = 1\tag{3}
$$

where $B(a) = B_0(a)e^{-p\int_0^a F_0(x)dx}$ is function of survival $B(a) = B_0(a)e^{-p\int_a^b F_0(x)dx}$ is function of survival rate of $(a) = B_0(a)e^{-b}$ is function of $\sum_{k=1}^{\infty}$ is function of survival rate of the population.

The proof of the theorem. We consider the case) from (1). The first equation) we shall increase on \cdot N^{p-1} then we shall receive identity $N^p = F_0(a)N^p$. The given equation we shall increase $\frac{1}{p}$ $\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right)N^p = F_0(a)N^p$. The given equation we ∂t ∂a \cdots $-\frac{1}{2}(\frac{\partial}{\partial x}+\frac{\partial}{\partial y})N^p = F_0(a)N^p$. The given equation we shall increase on function $\{ = \{(a) \geq 0, \text{ and result we shall integrate on } a \in [0, \infty) \}.$ Further, simple transformations we shall receive:

$$
-\frac{1}{p}\left\{\left[N^{p}\right]_{0}^{\infty}+\frac{1}{p}\int_{0}^{\infty}\frac{d\{\left\}}{da}N^{p}da-\frac{1}{p}\frac{d}{dt}\int_{0}^{\infty}\{\left(a)N^{p}da+\int_{0}^{\infty}\{\left[F_{0}(a)N^{p}da=0.\right|\right.\}Taking into account
$$

a boundary condition at *a* = 0, adding and subtracting a member $\frac{u}{p} \int_{0}^{\infty} \{ (a)N^{p} da \text{ in}$ (a) *N*^{*p*}**da** in p_{0}^{0} $\bigcup_{i=1}^{\infty}$ { (a) N^p da in

the left part of last identity, and also choosing function $\{ = \{ (a) \text{ as the solution} \}$

of a problem
$$
\begin{cases} \frac{d\{}{da} = (\mathsf{u} + pF_0(a)) \{ (a) - B_0(a) \{ (0), \ i.e.} \} \\ \{ (\infty) = 0 \end{cases}
$$

 $(a) = \{(0) | B_0(g)e$ a dg, we have the equ $(g-a)-p$ $F_0(\epsilon) d\epsilon$ 0 $\{(a) = \{(0) \mid B_0(q)e^{-a(y-y)p}\}$ dg, we have the equation of typ u (g – a) – p $\int_{0}^{9} F_{0}(\cdot) d\cdot$ a = { (0) $\int_a^{\infty} B_0(g)e^{-u(g-a)-p\int_a^b F_0(x)dx} dg$, we have the equation of type ϵ *a* $=\left\{\n\begin{array}{l}\n(0) \int_{0}^{\infty} B_0(g)e^{-u(g-a)-p\int_a^b F_0(x)dx} dg, \text{ we have the}\n\end{array}\n\right.$ $\int B_0(g)e$ *dg*, we have the equation of type equation Malthus

 $\sum_{i=1}^{p}$ *p* From (vyv) at *p* L^p . From (xxx) at $a=0$ we shall re *dt* $\frac{dL^p}{dt}$ = uL^p. From (xxx) at $a = 0$ we shall receive $\int_{a}^{\infty} B_0(a)e^{-4at^2-p\int_{0}^{a}F_0(s)ds} da = 1$. Having $\int\limits_{0}^{\infty} B_0(a^{\prime}) e^{-\frac{1}{2}(a^{\prime}-p)\int_{0}^{R_0(\varsigma)d\varsigma}} da^{\prime} = 1$. Having $u_0(a)$ c $au-1$. ' $e^{t^2-p\int_0^1 F_0(x)dx}$
da -1 Hay $B_0(a)e$ *da* =1. Having $a^{a^1-p}\int_0^a F_0(s)ds$ $da = 1$. Having

entered replacement $B(a) = B_0(a)e^{-\int_a^a F_0(x)dx}$ we receive the proof of the $B(a) = B_0(a)e^{-b}$ we receive the proof of the theorem $\mathcal{A}(a) = B_0(a)e^{-b}$ we receive the j $\frac{1}{x}$ we receive the proof of the theorem.

Definition. *Model (1), (2) we shall name power model, and the problem appropriate to this model power (p=2). Function (3) we shall name potential of the population.* **join.** *Model* (1), (2) we shall name power model, and the problem
 o this model power (p=2). Function (3) we shall name potential of
 n.
 ynomials model. The decision of a power problem is represented
 $e^{c_f t} \cos(s_f$ *x x x z* (@)³

b). Polynomials model. The decision of a power problem is represented as

$$
L(t)^p = \sum_{j=0}^{\infty} c_j^{p} e^{r_j t} \cos(s_j t), \text{ i.e. } L(t) = \left(\sum_{j=0}^{\infty} c_j^{p} e^{r_j t} \cos(s_j t)\right)^{1/p}, \text{ where factors } c_j \text{ are}
$$

defined from a condition $L(t)^p = \sum_{r=0}^{\infty} c_r^p$, at p=2 we have *number tree model in* $=\sum_{j=0}c_j^{\ p}$, at p=2 we have *number tre* $(t)^p = \sum_{i} c_i^p$, at p=2 we have *j* $p_{\text{at}} - 2$ we have $L(t)^p = \sum c_j^p$, at p=2 we have *number tree model in*

the form of: L^{-2} (0) = $\sum_{j=0}^{\infty} c^{-2} j \text{ or } x_1^{n} + x_2^{n} + ... + x_m^{n} = z^{n}$ (@)₃ 2^2 (0) = \sum c 2^2 or $x_1^{n} + x_2^{n} +$ $j = 0$

and sizes $r_j = s_j$ - are decisions of system $\int_0^{\infty} e^{i a t} e^{-i t} dt$ $\begin{cases} \int_{0}^{R} B(u) e^{-u} \sin \theta u \, du = 0 \end{cases}$ $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{pmatrix} \infty & & & \\ 0 & \infty & & \\ 0 & \infty & & \end{pmatrix}$ $= 0$ $= 1$ $\int B(a) e^{-r/a} \sin S_a a da = 0$ J $\int_{0}^{\infty} B(a) e^{-\Gamma_{j} a} \cos S_{j} a da = 1$
 $\int_{0}^{\infty} B(a) e^{-\Gamma_{j} a} \sin S_{j} a da = 1$ 0 0 $(a) e^{-1} i^{a} \sin S_{i} a da = 0$ $(a) e^{-1}$ ^d cos S _i a da = 1 *B* $(a) e^{-1}$ *i* **a s n s** *j a da* = 0 *B* $(a) e^{-x}$ *i* **cos S** *i a da* = 1 ^{*a*} sin S $_{i}$ *a* da = 0 ^{*a*} cos S_{*j}a da* = 1</sub> j^u ain c j^u and ∞ S $_a$ da = 0 S , a da = 1 $\Gamma_i a$ \longrightarrow $\Gamma_i a$ \longrightarrow Γ_i $\Gamma_i a$ $\qquad \qquad$ \qquad \qquad

The remark. For definition of a population in view of age structure we shall receive the following formula $N(a,t) = \frac{1}{a} \frac{(a)}{b} L(t)$ where { (*a*) it is (a) da $(a,t) = \frac{\int (a)}{a} L(t)$ where $\int (a)$ it 0 $^{2}(a)$ da $L(t)$ where $\{ (a)$ it is *a da* $N(a,t) = \sqrt{\int_{-\infty}^{\infty} \int_{0}^{a} \zeta^{2}(a) da} L(t)$ where $\{ (a)$ it is

defined under the formula and { (0) from a condition \int_{0}^{∞} { (a) da = 1. $\{(a) \, da = 1 \,.$

c). Model in view of age and spatial distribution. As is known, the concept of potential of the population was entered in works [1-6] in case of time, age distribution. Distinctive feature of the present work is that during change of manpower spatial parameters are taken into account. We shall enter function

$$
L(t) = \left(\int_{0}^{\infty} \int_{0}^{L} \left\{ (x, a, t) N^{p}(x, a, t) dx da \right\}^{1/p},\right.
$$
 (4)

where function $(.)$ characterizes the function describing serviceability, erudition of the population and $\{ = \{(x, a, t) \ge 0 \mid \int_0^{\infty} \{ (a) da = 1 \}, \ N = N(x, a, t) \}$ is a 0 population of age *a* in a point $x \in [0, L]$ at the moment time *t* also satisfies (1),).

$$
\begin{cases}\n\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a} + r\frac{\partial}{\partial x}\right)N = -F_0(a)N, \ 0 < a < \infty, 0 < t < t_k \\
N(x, a, 0) = N_0(x, a), \quad 0 \le a < \infty, \ 0 < x < L, \\
N(x, 0, t) = \sqrt[p]{\int_0^\infty B_0(a)N^p(x, a, t) da}, \ 0 < x < L, \\
N\Big|_{x=0} = 0 = N\Big|_{x=0},\n\end{cases}
$$

where t is time, *a*-age, x is spatial coordinate, $r=r(x)$ is the given function describing speeds of change of number on a direction *x*, $F_0(a)$ is a mortality rate coefficient $B_0(a)$ is factor of birth rate. Using our the theorem for definition of function $(.)$ from $(.)$ we have next equation

$$
\{(x,a,t) = \int_{a}^{\infty} B_o(\kappa) e^{-\int_{a}^{b} F_o(y) dy + u(a-\kappa)} \{ (x+r(g-a),0,t-a+\kappa) d\kappa \}
$$
 (5)

In (4) we shall put $a = 0$ then having taken, $\{(x, 0, t) = ce^{rt + 5a + \lambda x} \text{ we have the }$ equation of type $\frac{dE}{dr} = uL$, u : $\int B(a)e^{-u}da = 1$, where } = u + s + x r. = uL, $u : \int_{0}^{\infty} B(a)e^{-ua} da = 1$, where $\} = u + s + x r$. *L*, $u: [B(a)e^{-ua}da = 1$, where } = $u + s + x r$. dt , $\frac{1}{2}$ $\frac{dL}{dt} = uL$, $u : \int_{0}^{\infty} B(a)e^{-ua}da = 1$, where $\} = u + s + x r$.

d). **Model of the population for** *n* **countries in view of age distribution.**

Let's assume, that the population n of the countries satisfies to the equation

$$
\begin{cases} \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a} + r\frac{\partial}{\partial x}\right) N = -F_0 N, \quad 0 < a < \infty, 0 < t < t_k \text{, } x \in G = \bigcup_{i=1}^{n} G_i, G_i \in E^m, \\ N(x, a, 0) = N_0(x, a), \quad 0 < a < \infty, \quad x \in G = G + S, \\ N(x, 0, t) = \int_{0}^{\infty} B_0(a) N(x, a, t) \, da, x \in G, \end{cases}
$$

where $B_0(a)$, F_0 are the given matrixes *n* of the order with elements

$$
F_0 = \begin{pmatrix} f_{11} & f_{12} \dots & f_{1n} \\ f_{21} & f_{22} & f_{2n} \\ \dots & \dots & \dots \\ f_{n1} & f_{n2} & f_{nn} \end{pmatrix} \quad B_0 = \begin{pmatrix} b_{11} & b_{12} \dots & b_{1n} \\ b_{21} & b_{22} & b_{2n} \\ \dots & \dots & \dots \\ b_{n1} & b_{n2} & b_{nn} \end{pmatrix}, \quad N_0(x, a) \text{ is an initial number of}
$$

populations. Let's enter function $L(t) = \iint \{ (x, a, t)N (x, a, t)dxda, \text{ and } \frac{dL}{dt} = uL,$ $L(t) = \int_{0}^{\infty} \int_{G} \{ (x, a, t)N (x, a, t) dx da, \}$ and $\frac{dL}{dt} = uL$, where { $(.)$, *n* **-** a measured vector of function, { $=(x, a, t) \ge 0$, $\int_{0}^{\infty} \int_{G} \{ (x, a, t)N (x, a, t)dxda =1, \{ (x, a, t) = \int_{a}^{\infty} e^{ \int_{a}^{t} F_{o}(y)dy + u(a-x) dx} \}$ (x, a, t) N (x, a, t) dxda =1, { (x, a, t) } *G* $\{(x, a, t) \mid N \ (x, a, t) \text{d}x \text{d}a = 1, \ \{(x, a, t) = \int_{a}^{b} e^{at} \}$ $B_{0}(0)\{(x + r(0-a)) (t-a) + c\}$ where $\int_{0}^{\infty} F^*_{\rho}(y) dy + u(a \to 0)$ $f(x, a, t) = \int_{a}^{\infty} e^{-\int_{a}^{t} F_{a}(y) dy + u(a-x)} B^{*}(y) \{ (x + r(g-a)) (t-a+c) dx \}$, where $=\int_{a}^{\infty} e^{-\int F_{o}(y)dy+u(a-x)} B_{0}(y)\{(x+r(g-a))y+c-a+c\}dx$, where $*$ are conjugates to ℓ $_0$ are conjugates to B_0^* , F_0^* are conjugates to B_0 , F_0 matrices. Hence we have next equation

u :det(
$$
\int_{0}^{\infty} B(a)e^{-\lambda a}da - I
$$
) = 0, and $L(t) = \sum_{j=0}^{\infty} c_j e^{-\int_{0}^{t} t^j \cos(s_j t)}$, where $\} = u + s + x r$.

e). Computer experiments. For the first series of computer experiments initial functions we take in the following kind:

$$
B_0(a) = \begin{cases} 0, & a < a \text{ min} \\ b, & a \text{ min} \le a \le a \text{ max} \\ 0, & a \ge a \text{ max} \end{cases}, \quad F_0(a) \equiv F_0, & a \text{ min} = 0, & a \text{ max} = 90.
$$

For definition of factors of decomposition (@) we shall write as $=\sum_{j=1}^{m}$ *c* \sum_{jm}^{2} $L \frac{2}{m}$ (0) = $\sum_{j=1}^{m} c \frac{2}{jm}$ \sum_{m}^{2} (0) = \sum_{m}^{2} c \sum_{m}^{2}

where $m=1,2,3$..., and it is solved this equation on the basis of initial the equation $L_2^2(0) = c_{12}^2 + c_{22}^2$ with the help of t $2 + a^2$ with the help $12 + 22$ with the help $L_2^2(0) = c_{12}^2 + c_{22}^2$ with the help of transformation offered by us. Then on the basis of the decision $L_2^2(0) = c_{12}^2 + c_{22}^2$ we shall receive $2 + a^2$ we shall re $12 \tlog_2$ we shall let $L_2^2(0) = c_{12}^2 + c_{22}^2$ we shall receive the decision the equation $L_3^2(0) = C_{13}^2 + C_{23}^2 + C_{33}^2$ etc. up to the de $2 \cdot C^2$ atc up to the $_{23}$ \sim $_{33}$ cic. up to the $2 \cdot C^2 \cdot C^2$ atc un $13 \cdot \text{C}_{23} \cdot \text{C}_{33}$ civ. up $L_3^2(0) = C_{13}^2 + C_{23}^2 + C_{33}^2$ etc. up to the decision of the required equation $(0) = \sum C_i^2, ..., m = 2,3...$. The written all $L_m^2(0) = \sum_{j=1}^m C_j^2, ..., m = 2, 3...$ The written algorithm was p *j* $L_m^2(0) = \sum C_j^2$, *m* = 2,3... The written algorithm was programmed in language Borland Delphi and a series of computing experiments is carried out.

§5. Polynomials Models of Development of Losses in the Worst Condition by Kinds with Long Settlement - a modification method of the nearest neighbor

Now we consider questions of construction and investigating a new method of calculation a size of losses by kinds with long settlement: a modification method of the nearest neighbor. The proposition method is to define a size of losses in the worst condition of system and is based on using of so-called model of numbers tree. By using this method for a model, the data showed and carried out some computational experiments. It is noticed that the propose method basis on numbers tree model, is a simple and universal method for definition of value of Losses in the Worst Condition by Kinds with Long Settlement and it is easy programmed on all computer languages. The receiving formulas are controlled our calculations. As the methodology actuaries calculations uses the probability theory, given and the long-term statistical data, financial calculations that on faculty are in full read to a demography rates connected with System mathematical and the statistical regularities establishing mutual relation between the insurer and the insurant. They reflect as mathematical formulas the mechanism of formation (education) and an expenditure of insurance fund in long-term insurance operations. To them also carry calculations of tariffs on any kind of insurance: life's pensions, from accidents, property, work capacity. The methodology actuary's calculations use the probability theory, given to demography and the long-term statistical data, financial calculations. By means of the last in tariffs the income which is received by the insurer from use as credit resources of the accumulated payments of insurance is taken into account. Except for that rates on actuaries are read to calculations which is connected to one of the widespread problems(tasks) of such -statistical calculations connected to definition of norms and conditions of insurance, is, that the sum of insurance payments minus relying payments guaranteed reception by insurance firm (or the state organization) expected results. Making "tree" of decisions, it is necessary to draw "trunk" and the "branches" displaying structure of a problem. "Trees" from left to right settle down."Branches" designate possible alternative decisions which can be accepted, and the possible outcomes arising as a result of these decisions. On the circuit we use two kinds of "branches": The first - the dashed lines connecting squares possible decision, the second - the continuous lines connecting circles of possible outcomes. Square "units" designate places where it is made a decision, round "units" - occurrence of outcomes. As accepting the

decision cannot influence occurrence of outcomes, it needs to calculate probability of their occurrence only. When all decisions and their outcomes are specified on "tree", each of variants is counted, and in the end its monetary income is put down. All charges caused by the decision, are put down appropriate "branch". As is known, the forecast of the future losses, using observably average development of the nearest neighbors, usually define under the following circuit:

On the basis of given to the circuit the model of the nearest neighbor assumes the following (*ICA 2010, CA, Cape Town*): $Y = \sum_{i=1}^{k} S_i Y_i$, where $S_i \ge 0$ $Y = \sum_{i=1}^{n} \tilde{S}_i Y_i$, where $\tilde{S}_i \ge 0, i = 1,...,k;$ $\tilde{S}_i Y_i$, where $\tilde{S}_i \ge 0, i = 1,..., k;$ 1 .We shall consider more $\sum_{i=1}$ is S_i = 1 . We shall consider more general model, than this *k i*=1 $\tilde{S}_i = 1$ We shall consider more general model, than this model: *s* $s \mid \dots$ $\sum_{i=1}^{n}$ μ , where $\sum_i J_i$ k $\qquad \qquad \bigg\}^{1/3}$ *i*=1 / $Y_k = \left| \sum_i \mathsf{S}_i Y_{ik} \right|^s$, where $\sum_i \mathsf{S}_i^{\frac{1}{n-s}} = 1$, $n > s > 0$, S_i $1/s$ -1 j $i=1$ where $\sum \tilde{S}_i^{\frac{n}{n-s}} = 1$, $n > 1$ $\frac{1}{i-1}$ $\left(\sum_{i=1}^{k} \tilde{S}_{i} Y_{ik}^{-s}\right)^{1/3}$, where $\sum_{i=1}^{k} \tilde{S}_{i}^{\frac{n}{n-s}} = 1, n > 1$ $=\left(\sum_{i=1}^k \tilde{S}_i Y_{ik}^s\right)^{1/3}$, where $\sum_{i=1}^k \tilde{S}_i^{\frac{n}{n-s}} = 1$, $n > s > 0$, $\tilde{S}_i \ge 0$, $i = 1,...,k$; $k = 2,3$, Having $\sum_{n=1}^{k} \xi \frac{n}{n-s} - 1$ $n > s > 0$ *i*=1 $\tilde{S}_i^{\frac{n}{n-s}} = 1, n > s > 0, \, \tilde{S}_i \geq 0, i = 1, \dots, k; k = 2, 3, \dots$ Having entered a designation $X_{ik} = \tilde{S}_i^{1/s} Y_{ik}$ from this model we have

$$
Y_{k} = \left(\sum_{i=1}^{k} X^{s_{ik}}\right)^{1/s}
$$
 or
\n
$$
Y_{k} = \sum_{i=1}^{k} X^{s_{ik}}
$$
 (@)

and $\sum_{k} \left(\frac{X_{ik}}{Y_{k}} \right)^{n-s} = 1$, $n > s > 0$, $k = 2, 3, 4, \ldots$. The firs $\sum_{i=1}^{\lfloor \frac{X_{ik}}{Y_{ik}} \rfloor^{\frac{n}{n-s}} = 1, \quad n > s > 0, \ k = 2, 3, 4, \ldots$. The first equation is the equation of \sum_{k}^{k} $\left(X_{ik}\right)_{n=s}^{ns}$ 1 $n > s > 0$ $\sum_{i=1}^N (Y_{ik} - I) = 1,$ \ldots 520, $\kappa =$ X_{ik} $\Big|_{n-s}^{ns} = 1$ $n > s > 0$ $k=2$ $\frac{f(x)}{f(x)}$ $\frac{f(x)}{f(x)} = 1$, $n > s > 0$, $k = 2, 3, 4, \ldots$. The first equation is the equation of a degree *s* with *k+1* unknown and has infinite number of decisions. For allocation of the necessary decisions it is necessary that we use echo the equation (@). Exercise

For allocation

ion (@).
 $\sum_{1}^{\infty} \sum_{i=1}^{n} 1, n >$

lue that there
 $\sum_{i=1}^{\infty}$
 $\sum_{i=1}^{\infty} 1, n >$
 $\sum_{i=1}^{\infty} 1, n >$
 $\sum_{i=1}^{\infty} 1, n >$ and $\sum_{i=1}^{k} \left(\frac{x_{ik}}{Y_{ik}}\right)^{\frac{2\pi}{n-2}} = 1$, $n > s > 0$, $k = 2, 3, 4, \dots$. The first legree *s* with $k+1$ unknown and has infinite nu *Model of the worst development of losse* $> 0.5, \ge 0, i = 1, \dots, k$ right part of the equ

Model of the worst development of losses: For everything $\sum \tilde{S}_i^{\frac{n}{n-s}} = 1$, $n >$ $\sum_{i=1}$ $\mathsf{\breve{S}}_{i}^{\frac{n}{n-s}}$ = 1, $n >$ $\sum_{n=1}^{k} \xi \frac{n}{n-s} = 1$ $n >$ *i*=1 $\sum_{i=1}^{\infty} \frac{n}{n-s} = 1, n > 1$

 1 $/$

 $s > 0$ $\zeta \geq 0$, $i = 1,...,k$ right part of the equation has the maximal value that there corresponds the worst condition of system, i.e. $Y_k = \max_{\mathbf{X}} \left(\sum_{i=1}^{k} \mathbf{S}_i Y_{ik}^s \right)^{1/s}$ *s i k* k $\qquad \qquad \bigg\}^{1/3}$ $Y_k = \max_{\mathbf{\hat{S}} \in M} \left(\sum_{i=1}^N \mathbf{\tilde{S}}_i Y_i \right)^s$ $1 / s$ $\max \left(\sum_{i=1}^{k} \tilde{S}_{i} Y_{i,k} \right)^{1/2}$ \int $=\max_{S \in M} \left(\sum_{i=1}^k \tilde{S}_i Y_i \right)^{1/3}$ *s* θ *S*_{*i*} \ge 0, *i* = 1,...,*k* right part of the e
rresponds the worst condition of syst
nere $M = \left\{ \check{S} : 0 \le \check{S}_j \le 1, \sum_{j=1}^k \check{S}_j^{\frac{n}{n-s}} = 1, n \right\}$
e equation (4) we shall call "*Model a*
 $\underset{k}{\overset{s}{\phantom$

where $M = \{5$ \int $\left\{ \cdot \right\}$ $\mathbf{1}$ $\frac{1}{j=1}$ $=\left\{\tilde{S}: 0 \leq \tilde{S}_j \leq 1, \sum_{j=1}^k \tilde{S}_j^{\frac{n}{n-s}} = 1, n > s, s > 0, k > 1\right\}.$ $M = \left\{ \check{S} : 0 \le \check{S}_j \le 1, \sum_{j=1}^k \check{S}_j^{\frac{n}{n-s}} = 1, n > s, s > 0, k > 1 \right\}.$

The equation (4) we shall call "*Model of the worst development of losses"*

$$
z_k^s = \sum_{i=1}^k x_{ik}^s \qquad \qquad (\mathcal{Q})_5
$$

§6. Extreme Economics and Economic Crisis

1. Model of an economic crisis. We shall consider following modeling Economy offered in works:

$$
S > 0 \text{ S}_t \ge 0, i = 1, \dots, k \text{ right part of the equation has the maximal value that therecorresponds the worst condition of system, i.e. $Y_k = \max_{S \in M} \left(\sum_{i=1}^k \text{S}_i Y_{i,k} \right)^{1/s}$
where $M = \left\{ \text{S} : 0 \le \text{S}_t \le 1, \sum_{j=1}^k \text{S}_j^{\frac{n}{n-s}} = 1, n > s, s > 0, k > 1 \right\}$.
The equation (4) we shall call "Model of the worst development of losses"
 $Z_k^s = \sum_{i=1}^k x^s_{ik}$ ($\text{Q} \text{)}_s$
86. **Extreme Economics and Economic Crisis**
1. Model of an economic crisis. We shall consider following modeling
Economy offered in works:

$$
\begin{cases}\n\frac{dK}{dt} = v \text{ A } f(K, L), 0 < t \le t_k, K(0) = K_0, \\
\frac{dL}{dt} = u L, L(0) = L_0, \\
u : \int_0^{u_{\text{max}}} B(a) e^{-u a} da = 1, u \in (-\infty, \infty), \\
u : \int_0^{u_{\text{max}}} B(A) e^{-u a} da = 1, u \in (-\infty, \infty), \\
\frac{dA}{dt} = a_0 A - a_1 A^2, A(0) = A_0,\n\end{cases}
$$
$$

where $K=K(t)$ *-* size of the capital at the moment of time *t*, $L=L(t)$ *-* size of a manpower, \vee a share of the national income - *Y* going on capital investments, Csize of consumption, *A=A (t) is* a technological level, *B-B (a) function* of
stability of a manpower determined as $B(a)=B_0(a)e^{-b}$, $B_0(a)$ function of $\int f_0(\cdot) d\cdot$ $B(a)=B_0(a)e^{-b}$, $B_0(a)$ function of $-\int_{0}^{a} f_{0}(\epsilon) d\epsilon$ $(a) = B_0(a)e^{-\int_a^b f(x)dx}$, $B_0(a)$ function of $0(\mathbf{u})^{\mathbf{c}}$, $\mathbf{D}_0(\mathbf{u})$ ϵ ^{old}, B₀ (a) function of birth rate of manpower resources, F0 (a) - function of death rate. It is necessary to note, that function $L=L(t)$ is represented as

$$
L(t) = \int_0^{a_{\text{max}}} \{ (a, t) N(a, t) da,
$$
 (2)

where $N = N(a,t)$ is the decision of a problem: $(a)N$, $0 < t \leq t_0$, $0 \leq a \leq a_{\text{max}}$ $(a,0) = N_0(a)$ $N(0,t) = \int_{0}^{t} B_0(\epsilon) N(\epsilon, t) d\epsilon$ $\begin{pmatrix} 1 & 0 \\ N(\pi, 0) & N(\pi) \end{pmatrix}$ $\{N(a,0) = N_0(a)\}$ (a, a) $= |B_0(\epsilon)N(\epsilon,t)ds|$ $=N_0(a)$ $N = -F_0(a)N$, $0 < t \le t_0$, $0 \le a \le a_{\text{max}}$ \int 0×1 \int $0 \times$ $\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial t}\right)N = -F_0(a)N, \quad 0 < t \leq t_0,$ $\left(\partial t \quad \partial a\right)$ $\left(0 \right)$ $\begin{pmatrix} \partial & \partial \end{pmatrix}$ $\mathbf{v} = \mathbf{E}(\mathbf{v})\mathbf{v} = 0$ ∂a \int ∂ $\left(\frac{1}{2} \right)$ $+\frac{\partial}{\partial t}N=-F_0(a)N, \quad 0 < t \leq t_0, \quad 0 \leq a \leq$ ∂t ∂a \int $v(t)dt$ ∂t ∂t ∂t $\begin{pmatrix} \frac{\partial}{\partial} & \frac{\partial}{\partial} \end{pmatrix}$ $\int\limits_0^{\text{max}}\text{B}_0(\textbf{k})N(\textbf{k},t)d\textbf{k}$ 0 $0 \vee \vee \vee \vee \vee \vee \vee$ 0^{u} $0 \left(u \mu \right)$, $0 \leq t \leq t_0$, $0 \leq u \leq u_{\text{max}}$ $(0, t) = \frac{1}{2} B_0(\epsilon) N(\epsilon, t) d\epsilon$ $(0) = N_0(a)$ $0 < t \le t_0$, $0 \le a \le a_{\text{max}}$ a_{max} $N(0,t) = \prod_{i=0}^{n} (k_i) N(k_i, t) d^{t}$ $N(a,0) = N_0(a)$ $N = -F_0(a)N$, $0 < t \le t_0$, $0 \le a \le a_{\text{max}}$ *t* ∂a *d* θ *d* $\left\langle \cdot\right\rangle$ N $\left(\cdot\right),t\right)$ d $\left\langle \cdot\right\rangle$ *N* = *N*(*a,t*) is the decision of a problem:
 $\frac{\partial}{\partial u}|N = -F_0(u)N$. $0 < t \le t_0$, $0 \le a \le a_{max}$
 $0|N - F_0(u)|$
 $0|N_0(u)|$
 $0|-\int_0^u P_0(u)$
 $0|N_1(u)|^2$
 $0|-\int_0^u P_0(u)$
 $0|N_2(u)|^2$
 $0|N_1(u)|^2$
 $0|N_2(u)|^2$
 Definition 1

Here $\{ = \{(a,t) \text{ is the decision of the connected problem and refers to potential to } \}$ function of manpower resources.

Definition 1. Let entrance functions of a problem (1) are given in ranges of definition of the parameters, and also known entrance parameters K_0 , L_0 , A_0 , v $f(0)$ some production function then a vector function (K, L, Y, C, A) determined as the decision of a problem(task) (1) we name modeling economy the appropriate production function f (K, L). Thus production function $A \cdot f(K, L)$ we shall name modeling to manufactures. nction of manpower resources.
 Definition 1. Let entrance functions of a problem (1) are given in ranges

definition of the parameters, and also known entrance parameters
 L_0 , L_0 , A_0 , V (0) some production fu **notion of manpower resources.**
 Definition 1. Let entrance functions of a problem (1) are given definition of the parameters, and also known entrance definition of the parameters, and also known entrance termined as th

Definition 2. We shall tell, that the modeling economy (1) is in a condition of crisis if there are such constant positive numbers K^* , L^* and μ for which have places of an inequality **Definition 2.** We shall tell, that the modeling ecotion of crisis if there are such constant positive numbers
 \mathbf{h} have places of an inequality
 \mathbf{K} , L^* \leq \mathbf{Y}^{\dagger} $(\mathbf{K}^*, L^*) \leq \mathbf{Y}^{\dagger}$ $(\mathbf{K$ propriate production function $f(K, L)$. Thus product
all name modeling to manufactures.
Definition 2. We shall tell, that the modeling
tion of crisis if there are such constant positive num
n have places of an inequality

$$
Y^{\dagger} (K, L^*) \leq Y^{\dagger} (K^*, L^*) \leq Y^{\dagger} (K^*, L), \tag{3}
$$

what decision (1) satisfying conditions would not be

$$
\frac{1}{t} \int_{0}^{t} K(t) dt \leq K^{*}, \qquad \frac{1}{t} \int_{0}^{t} L(t) dt \geq L^{*}, \qquad (4)
$$

where $Y^{\dagger} = \frac{1}{I} \int_{I}^{I} A(t) f(K, L) dt$. For example, if we $Y^{\dagger} = \frac{1}{\dagger} \int_{0}^{1} A(t) f(K, L) dt$. For example, if we shall consider man \ddagger and the set of th $\frac{1}{\epsilon}$ $\frac{1}{\epsilon}$ \int $A(\lambda) f(\mathbf{v} \cdot \mathbf{r})$. For example, if we shall consider manufactures Cobb-Douglas $f(K,L)=f_0\left|\frac{K}{K}\right|+\left|\frac{L}{K}\right|$ where r a c $r \sim 1-r$ where a degree of μ $\begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$ $\left(\overline{L_0}\right)$ where a degree of $\left(L \right)$ $\begin{array}{ccc} 1 & 1 & 1 \end{array}$ $\int \left(\overline{L_0}\right)$ where a degree $\left(L \right)$ $\left(\overline{K_0}\right)\left(\overline{L_0}\right)$ where a defined a set $= f_0 \left(\frac{K}{\epsilon} \right)^r \left(\frac{L}{\epsilon} \right)^{1-r}$ where r a degree of $0 \bigvee C_0$ $(L) = f_0\left(\frac{K}{K_0}\right)\left(\frac{L}{L_0}\right)$ where r a degree of use of a m. K_0 \bigcup L_0 \bigcup $f(K,L) = f_0\left(\frac{K}{K}\right) \left(\frac{L}{K}\right)$ where r a degree of use of a manpower during manufacture, $0 < r < 1$. It is easy to see, that if there is a pair (K^*, L^*) satisfy (3) $\frac{1}{\tau} \int K(t) dt \leq K^*$, $\frac{1}{\tau} \int L(t) dt \geq L^*$, i.e. has places of an in $\int_{0}^{R} K(t)dt \leq K^*$, $\frac{1}{\tau} \int_{0}^{L} L(t)dt \geq L^*$, i.e. has places of an inequality (4). \ddagger and the contract of \ddagger and \ddagger $\frac{1}{4}$ \int_{0}^{1} - (*r*) = -- \int_{0}^{1} + \int_{0}^{1} - (*r*) = = - , nor mas praces 5. i.e. has places of an inequality (4). uring manufacture, $0 < r < 1$
atisfy (3) $\frac{1}{\tau} \int_0^{\tau} K(t) dt \le K^*$, $\frac{1}{\tau} \int_0^{\tau} L$
The statement 1. For and the condition (3) is necessary
 $\frac{f}{x} > 0$, $\frac{\partial f}{\partial t} > 0$, and $0 < A_{\min} < A(t)$ uring manufacture, $0 < r <$

atisfy (3) $\frac{1}{\tau} \int_0^{\tau} K(t) dt \le K^*$, $\frac{1}{\tau} \int_{\tau}^{\tau}$
 The statement 1. Fore condition (3) is necessarion
 $\frac{f}{x} > 0$, $\frac{\partial f}{\partial t} > 0$, and $0 < A_{\min} < A$
 $\frac{f}{x}$, $\frac{f}{x} > 0$, $\frac{\partial$ 38

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during manufacture, $0 < r < 1$. It is easy to see, that if there is a pair (*k*

satisfy (3) $\frac{1}{4} \int_0^1 K(t)dt \le K^*$, $\frac{1}{4} \int_0^1 L(t)dt \ge L^*$, i.e. has places of an inequality (4).
 The statement 1. For anyone m 38

ing manufacture, $0 < r < 1$. It is easy to see, that if there is a pair (K^*, s)

sfy (3) $\frac{1}{\frac{1}{s}} \int_0^s K(t)dt \le K^*, \frac{1}{\frac{1}{s}} \int_0^1 L(t)dt \ge L^*,$ i.e. has places of an inequality (4).
 The statement 1. For anyone mode 38

antisfy (3) $\frac{1}{4} \int_0^1 K(t)dt \le K^*$, $\frac{1}{4} \int_0^1 L(t)dt \ge L^*$, i.e. has places of an inequality (4).
 The statement 1. For anyone modeling manufacture of type $Y = Af(t)$
 The statement 1. For anyone modeling manufac if itsfy (3) $\frac{1}{\tau} \int_{0}^{\tau} K(t) dt \le K^*$, $\frac{1}{\tau} \int_{0}^{\tau} I(t) dt \le K^*$, $\frac{1}{\tau} \int_{0}^{\tau} I(t) dt$
 The statement 1. For

e condition (3) is necessary
 $\frac{f}{K} > 0$, $\frac{\partial f}{\partial t} > 0$, and $0 < A_{\min} < A(t)$
 $\frac{f}{\tau} \left(k, L^* \right$

The statement 1. For anyone modeling manufacture of type $Y = Af(K, L)$ the condition (3) is necessary and sufficient. Really, by definition of production

$$
\frac{\partial f}{\partial x} > 0, \frac{\partial f}{\partial l} > 0, \text{ and } 0 < A_{\min} < A(t) < A_{\max}, \text{ we have}
$$

38
\nduring manufacture,
$$
0 < r < 1
$$
. It is easy to see, that if there is a pair (K^*, L^*)
\nsatisfy (3) $\frac{1}{t} \int_0^t K(t) d \le K^*, \frac{1}{t} \int_0^t L(t) dt \ge L^*$, i.e. has places of an inequality (4).
\n**The statement 1.** For anyone modeling manufacture of type $Y = Af(K, L)$
\nthe condition (3) is necessary and sufficient. Really, by definition of production
\n $\frac{\partial f}{\partial x} > 0$, $\frac{\partial f}{\partial l} > 0$, and $0 < A_{min} < A(t) < A_{max}$, we have
\n $f^t (k, L^*) - f^t (k^*, L^*) = \frac{1}{t} \int_0^t A(t) [f (k, L^*) - f (k^*, L^*)] dt =$
\n $= \frac{1}{t} \int_0^t A(t) \frac{\partial f}{\partial K} |_{K, L} (K - K^*) dt$
\nFrom here, if are executed condition (3), $f^t (K, L^*) - f^t (K^*, L^*) \le 0$ and

From here, if are executed condition (3), $f^{\dagger}(K, L^*) - f^{\dagger}(K^*, L^*) \leq 0$ and $\int_0^{\tau} K(t)dt \leq K^*$. Similarly, using a condition f $\frac{1}{\pi} \int_0^{\pi} K(t) dt \le K^*$. Similarly, using a condition $f^{\dagger}(K^*, L^*) - f^{\dagger}(K^*, L) \le 0$, we shall receive $\frac{1}{t} \int_0^t L(t) dt \ge L^*$. The proof of the given inequa $\frac{1}{\tau} \int_0^t L(t) dt \ge L^*$. The proof of the given inequality also follows from an inequality (5). Thus, necessary and sufficient conditions of crisis of modeling economy are inequalities (4). It is necessary to note, that numbers (K^*, L^*) are defined from the decision of the differential equation. For a presence of the decision of the equation $\frac{dL}{L} = u L$, we should solve the equa *dt* $\frac{dL}{dt}$ = u L, we should solve the equations $\mathbf{B}(a)e^{-\mathbf{u}a}da = 1$ all over again. Generally, if I $\int_0^{a_{\text{max}}} B(a)e^{-ua} da = 1$ all over again. Generally, if $B(a) \ge 0$, this equation has one maximal material root $u = u_{max}$ and accounting numbers of in a complex connected roots of type $u_j = r_j \pm is_j$ so we have: $L_k(t) = c_k e^{u_k t}$, $k = 0,1,2...$, and $k_k(t) = c_k e^{u_k t}$, $k = 0,1,2...$, and therefore $L(t) = L_0 e^{u_{\text{max}}t} + \sum_{i=0}^{\infty} c_i e^{v_i t} \cos s_i t$, $0 \le t \le t_k$, where c_i are Fourier coefficients $= L_0 e^{u_{\text{max}}t} + \sum_{i=1} c_j e^{v_i t} \cos s_j t$, $0 \le t \le t_k$, where c_j are Fourier coefficients $0 \leq t \leq t_k$ or $\sum c_j e^{t_j t} \cos s_j t$, $0 \leq t \leq t_k$, where c_j are Fourier coeff *i*=1 ν_j λ_j λ_k , where λ_j μ λ λ_j t ₁₁₁₀ $L(t) = L_0 e^{u_{\text{max}}t} + \sum_{i} c_i e^{v_i t} \cos s_i t$, $0 \le t \le t_k$, where c_i are Fourier coefficients of the given decomposition. For legality of the given decomposition, we should consider a class of function $B(a) \ge 0$ for which system of function $\{\cos s, t\}$ is . Numbers r_j also s_j we shall define from system

$$
\begin{cases}\n\int_0^{a_{\text{max}}} B(a)e^{-r/a} \cos S_d da = 1 \\
\int_0^{a_{\text{max}}} B(a)e^{r/a} \sin S_d da = 0\n\end{cases}
$$

From here, it is easy to see, that $\sin(s_j \ a) = 0$, i.e. $s_j = \frac{2f}{j}$, $j = 0,1,2,...$ and roots $=\frac{2j}{a_{\text{max}}}$, $j = 0,1,2,...$ and roots *j* . 010 and ro *j* $s_i = \frac{2f}{j}$, $j = 0,1,2,...$ and roots $\left| \frac{\mathsf{r}_j}{\mathsf{u}_{\max}} \right|, \left| \frac{\mathsf{c}}{\mathsf{u}_{\max}} \right|, \left| \frac{\mathsf{c}}{\mathsf{u}_{\max}} \right|, \left| \frac{\mathsf{c}}{\mathsf{a}} \right| = \frac{1}{a} \ln(B_j), \left| \frac{\mathsf{c}}{\mathsf{a}} \right|, \left| \frac{\mathsf{d}}{\mathsf{a}} \right| = 1,$

$$
\Gamma_j < 0
$$
 at $B_j < 1$, $\Gamma_j > 0$ and $B_j > 1$, where $B_j = \int_0^{a_{\text{max}}} B(a) \cos(\frac{2f_j}{a_{\text{max}}}) da$.

The statement 2. For the uniform law of distribution

$$
B(a) = \begin{cases} 0, & a \le 0 \\ \frac{1}{a_{\max}}, & 0 < a \le a_{\max} \\ 0, & a > a_{\max} \end{cases}
$$

we have $r_j < 0$ *at* $B_j = 0$, $L(t) = \sum_{j=1}^{\infty} c_j e^{r_j t} \cos \frac{\partial^2 f_j}{\partial t^2} t$, $L^* = \frac{1}{t} \int_0^t L(t) dt$ $=\sum_{c_i}e^{r_i t}\cos\left(\frac{2f_j}{t}\right), \quad L^*=\frac{1}{t}\int_0^t L(t)dt$ \int_{-1}^{1} degrees α_{max} , $\qquad \qquad \qquad \qquad$ $\qquad \qquad$ \q $L(t) = \sum_{j=1}^{t} c_j e^{r_j t} \cos \frac{2f_j t}{a_{\max}} t$, $L^* = \frac{1}{t} \int_0^t L(t) dt$ \int_{t}^{t} *j j j j* \int_{t}^{t} *j* \int_{t}^{t} *j* \int_{t}^{t} iii $e^{i \int f} \cos \left(\frac{2f}{a_{\max}}t\right),$

The statement 3. Let B (a) function of distribution of the normal law is defined as the normal law of distribution i.e. $B(a) = \frac{1}{\sqrt{a^2 + 4}} e^{-2t^2}$ then μ_i are $(a-a_0)$ $\frac{a-a_{0}}{212}$ then \overline{a} are $2f$ \uparrow $\frac{1}{e^{-\frac{(a-a_0)}{2\pi}}}$ then II are f † $\qquad \qquad$ $a-a_0$) a = $\frac{1}{\sqrt{a}}$ e ^{2†2} then u_i are $B(a) = \frac{1}{\sqrt{25}} e^{-\frac{(a-a_0)}{2T^2}}$ then μ_j are defined from the decision of the equation $F\left|\frac{a_0}{r}-\frac{1}{r}u\right|=e^{\frac{a_0}{r}-\frac{1}{r}}$ where a_0 , t $\begin{pmatrix} 1 \\ 2 \end{pmatrix} = e^{u a_0 - \frac{1^2 u^2}{2}}$ where a_0 , t $\left(1-\frac{1}{2}\right)$ and $\left(1-\frac{1}{2}\right)$ $= e^{\frac{u_{a_0} - \frac{u}{2}}{2}}$ where a_0 , \uparrow and $\frac{1^2u^2}{2}$ 1 $\left(\frac{1}{1}-1\right) = e$ where $\left(\frac{a_0}{a_0} - \tau u\right) = e^{u a_0 - \frac{\tau^2 u^2}{2}}$ where a_0 , τ $F\left[\frac{a_0}{t} - \tau u\right] = e^{a_0^2}$ where a_0 , τ $,†$ F_j < 0 at B_j < 1, F_j > 0 and B_j >1, where $B_j = \int_0^{\infty} B(a) \cos(\frac{a}{a_{max}}a) da$.

The statement 2. For the uniform law of distribution
 $B(a) = \begin{cases} 0, & a \le 0 \\ \frac{1}{a_{max}}, & 0 < a \le a_{max} \end{cases}$

we have F_j < 0 at $B_j = 0$, $L(t) = \$

 \uparrow , \uparrow , \downarrow $\frac{1}{2}$ tu + $\frac{a}{1}$, $a = a_0 - 1^2$ u + t , and $t = \pm 10 + \frac{a - a_0}{t}, \quad a = a_0 - \pm 20 + t, \text{ and}$ $=$ $\frac{1}{2}$ dt, $-$ u a_0 + $\frac{1}{2}$ u^2 $\frac{1}{2}$ u $t - \frac{1}{2}$ $\frac{u^2}{a_0}$ $\frac{1}{2}$ $\frac{1}{2}$ 2 2 \pm ,-u a_0 +t²u²-tut- $\frac{(ut-tu)^2}{2}$ =-t a_0 +t²u²-utt $\frac{t^2}{2}$ + ttu- $\frac{t^2u^2}{2}$ =-u a_0 $0¹$ and $0¹$ 2 and λ^2 $2u^2$ + $u \cdot (u^2 - 1) = 4a$ $0¹$ and α $dx = \frac{1}{2} dt$, $du = \frac{du}{dt} + \frac{2u^2 - 1}{u^2} = -\frac{1}{2} a_0 + \frac{1}{2} u^2 - u \cdot \frac{t^2}{t^2} + t \cdot \frac{1}{2} u - \frac{1^2 u^2}{t^2} = -\frac{1}{2} a_0 + \frac{1}{2} \cdot \frac{1}{2} u^2 - \frac{t^2}{t^2}$ 2 2 2 2 2 2 2 $2 \cdot 2 \cdot 4$ $\begin{array}{ccc} 0 & 0 \\ 0 & 0 \end{array}$ $t + u - \frac{1^2 u^2}{2} = -u a_0 + \frac{1}{2} t^2 u^2 - \frac{t^2}{2}$ Then we have $\frac{1}{\sqrt{2f}} \int_{-\frac{a_0}{t} + t u}^{\infty} e^{-\frac{t^2}{2}} dt \cdot e^{-u a_0 + \frac{t^2 u^2}{2}} = 1$, $\frac{1}{\sqrt{2f}} \int_{-\infty}^{t u - \frac{a_0}{t}} e^{-\frac{t^2}{2}} dt =$ 0^{+} 1 1 0_{+11} \ldots \ldots 2 $+2\cdot 2$ $\int_{a_0}^{\infty}$ $e^{-\frac{1}{2}} dt \cdot e^{-u a_0 + \frac{u a_0}{2}} = 1$, $\frac{1}{\sqrt{u}} \int_{0}^{t u - \frac{u a_0}{2}} e^{-\frac{u}{2}} dt = e^{u a_0 - \frac{u a_0}{2}}$, as was to $\int_{-a_{0}+1}^{\infty} e^{-\frac{t^2}{2}} dt \cdot e^{-u a_0 + \frac{t^2 u^2}{2}} = 1, \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{1} t^{\frac{a_0}{2}} e^{-\frac{t^2}{2}} dt = e$ $f \frac{J-\alpha_0}{t}$ +tu $\sqrt{2}f$ $J-\infty$ $a_0 + \frac{a_0}{a_0}$ 1 $a_0 + \frac{a_0}{a_0}$ a_{0} e $u \cdot e$ $$ t^2 t^2 t^3 $e^{-\frac{t^2}{2}} dt \cdot e^{-u a_0 + \frac{t^2 u^2}{2}} = 1$, $\frac{1}{\sqrt{2f}} \int_{-\infty}^{t u - \frac{a_0}{t}} e^{-\frac{t^2}{2}} dt = e^{u a_0 - \frac{t^2 u^2}{2}}$, as was to be $\int \tan^{-1} \frac{a_0}{\sqrt{a_0^2 - 2}} \, dt$ = $e^{\int \frac{1}{a_0} \frac{1^2 u^2}{2}}$ as was to be $f \rightarrow -\infty$ $-\frac{a_0}{a_1} - \frac{b_1}{a_0}$ u $a_0 - \frac{a_1}{a_0}$ $\int_{-\infty}^{t} \frac{a_0}{t} e^{-\frac{t^2}{2}} dt = e^{-\frac{u}{2} a_0 - \frac{t^2 u^2}{2}}$, as was to be $e^{-2} dt = e^{a a_0 - 2}$, as was to be shown we shall notice, that if to enter replacement $x = \frac{a_0}{t} - u_t$, that we shall \uparrow $x = \frac{a_0}{t} - u_t$, that we shall receive $f(x) = e^{-2(x-1)^2}, 0 \le f(x) \le 1, x^2 - \frac{a_0}{1^2} \ge 0, |x| \ge \frac{a_0}{1^2},$ 2 $\frac{1}{2} \left[x^2 - \frac{a_0}{t^2} \right]$ 0 < $f(x)$ < 1 $x^2 - \frac{a_0^2}{t^2} > 0$ $|x| > \frac{a_0}{t^2}$ $1\begin{pmatrix} 2 & a_0^2 \end{pmatrix}$ \int_0^2 , $0 \le f(x) \le 1, x^2 - \frac{a_0}{x^2} \ge 0, |x| \ge \frac{a_0}{x},$ $f(x) = e^{-\frac{1}{2}\left(x^2 - \frac{a_0^2}{t^2}\right)}, 0 \leq f(x) \leq 1, x^2 - \frac{a_0^2}{t^2} \geq 0, |x| \geq \frac{a_0}{t^2},$ $x^2 - \frac{a_0^2}{a_0^2}$ $= e^{-\frac{1}{2}(x-\frac{1}{1-}y)}, 0 \leq f(x) \leq 1, x^2 - \frac{a_0^2}{2} \geq 0, |x| \geq \frac{a_0}{2},$ $\sqrt{2}$ $\begin{pmatrix} x & -\frac{1}{2} \\ 1 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{pmatrix}$ $-\frac{1}{a} \left(x^2 - \frac{a_0^2}{a_0^2} \right)$,

and therefore $x = \frac{a_0}{t} - u \uparrow \le -\frac{a_0}{t}$, $u \ge \frac{a_0}{t^2}$, $u \prec \infty$. Similarly, we shall receive \uparrow ² $u \ge \frac{u_0}{v_0}$, $u \prec \infty$. Similarly, we shall \uparrow , \uparrow \uparrow ut $\leq -\frac{u_0}{v_0}$, $u \geq \frac{u_0}{v_0}$, $u \prec \infty$. Similarly, we sh $x = \frac{a_0}{\tau} - u\tau \le -\frac{a_0}{\tau}$, $u \ge \frac{a_0}{\tau^2}$, $u \prec \infty$. Similarly, we shall receive

 $=\frac{a_0}{\lambda}-u$ $\uparrow \geq \frac{a_0}{\lambda}$, $u \leq 0$, $u \succ -\infty$. Thus $u \geq u_{\min} = \frac{a_0}{\lambda^2}$, is unique material a root ut $\geq \frac{u_0}{u_0}$, $u \leq 0$, $u \succ -\infty$. Thus $u \geq u_{min} = \frac{u_0}{u_0}$ $x = \frac{a_0}{t} - u$ $\neq \frac{a_0}{t}$, $u \le 0$, $u \succ -\infty$. Thus $u \ge u_{\min} = \frac{a_0}{t^2}$, is unique materi $u \ge u_{\min} = \frac{a_0}{t^2}$, is unique material a root of the equation $\int_{a}^{\pi_{\text{max}}} B(a)e^{-ua} da = 1$ in the first and $u \le u_{\text{max}} = 0$, in $\int_0^{a_{\text{max}}} B(a)e^{-ua} da = 1$ in the first and $u \le u_{\text{max}} = 0$, in the second cases. These values are used at definition of crisis values.

2. Polynomials model of economic crisis. We shall consider

$$
y^{\dagger} = y^{\dagger} (|, l^*) = \frac{1}{\dagger} \int_{0}^{\dagger} A(t) f(|, l^*) dt
$$

Case 1.As $y^t = \frac{1}{t} \sum_{i=0}^{m} \int_{t_i}^{t_{i+1}} A(t) f(t, t^*) dt = \frac{1}{t} \sum_{i=0}^{m} \Gamma_i \int_{t_i}^{t_{i+1}} f(t, t^*) dt$, *i* $\sum_{i=1}^{m} \int_{0}^{t_{i+1}} A(t) f(t) dt = \frac{1}{t} \sum_{i=1}^{m} \sum_{i=1}^{m} f(t) f(t) dt$ $i=0$ t_i t_{i+1} $\sum_{i=0}^m \int\limits_{t_i}^{t_{i+1}} A(t) f\Big(\|\ , I^*\Big) dt = \frac{1}{\ddagger} \sum_{i=0}^m \Gamma_i \int\limits_{t_i}^{t_{i+1}} f\Big(\|\ , I^*\Big) dt\;,$ $=0$ $\qquad t_i$ $\hat{t} = \frac{1}{\tau} \sum_{i=0}^{m} \int_{t_i}^{t_{i+1}} \! \text{A}(t) f\!\left(|\right|,l^*) dt = \frac{1}{\tau} \sum_{i=0}^{m} \Gamma_i \int_{t_i}^{t_{i+1}} \! \int_{t_i}^{t_{i+1}} \! \text{A}(t) \, dt \;,$ 0 t_i + $i=0$ t_i $\frac{1}{t} \sum_{i=0}^{t} \int_{t} A(t) f(t) dt = \frac{1}{t} \sum_{i=0}^{t} \Gamma_{i} \int_{t} f(t) dt$ $\mathcal{L}^{\dagger} = \frac{1}{I} \sum_{i=1}^{I} \left[A(t) f(t) \right, l^{*} \, dt = \frac{1}{I} \sum_{i=1}^{I} \sum_{i=1}^{I} \left[f(t) \right], l^{*} \, dt$

where $r_i = A(\epsilon)$, $t_i \le \epsilon \le t_{i+1}$, we shall take $r_i \in M = \left\{ r_i : \sum_{i=1}^n r_i^{\frac{1}{n-1}} = A_0^{\frac{1}{n-1}} \right\}$, and $\begin{array}{ccc} \end{array}$ $\left(\begin{array}{ccc} \cdot & \bullet & \bullet & \bullet \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right)$ $\left\{r_i: \sum r_i^{\overline{n-1}} = A_0^{\overline{n-1}}\right\}$, and $A \in M = \left\{r_i : \sum_{i=1}^{m} r_i^{\frac{n}{n-1}} = A_0^{\frac{n}{n-1}}\right\}, \text{ and}$ 0^{n-1} () and 1 : $\sum_{n=1}^{m} \Gamma_{n}^{\frac{n}{n-1}} = A_0^{\frac{n}{n-1}}$, and $\sum_{i=1}^{n} \Gamma_i^{n-1} = A_0^{n-1}$, and $\Gamma_i \in M = \left\{ \Gamma_i : \sum_{i=1}^{m} \Gamma_i \frac{n}{n-1} = A_0 \frac{n}{n-1} \right\}, \text{ and}$ designating $f_i = \frac{1}{\tau} \int_{t_i}^{t_{i+1}} f(|,t^*) dt$, we shall receive $y^{\tau} = \sum_{i=1}^{m} \tau_i f_i$ $\frac{1}{\tau} \int_{t_i}^{t_{i+1}} f(|,t^*) dt$, we shall receive $y^{\tau} = \sum_{i=1}^{m} \Gamma_i f_i$. Now we shall *i*=1 $y^{\dagger} = \sum r_i f_i$. Now we shall 1 $f^{\dagger} = \sum r_i f_i$. Now we shall $y^4 = y^4 \left(\frac{1}{2} x^4 \right) = \frac{1}{4} \int_0^1 A(t) f\left(\frac{1}{2} x^4 \right) dt$
 $\text{Case I.As } y^4 = \frac{1}{4} \sum_{i=0}^{n} \int_0^x A(t) f\left(\frac{1}{2} x^4 \right) dt = \frac{1}{4} \sum_{i=0}^{n} r_i - \int_0^{n} f\left(\frac{1}{2} x^4 \right) dt$,
 $\text{where } r_i = A(s), t_i \le s \le t_{i+1}$, we shall take $r_i \in M = \left\$ $\Gamma_i \in M = \left\{ r_i : \sum_{i=1}^{m} r_i^{\frac{n}{n-1}} = A_0 \right\}$

ive $y^t = \sum_{i=1}^{m} r_i f_i$. Now
 M, i.e.

ons of a maximum of f
 f_i^r , $\frac{n}{i}$,
 $i = 1, m$

technology. How to e

f model a tree of numbers of the right pair i^* i^* $M = \left\{ r_i : \sum_{i=1}^{m} r_i \sum_{i=1}^{n} c_i \sum_{j=1}^{n} c_j \sum_{j=1}^{n} r_i f_i$. Now we shall
 \vdots
 $i = 1, m$
 $i = 1, m$

hnology. How to exhaust an odel a tree of numbers. Each

them) are decomposed as the

them) are decompose $f(\vert x^*)dt = \frac{1}{\tau} \sum_{i=0}^{n} r_i \int_{t_i}^{t_i} f(\vert x^*) dt$,
 $f(\vert x^*)dt = \frac{1}{\tau} \sum_{i=0}^{n} r_i \int_{t_i}^{t_i} f(\vert x^*) dt$,
 $f(\vert x^*)dt$, we shall receive $y^* = \sum_{i=1}^{m} r_i f_i$. Now we shall
 $f(\vert x^*)dt$, we shall receive $y^* = \sum_{i=1}^{m} r_i f_i$. Now we $\sum_{i=0}^{m} \Gamma_i \int_{t_i}^{t_{i+1}} f(t_i, t^*) dt$,

all take $\Gamma_i \in M = \left\{ \Gamma_i : \sum_{i=1}^{m} \Gamma_i \frac{n}{t^{n-1}} = A_0 \frac{n}{n-1} \right\}$, and

shall receive $y^i = \sum_{i=1}^{m} \Gamma_i f_i$. Now we shall

on set of M, i.e.

ng conditions of a maximum of function

consider a problem of maximization on set of M, i.e.

max \sum_{i} $\Gamma_{i} f_{i}$. Using condition = $\max_{r \in M}$ $\sum_{i=1}^{m}$ r_i f_i . Using conditions y^{\dagger} = $\max_{\substack{r \in M}} \sum_{i=1}^r r_i f_i$. Using conditions of a maximum of function *m* of variables from here we have

consider a problem of maximization on set of M, i.e.
\n
$$
y^{\dagger} = \max_{r \in M} \sum_{i=1}^{m} r_i f_i
$$
. Using conditions of a maximum of function
\nof variables from here we have
\n $\begin{bmatrix} x^{\dagger} \end{bmatrix}^n = A_0 \sum_{i=1}^{m} f_i^n$,
\n $r_i^{\frac{n}{n-1}} = A_0^{\frac{n}{n-1}} \frac{f_i}{f_i^n + f_2^n + ... f_n^n}$, $i = 1, m$
\nThus maximization occurs on parameter of technology. How to exhaust
\nalignment, we represent the basic equation of model a tree of numbers. If
\nmember of the sum of the right part (or some of them) are decomposed as

Thus maximization occurs on parameter of technology. How to exhaust an alignment, we represent the basic equation of model a tree of numbers. Each member of the sum of the right part (or some of them) are decomposed as the sum less composed in the same are sedate, and members of the right parts of last sums in turn are again decomposed as the sum less composed in the same are sedate. **f** $\frac{f}{t} \sum_{i=0}^{n} \binom{f}{i}$ *f* $\binom{f}{i}$ *f* $\binom{f}{i}$ *f* $\binom{f}{i}$
 shall take $\Gamma_i \in M = \left\{ \Gamma_i : \sum_{i=1}^{m} \Gamma_i f_i \right\}$
 b shall receive $y^{\dagger} = \sum_{i=1}^{m} \Gamma_i f_i$
 b on on set of M, i.e.
 l sing conditions of

n

Let y^{\dagger} , f_i are the decision the equation a tree of numbers then we shall define 1 $+1$ and \sim -1 and -1 and -1 *i*+1 (

sizes | and *l* from a condition
$$
\int_{t_{i+1}}^{t_{i+1}} f(|,t^*) dt = f_i, \quad \frac{1}{\pm} \int_0^{\pm} f(|^*,t) dt = x^{\pm}.
$$

Case 2. In this case we maximize sizes y^{\dagger} on size of the capital.

Case 2. In this case we maximize sizes
$$
y^i
$$
 on size of the capital
\n
$$
y^i = \frac{1}{i} \sum f_i \int_{i_i}^{t_{i+1}} A(t) dt
$$
, where $f_i = f(|(x)|t^*)$ $t_i \le x \le t_{i+1}$. Let's enter $A_i = \frac{1}{i} \int_{i}^{t_{in}} A(t) dt$
\nand let the set M is defined as $M = \left\{ f_i : \sum_{i=1}^{m} f_i^{-m} = f_0^{\frac{n}{n-1}} \right\}$, then considering a
\nproblem $y^i = \max_{f_i \in M} \sum_{i=1}^{m} f_i A_i$. Let's receive the equation
\n
$$
y^{i^*} = f_0 \sum_{i=1}^{m} A_i^{n}, \qquad (\text{© })_6
$$

\n
$$
f_i^{\frac{n}{n-1}} = f_0^{\frac{n}{n-1}} \frac{A_i^{n}}{A_i^{n} + A_2^{n} + ... A_m^{n}}, \quad i = 1, m
$$

\nThis equation such is the basic equation a tree of numbers. Solving it we find
\n x^i, A_i and then sizes f_i , the sizes capitals and general forces.
\nS7. Model of monetary circulation

and let the set M is defined as $M = \left\{ f_i : \sum_{i=1}^{m} f_i^{\frac{n}{n-1}} = f_0^{\frac{n}{n-1}} \right\}$, then considering $i=1$ $\qquad \qquad$ \qquad $1-f^{n-1}$, then cons \int $\}$, then considering a $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $\{f_i: \sum f_i^{n-1} = f_0^{n-1}\}\,$, then consider $M = \{f_i : \sum f_i^{n-1} = f_0^{n-1}\}\,$, then considering a $f_i: \sum_{i=1}^n f_i^{\overline{n-1}} = f_0^{\overline{n-1}} \left| \right\rangle$, then considering a

problem $y^{\dagger} = \max_{f_i \in M} \sum_{i=1}^{m} f_i A_i$. Let's receive the equation $i=1$ y^{\dagger} = $\max_{f_i \in M}$ $\sum_{i=1}^{N}$ f_i *A*_{*i*}. Let's rec \int_{0}^{1} = max $\sum f_i$ A \int_{i} . Let's receive the equation

$$
y^{+}
$$
\nand let the set M is defined as $M = \left\{ f_i : \sum_{i=1}^{m} f_i^{\frac{n}{n-1}} = f_0^{\frac{n}{n-1}} \right\}$, then consist
\nproblem $y^{\pm} = \max_{f_i \in M} \sum_{i=1}^{m} f_i A_i$. Let's receive the equation
\n
$$
y^{\pm} = f_0 \sum_{i=1}^{m} A_i^n,
$$
\n(ω)_{*i*}
\n $f_i^{\frac{n}{n-1}} = f_0^{\frac{n}{n-1}} \frac{A_i^n}{A_i^n + A_2^n + ... A_m^n}, i = 1, m$
\nThis equation such is the basic equation a tree of numbers. Solving it
\n k^{\pm} , A_{*i*} and then sizes f_i , . the sizes capitals and general forces.

$$
f_i^{\frac{n}{n-1}} = f_0^{\frac{n}{n-1}} \frac{A_i^n}{A_1^n + A_2^n + ... A_m^n}, \quad i = 1, m
$$

This equation such is the basic equation a tree of numbers. Solving it we find x^{\dagger} , A_i and then sizes f_i , the sizes capitals and general forces.

§7. Model of monetary circulation

Monetary circulation, as is known, according to the quantitative theory of money for cumulative demand (dependence between quantity of made production on which purchasing demand is showed and general a price level) takes place:

$$
M V = P Y \tag{1}
$$

Here *M* is the offer of money, *V* is a speed of the manipulation of money, a *P* is a price level, and *Y*ia the quantity of the made goods and services. This equation approves, that the offer of money defines volume of manufacture in nominal expression which in turn, depends from price levels and quantity of made production: $= 0$, $0 = \frac{1}{V}$. From here $= 0 \frac{1}{y}$, $K = \frac{M}{V}$, and therefore *V* $\frac{1}{y}$, $K = \frac{M}{V}$, and therefore between a price level and to volumes of manufacture there is an inverse relationship. As the volume of manufacture is defined by various kinds made production $Y = (1, 2, \ldots, n)$ and the vector of price level is connected to it $P = (1, 2, ..., n)$ the basic equation will be defined(determined) in the following kind: $(Y,) = V$, where $(p, y) = \sum_{i=1}^{u} P_i$, Besides we sl *i*=1 *P_i i* **Besides** 1 and 1 and 1 and 1 and 1 and 1 *.* Besides we shall assume, that price levels and volume of manufacture , at are functions of some parameter $f = (t, r, e, x)$, where t-time, r-the real rate of interest, -the exchange

rate, -the spatial factor. Then basically the equation (1) scalar product $(,)$ is defined as: $(p, y) = \sum_{i=1}^{n} \int_{\text{emin}}^{\text{max}} \int_{G} P_i(t, r, e, x) y_i(t, r, e, x) dr dx$ $E(x, y) = \sum_{i=1}^{n} \int_{\text{emin}}^{\text{max}} \int_{G} P_i(t, r, e, x) y_i(t, r, e, x) dr dx$ $(p, y) = \sum \left[\int P_i(t, r, e, x) y_i(t, r, e, x) dr + c \right]$

If will designate through $P_{min}(t)$ and $P_{max}(t)$ - accordingly minimal and maximum levels of the prices at the moment of time t from the basic equation (1) we shall receive an inequality: $P_{min}(t)y(t) \le M(t)v \le P_{max}(t)y(t), 0 \le t \le t_k$, where $y(t) = \sum_{i} \int_{e}^{e^{max}} \int_{R} y_i(t, r, e, x) dr d e dx$ is total amount of manufact *e* $r \max$ $f(t) = \sum_{i} \int_{0}^{r_{\text{max}}} \int_{e^{\min}}^{e^{\max}} \int_{R} y_i(t, r, e, x) drde dx$ is total amount of manufacture. Naturally, minimal and to maximum levels of the prices the minimal and maximal offers of money accordingly answer. Then $P_{min}(t) =$ *y* $V(t) = \frac{VM_{\text{max}}(t)}{M}$. From $y(t)$ **max** *y y* $V(t) = \frac{VM_{min}(t)}{M}$, $P_{max}(t) = \frac{VM_{max}(t)}{M}$. From (t) max y y $P_{min}(t) = \frac{VM_{min}(t)}{M_{max}(t)}$, $P_{max}(t) = \frac{VM_{max}(t)}{M_{max}(t)}$. From here $\frac{M_{min}(t)}{D_{min}(t)}$ (t) (t) The ettitude of t (t) $P_{max}(t)$ $\frac{m_{\min}(t)}{m_{\min}(t)} = \frac{M_{\max}(t)}{P_{\max}(t)}$. The attitude *t* $M_{\text{max}}(t)$ The ettitude of the minimal effect of mon *t*) $P_{\text{max}}(t)$ $M_{min}(t)$ $M_{max}(t)$ **The ettitude of the minimal effect** $\frac{W_{min}(t)}{P_{min}(t)} = \frac{W_{max}(t)}{P_{max}(t)}$. The attitude of the minimal offer of money on a minimum level of the prices equally to the attitude of the maximal offer of money on maximal price levels. This attitude refers to as a stock of money. Thus, at a constancy of volume of manufacture on parameters (r, e, x) stocks of money does not change. Using the theorem of average for average value price levels on (r, e, x) we have: $P(t) = \frac{vW}{\sqrt{2}}$, $P_{min}(t) \le P_{cp}(t)$, and therefore, $f(t) = \frac{V}{y(t)}$, $P_{min}(t) \le P_{cp}(t)$, and therefor $P(t) = \frac{VM}{P(t)}$, $P_{min}(t) \leq P_{cp}(t)$, and therefore, $M_{min}(t) \le M_{cp}(t) \le M_{max}(t)$. The received results are fair for average values, M, at on time in the considered time interval of supervision. *i.e* $P_{min} \leq P_{cp} \leq P_{max}$, $\overline{M}_{min} \leq \overline{M}_{cp} \leq \overline{M}_{max}$, where feature above in sizes means averaging on time of values of these sizes, for example, $\overline{p} = \frac{1}{t_k} \int_0^k P(t) dt$.

Fig 3.covers, all possible cases which can arise in reality

Depending upon that in which part of figure to be point $_0 = M$ ($_0$, $_0$), those who offers money, it is necessary for a society, conduct appropriate a policy of change of value of volume of manufacture and price levels. For example, for a constant level of volume of manufacture θ_0 of the price can vary from a minimum level up to maximal. Similarly, we can hold price levels on some favorably all level 0, and volume of manufacture to reduce or increase (from $_{min}$ up to $_{max}$). In result, the reasonable policy under the attitude the offer of money is defined. At any price level, the offer will result increase in increase of a stock of money and reduction will result the offer of money in its reduction. In the first case when the volume of manufacture is increased, and in the second is decreases. If the economy in the beginning of supervision is in condition θ_0 that at decrease of cumulative demand connected with reduction from the offer of money there is a transition from point θ to point D in which the volume of manufacture of below real level, and then in process of reduction of prices occurs growth of economy up to level 0. In the same figure other picture is observed also. At volume of manufacture equal 0, all over again a price level it is increased up to $_{\text{max}}$, that is up to a point And, and then it is smoothly reduced up to a point In. In result, there is a jump in economy, which is manufactures to

become maximal. As $MV = P$, $\frac{dP}{dt}y + \frac{dy}{dt}P = \frac{dM}{dt}V + \frac{dV}{dt}M$, also we shall enter *d* $V + \frac{dV}{dV}$ *M*, also we shall enter $d\ddagger$ $d\ddagger$, $d\ddagger$ $dM_{\rm U}$ $dV_{\rm M}$ also we shall ontar $d\ddagger$ $d\ddagger$ $d\ddagger$ $\frac{dP}{dt}y + \frac{dy}{dt}P = \frac{dM}{dt}V + \frac{dV}{dt}M$, also we shall enter designations $v_0 = \frac{uv}{dt}$, $v_1 = \frac{uv}{dt}$, $\overline{v} = v_0 \cdot V + v_1 \cdot M$, we have $d\ddagger$, we $dV =$ *d v v v M v d* $v_1 = \frac{uv}{v_1}$, $\overline{v} = v_0 \cdot V + v_1 \cdot N$ $d\ddagger$ $d\dd$ dM $dV =$ $V + V$ $v_0 = \frac{am}{d\bar{\tau}}, v_1 = \frac{av}{d\bar{\tau}}, \bar{v} = v_0 \cdot V + v_1 \cdot M$, we have \overline{v} and from here, we shall rec *d* \uparrow *y* and *h* of *nei*, *we shall* $dy = 1 = 1$ *d y* $\frac{dP}{dt}$ = $-\frac{1}{2} \frac{dy}{dt}$ P + $\frac{1}{2} \frac{dy}{dt}$ and from here, we shall rec $\frac{1}{x}$ $\frac{1}{y}$ $\frac{1}{y}$ $\frac{1}{y}$ $\frac{1}{y}$ $\frac{1}{y}$ $\frac{1}{y}$ and from here, we shall receive $\frac{1}{2} \frac{dA}{dt} f - \frac{A}{2} \frac{\partial f}{\partial x} \cdot \frac{dK}{dt} - \frac{A}{2} \frac{\partial f}{\partial x} \frac{dL}{dt} + \frac{1}{2} \bar{v}$. Considering values $\frac{dA}{dt}$, we really $\frac{df}{dt}$ $\frac{dL}{dt}$ + $\frac{1}{y}$ \bar{v} . Considering values $\frac{dA}{dt}$, we $y \partial L d$ y *A* ∂f *dL* $1 =$ Considering values *dA* we want $d\uparrow$ y $\partial L d\uparrow$ y *f dK A* ∂f *dL* $1 =$ *Considering vol* $y \partial K$ d^{\uparrow} $y \partial L$ d^{\uparrow} y $\frac{dA}{dt}f - \frac{A}{y}\frac{\partial f}{\partial K} \cdot \frac{dK}{dt} - \frac{A}{y}\frac{\partial f}{\partial L}\frac{dL}{dt} + \frac{1}{y}\overline{v}$. Considering value $d\ddagger$ *y* $d\ddagger$ *y* ∂K $d\ddagger$ *y* ∂L $d\ddagger$ *y* $\frac{dP}{dt} = -\frac{1}{2}\frac{dA}{dt}f - \frac{A}{dt}\frac{\partial f}{\partial x} \cdot \frac{dK}{dt} - \frac{A}{dt}\frac{\partial f}{\partial y} \frac{dL}{dt} + \frac{1}{2}\bar{v}$. Considering values $\frac{dA}{dt}$, we rece $\partial L \, d\ddagger$ y $d\ddagger$ $\frac{P}{T} = -\frac{1}{y}\frac{dA}{dt}f - \frac{A}{y}\frac{\partial f}{\partial K} \cdot \frac{dK}{dt} - \frac{A}{y}\frac{\partial f}{\partial L}\frac{dL}{dt} + \frac{1}{y}\overline{v}$. Considering values $\frac{dA}{dt}$, we rece $\frac{dP}{dt} = -\frac{1}{y}\frac{dA}{dt}f - \frac{A}{y}\frac{\partial f}{\partial K} \cdot \frac{dK}{dt} - \frac{A}{y}\frac{\partial f}{\partial L}\frac{dL}{dt} + \frac{1}{y}\overline{v}$. Considering values $\frac{dA}{dt}$, we receive manutacture of below real level, and then in process of reduction of prices
cocurs growth of economy up to level 0. In the same figure other picture is
bbserved also. At volume of manufacture equal 0, all over again a pri beserved also. At volume of manufacture equal 0, all over again a price is increased up to $\frac{dP}{dx}$ y $\frac{dP}{dt}$ y $\frac{dP}{dt}$ y $\frac{dV}{dt}$ p = $\frac{dV}{dt}$ y $\frac{dV}{dt}$ p = $\frac{dV}{dt}$ y $\frac{dV}{dt}$ p = $\frac{dV}{dt}$ y + $\frac{d$ nufacture of below real level, and then in process of reduction of prices
curs growth of economy up to level 0. In the same figure other picture is
exerved also. At volume of manufacture equal 0, all over again a price le + y a + y

y
 $\frac{dy}{dt} = -\frac{1}{y}\frac{dA}{dt}f - \frac{A}{y}\frac{\partial f}{\partial K} \cdot \frac{dK}{dt} - \frac{A}{y}\frac{\partial f}{\partial L}\frac{dL}{dt} + \frac{1}{y}\overline{v}$. Cons

e **price equation:**
 $\frac{P}{dt} = -(1 - V - MPC)^{-1}\frac{u}{y} \cdot P + \frac{1}{y}$

here $\frac{dP}{dt} = \frac{\partial P}{\partial t} + x_0 \frac{\partial P}{\partial r} + \frac{\partial P}{\partial$ *t* y *d t y* ^{*t ma distributed* y $d\theta$ *t* $-\frac{1}{y} \frac{dA}{dt} f - \frac{A}{y} \frac{\partial f}{\partial K} \cdot \frac{dK}{dt} - \frac{A}{y} \frac{\partial f}{\partial L} \frac{dL}{dt} + \frac{1}{y} \overline{v}$. Considering
 price equation:
 P = -(1 - v - *MPC*)⁻¹ $\frac{u}{y}$ · P +} $\frac{dP}{dt} = -\frac{1}{y} \frac{dy}{d\tau} P + \frac{1}{y} \overline{v}$ and from here, we shall receive
 $\frac{dP}{dt} = -\frac{1}{y} \frac{dA}{dt} f - \frac{\Delta}{y} \frac{\partial f}{\partial x} \cdot \frac{dK}{dt} - \frac{\Delta}{y} \frac{\partial f}{\partial z} \frac{dL}{dt} + \frac{1}{y} \overline{v}$. Considering values $\frac{dA}{dt}$, we receive

the $\frac{dL}{dt} + \frac{1}{y} \overline{v}$. Considering values $\frac{dA}{dt}$, we receive
 $\frac{d}{dt} \cdot \overline{P} + \frac{\overline{v}}{y}$, $P(0) = P_0$,
 $\sum_{i} v_i \frac{\partial P}{\partial x_i}$.

ion of the quantitative theory of money. As,
 $\frac{A}{dt}$ also $v_1 = \frac{dV}{dt}$, that in *y*
 $\frac{L}{t} + \frac{1}{y} \overline{v}$. Considering values $\frac{dA}{dt}$, we receive
 $\frac{dA}{dt} \cdot \mathbf{P} + \frac{\overline{v}}{y}$, $P(0) = P_0$,
 $v_i \frac{\partial P}{\partial x_i}$.

on of the quantitative theory of money. As, into

also $v_i = \frac{dV}{dt}$, that in our arr $rac{1}{y} \overline{v}$ and from here, we shall receive
 $rac{dL}{dt} + \frac{1}{y} \overline{v}$. Considering values $\frac{dA}{dt}$, we receive
 $y^{-1} \frac{u}{y} \cdot P + \frac{\overline{v}}{y}$, $P(0) = P_0$,
 $\sum v_i \frac{\partial P}{\partial x_i}$.

Ation of the quantitative theory of money.

the **price equation**:

$$
\frac{dP}{d\ddagger} = -(1 - V - MPC)^{-1} \frac{u}{y} \cdot P + \frac{\overline{v}}{y}, P(0) = P_0,
$$

where $\frac{dP}{dt} = \frac{\partial P}{\partial t} + X_0 \frac{\partial P}{\partial r} + \frac{\partial P}{\partial x_1} + \sum v_i \frac{\partial P}{\partial x_i}$. ∂P ∂ α α β α β $+\frac{\partial P}{\partial x}x_1+\sum v_i\frac{\partial P}{\partial y_i}.$ ∂r ∂ \int \int ∂x_i $+X_0 \frac{\partial P}{\partial x} + \frac{\partial P}{\partial y} + \sum v_i \frac{\partial P}{\partial y}.$ ∂t ∂r ∂ \int ∂x_i $\frac{P}{dt} = \frac{\partial P}{\partial x} + X_0 \frac{\partial P}{\partial y} + \frac{\partial P}{\partial z} + X_1 + \sum_i v_i \frac{\partial P}{\partial y_i}.$ *i* x_i $d\ddagger$ ∂t ∂r ∂ ¹ Δ^{n} ∂x_i $\frac{dP}{dt} = \frac{\partial P}{\partial t} + X_0 \frac{\partial P}{\partial t} + \frac{\partial P}{\partial t}X_1 + \sum v_i \frac{\partial P}{\partial t}.$ \ddagger ∂t ∂r ∂ \ddots ∂x_i

This equation is the basic equation of the quantitative theory of money. As, into a designation for \overline{v} enter $v_0 = \frac{u v_1}{v_1}$ also $v_1 = \frac{u v_2}{v_1}$, $d\ddagger$ $d\ddagger$, $d\ddagger$ $v_0 = \frac{dM}{dt}$ also $v_1 = \frac{dV}{dt}$, that in our arrangement, there is $\frac{dP}{dt} = \frac{\partial P}{\partial t} + x_0 \frac{\partial P}{\partial r} + \frac{\partial P}{\partial x} x_1 + \sum v_i \frac{\partial P}{\partial x_i}.$

vation is the basic equation of the quantitative theory of money. As, into

vation for \overline{v} enter $v_0 = \frac{dM}{dt}$ also $v_1 = \frac{dV}{dt}$, that in our arra $(1 - V - MPC)^{-1} \frac{u}{y} \cdot P + \frac{v}{y}$
 $= \frac{\partial P}{\partial t} + x_0 \frac{\partial P}{\partial r} + \frac{\partial P}{\partial x_1} + \sum v_i \frac{\partial P}{\partial x_i}.$

tion is the basic equation of the quadron for \overline{v} enter $v_0 = \frac{dM}{dt}$ also $v_1 = \frac{dV}{dt}$
 $= \frac{dv}{v_0} + \frac{\overline{v}}{v_0} (e^{\frac{-V_$ $- (1 - V - MPC) - \frac{1}{v} \cdot P + \frac{1}{v}$
= $\frac{\partial P}{\partial t} + x_0 \frac{\partial P}{\partial r} + \frac{\partial P}{\partial x_1} + \sum_{i} v_i \frac{\partial P}{\partial x_i}$.
tion is the basic equation of the qu
ion for \overline{v} enter $v_0 = \frac{dM}{dt}$ also $v_1 = \frac{d}{dt}$
 $\frac{v_{0u}}{v} + \frac{\overline{v}}{v_0 u} (-e^{\frac{-v_$ + $\frac{1}{y}$, $P(0) = P_0$,
quantitative theory of money. As, into
 $\frac{dV}{dt}$, that in our arrangement, there is
 $\frac{1}{(1-v-MPC)^{-1}\frac{u}{y}d\epsilon} + \int_0^t \frac{\overline{v}}{y} e^{-\int_0^t (1-v-MPC)^{-1}\frac{u}{y}d\epsilon} d\epsilon$ must P $(P(0) = P_0,$

must observe the ory of money. As, into
 P , that in our arrangement, there is
 P
 P _{MPC $D^{-1} \frac{u}{y} d \leftarrow \int_0^t \frac{\overline{v}}{y} e^{-\int_0^t (1-v-MPC)^{-1} \frac{u}{y} d \left(\frac{u}{v} \right)}$
 $\frac{d}{v}$} $\frac{\overline{v}}{y}$, $P(0) = P_0$,

uantitative theory of money. As, into
 $\frac{dV}{dt}$, that in our arrangement, there is
 $\frac{dV}{dt}$, $\lim_{y \to MPC} \int_{-\frac{V}{y}}^{t} \frac{dV}{dt} e^{-\int_{0}^{t} (1-v-MPC)^{-1} \frac{u}{y} dx}$ $P(0) = P_0$,
ative theory of money. As, into
aat in our arrangement, there is
 $\int_{-\frac{1}{y}dx}^{\frac{1}{y}-\frac{1}{y}} + \int_0^t \frac{\overline{v}}{y} e^{-\int_{-\frac{1}{y}dx}^{-\frac{1}{y}-\frac{1}{y}} dx} dx$ $P + \frac{V}{y}$, $P(0) = P_0$,

le quantitative theory of money. As, into
 $P_1 = \frac{dV}{dt}$, that in our arrangement, there is
 $-\int_0^t (1-v-MPC)^{-1}\frac{u}{y}d\zeta$
 $+\int_0^t \frac{\overline{V}}{y}e^{-\int_{\zeta}^t (1-v-MPC)^{-1}\frac{u}{y}d\zeta}d\zeta$

$$
P(t) = P_0 e^{-\frac{V_{0u}}{y}} + \frac{\overline{v}}{V_0 u} \left(-e^{-\frac{V_0 u}{y}} + 1\right), \quad P(t) = P_0 e^{-\int_0^t (1 - v - MPC)^{-1} \frac{u}{y} ds} + \int_0^t \frac{\overline{v}}{y} e^{-\int_s^t (1 - v - MPC)^{-1} \frac{u}{y} ds} ds
$$

a choice of their change, that is change of rates of the offer of money and speed of the reference of money. These rates are allowable by management, and they are defined from the decision of some of a typical problem of optimum control. At $t = t$, from receiving equation we shall receive the formulas: of the offer of money and speed
wable by management, and they
a typical problem of optimum
all receive the formulas:
 $v_0 = (1 - v - MPC)^{-1} > 0$
levels at a constancy of other

From which at a constancy u, \overline{v} , *A*, *y* we have: $v_0 = (1 - v - MPC)^{-1} > 0$

This formula characterizes time change price levels at a constancy of other parameters (see fig. 4).

Fig4. Dependence price levels from the real rate of interest at constant parameters $\frac{du}{dt} = const$ *dr* $(P_1 = \frac{\overline{v}}{v_0 X u}, X_0 = \frac{dr}{dt} = const)$

Fig 5. Dependence price levels from the real rate of interest at constant parameters

If $\ddagger = (t, r)$, that we receive the equations in private derivatives of 1-st order of next type:

$$
\frac{\partial P}{\partial t} + X_0 \frac{\partial P}{\partial r} = \frac{V_0 u}{y} P + \frac{\overline{v}}{y}.
$$

At $t \to \infty$, the solution of this equation (with a condition $P_{r=0} = P_1$) is submitted on Fig 3. For solution of this equation we shall set also initial conditions $P_{t=0} = P_0(r)$ and boundary conditions such as formation (education) price levels depending on parameter r, that is

$$
P_{r=r \max} = \int_0^{r \max} \{ (r)P(r,t)dr. \text{ Here } \{ (r) \ge 0, \int_0^{r \max} \{ (s)dt \} = 1, x_0 = \frac{dr}{dt} = const.
$$

The received problem represents an example of problems with functional entry conditions which are entered and investigated in works of the author [14]. It is easy to see that the solution of equation in this case represents as:

$$
P(r,t) = P(0,t-\frac{r}{x_0})e^{v_0\int_r^r \frac{\max u}{y}} dz - \int_r^r \frac{\sum v}{y} e^{\int_r^r v_0\frac{u}{y}du} dz
$$

Function $-(t) = P(0,t)$ we shall define from a boundary condition of formation (education) of the prices, that is

$$
-(t)=\int\limits_{0}^{r\max}\left\{\left(r\right)e^{v_0}\int\limits_{r}^{\max}\frac{u}{r}d\left(r\right)^2-\left(t-\frac{V}{\chi}\right)d\left(r\right)+f_0(t),\text{ where }f_0(t)=-\int\limits_{0}^{r\max}\int\limits_{r}^{\max}\frac{\overline{v}}{y}e^{\int\limits_{r}^{\max}\frac{u}{r}d\left(r\right)}d\left(r\right).
$$

The equations restoration represents non-uniform integrated the equations of type. At $t \rightarrow \infty$ we shall receive:

$$
P(r) = P(r_{\max})e^{v_0x_0^{-1}\int_r^r \frac{\pi a x}{Y} \frac{u}{Y}d\zeta} - \frac{1}{x_0}\int_r^r \frac{\frac{\pi a x}{Y}}{\frac{\pi}{Y}} \frac{\frac{\pi}{V}e^{\frac{v_0}{x_0}\int_r^s \frac{u}{Y}du}}{\frac{\pi}{Y}} d\zeta,
$$

where

$$
P(r_{\max}) = \frac{-\frac{1}{x_0} \int_{0}^{r_{\max}} \int_{r}^{\max} \frac{u}{\gamma} e^{\frac{V_0}{x_0} \int_{r}^{r} \frac{u}{r} du}}{1 - \int_{0}^{r_{\max}} \left\{ (r) \frac{v_0}{x_0} \int_{r}^{r_{\max}} \frac{u}{r} du \right\}} \ge 0
$$

From this formula, at constants Y, u^{\perp} , we ha *v* $\frac{1}{2}$, we have

 $(r_{\text{max}}-r)$ $\frac{v}{\omega} + \left[P(r_{\text{max}}) - \frac{v}{v_0 u} \right] \ell^{\frac{v_{\text{max}} - v_{\text{max}} - v}{2}}$. The rece 0^{μ} (r - r) $(r) = \frac{\overline{v}}{r} + \left[P(r_{\text{max}}) - \frac{\overline{v}}{r} \right] \left[e^{\frac{V_0 u}{x_0 r}(r_{\text{max}} - r)} \right]$. The received \boldsymbol{u} | r_{max}) – $\frac{\overline{v}}{v}$ $\left| \ell^{\frac{0.0}{x_0 y}(r_{\text{max}} - r)} \right|$. The received for $u \quad \begin{array}{c} \text{max} \\ \text{max} \end{array}$ \overline{v} = $\frac{\overline{v}}{\overline{v}}$ + $\overline{p}(r_{\text{max}})$ - $\frac{\overline{v}}{\overline{v}}$ $\left| \ell^{\frac{v_0}{x_0} (r_{\text{max}} - r)} \right|$. The received for ℓ^{n_0} . The received \mathbf{J} and \mathbf{J} are all \mathbf{J} and \mathbf{J} $\int_{0}^{\infty} \frac{v_0 u}{x_0 y} (r_{\text{max}} - r)$ $P(r_{\text{max}}) - \frac{\mu_{01}}{v_{\text{max}}}$. If $P(r) = \frac{\overline{v}}{r} + \left[P(r_{\text{max}}) - \frac{\overline{v}}{r} \right] \ell^{\frac{V_0 u}{\lambda_0 r}(r_{\text{max}} - r)}$. The received formu $V_0 u$ $V_0 u$ $\ell^{x_0 Y}$ and $\ell^{x_0 Y}$. The received formula is interpreted as the

following figure.

Fig.6.*Dependence price levels from the real rate of interest at constant parameters* $\left(P_0 = P(r_{\text{max}}), x_0 = \frac{dr}{dt} = const \right).$ \int λ r_{max}), $x_0 = \frac{dr}{t} = const$.

Let's notice, that figures 1,2 are identical, though is present different boundary conditions. Similarly, we consider the case, when $t = l, t = x$ and $\dagger = (t, r, e, x).$

§8. Model of interaction of countries with different economic level of development

It is known, that in epoch of stable and steady growth of economics the method on the basis of statistical processing various economic parameters yielded normal results. But the situation has sharply changed with the beginning of an economic crisis. The forecast on the basis of the analysis of long-term tendencies became unusable. Burst world financial and economic crisis, which approach became for many unexpectedness, has sharply raised the question about opportunities of economic forecasting, about ability of a science adequately to describe complex social processes and to predict their development. And the decision in such cases is with the help of other methods.

Methods of nonlinear dynamics (changes) concern to these methods, economic synergetrics, aimed on the description of none equilibrium processes, on the analysis laws destructions old and formations of new social and economic structures. Thus, we consider the use of nonlinear dynamics for research of economic and sociopolitical processes in scale the separate country and the world as a whole, the basic attention

having given the analysis: laws of formation of steady social and economic structures; laws of transients, crises, and phase transitions from one structure to another. For development of economy of the countries, there are countries with HP and the countries with LP with following the circuit of development:

Volume of Production

Fig 7.The countries with high production (HP) and low production (LP).

and mathematical model of type

2 1 *i i i ij i j ij j* 1..., ; ⁰ , 1, 1, *j j* 2 1 (), *n i i i ij i j ij j j*

 $j = 1$, will *j* $i = 1...$, *n*; $0 \le t \le t_k$, $\sum \Gamma_j = 1$, $\sum \Gamma_j = 1$, where $x_i, y_i, i = 1,..., n$; - gross national product per capita cooperating countries with high production (*HP*) and low production (LP) , a_{ii} - the factor describing intensity of interaction, x $_0$, y $_0$ initial conditions $(x_{i0} > y_{i0}$, that is y_i - countries - leaders, x_i - catching up countries).

Example. We shall consider a case when we have system "one catching up country with two leaders" then the written model is higher looks like

Example. We shall consider a case when we have system "one catching up
country with two leaders" then the written model is higher looks like

$$
\frac{dx_i}{dt} = ax^2 - (d_1x - s_1d_2y - x_1d_3z),
$$

$$
\frac{dy_i}{dt} = by^2 + (\Gamma_1d_1x - d_2y + x_2d_3z),
$$

$$
\frac{dz_i}{dt} = cz^2 + (\Gamma_2d_1x + s_2d_2y - d_3z),
$$
where $x(0)=x_0$ $y(0)=y_0$, x_0 $z(0)=z_0$, y_0 y_0 y_0 z_0 are parameters of the equation. It is easy to see, that in the field of values

where *x(0)=x0< y(0)=y0, x0< z(0)=z0*, *^j,βj,γj,a,b,c,d^j* are parameters of the equation. It is easy to see, that in the field of values

$$
(x,y,z)
$$
 0:
\n₁ $d_2y + {}_1d_3z$ $d_1^2/4a$,
\n₁ $d_1y + {}_2d_3z$ $d_2^2/4b$,
\n₂ $d_1x + {}_2d_2y$ $d_3^2/4c$

the given system has two equilibrium conditions of type fig. 1.

Using differential system of this example it is possible to show, that all of its solutions satisfy to a condition for total national income

$$
x+y+z=\int_{0}^{t} (ax^{2}+by^{2}+cz^{2})dt+C_{0},
$$

where C_0 - means the left part (2) at $t=0$, i.e. the Gross National Product (GNP) countries at the initial moment of time. It is necessary to note, that the left part (2) represents GNP all countries at the moment of time t, and the right part (integral) is energy of system (« energy of economy »). For the decision of the received problem we shall take advantage system Borland DELPI c modeling given on

Fig.8. Results of computer experiments at a =0,1. Fig. 9. Results of computer experiments at ^a

=0,5.

fig. 8 then we shall receive results were catching up country very strongly lags behind the countries of leaders. Let's operate now the given system with the help of parameter, *a* from an interval **(0,1).** We shall **t**ake for example parameter *a* **=0.5** then we have *(fig 9).*Thus, as a result of successful management with the help of parameter **a**, we shall receive transitions of the country from crisis state approximately in 2047 on a highway of advanced production. =**0,5.**

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§9. MODEL OF POPULATION TURBULENCE AND CATASTROPHE IN EXTREME ECONOMICS

This item deals with a study of turbulence models of population with the time-space distribution of age-related changes in the parameters (diffusion coefficient) in a defined area of nonlinear equations. Consider a model population with the time-space-age distributions : In 2047 on a inguway of advanced production.
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§9. MODEL OF POPULATION TURBULENCE AND CATASTROPHE IN
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time-space distribution of age-related changes in the parameters (diffusion
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population with the time-space-age distributions :

$$
\left\{\frac{\partial N}{\partial t} + \frac{\partial N}{\partial a} + \sum_j \int_j \frac{\partial N}{\partial x_j} = F_0(a)N + \sum_j D_j \frac{\partial^2 N}{\partial x_j^2}, 0 < x_j < L_j, 0 < a \le a_{\text{max}}, 0 < t \le t_k,
$$

$$
N(x, a, 0) = N_0(x, a), 0 \le x \le L_j, 0 \le a \le a_{\text{max}},
$$

$$
N(x, 0, t) = \int_0^{a_{\text{max}}} B_0(x)N(x, \langle t, t \rangle) d\langle, 0 \le x_j \le L_j, 0 \le t \le t_k,
$$

(1)

$$
\left\{\frac{\partial N}{\partial x} - \Gamma_j N\Big|_{x_j = (0, t_j)},
$$

where $N = N(x, a, t)$ is the size of the population at the point x, age a, at time t,
 $F_0 = F_0(a)$ is death rate, $B_0 = B(a)$ is birth rate, $N_0 = N_0(x, a)$ is the population

where $N = N(x, a, t)$ is the size of the population at the point x, age a, at time t,

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size at the initial time. On the base of our replacement $\left| \frac{\partial f}{\partial x_i} \right|$, *a* $\big)$ $D_i \stackrel{\sim}{\rightarrow} 4D_i$ $x_i \in \left[\begin{array}{c} 2a \\ i \end{array}\right]$ $U(x,a,t) = \{ (x,a,t) \exp \mid F_0(\epsilon) d\epsilon + \sum_{i=1}^n \left[\frac{y_i}{a+b} - \sum_{i=1}^n \frac{y_i}{a+b} \right],$ $a^{'} = a, t^{'} = a + \ddagger, \{ (\chi, a, \ddagger) = N(\chi, a, a + \ddagger), \}$ *a j* ω_j *j* $\neg \omega_j$ *j* j j $\boldsymbol{\tau} \boldsymbol{\nu}_j$ *j* \sum j j \int ^{r₀(\sqrt{u} + \angle ^j_{2D} - \angle} $\left| \cdot \right|$ \int $\sqrt{2}$ $\begin{pmatrix} 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 \end{pmatrix}$ $=\{ (x,a,\ddagger)\exp\left(\int_a^a F_0(x)dx + \sum_{i=1}^{\infty}\left[\frac{x_i}{2D} - \sum_{i=1}^{\infty}\frac{1}{4D}\right],\right\}$ 0 J \sim 2α $\frac{1}{i}$ $\left(\frac{1}{i} a_2 \right)$ $\frac{1}{i} a_1 \left(\frac{1}{i} a_2 \right)$ $\frac{1}{i} a_2 \left(\frac{1}{i} a_2 \right)$ $\left[\begin{array}{c} 2a \\ i \end{array}\right]$ \downarrow) = { (x, a, \downarrow) exp| | F_0 (x) d x + \sum [$\frac{a}{2}$ = \sum $\frac{c}{2}$ | $\frac{c}{2}$ |,

instead of (1) we obtain the problem:

$$
\begin{cases}\n\frac{\partial u}{\partial a} = \sum_{j} D_{j} \frac{\partial^{2} u}{\partial x_{j}^{2}}, & 0 \leq x_{j} \leq L_{j}, \ 0 < a \leq a_{max}, \ 0 < t \leq t_{k} \\
U(x, 0, t) = \int_{0}^{a_{max}} B_{0}(\langle v, t, x_{j}, t_{k} \rangle) d\langle u, \frac{\partial u}{\partial x_{j}} \bigg|_{x_{j} = 0} = 0.\n\end{cases}
$$
\n(2)

Suppose that. $D_j = Dr_j$, $\Gamma_j \in M$, where $M = \left\{ \Gamma : \Gamma_0 = (\Gamma_1, ..., \Gamma_m), \sum_j \Gamma_j^{n-s} = 1 \right\}$ $\left\{ \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right\}$ \mathbf{r} $=\left\{r : r_{0} = (r_{1},...,r_{m}), \sum r^{\frac{n}{n-s}} = 1\right\}$ $-s$ -1 $M = \left\{ r : r_0 = (r_1, ..., r_m), \sum_j r^{\frac{n}{n-s}} = 1 \right\}$ $,...,r_{m}$, $\sum_{i} r_{i}^{n-s} = 1$

Using the idea of the method of separation of variables for the problems (2) formulate two classes of possible solutions:

1). Class of simple solutions $D_j \frac{\partial^2 u}{\partial x_i^2} = C_j$, $\frac{\partial u}{\partial a} = C$. *j* $=C_i, \frac{\partial u}{\partial t} = C.$ ∂x_i^2 and ∂a $\partial^2 u$ ∂u ∂u 2 υ_j , υ_i $2\ldots$ $2\ldots$

2). Class of exponential solutions $D_j \frac{\partial^2 u}{\partial x^2} = C_j u$, $\frac{\partial u}{\partial q} = Cu$, which D defin *Cu*, which D defines the α and α is the set of α $D_j \frac{\partial^2 u}{\partial^2} = C_j u$, $\frac{\partial u}{\partial x} = Cu$, which D defines the *j* $j\frac{\partial u}{\partial \theta_i^2} = C_j u$, $\frac{\partial u}{\partial a} = Cu$, which D defines the $=C_{i}u, \frac{\partial u}{\partial t} = Cu$, which D defines the ∂_i^2 ∂a ∂b ∂b $\frac{\partial^2 u}{\partial x^2} = C_i u$, $\frac{\partial u}{\partial x} = C u$, which D defines the representation $D_j = Dr_j$, $j = 1, 2, 0 \le r_j \le 1$.

The definition. A population "turbulence" in the framework of the model (2) (or (1)), we call this state population, which at a certain value of the vector *r*, $r = (r_1, ..., r_m) \in M$, the value

$$
\left(\sum_{j=1}^{m} \Gamma_j \left(\frac{\partial^2 u}{\partial x_j^2}\right)^s\right)^{1/s}, \qquad s > 0,
$$
\n(3)

takes its maximum value, i.e.

$$
\frac{\partial u}{\partial a} = \max_{r \in M} \sum_{j=1}^{m} r_j \frac{\partial^2 u}{\partial x_j^2}.
$$
 (4)

The following theorem:

Theorem 1.The Equation $Z = max(\Gamma, X^s)^{1/s}$ and the equation $Z = \max_{\Gamma \in M} (\Gamma, X^s)^{1/s}$ and the equation $Z^n = \sum_{j=1}^s X^n_j$ *m* 1 are equivalent. Now we consider the equation

$$
\frac{\partial u}{\partial a} = \max_{r \in M} \left(r \left(\frac{\partial^2 u}{\partial x^2} \right)^s \right)^{1/s}
$$
(5)

Theorem 2. Let conditions 1.2 then there is a value of M for which the equation (5) and the equation

$$
\left(\frac{\partial u}{\partial a}\right)^n = \sum_{j=1}^m \left(D\frac{\partial^2 u}{\partial x_j^2}\right)^n\tag{6}
$$

equivalent. Indeed, under the conditions of 1.2 (5) is transformed into the equation of the type that corresponds to the maximization of the functional (3). Thus, equation (6) $Z^n = \sum_i x_i^n$, is the equation of the "turbulence", i.e addressed by the functional (3) takes the maximum value. Solve the equation (6) in the class of possible simple solutions: $\frac{\partial u}{\partial a} = C$, $\frac{\partial u}{\partial x_i} = C_j$, $\sum_{j=1} C_j = C$. $=C, \frac{\partial^2 u}{\partial x^2} = C_i, \sum_{i=1}^m C_i = C.$ ∂a ∂x_i ∂y_i ∂u $\partial^2 u$ $\partial \overline{u}$ $\partial \overline{u}$ $\partial \overline{u}$ *j* j' \sum_j \sum_j \sum_j \sum_j $\frac{\partial u}{\partial a} = C, \ \frac{\partial^2 u}{\partial x_j} = C_j, \ \ \sum_{j=1}^m C_j = C.$ 1 and 1 and 1 and 1 and 1 and 1 $2 \ldots$ m

Theorem 3. The solution of equation (6) in the class of possible simple solutions can be represented as $x_i x_j + \sum_{i=1}^{m} C_i \frac{x^2}{2}$, $\uparrow = t - a$. (7) $\left| x_i \partial x_i \right|$ $\left| \begin{array}{cc} 1 & i \end{array} \right|$ $\left| \begin{array}{cc} 2 & i \end{array} \right|$ $x_i + \sum_{i=1}^{\infty} \frac{\partial^2 u}{\partial x_i}$ $x_i x_j + \sum_{i=1}^{m} C_i \frac{x^2}{\partial x_i}$ $t = t - a$. (7) x_i $\frac{1}{i+i} \partial x_i \partial x_i$ $\frac{1}{i+i} \partial x_i$ $U(x, a, \ddagger) = U(0, a, \ddagger) + \sum_{i=1}^{m} \frac{\partial u}{\partial x_i} \begin{vmatrix} x_i + \sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2} & x_i x_i + \sum_{i=1}^{m} C_i \frac{x_i^2}{\partial x_i^2} & \ddots & \end{vmatrix} = t - a.$ (7) $j=i$ \qquad i^{i} *j* \sum_{j} j \sum_{j} j \sum_{j} $x=0$ $y=e$ $j \neq j$ ${}^{i}C\lambda_{i} {}^{i}C\lambda_{j} \Big|_{x=0}$ $j =$ *j* $\sum_{i=1}^{m} \frac{\partial u}{\partial x_i}\Big|_{x=0} x_j + \sum_{j\neq j} \frac{\partial^2 u}{\partial x_i \partial x_j}\Big|_{x=0} x_i x_j$ $+\sum C_i \frac{x}{z}$, $\ddagger = t - a$. (7) $\partial x_i \partial x_j$ \cdots \cdots \cdots \cdots \cdots $+\sum_{i=1}^{\infty} \frac{\partial^2 u}{\partial x_i}$ $x_i x_j + \sum_{i=1}^{m} C_i \frac{x^2}{\partial x_i}$, $t = t - a$. (\downarrow $= U(0, a, \downarrow) + \sum_{i=1}^{m} \frac{\partial u}{\partial x_i} \left| \left| x_j + \sum_{j \neq j} \frac{\partial^2 u}{\partial x_i \partial x_j} \right| \right| x_i x_j + \sum_{j=i}^{m} C_j \frac{x^2}{2}, \downarrow = t - a.$ (7) $=0$ $1-t$ $=0$ J^+J l $J|_{x=0}$ $0, a, \frac{1}{2}$ + $\sum_{i=1}^{\infty} |x_i + \sum_{i=1}^{\infty} \frac{c}{2} x_i | x_i$ $2\frac{1}{2}$ $m \frac{2}{2}$ $0 \qquad \qquad J^{-\iota}$ 0 $J^{\neq}J$ l $J|_{x=0}$ $\begin{vmatrix} 1 & \mathcal{U}\mathcal{N}_i \\ 1 & 0 \end{vmatrix}$ $\begin{vmatrix} j \neq j & \mathcal{U}\mathcal{N}_i \\ 0 & j \end{vmatrix}$ (7)

Proof. For simplicity, the proof proceeds in the case where $m = 2$ and $D = 1$.

Consider the class of simple solutions $\frac{\partial u}{\partial a} = C$, $\frac{\partial^2 u}{\partial x_1^2} = 0$, $\frac{\partial^2 u}{\partial x_2^2} = 0$. 2 2_{11} 2, λx^2 1 α_2 2. a^2 . $=0.$ ∂x_2^2 $= 0, \frac{\partial^2 u}{\partial x^2} = 0.$ ∂x_1^2 ∂x_2^2 $=C, \frac{\partial^2 u}{\partial x^2} = 0, \frac{\partial^2 u}{\partial y^2} = 0.$ ∂a ∂x_1^2 ∂x_2^2 \cdots ∂u $\partial^2 u$ $\partial^2 u$ ∂

It is easy to see that it follows $\frac{\partial u}{\partial q} = C$, $\frac{\partial u}{\partial r^2} = 0$, $\frac{\partial u}{\partial r^2} = 0$. Delivering this into 2 2_{11} 2, $\frac{3}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{1}{2}$ 1 α_2 2. λ^2 $= 0$. Delivering this into ∂x_i^2 $= 0$, $\frac{\partial^2 u}{\partial x^2} = 0$. Delivering this into ∂x_1^2 ∂x_2^2 \cdots ∂x_n^2 $=C, \frac{\partial^2 u}{\partial x^2} = 0, \frac{\partial^2 u}{\partial y^2} = 0$. Delivering this ∂a , ∂x_1^2 , ∂x_2^2 , ∂x_3^2 ∂u $\partial^2 u$ $\partial^2 u$ ∂ Deliver x_2^2 \cdots = \cdots \cdots \cdots \cdots \cdots \cdots u_{o} Delivering this into x_1^2 ∂x_2^2 \cdots \cdots *C*, $\frac{\partial^2 u}{\partial x^2} = 0$, $\frac{\partial^2 u}{\partial y^2} = 0$. Delivering this into a ∂x_1^2 ∂x_2^2 ∂x_3^2 $\frac{\partial^2 u}{\partial x^2} = C$, $\frac{\partial^2 u}{\partial y^2} = 0$. Delivering this into condition, $\frac{\partial^2 u}{\partial x_2^2} = 0$, we obtain (7). Formula (7) con 2 2_{11} $= 0$, we obtain (7). Formula (7) corre ∂x_2^2 $\frac{\partial^2 u}{\partial x^2}$ = 0, we obtain (7). Formula (7) corresponds to the maximization problem (3) that sum of the second derivatives, which corresponds to the "population of turbulence." Now consider the case where the population in the

process of turbulence at which point the field reached the maximum number of populations. From $\max_{(x_1, x_2)} U(x_1, x_2, a, \ddagger)$, we have: (x_1, x_2)

$$
\begin{cases}\n\frac{\partial u}{\partial x_1} = \frac{\partial u}{\partial x_1}\Big|_{(x_1, x_2) = 0} + \frac{\partial^2 u}{\partial x_1 \partial x_2}\Big|_{(x_1, x_2) = 0} x_2 + C_1 x_1 = 0 \\
\frac{\partial u}{\partial x_2} = \frac{\partial u}{\partial x_2}\Big|_{(x_1, x_2) = 0} + \frac{\partial^2 u}{\partial x_1 \partial x_2}\Big|_{(x_1, x_2) = 0} x_1 + C_2 x_2 = 0\n\end{cases}
$$

Hence

$$
x_1^0 = \frac{\left(\frac{\partial^2 u}{\partial x_1 \partial x_2}\Big|_0\right) - C_2 \frac{\partial u}{\partial x_1}\Big|_0}{C_1 C_2 - \left(\frac{\partial^2 u}{\partial x_1 \partial x_2}\right)^2\Big|_0^2}, \qquad x_2^0 = \frac{\frac{\partial^2 u}{\partial x_1 \partial x_2}\Big|_0 \cdot \frac{\partial u}{\partial x_1}\Big|_0 - \frac{\partial u}{\partial x_2}\Big|_0 \cdot C_1}{C_1 C_2 - \left(\frac{\partial^2 u}{\partial x_1 \partial x_2}\Big|_0\right)^2}.
$$

Compute the matrix of second derivatives

$$
\frac{\partial^2 u}{\partial x_1^2} = C_1, \quad \frac{\partial^2 u}{\partial x_2 \partial x_1} = \frac{\partial^2 u}{\partial x_1 \partial x_2}\bigg|_0, \quad \frac{\partial^2 u}{\partial x_1 \partial x_2} = \frac{\partial^2 u}{\partial x_1 \partial x_2}\bigg|_0, \quad \frac{\partial^2 u}{\partial x_2^2} = C_2.
$$

Thus, if $(C_1; C_2) > 0$, and $C_1C_2 > \left(\frac{\partial^2 u}{\partial x_1 \partial x_2}\right)^2$, $(C_1) > 0$, then at the point (x_1^0, x_2^0) 0 $1^{U\lambda_2}$ $\left| \right|$ 2. \vert \vert $\left\{C_1C_2 > \left(\frac{\partial u}{\partial x_1 \partial x_2}\right)\right\}$, $(C_1) > 0$, then at the point $\left(x_1^0, x_2^0\right)$ $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (c) of the state $\left(\frac{\partial x_1}{\partial x_1 \partial x_2}\right)$, $(C_1) > 0$, then $\begin{pmatrix} \frac{\partial^2 u}{\partial x^2} & \frac{\partial$ $\partial x_1 \partial x_2$ | \langle \cdot \rangle | \langle \cdot | \rangle + \r $\left[\infty\right] \left[\infty\right]$ (C₁) > 0, then at the point (x_1^0, x_2^0) 2 / $\left(x_1^0, x_2^0\right)$

function $u = u(x_1, x_2, a, \ddagger)$ defined by (7) reaches its maximum.

Remark. In (7) all the coefficients $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $j=1,2$ depend on 1^{λ_2} | 0 $2\ldots$ $\qquad \qquad \ldots$ $\left[\frac{\partial^2 u}{\partial x_j}\right]_0$, $\left[\frac{\partial^2 u}{\partial x_1 x_2}\right]_0$, $j = \overline{1,2}$ depend on $\partial^2 u$ $\Big|$ \Big ∂x_i , $\partial x_1 x_2$, $\partial x_3 x_3$ $\frac{\partial u}{\partial x}$, $\frac{\partial^2 u}{\partial y}$, $j = 1,2$ depend on

 $(a, t - a)$, i.e, in fact, they are functions and they are determined by the initial conditions $(u|_{t=0}, u|_{t=0})$ Representation (7) can be written as the following polynomial:

$$
u = u_0 + u_1 x_1 + u_2 x_2 + u_3 x_1 x_2 + \frac{C_1}{2} x_1^2 + \frac{C_2}{2} x_2^2,
$$
 (7')

where
$$
u_0 = u(0, a, \ddagger)
$$
, $u_1 = \frac{\partial u}{\partial x_1}\bigg|_0$, $u_1 = \frac{\partial u}{\partial x_1}\bigg|_0$, $u_2 = \frac{\partial u}{\partial x_2}\bigg|_0$, $u_3 = \frac{\partial^2 u}{\partial x_1 \partial x_2}\bigg|_0$, $C_1^n + C_2^n = C^n$, $n > 1$.

From this polynomial, it follows that in the case of turbulence population behavior "forgotten" the initial conditions.

Theorem 4. Let the representation (7 ') u_{*j}*, $j = 0,1,2,3$ is a known quantity</sub> and C_1 , C_2 is a solution $C_1^n + C_2^n = C^n$ for some $n > 1$ and $C \ge 4u_3^{2n}$, then every process of population turbulence described function (7') makes an emergency landing at only two points (C_1^+, C_2^+) and (C_1^-, C_2^-) , *,* $\frac{\partial}{\partial x_1 \partial x_2} \bigg|_0 = \frac{\partial}{\partial x_2 \partial x_1} \bigg|_0$. $|u|$ $\mathcal{L}_1 \cdot \mathcal{L}_2 = \left| \frac{C^n - \sqrt{C^n - 4u^{2n}}}{2} \right|$, $U_3 = \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$. $C_1^{\gamma_1}$ | 2 | $\partial x_1 \partial x_2 \Big|_0 \partial x_2 \partial x_1 \Big|_0$ *,* $C_1 = \frac{u_3^2}{2}$, $C_2 = \left(\frac{C^n - \sqrt{C^n - 4u^{2n}}}{2} \right)$, $U_3 = \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$. $C^{n} + \sqrt{C^{n} - 4u_{3}^{2n}}$ $\qquad \qquad$ u_{3}^{2} $\qquad \qquad$ $\qquad \qquad$ $\qquad C^{n} - \sqrt{C^{n} - 4u^{2n}}$ $\qquad \qquad$ $\qquad \qquad$ $\partial^{2}u$ $C_1^* = \frac{\sigma^2 + \sqrt{2\sigma^2 - m_3}}{2}$, $C_1^* = \frac{m_3}{2}$, $C_2^* = \frac{\sigma^2 - \sqrt{2\sigma^2 - m_1}}{2}$, U_3 $C_2^{+,-1}$ | 2 | $C_1^{-,-1}$ | 2 | $\left($ $C_1^+ = \frac{u_3^2}{2}$, $C_1^+ = \left[\frac{C^n + \sqrt{C^n - 4u_3^{2n}}}{2} \right]^{1/n}$, $C_1^- = \frac{u_3^2}{2}$, $C_2^- = \left[\frac{C^n - \sqrt{C^n - 4u^{2n}}}{2} \right]^{1/n}$, $U_3 = \frac{\partial^2 u}{\partial x^2} \bigg|_{x=0} = \frac{\partial^2 u}{\partial y^2} \bigg|$. $2^{\mathbf{c} \cdot \mathbf{c}_1}$ | 0 2^{10} 2^{10} 2^{10} 2.1 2^2 , 1 $3 - 2.2$ $- 2.2$ $\sqrt{2n}$ $\sqrt{1/n}$ $\approx 2 \cdot 1$ $\approx 2 \cdot 1$ 2 $\sqrt{2}$ $1 \quad \sqrt{2}$ 2 \bigcap^n \bigcap^n \bigcap^{n} \bigcap^{n} $T_1 = \frac{u_3}{C_1}, C_2 = \frac{C_1}{C_2}$ $\frac{1}{2n}$ $\bigg\{ \frac{1}{2n} \bigg\}$ $\bigg\{ \frac{1}{2n} \bigg\}$ $3 \left[\begin{array}{ccc} 3 & - & u_3 & - \end{array} \right]$ $1 \begin{array}{ccc} 1 & 1 \end{array}$ 2 $\qquad \qquad \blacksquare$ 2 $\int C^n + \int C^n - 4u^{2n}$ $\left[\frac{C_1^*}{C_1^*},C_1^*=\left[\frac{C_1^*}{C_1},C_1^*=\frac{C_2^*}{C_1}\right],C_1^*=\frac{C_3^*}{C_1^*},C_2^*=\left[\frac{C_1^*}{C_1},C_2^*=\frac{C_1^*}{C_1}\right],U_3^*=\frac{C_2^*}{\partial x_1\partial x_2}\right]_0^1=\frac{C_3^*}{\partial x_2\partial x_1}\left[\frac{C_3^*}{C_1},C_2^*=\frac{C_4^*}{C_1}\right]_0^1$ $4u^{2n}$ $\begin{array}{cc} & \partial^2 u & \partial^2 u \end{array}$ 2 $\left(\begin{array}{ccc} \begin{array}{ccc} \end{array} \begin{array} \begin{array} \end{array} \begin{array} \end{array} \begin{array} \begin{array} \end{array} \begin{array} \end{array} \begin{array} \begin{array} \end{array} \$ $4u_3^{2n}$ u_3^2 c_5 $(c^n - \sqrt{c^n - 4u^{2n}})$ c_5 $\partial x, \partial x_1$ $=\frac{\partial^2 u}{\partial x^2}$. $\partial x_i \partial x_j\Big|_{\alpha} \partial x_j \partial x_i\Big|_{\alpha}$ $\left\{\right\}$, $U_3 = \frac{\partial^2 u}{\partial x_1 \partial x_2}\Big|_0 = \frac{\partial^2 u}{\partial x_2 \partial x_1}\Big|_0$. $\partial^2 u$ $\partial^2 u$ $\partial^2 u$ $\left(\frac{\overline{a}}{2}\right)^{\frac{1}{2}}, \frac{U_3}{\overline{a}} = \frac{\partial}{\partial x_1 \partial x_2}\Big|_0 = \frac{\partial}{\partial x_1 \partial x_2}\Big|_0$ \int_0^{∞} , $C_1 = \frac{u_3^2}{C_1}$, $C_2 = \left(\frac{C^n - \sqrt{C^n - 4u^{2n}}}{2}\right)^{1/n}$, $U_3 = \frac{\partial^2 u}{\partial x_1 \partial x_2}\Big|_0^{\infty} = \frac{\partial^2 u}{\partial x_2 \partial x_1}\Big|_0^{\infty}$. $u^2 = \left(C^n - \sqrt{C^n - 4u^{2n}}\right)^{1/n}$ $\left(\frac{2}{2}\right)^{4}$, $C_{1} - \frac{1}{C_{1}}$, $C_{2} - \left(\frac{1}{C_{2}}\right)^{2}$ $\left(C^{n}+\sqrt{C^{n}-4u_{3}^{2n}}\right)^{1/n}$ u_{3}^{2} $\qquad\left(C^{n}-\sqrt{C^{n}-4u^{2n}}\right)^{1/n}$ $\qquad\qquad\tilde{c}$ $=\frac{u_3}{u_3}$, $C_1 = \frac{C_1 + C_2 - u_3}{2}$, $C_1 = \frac{u_3}{2}$, $C_2 = \frac{C_2 - C_3 - u_1}{2}$, U_3 \sim \sim \sim \sim \sim \sim $\begin{array}{cc} \begin{array}{cccc} + \end{array} & \begin{array}{cccc} \end{array} & \begin{array$ $+$, \sim 1 | \sim | μ u_3 μ σ σ where the proof follows from the definition of disasters: $\frac{\partial u}{\partial x_i} = 0$, $i = 1, 2$, $\det K = 0$, ∂u 0 12 $1/L$ 0 $\sqrt{2}$ $\partial^2 u$ | | $2 \cdot \cdot$ \cdot

where
$$
K = \begin{bmatrix} C_1 & \frac{\partial^2 u}{\partial x_1 \partial x_2} \Big|_0 \\ \frac{\partial^2 u}{\partial x_2 \partial x_1} \Big|_0 & C_2 \end{bmatrix}
$$
.

§10. Computational experiments

Crisis. In next figure we show results of computational experiments with economical crisis parameters and we defined the structure of the crisis.

Fig. 10.The structure of the crisis

Population turbulence. Now, here are some computer simulations with the following model data: $u0 = 5000$; $u1 = 0.1$; $u2 = 0.1$; $u3 = -0.1$; $n = 2$; $c = 1$; $x = -1$: .1: 1; $y = -1$: .1: 1; c1 = ((c ^ n-(c ^ (2 * n) -4 * u3 ^ 2) ^ (1/2)) / 2) ^ (1 / n); c2 \star c2 = u3 \land 2; c1 \land n + c2 \land n = c \land n. For these data, the following are some of the results of numerical experiments.

From the given results it is visible, that at change parameters there is "turbulence" population number.

Experiments with capital size in different case. *Now we consider some results of computational experiments with capital size in time extreme regime at next model data =0.7,a0=1,a1=0.000001,a2=0.00000001,*

f0=1,de=-0.5,be=10, l0=70 al=0.1al=0.2

54

Fig. 12.Dependence of the capital size on time for different value parameters (case of a)-d))

The Conclusion

Any type of Economics in different periods may by pass through the development crises. In a broad sense, it is a process that threatens the existence of the Extreme Economics in some organizations. Crises often come to the organization's management suddenly. However, in practice, the emergence of the crisis shows many symptoms: loss of income from the sale of goods (works, services), reduction of other indicators of financial and economic activities of the organization. Many companies are not able to overcome such crises develop and leave the market. The very possibility of a crisis is defined risky development of any organization that is always there. Therefore, it is advisable

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to consider the typology of crises in the development of the organization, the causes and patterns of their occurrence, the criteria specific to the crisis. Any complex objects (country, organization) in the different periods may by-pass through the development crises. In a broad sense, it is a process that threatens the existence of the organization. Crises often come to the organization's management suddenly. However, in practice, the emergence of the crisis shows many symptoms: loss of income from the sale of goods (works, services), reduction of other indicators of financial and economic activities of the organization. Many companies are not able to overcome such crises develop and leave the market. The maximum possibility of a crisis is defined risky development of any organization that is always there. Therefore, it is advisable to consider the typology of crises in the development of the organization, the causes and patterns of their occurrence, the criteria specific to the crisis.

Appendix 1. Polynomial model information security

It is possible it used at protection of the information or a kind

$$
N \t m \t - Y \t \sum_{i=3}^{m} X \t m - i Z \t i - 2 = X \t (m - 1) + YX \t m - 2
$$

where $Z=X+Y$ and all decisions of the equation $x^n + y^n = z^p$ at $p=n$ are represented as

$$
x = zt^{\frac{1}{n}}, \quad y = z(1-t)^{\frac{1}{n}}, \quad p = n, \quad t \in (0,1).
$$

 $P_m^p(x, y) = N_m^p$ is a polynomial of a degree *(m-1)* on x_n and it is a class curve higher degrees which are very well used at protection of the information. For example, we shall write a class of such curves in special cases with the help of displays:

$$
N_m = z^{m-1}
$$
, $a_{1m} = x^{m-1}$, $a_{2m} = yx^{m-2}$, $a_{im} = yz^{\frac{i-2}{n}}x^{m-i}$, $i = 1, ..., m$; I.e. curves

 $P_{2}(x, y) = X + Y$, $P_{3}^{p}(x, y) = X^{2} + YX + YZ$ and $P_{4}^{p}(x, y) = X^{3} + YX^{2} + YZX + YZ^{2}$, ..., and also type $N_4^2 = X^3 + YX^2 + YZX + Z^2Y$, $Z=X+Y$, which belongs to a class elliptic curves. Pairs (x, y) is usual to name "point" which can "be put" about other similar point of an elliptic curve. « The sum » two points, in turn, too "lays" on an elliptic curve. Except for the points laying on an elliptic curve, « the zero point » is considered also. The sum of two points A with coordinates (XA is considered, that, YA) and B with coordinates (XB, YB) is equal O, if $XA =$

 $XB, YA = - YB \pmod{p}$. The zero point does not lay on an elliptic curve, but, nevertheless, participates in calculations; it can be considered as indefinitely removed from a curve. Set of points of an elliptic curve together with a zero point and with the entered operation of addition, we shall name them "group". For each elliptic curve the number of points in group certainly, but is great enough. The important role in algorithms of the signature with use of elliptic curves is played with "multiple" points. Point Q refers to as a point of frequency rate k if for some point P k time it is executed equality: $P = Q + Q + Q + ... + Q =$ kQ. If for some point P there is such number k, that $kP = 0$, this number is named the order of point P. Multiple points of an elliptic curve are analogue of degrees of numbers in a simple field. The problem of calculation of frequency rate of a point is equivalent to a problem of calculation of the discrete logarithm. Follows noticed that reliability of the digital signature is based on complexity of calculation of "frequency rate" of a point of an elliptic curve and. Though equivalence of a problem discrete logarithm and problems of calculation of frequency rate also is proved, the second has the big complexity. For this reason at construction of algorithms of the signature in group of points of an elliptic curve appeared possible to do without shorter keys in comparison with a simple field at maintenance of the greater stability. A confidential key, consider some random number. The open key considers coordinates of some point on elliptic curve P which is defined as $P = xQ$ where Q - special image the chosen point of an elliptic curve named « a base point. Coordinates of point Q together with factors of the equation specifying a curve, are parameters of the circuit of the signature and they should be known to all participants of an exchange messages. From here follows, that anyone "modern" cryptosystem can "be shifted" on elliptic curves.

Appendix 2. Algebraic representation of tree numbers of some differential models

The Model of Tree Numbers

Definition of numbers tree and its representation**.** *Let N is some natural number. We shall tell, that the number N forms a tree of numbers if there will be natural numbers* $n, m \geq 2$ *and integers* $a_1, a_2, ..., a_m$ *for which*

$$
N^n = a_1^n + a_2^n + \dots + a_m^n \t{1}
$$

and in turn some a ^j (or all) represented as

$$
a_j^n = a_{1j}^n + a_{2j}^n + \dots + a_{m_1j}^n, \ m_1 \le m \tag{2}
$$

and some aij of (2) also can be submitted as

$$
a_{ij}^n = a_{1ij}^n + \dots + a_{m_2ij}^n, \ \ m_2 \leq m_1, \dots,
$$

and at last decomposition takes place

$$
a_{ijj_1...j_{m_k}}^n = a_{1ijj_1...j_{m_k}}^n + a_{2ijj_1...j_k}^n, \qquad (3)
$$

in which members of the right part (3) cannot beat are submitted as the final sum composed n-th degrees of some integers so-called by a basis (basis) of a tree.

Conceptual Model of Numbers Tree in general case is given in fig. 1.

Fig.1. Conceptual Model of Numbers Tree in general case

So last a level the tree consists of the sum such as (3).base elements can enter into each level a tree, therefore from each level we take only those elements,

which not as (2), (3). Then in result the number N *is uniquely represented* as

$$
N^{n} = \sum_{j_q} k_{j_q} a_{ijj_1...j_q}^{n}, \qquad (4)
$$

where k_{j_q} - number of occurrence of a basic element $a_{ijj_1...j_q}$ in a tree of numbers.

We shall consider a problem for number *N* and a vector $a = (a_1, \dots, a_k)$, $k \ge 2$ from the equation

$$
N = \max_{\Gamma \in M} (\Gamma, a), \text{ where } M = \left\{ (\Gamma_1, ..., \Gamma_k) = \Gamma; \sum_{j=1}^k \Gamma_j^{\frac{n}{n-s}} = 1, n > s > 0, 0 < \Gamma_j < 1 \right\}.
$$

The set M represents a measured curvilinear spheroid and at $s = 1$, $n = 2$ turns in usual m-a measured spheroid. Using theorems 1 from work [12] we shall receive:

$$
\begin{cases}\nN^n = a_1^n + \dots + a_k^n, \\
N_k^n = a_{1k}^n + \dots + a_{kk}^n, \quad k \ge 2\n\end{cases}
$$
\n(4')

Thus, this equation is optimum in sense (4), and the tree of numbers appropriate by this equation represented on rice. 1 also is an optimum tree.

Now, we consider questions of the best representation some character of any objects (elements) with the help of some properties (elements) given object and its applications in some cases where distributions processes of growth processes, heats and waves processes, diffusion processes, at some parameters values take place in the maximal regimes. Extreme regimes of such physical processes are arising in the case when the values of their parameters are chaining in some given set. They can be an accumulation of the warmth, particles, wave energy in some areas where the considered physical processes are arising. Series computational experiments also carried out with models data for some considering heat transfer processes under complex conditions and in extreme regime. Let and *L* - some normalized spaces and the set of *M* is given Let's assume, that for any objects (elements) $z \in H$ and any properties of objects

$$
x_j \in L
$$
, $j = \overline{1,m}$ norms $P = ||z||_H$, $h_j = ||x_j||_L$, $j = \overline{1,m}$, $m > 1$ and also ways of display

of properties of the considered objects are determined. For example, we can define their norms in the appropriate spaces, i.e.

$$
P = -(\Gamma) = \left(\int_{T}^{m} \left(\sum_{j=1}^{m} \Gamma_{j} \middle| x_{j}\right)^{s}\right)^{\frac{n}{s}} dt\right)^{1/n}, \quad \left\|x_{j}\right\|_{L_{m}^{n}(T)} = \left(\int_{T}^{m} \sum_{j=1}^{m} \left|x_{j}\right|^{n} dt\right)^{1/n} < \infty.
$$

Definition. We shall tell, that any object (element) $z \in H$ is in the best way submitted with the help of some properties (elements) $x_i \in L$, $j = \overline{1,m}$, if for some element $r^0 \in M$ fairly a parity s ^{1/s} For anyone $s > 0$ h^{s} ^{1/s} = $\left(r^{0} h_{1}^{s} + ... + r^{0} h_{m}^{s}\right)^{1/s}$ For anyone s > 0 and *M* (11 *m m)* $P = \max (r, h^s)^{1/s} = (r_1^0 h_1^s + ... + r_m^0 h_m^s)^{1/s}$ For anyone $s > 0$ $\binom{0}{1}h_1^S + ... + \binom{0}{m}h_m^S$ For anyone $\max_{i \in M} (r, h^s)^{1/s} = \left(r_1^0 h_1^s + ... + r_m^0 h_m^s \right)^{1/s}$ For anyone s $\left(r_1^0 h_1^s + ... + r_{m}^0 h_m^s\right)^{1/3}$ For anyone s: $=\left(r_1^0 h_1^s + ... + r_0^0 h_s^s\right)^{1/3}$ For anyone $s > 0$ $\in M$ (11 m m) = max $(r,h^s)^{1/s} = [r_1^0h_1^s + ... + r_n^0h_n^s]$ For anyone $s>0$ and $\Gamma \in M$ (11 *m m)* For anyone $s > 0$ and $s < n < \infty$. .

The Theorem 1. That the object (element) $z \in H$ the best is submitted (maximal) image through properties $x_i \in L$, $j = \overline{1,m}$, it was necessary and enough that had places about parity:

$$
r \frac{n}{j n - s} = \frac{h_j^n}{\sum_{\substack{m \\ j = 1}}^n h_j^n}, j = \overline{1, m} \qquad , n > s > 0 \, , \ P^n = \sum_{\substack{m \\ j = 1}}^m h_j^n
$$

Example 1. *Distribution problem of resources*. As an example it is possible to result a problem of representation of a monetary stream, or a word at coding etc. as the sum m numbers:

$$
\left(\sum_{j=1}^{m} r_j h_j^s\right)^{\frac{1}{s}}
$$
 then $P = \max(r, h^s)^{1/s}$ we shall receive the equation

$$
P^n = \sum_{j=1}^{m} h_j^n.
$$

Last equation for anyone $m > 2$, $n > 1$ has accounting number of integer decisions h_1 , h_2 ,, h_m and P.

Example 2. *Growth of plant*. Let us consider model of growth of plants with $m, m \geq 2$ parts (for example, roots, trunk, leaves, buds). Let first part makes means of production biomass x_1 , which can be spent for development of all other parts of plant. Let $x_i(t)$, $j = \overline{2,m}$ are biomass of part now of time t. Development of plant we shall set to the following model:

$$
\dot{x}_1 = \Gamma_1 f(x_1, l), \quad \dot{x}_j = \Gamma_j x_j, \quad x_1(0) = x_1^0, \quad x_j(0) = x_j^0, \quad j = 2, m \quad ,
$$

$$
I(r) = \{ (x(t_{t_k}), t_k) - \max , r = (r_1, ..., r_m) \in M, 0 \le t \le t_k,
$$

where $x_j^0 \ge 0$, $j = 1$, *m* are given numbers, $f = f(.)$ the law of formation of a new biomass, *l* - size of photosynthesis parameters. On the base of our theorem Bellman function

correspondents to optimal control problem satisfy next equation

$$
-\frac{\partial z}{\partial t} = \max_{\Gamma \in M} \left\{ \left(\Gamma, x_1 \frac{\partial z}{\partial x} \right) \right\} \text{ i.e.} \left(\frac{\partial z}{\partial t} \right)^n = \sum_{j=1}^m \left(x_1 \frac{\partial z}{\partial x_j} \right)^n
$$

or $-(x_1, x_2, \dots, x_m, t) = z_0 + Ct + \ln \left(\frac{x_1}{x_1^0} \right)^{C_1} \left(\frac{x_2}{x_2^0} \right)^{C_1} \dots \left(\frac{x_m}{x_m^0} \right)^{C_m} \left| , C_1^1 + C_2^1 + ... + C_m^1 = C^n \right\}$

Example 3. *Differential equations with extreme properties*. Many processes (distributions processes of heats and waves, diffusion processes) belong to so-called model equations with extreme properties. In the general case such equations may be represented in the form of:

$$
Lu = \max_{\Gamma \in M} \left\{ \sum_{j=1}^{m} \Gamma_j \left(L_j u \right)^s \right\}^{\frac{1}{s}} \quad \text{or} \quad (Lu)^n = \sum_{j=1}^{m} \left(L_j u \right)^n \quad , \ n > s > o, \text{ here } \quad L, \ L_j
$$

are given operators, which characterize the considered physical processes.

Let it is given area $G \subseteq E^m$ with border : $= \bigcup_{i}$, and then it is known, that the temperature mode inside area G and wave process satisfy to next equations[2]: $\frac{\partial u}{\partial x} = \sum_{n=1}^{m} \left[\frac{\partial u}{\partial x_n} \right], \quad x \in G$ $\sum_{i=1}^{\infty} \frac{c}{\partial x_j} \left(\begin{array}{c} k \\ j \frac{\partial x}{\partial x_j} \end{array} \right)$, $\sum_{i=1}^{\infty} \frac{c}{i}$, $\sum_{i=1}^{\infty}$, $\sum_{i=1}^{\infty}$ $\sqrt{2}$ $\left(\begin{array}{cc}J\ \partial x_j\end{array}\right)\ \ \ \ \ o\ \ \leq\ \ t\ \leq\ t_k$ $\left(\begin{array}{c}1\\2\end{array}\right)$ ∂x $\qquad \qquad$ $\qquad \qquad$ $\qquad \qquad$ $\partial \leq t \leq t$ ∂u \qquad $\frac{\partial u}{\partial t} = \sum_{i=1}^{m} \frac{\partial}{\partial x_i} k_j \frac{\partial u}{\partial x_i}$, $x \in G$, ∂u *m* ∂ ∂ ∂u $\frac{u}{t} = \sum_{j=1}^{m} \frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right)$, $x \in G$, $x \leq t$, $x \leq t$, $x \leq t$, $x \in G$,

$$
\frac{\partial^2 u}{\partial t^2} = \sum_{j=1}^m \frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right), \ x \in G, \ o < t \le t_k. \text{Here}
$$

 $k_j = k_j(\cdot)$, $j = 1, m$ are coefficients of heat conductivity (or wave velocity) characterizing a version of environment. We shall make the following assumption. Let the environment of heat conductivity is those, that

$$
\frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right) = r_j \frac{\partial}{\partial x_j} \left(k \frac{\partial u}{\partial x_j} \right), j = \overline{1, m}, \text{ where } r_j(\cdot) \ge 0, \sum_{j=1}^m r_j \frac{n}{n-s} = 1, n > s > 0, m > 1,
$$

 $r = (r_1...r_m) \in M$. The equation of heat conductivity (1) corresponds (meets) a case when s=1, i.e. heat exchange occurs under the usual law of Newton. Therefore at a choice of a set of parameters $r = (r_1 ... r_m)$ from *M* at *s*=1. Then we have the equations with extreme properties

$$
\frac{\partial u}{\partial t} = \max_{\Gamma \in M \mid s = 1} \sum_{j = 1}^{m} \Gamma_j \frac{\partial}{\partial x_j} \left(K \cdot \frac{\partial u}{\partial x_j} \right), \quad \text{and} \quad \frac{\partial^2 u}{\partial t^2} = \max_{\Gamma \in M \mid s = 1} \sum_{j = 1}^{m} \Gamma_j \frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j} \right), \quad x \in G, \ o < t \le t_k \tag{2}
$$

And therefore, on the basis of the basic theorem we shall receive the nonlinear equations of type

$$
\left(\frac{\partial u}{\partial t}\right)^n = \sum_{j=1}^m \left[\frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j}\right)\right]^n, \quad x \in G \quad \left(\frac{\partial^2 u}{\partial t^2}\right)^n = \sum_{j=1}^m \left[\frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j}\right)\right]^n, \quad x \in G, \quad o \le t \le t_k
$$
 (3)

The right parts of last equations corresponds to a maximum quantity of heat and a wave in area G, formed as a result of maximization of the previous equations (2) on a set $r \in M|_{s=1}$. For the solutions of last equations (for example, the first equation (3)) is necessary to set a class of possible solutions or simple type,

$$
\frac{\partial u}{\partial t} = c \, , \, \frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j} \right) = c_j \, , \, j = \overline{1, m} \, , \text{ or exponent type } \frac{\partial u}{\partial t} = c u \, , \, \frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j} \right) = c_j u, \, j = \overline{1, m} \, ,
$$
\n
$$
\text{where } c_j, \, c \text{ are solutions of the coordination equation: } \sum_{j=1}^{m} c_j^n = c^n \, .
$$

The Lemma 1. Let area G is a rectangular with the sides: l_1 , l_2 , And in the first equation (3) is given (similarly for the second), and also are given initial and boundary conditions: $u_{t=0}^{i} = u_0(x_1, x_2) (x_1, x_2) \in \overline{G}$, $u_{x_i}^{i} = 0 = u_{x_i}^{i} = 1$, $i = 1, 2$. Then the $\sum_{j=0}^{\infty}$ *l* $\left| \sum_{j=1}^{\infty} f_j \right|$ *j* = 1,2. Then the u_{x} \sim 0 = u_{x} \sim j \sim j \sim j \sim \sim \sim \sim \sim \sim \sim solution is represented in the following kind: $(x_1, x_2, t) = \frac{2}{\sqrt{l_1 l_2}} \sum_{n_1, n_2 = 1} D_{n_1 n_1} e^{-\frac{n_1 n_2}{l_1} x_2} \sin \frac{n_1 n_2}{l_1} x_1 \sin \frac{n_2 n_2}{l_2} x_2$, where $D_{n_1 n_2}$ are coeffic $\int_{1}^{1} 2 \sin \frac{f(t)}{t} x_1 \sin \frac{f(t)}{t} x_2$, where $D_{n,n}$ are l_1^2 $n_1, n_2 = 1$ l_1^2 l_1^2 l_2^2 l_1^2 l_2^2 l_1^2 $L_1, x_2, t = \frac{2}{\sqrt{l_1 l_2}} \sum_{n \text{ n } n = 1} D_{n_1 n_1} e^{-n_1 n_2} \sin \frac{f n_1}{l_1} x_1 \sin \frac{f n_2}{l_2} x_2$, where $D_{n_1 n_2}$ are co n_2 where D are coef $\frac{1}{l_1}$ _{x_1} sin $\frac{2}{l_2}$ x_2 , where $D_{n_1 n_2}$ a $c_{n_1n_2}^c$ *f*_n *f*_n *f*_n *f*_n *where D* are $e^{-1/2}$ sin⁻¹ x_1 sin² x_2 , where $\sum_{n_1, n_2 = 1}^{\infty} D_{n_1 n_1} e^{-1/2} \sin \frac{1}{l_1} x_1 \sin \frac{2}{l_2} x_2$, where $D_{n_1 n_2}$ are $l_1 l_2 n_1 = 1$ $n_1 n_1$ $u(x_1, x_2, t) = \frac{2}{\sqrt{2\pi}} \sum D_{n} e^{-1/2} \sin \frac{1}{\sqrt{2\pi}}$ fn_1 fn_2 where D are coeffici- $\sum D_{n,n} e^{-1/2} \sin \frac{1}{x} x_1 \sin \frac{2}{x_2} x_2$, where $D_{n,n}$ $=1$ $\binom{n}{1}$ $\binom{1}{1}$ $\binom{1}{2}$ $\binom{2}{1}$ $=\frac{2}{\sqrt{2}}$ Σ $D_{\text{max}}e^{-n_1n_2}\sin\frac{f_n}{x_1}\sin\frac{f_n}{x_2}\cos\theta$ where $D_{n,n}$ $D_{n_1 n_2}$ are coefficients Fourier of function u_0 (x_1, x_2) , and parameters $c_{n_1, n_2} = c$ are the solutions of the $1^{n}2$ are the solutions of the coordination equation $c_1^n + c_2^n = c^n$, $c_{n,n_0} = n \left| \frac{f^n n_1}{l} \right| + \left| \frac{f^n 2}{l} \right|$, $n_j = 2,3,..., j = \overline{1,2}$. 2^{\prime} $\frac{2}{2}$ $\frac{n}{2}$ $\frac{-23}{12}$ $\frac{-12}{12}$ $1)$ (2) $n_1 n_2 = n \sqrt{\frac{J n_1}{l_1}} + \frac{J n_2}{l_2}$, $n_j = 2, 3, ..., j = 1, 2$. $\left(\overline{l_2}\right)$, $n_j = 2, 3, ..., j - 1, 2$ $(f n_{\alpha})^n$ $\qquad \qquad \qquad$ $+\left(\frac{2}{l_2}\right)$, $n_j = 2,3,...,j = 1,2$. $\int_{1}^{n} (f n_{\alpha})^{n}$ $\left(\overline{l_1}\right)$ $\left(\overline{l_2}\right)$, n_j , \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow $(f n)^n$ $(f n^2)^n$ $\qquad \qquad \qquad$ $= n_{\text{v}} \left| \frac{1}{l} \right| + \left| \frac{2}{l_{\text{o}}} \right|, n_{j} = 2, 3, ..., j = 1, 2$ $c_{n_1 n_2} = n \left(\frac{f n_1}{l_1} \right)^n + \left(\frac{f n_2}{l_2} \right)^n$, $n_j = 2, 3, ..., j = \overline{1,2}$.

The Lemma 2. Let $G \subset E^2$ is a rectangular with the sides l_1, l_2 and next initial and boundary conditions are given $\int u \, dt = 0$ = u_0 $\left(x_1, x_2\right)$, $u_t \, dt = 0$ = u_2 $\left(x_1, x_2\right)$, $u_t \, dt = 0$ = 0. Then the solution of the

second equation (3) is represented as

$$
u(x_1, x_2, t) = \sum_{n_1 n_2 = 1} \left(A_{n_1 n_2} \cos \sqrt{c_{n_1 n_2}} t + B_{n_1 n_2} \sin \sqrt{c_{n_1 n_2}} t \right) \qquad \sin \frac{(2n_1 + 1) f x_1}{2l_1} \sin \frac{(2n_2 + 1) f x_2}{2l_2} ,
$$

where $c_{n_1 n_2} = \sqrt{\left(\frac{(2n_1 + 1) f x_1}{2l_1} \right)^n + \left(\frac{(2n_2 + 1) f x_2}{2l_2} \right)^n}$, $A_{n_1 n_2}$, $B_{n_1 n_2}$ are defined

from initial conditions.

Example 4. *The function .* Let there is a bunch of particles with the certain pulse and some energy. It is known, that such bunch of particles is described by wave function \mathbb{E} , i.e. amplitude of probability. This wave falls on the screen in a crack, and is further from these cracks leaves as a spherical wave and on the following screen, these waves are unreferenced. Summing wave from the top and bottom cracks of the screen is organized as a wave $\mathbb{E} = \sum_{j=1}^{k} \Gamma_{j,k} \mathbb{E}_{jk}$, *j* 1 and 1 where $k = 2$ for one particle and $k = 2^m$ for m particles, r_{jk} accordingly shares of waves leaving cracks in general summing waves. We shall assume, that $\sum_{j=1}^{k} r^{\frac{n}{n-s}}_{jk} = 1$, where $r^{n}{}_{jk}$ the appropriate pr $\int_{i}^{\overline{n-s}}$ =1, where \int_{i}^{n} the appropriate probabilities, *n*,*s* parameters of the 1 environment of distribution (for example $n = 2$, $s = 1$). Then considering integrated approach wave function, i.e. $\mathbb{E} = P + iP^*, \mathbb{E}_{ik} = h_{ik} + ih_{ik}$ on the basis of a principle of optimum representation we have: $= \sum_{j=1}^{k} h_{jk}^{n}$, $extrP_{k}^{*s} \cdot extrP_{k}^{n-s} = \sum_{j=1}^{k} h_{jk}^{*s} h_{jk}^{n-s}$, where $extr=(max \ at \ n>s, and \ min \ at \ b)$ j_k , where $\epsilon \lambda u$ – μ $s \cdot \text{extr}P_k^{n-s} = \sum h_{jk}^{*s} h_{jk}^{n-s}$, where $\text{extr} =$ k $\epsilon \lambda \mathbf{u} \mathbf{u}_k$ $-\sum_{j} n_{j}$ $ext{rP}_k^n = \sum_{j=1}^k h_{jk}^n$, $ext{rP}_k^{*s} \cdot ext{rP}_k^{n-s} = \sum_{j=1}^k h_{jk}^{*s} h_{jk}^{n-s}$, where $ext{r} = (max \text{ at n>s, and } n \leq k$ 1 *s $_{\text{ext}}P^{n-s} = \sum k^{*s} h^{n-s}$ where $_{\text{ext}}(r)$ 1 , $extrP_k^{*}$ \cdot $extrP_k^{*}$ \cdot $extrP_k^{n-s}$ = $\sum h_{ik}^{*}$ h_{ik}^{n-s} , where $extr=(max \ at \ n>s,$ and min at n<s). The first formula is characterized probability of particles detection, and the second is interference picture. Using \sim - transformation $h_{ik+1} = x h_{ik}, h_{ik+1} = y e x t P_k$, $P_{ik+1} = z e x t P_k$ we are easily established $\sum_{k=1}^{N}$ = z extr P_k we are easily established connections by conditions for various particles. Here (x, y, z) is some solutions of the equation $x^n + y^n = z^n$.

Example 5. *Some properties of* \sim *- function and it application.* Let's consider so-called \sim -function:

$$
-(\Gamma) = \left(\Gamma x^S + \left(1 - \Gamma \frac{n}{n-s}\right)^{1/s} y^S\right)^{1/s}, \quad s > 0,
$$

where *, y* - positive numbers, *n*, *s* - natural numbers, $0 < r < 1$.

Properties 1. Function ∼(r), 0 < r < 1 in a point r * =
$$
\left[\frac{x^n}{x^n + y^n}\right]^{\frac{n-s}{n}}
$$
 at $s < n$ has maximal, and at $s > n$ minimal values equal $z = \left(x^n + y^n\right)^{\frac{1}{n}}$,

i.e. $x^n + y^n = z^n$, where $z = \max_{x \in \mathcal{X}} z(x)$ at $s < n$ and $z = \min_{x \in \mathcal{X}} z(x)$ $0 < r < 1$ 0 $< r < 1$ *z* = max \sim (*x*) at s < *n* and *z* = min \sim (*x*) at s > $<$ r $<$ 1 0 $<$ r $<$ 1 0 $<$ r $<$ 1 $=$ max $\sim(x)$ at $s < n$ and $z =$ min $\sim(x)$ at $s > n$. $0 < r < 1$ $z = \min$ $\sim(x)$ at $s > n$. $\langle r \rangle$ $\langle 1 \rangle$ $=$ min $\sim(x)$ at $s > n$.

Really, as

$$
\frac{d}{dr} = \frac{1}{s} \left[r x^s + \left(1 - r \frac{n}{n-s} \right) \frac{n-s}{n} y^s \right]^{-1} \cdot \left[x^s - r \frac{s}{n-s} \left(1 - r \frac{n}{n-s} \right) - \frac{s}{n} y^s \right]
$$

and

$$
\frac{d^{2}_{\infty}}{dr^{2}} = \frac{1}{s} \left(\frac{1}{s} - 1 \right) \left[r x^{s} + \left(1 - r^{\frac{n}{n-s}} \right)^{\frac{n-s}{n}} \cdot y^{s} \right]^{1/2} \left[x^{s} - r^{\frac{s}{n-s}} \left(1 - r^{\frac{n}{n-s}} \right)^{-\frac{s}{n}} \cdot y^{s} \right]^{2}
$$

$$
- \frac{1}{s^{s}} \left[r^{\frac{s}{n-s}} - 1 \left(1 - r^{\frac{n}{n-s}} \right)^{-\frac{s}{n}} + \left(1 - r^{\frac{n}{n-s}} \right)^{-\frac{s}{n}} - 1 \cdot r^{\frac{2s}{n-s}} \right] \cdot y^{s}
$$

$$
\left[rx^{s} + \left(1 - r^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}} \cdot y^{s}\right]^{1-s}, \text{ that from a condition } \frac{d}{dr}\Big|_{r} = 0 \text{ we have}
$$

$$
r^* = \left[\frac{x^n}{x^n + y^n}\right]^{\frac{n-s}{n}}, 0 < r^* < 1. \text{ It is easy to see, that } \frac{d^2 z}{dr^2}\Big|_{r^*} < 0, \text{ at } s < n \text{ and}
$$
\n
$$
\left.\frac{d^2 z}{dr^2}\right|_{r^*} > 0, \text{ at } s > n. \text{ Hence, the point } r^*, 0 < r^* < 1 \text{ at } s < n \text{ is unique}
$$

point of a maximum of function $\tilde{ }$: $z =$ max $-(x) = -(r^*)$ and at $s < n$ is a $0 < r < 1$ max $\sim(x) = \sim(r^*)$ and at $s < n$ is a $r < 1$ $=$ \sim (r^{*}) and at s < n² is a $\langle r \rangle$ $\langle 1 \rangle$ *z* = max \sim (x) = \sim (r^*) and at $s < n$ is a unique point of a minimum of function $\tilde{ } : z = \min_{x \in X} f(x) = f(x^*)$. Besides $0 < r < 1$ $min \sim(x) = \sim(r^*)$. Besides $r < 1$ $=$ \sim (Γ *). Besides $\langle r \rangle$ $\langle 1 \$ $z = \min$ \sim $(x) = \sim$ $(r*)$. Besides $z = (x^n + y^n)^{n}$ and $x^n + y^n = z^n$. $\frac{1}{\sqrt{2}}$ $\int u \text{ and } x'' + y'' = z''$. $=\left(x^n + y^n\right)$ *n* and $x^n + y^n = z^n$.

Properties 2. If (x, y, z) is the decision of the equation $x^n + y^n = z^n$ at some n the point $r =$ *n* $n - s$ $x^n + y^n$ $\begin{array}{c}\n\frac{n-s}{n}\n\end{array}$
 x^{*n*} **dece** unique noint outnome of the \sim is a unique point α \int $\sqrt{2}$ $\left| \frac{n}{x^n + y^n} \right|$ is a diffuse $(x'' + y'')$ $\begin{pmatrix} n \end{pmatrix}$ $+ y^n$) $r = \frac{x}{\sqrt{2\pi}}$ is a unique point extreme of the functions \sim (r), 0 < r < 1.

Really, let (x, y, z) is the decision of the equation $x^n + y^n = z^n$. As $x^{n-s} \cdot x^s + y^{n-s}y^s - z^{n-s}z^s$, having entered designations

 $n-s$ *z*) we shall receive sy y^{n-s} $(y)^{n-s}$ z) $\langle z \rangle$ we shall $f(x)$ ^{n-s} (y)^{n-s} The contract of the contract of the $\frac{1}{2}$ we shall receive sys $\left(\frac{y}{x}\right)^{n-s}$ $\langle z \rangle$ we shall receive s $\begin{pmatrix} -s \\ s \end{pmatrix}$, $s = \left(\frac{y}{s}\right)^{n-s}$ $| , S = | \stackrel{y}{-} |$ $\left(\begin{array}{cc} z \end{array} \right)$ we shall $\left(\frac{x}{x}\right)^{n-s}$, $S = \left(\frac{y}{x}\right)^{n-s}$ (z) (z) we sh $\Gamma = \left(\frac{x}{2}\right)^{n-3}, S = \left(\frac{y}{2}\right)^{n-3}$, we shall receive system of the equation be relative (x, y, z):

$$
\begin{cases}\n\Gamma \ x^s + S \ y^s = z^s, & r + S > 1, r + S < 2 \\
x^s - \Gamma^{\frac{s}{n-s}} \ z^s = 0 \\
y^s - S^{\frac{s}{n-s}} \ z^s = 0\n\end{cases}
$$

Last system is relative (x^s, y^s, z^s) has the unique decision (it is a positive) as the determinant of system is equal to zero det = $r^{\frac{n}{n-s}} + s^{\frac{n}{n-s}} - 1 = 0$. From here *n* $\left(\frac{n}{n-s}\right)^{\frac{n-s}{n}}$ and from 1-st eq $\left\{\n\begin{array}{ccc}\n\text{and from 1-st} & \text{equa}\n\end{array}\n\right\}$ \sqrt{n} $\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ and none i- $\left(\begin{array}{cc} n \\ n \end{array}\right)$ $\left(\begin{array}{cc} n \\ n \end{array}\right)$ $\left(\begin{array}{cc} n \\ n \end{array}\right)$ $\left(\begin{array}{cc} n \\ n \end{array}\right)$ $s = |1 - r^{n-s}|$ and from 1-st equation of system we shall receive value *s* $\begin{array}{c|c} n & \downarrow s & \downarrow n \end{array}$ $n-s$ \sqrt{ss} $z = \left(r x^s + \left(1 - r^{\frac{n}{n-s}} \right)^{\frac{n}{n}} y^s \right)$ and function $z = \left(r x^s + r^s \right)$ $\frac{1}{2}$ $1-r^{n-s}$ y^{s} and function ~= \mathbf{r} \mathcal{L} \bigvee is a set of \bigwedge $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ $\left(\begin{array}{cc} & n-s \\ & \end{array}\right)^{1/s}$ $\left| \begin{array}{c} y \\ z \end{array} \right|$ and function \sim \sqrt{n} 1.6 \cdots $\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ on $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ and funct $=\left[rx^{s}+\left(1-r^{\frac{n}{n-s}}\right)^{\frac{n}{n}}y^{s}\right]$ and function \sim = $\left[rx^{s}\right]$ $rx^{s} + \left(1 - r^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}} y^{s}$ and function $\left(1 - r^{\frac{n-s}{n}}\right)^{\frac{n-s}{n}} y^{s}$. $\begin{array}{c} n \\ \mathbf{r} \end{array}$ $n-s$ \sqrt{ss} $n \downarrow v^s$ $n-s$ $\sqrt{\frac{n}{n}}$ $x^s + 1 - r^{-n}$ y^s . **The Community of the Community** \int $\Big\}$ /s $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ $\left(\begin{array}{cc} & n-s \\ & \end{array}\right)^{n-s}$ $\begin{array}{c|c|c|c|c} & y & \cdot & \cdot & \cdot \\ \hline \end{array}$ \sqrt{n} $\begin{bmatrix} 1 & -1 & \cdots & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 &$ $=\left[\left\lceil rx^{s}+\left(1-r^{\frac{n-s}{n}}\right)^{\frac{n}{n}}y^{s}\right\rceil\right]$. $-s$ $\sqrt{2s}$ $-s$ $\sqrt{\frac{n}{n}}$ \sim = $\left[\left\lceil \int r x^s + 1 - \int r x^b \right\rceil \right]$,

Properties 3. All decisions of the equation $x^2 + y^2$ $x^n + y^n = z^n$ are represented in the following parametrical formulas: $x = z \frac{1}{t^n}$, $y = z(1-t)^{\frac{1}{n}}$, $0 < t < 1$, (*) $x = z t^n$, $y = z(1-t)^n$, $0 < t < 1$ (*) $(*)$

where *t* and z any positive numbers, and $t = r^{\frac{n-s}{n}}$, $0 < r < 1$, $z = k^{\frac{n}{n}}$ $t = r^{\frac{s}{n-s}}$, $0 < r < 1$, $z = k^{\frac{1}{n}}$. Really, using $= k^{\overline{n}}$. Really, using presentations $r^{n-s} = \frac{x}{x^n + y^n}$, $s^{n-s} = \frac{y}{x^n + y^n}$ we have homogeneous system $\frac{n}{n-s}$ = $\frac{y^n}{s}$ we have homogeneous system *n* $x^n + y^n$, $x^n + y^n$ we have hold $\frac{n}{n-s}$ $\frac{x^n}{s}$ $\frac{n}{n-s}$ $\frac{y^n}{s}$ we have ho *n* $+ y^n$ $s = \frac{y^{n}}{n}$ we have homogeneous sy $x^n + y^n$ $r^{n-s} = \frac{x^{n}}{n}$, $s^{n-s} = \frac{y^{n}}{n}$ we have homogeneous system of the algebraic equations be relative (x^n, y^n) :

$$
\left[\left(1 - r^{\frac{n}{n-s}}\right)x^n + r^{\frac{n}{n-s}}y^n = 0\right] \text{ or } \left[\left(1 - r^{\frac{n}{n-s}}\right)x^n + r^{\frac{n}{n-s}}y^n = 0\right]
$$
\n
$$
-\frac{n}{n-s}x^n + \left(1 - \frac{n}{n-s}\right)y^n = 0
$$
\n
$$
-\left(1 - r^{\frac{n}{n-s}}\right)x^n + r^{\frac{n}{n-s}}y^n = 0
$$

As the determinant of system is equal to zero it has the not trivial decision , $y^n = |1 - r^{n-s}|$, $0 < r < 1$. By virtue of uniformity of sys \int $\sqrt{2}$ $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (n) $=$ Γ^{n-s} , y^{n} $=$ $\left|1-\Gamma^{n-s}\right|$, $0 < \Gamma < 1$. By virtue of uniformity of system *n*) $y^{n} =$ $\begin{vmatrix} 1 - r^{n} s \end{vmatrix}$, $0 < r < 1$. By virtu *n* (*n*) $x^n = r^{n-s}$, $y^n = \left(1 - r^{n-s}\right)$, $0 < r < 1$. By virtue of uniformity of system, its

decision are represented as $x^n = k r^{n-s}$, $y^n = k \left| 1 - r^{n-s} \right|$, $k = const.$. From *n* $\big)$ $n-s$, $y^n = k \left(1 - r^{n-s}\right)$, $k = const.$ From *n* (*n*) $x^n = k r^{n-s}$, $y^n = k \mid 1-r^{n-s} \mid$, $k = const.$. From \int $\sqrt{2}$ $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (n) $= k r^{n-s}$, $y^n = k \left| 1 - r^{n-s} \right|$, $k = const.$. From

here $x = k^{n} \sqrt{n-s}$, $y = k^{n} |1 - \sqrt{n-s}|$, $z = k^{n}$ and having ent $x = k^{\frac{1}{\gamma_n}} \Gamma^{\frac{1}{n-s}}$, $y = k^{\frac{1}{\gamma_n}} \left(1 - \Gamma^{\frac{n}{n-s}}\right)^{\gamma_n}$, $z = k^{\frac{1}{\gamma_n}}$ and having entered de $\frac{1}{2}$ $\frac{1}{2} \int_{0}^{\frac{\pi}{n-s}} y = k^{\frac{1}{2} \left(\frac{1}{n-s} \right)^{\frac{n}{n-s}}}$, $z = k^{\frac{1}{2} \left(\frac{1}{n} \right)}$ and having entered designal $\begin{matrix} \n\sqrt{n} & & & \n\end{matrix}$ $\begin{matrix} \n\sqrt{n} & & \n\end{matrix}$ $\begin{matrix} \n\sqrt{n} & & \n\end{matrix}$ $\begin{matrix} \n\sqrt{n} & & \n\end{matrix}$ $\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, $\lambda = k$ and 1 $a = k^{\frac{1}{n}} \Gamma^{\frac{1}{n-s}}$, $y = k^{\frac{1}{n}} \left(1 - \Gamma^{\frac{n}{n-s}}\right)^{\frac{1}{n}}$, $z = k^{\frac{1}{n}}$ and having entered designations

 $n-s$ all decisions of t^2 *s* $t = \int_0^{\sqrt{n-s}}$ all decisions of the equation $x^n + y^n = z^n$ we shall copy as (*).

The remark*. For any decisions (*) are fair estimations*

1). $2z^{s} < x^{s} + y^{s} < 2 \frac{1-\frac{1}{n}}{n}z^{s}, \quad s < n,$
 2 $2 \frac{1-\frac{1}{n}}{n}z^{s} < x^{s} + y^{s} < z^{s},$ *s s s* $z^{s} < x^{s} + y^{s} < 2$ $n \, z^{s}$, $s < n$, 2). 2 $n \, z^{s} < x^{s} + y^{s} < z^{s}$, *i* $2x^{s} + y^{s} < 2$ $\frac{1-\frac{s}{n}}{z^{s}}, \quad s < n,$ 2). $2 \frac{1-\frac{s}{n}}{z^{s}} < x^{s} + y^{s} < z^{s}, \quad n > s, \text{ and}$ *s* $\langle x^s + y^s \rangle \langle z^s, n \rangle$ and $-\frac{1}{2}$ *and also for them takes place formulas:*

$$
\int_0^1 \frac{x^s}{y^s} dt = \frac{\left(\frac{s}{n}\right)f}{\sin\left(\frac{s}{n}\right)f} \qquad \qquad \int_0^1 \frac{y^s}{x^s} dt = \frac{\left(\frac{s}{n}\right)f}{\sin\left(\frac{s}{n}\right)f}
$$

Corollary*. At n > 2 decisions (*) are not the whole positive numbers. It is necessary to note, that at n=2 decisions (*) can be integers and no integers. For example, all decisions of type (*) at* 2 $(2+1)$ are magnetic managers and me $2j$ \int and integers is much an and t are integer s numbers d $\begin{pmatrix} 2 & 1 & 1 \ 1 & 1 & 1 \end{pmatrix}$ $\left(\overline{j^2+1}\right)$ are integer s num $\left(\begin{array}{c}2j\end{array}\right)^2\qquad,\qquad\qquad$ $+1$) and anogen strategies and anogen- $=$ $\frac{2J}{r^2}$ are integer's numbers and j^2+1 $\left[\right]$ \cdots $\left[\right]$ \cdots $\left[\right]$ $t = \left(\frac{2j}{n^2}\right)$ are integer's numbers and they are represented as: $x = j$, $y = \frac{j^2 - 1}{2}$, $z = \frac{j^2 + 1}{2}$ at odd j and $x = j$, $y = \frac{j^2 - 1}{2}$, $z = \frac{j^2 + 1}{2}$ *at odd j and* \int at even j, $j=$ (j^2+1) at even j, j= $\Big), z = (j^2 + 1)$ at even j, $j=1,2,3,4, ...$ $x = 2j$ $y = (j^2 - 1), z = (j^2 + 1)$ at even *j*, $j=1,2,3,4, ...$

2. Some examples of Numbers Tree

Now we shall consider now examples of Numbers Tree for different numbers.

1). Let $N = 25$, $n = 2$, then we have

Fig.2. Model of Numbers Tree for $N = 25$, $n = 2$

k) - means quantity (amount) of an element of the given top of a tree, and number on edges paw decomposition. From here follows, that representation (4) takes the following kind:

$$
25^2 = 9^2 + 2 \cdot 12^2 + 16^2.
$$

2). Now we shall consider number $N = 50$.

Fig.3. Model of Numbers Tree for $N = 50(50^2 - 18^2 + 2 \cdot 24^2 + 32^2)$

Fig.4. Model of Numbers Tree for $N = 75$ $(75^2 = 27^2 + 2.36^2 + 42^2)$

5). $N = 125$.

Fig.6. Model of Numbers Tree for $N=125$, $125^2 = 27^2 + 3 \cdot 36^2 + 3 \cdot 48^2 + 64^2$

Fig.7. Model of Numbers Tree for $N = 625$, $n = 2$

 $(625^2 - 81^2 + 4 \cdot 108^2 + 6 \cdot 144^2 + 4 \cdot 192^2 + 256^2)$

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7). Similarly, for $N = 3125$ we shall receive representation

 $3125^2 = 243^2 + 4.324^2 + 372^2 + 8.432^2 + 496^2 + 9.576^2 + 5.768^2 + 1024^2$
For cases when $n > 2$ not always it is possible to construct graceful examples. We shall result one example. Let $N = 35$, $n = 3$, then

Fig.9

From the given example follows, that we have not received a tree, but only it(him) We shall name such "trees" not growing trees. For growth of such tree it is necessary to make "cuttings" from another (for example) $n = 2$ a tree and to insert them into not growing trees. For example

Fig.10. $98 = 1^4 + 2^4 + 1^2 + 4^2 + 8^2$.

4. Economical interpretation, structure of a tree of solutions. The tree of decisions represents one of ways of splitting of set of the data on classes or categories. The root of a tree implicitly contains all classified data, and leaves - the certain classes after performance of classification. Intermediate units of a tree represent items (points) of decision making on a choice or performance of testing procedures with attributes of elements of the data which serve for the further division of the data in this unit. Usually] the tree of decisions is determined as structure which consists from Units - leaves, each of which represents the certain

class; Units of acceptance of decisions, the certain test procedures which should be executed in relation to one of values of attributes; the unit of acceptance of decisions is left with branches which quantity corresponds to quantity (amount) of possible(probable) outcomes of testing procedure. It is possible to consider a tree of decisions and from other point of view: intermediate units of a tree correspond to attributes of classified objects, and arches - to possible alternative values of these attributes. The example of a tree is submitted on figure. On this tree intermediate units represent attributes supervision, humidity, is windy. Leaves of a tree are marked by one of two classes P or . It is possible to count, that P corresponds to a class of positive copies consultation, and - to a class negative. For example, P can represent a class " to leave on walk ", and - the class " to sit at home ". Though it is obvious, that the tree of decisions is the way of representation which is distinct from inducing rules, the tree can compare the certain rule of classification which gives for each object having the appropriate set of attributes it is submitted by set of intermediate units of a tree), the decision to what from classes to attribute (relate) this object (a set of classes is submitted by set of values of leaves of a tree). In the given example the rule will carry objects to class P or .

References

- [1]. Akaev A. A. 2009. Modern financial and economic crisis in a view of the theory of innovative-technological progress of economy and management of innovative process. System monitoring: Global and regional Progress. D. A. Halturina and Century Korotayev. . 141–162.
- [2]. I. S. Malkov. 2010. Nonlinear Dynamics of the Nonlinear World. www.ya.ru.
- [3]. Ajvazjan S. A., V. M. Buhshtaber, I. S. Enjukov, L. D. Meshalkin. 1989. Applied statistics: classification and decrease (reduction) of dimension. - M.: The Finance and Statistics.
- [4]. Zhuravlev J. I., V.V. Rjazanov, Sen'ko O.V. 2006. "Recognition". Mathematical Methods. Programsystem. Practical Applications. - M.: The Phase. ISBN 5-7036-0108.
- [5]. Zagoruiko N. 1999. Applied methods of the analysis of the data and knowledge. - Novosibirsk: HIM(IT) the Siberian Branch of the Russian Academy of Science, ISBN 5-86134-060-9
- [6]. Hastie T., R. Tibshirani, J. Friedman. 2001. The Elements of Statistical Learning. - Springer. ISBN 0-387-95284.
- [7]. Vagin V.N., E. J. Golovina, A.A Zagorjanskaja, M. V. Fomina. 2004. Authentic and Plausible Conclusion in Intellectual Systems // Under V. N. Vagina's edition, D. A. Pospelova. - M.: Fizmatlit.704 p.
- [8]. Gelovani V.A., A. A. Bashlykov, V. B. Britkov, E. D. Vyazilov. 2001. Intellectual of system of support of acceptance of decisions in supernumerary situations with use of the information on a condition of natural //- .:Editorial of environment. 304p.
- [9]. Aamodt A., E. Plaza. 1994. Case-based reasoning: foundational issues, methodological variations, and system approaches // AI Communications. IOS Press. Vol. 7: 1. P. 39-59.
- [10]. Bashmakov A.I., 2005. I.A. Intellectual's Boots information technologies: Studies. The grant //-M.:MGTU. N.E. Baumana. 304 .
- [11]. Varshavskij P.R., A. P. Eremeev. 2006. The method of plausible reasoning on the basis of analogies and precedents for intellectual systems of support of acceptance of decisions // News of the artificial Intellect, 3, p.39-62.
- [12]. Warsaw P.R. 2006. Realization of a method of plausible reasoning on the basis of precedents for intellectual systems support of acceptance of decisions // Works of the tenth national conference on II with international participation - In 3 . .1. - M.: Fizmatlit. p. 303-311.
- [13]. Eremeev A., P. Varshavsky. 2007. Methods and Tools for Reasoning by Analogy in Intelligent Decision Support Systems // Proc. of the International Conference on Dependability of Computer Systems. Szklarska Poreba, Poland, 14-16 June, 2007, IEEE, P.161-168.
- [14]. M. Yunusi. 2009. Model economics. TNU. 129p. (www.yunusi.tj).
- [13]. M. Yunusi. 2005. Some Hypothetical Models of Real Spaces and the Phenomena Occurring in them. The Bulletin of National University, №3, a series of natural sciences. c.40-43. (ISSN 2074-1847).
- [15]. M. Yunusi. 2005. About the Equations with Extreme Properties and their Applications. Journal of Vestnik National University, series of Mathematics, № 2. p.168-177. (ISSN 2074-1847).
- [16]. M. Yunusi. 2009. Modeling of Numbers Tree and its Applications. Proceeding of International Conference on Mathematics and Information Security, p.33, p.55, Sohag, Egypt. 13-15 November 2009. http://www.icaimis2009.tk / speakers/).
- [17]. M. Yunusi. 2004. About One Class of the Modeling Equations with Extreme Property. Journal of Vestnik National University, series Mathematics, 1, p.128 -135 (ISSN 2074-1847).
- [18]. Yunusi, M. 2010. The Book Abstracts, ICM 2010, Hyderabad, India. p. 368.
- [19]. Yunusi, M. 1998. The Book Abstracts, ICM 1998, Berlin, Germany.
- [20]. Yunusi, M. 2002. The Book Abstracts, ICM 2002, Beijing, Chine, p. 385.
- [21]. Yunusi, M. 1999. The Book Abstracts, Edinburgh, p.330.
- [22]. Yunusi, M. 2000. About solutions of the equations $\sum_{j=1}^{m} X_j^n = Z^n$. Vestnik National University, № 4, . 3-8. (ISSN 2074-1847).
- [23]. Yunusi, M. 2008. Investigation of Some Nonlinear Singular Model Ecosystems and New Concerned Mathematical Problems. J. Ecological Modeling, Volume 216, Issue 2, 24 August 2008. p.172-177**.**
- [24]. Yunusi, M. 2001. Introduction to Model of Economics. Dushanbe, $-37p$.
- [25]. Yunusi, M.K., F.M. Yunusi and M.M. Yunusi. 2009. About The Best Model Production and Global Economy Connected to Them. Central Asia Journal of Information Technology. CAJIT, P. 131-137.
- [26]. Yunusi, M.K., F.M. Yunusi and M.K.Yunusi. 2009. Power model of growth of the population. Vestnik National University (Sci. Journal), 1(49), *P. 73-77.*
- (ISSN 2074-1847).
- [27]. Yunusi, M. 2012. Differential Equations with Extreme Properties. Vestnik National University (Sci. Journal), 1.1(77), P.3-12. (ISSN 2074-1847).
- [28]. Yunusi, M.K., J. Ganiev, S. Odinaeva. 2012. Mathematical Queries of an assessment Population Number. Vestnik National University (Sci. Journal), 3(85), P.3-15. (ISSN 2074-1847).
- [29]. Yunusi M. 2013. Polynomial representations of differential equations with extreme properties. Modern problems of function theory and differential equations. Mater.mezhd.nauch.konf.posv. 85 anniversary akad. AN RT L.G. Mikhailov. Dushanbe, p.160-162.
- [30]. Yunusi, M. 2012. Computational analysis of heat conduction in "pipeline ground" system and some corresponding models in extreme regime. In the

book « Heat-Physics researches and measurements in Energy and system at the control and quality management, production and services, Materials for International Heat-Physical Schools, 60 Years Celebration of Cafarova M.M., 8-13 October 2012, p.212-228.

- [31]. Yunusi M. 2012. Theory of differential equations with Extreme Properties and its Applications in Physics, Ecology and Economics. "International Science Conference" «Problem of Differential Equations» for 80 years celebrations of Juraeva A., Dushanbe, 2012, 108-110.
- [32]. Yunusi M. 2012. About Extreme and Algebraic Representations of Processes Describing Numbers Tree Models. $13th$ International Pure Mathematics Conference . (Abstracts), 1-3 September.2012, Islamabad, p.45.
- [33]. Yunusi M. 2012. Representations of Some Physical Processes in Extreme Regimes. Materials of National Conference, Modern Problems of Physics, for 70-years Celebrations of Tajik Professor, Boboeva T.B., Dushanbe, .33-34.
- [34]. Yunusi M. 2010. Models of Development of Losses in the Worst Condition by Kinds with Long Settlement - Modification Method of the Nearest Neighbour. International Congress Actuaries, March 7-12, Cape- Town, SA, 2010, p.23.
- [35]. Yunusi M*.* 2010. The formula: a tree of numbers for both its application in protection of the information and the analysis complex systems. Vestnik of National University, Special release is devoted to year of education and technical knowledge, .21-31. (ISSN 2074-1847).
- [36]. Yunusi M. 2011. Some Questions of Economic Crisis's and their Managements. Vestnik of National University, 6(70), the special issue is devoted to the 20 anniversary of Independence Day of Republic Tajikistan, c.3-11. (ISSN 2074-1847).
- [37]. Yunusi M. 2011. About Regulations of Unstable Structures of Regional Reserves Connected with Models of Protection of Rare and Disappearing Types. Vestnik of National University, 6(70), the special issue is devoted to the 20th anniversary of Independence Day of Republic Tajikistan, .11-17. (ISSN 2074-1847).
- [38]. Yunusi, M. 2012. Differential Equations with Extreme Properties. Vestnik Tajik National University, Nature Sci. 1(77), c.3-12. (ISSN 2074-1847).
- [39]. Yunusi M. 2001. The Model of inter-country's relations. Vestnik of Tajik State National University, N3, 2001, p.3-9. (ISSN 2074-1847).
- [40]. Yunusi M. 2000. Tajikistan by 2000 and Some Integration Questions: Designing and Modeling of Global Economy. Reprint of $8th$ International Congress of PWPA Seoul, Korea.18p.
- [41]. Yunusi M. 2000. Tajikistan The Book: Globalization of Economy. Proceeding of 8th International Congress of PWPA. Seoul, Korea, February p.10-14.
- [42]. Yunusi M. 2013. Theorem on representation of complex objects described by differential equations. Vestnik National University, 1/1 (102), p. 3-12. (ISSN 2074-1847).
- [43]. Yunusi M., Rizoev C. 2013. Optimal control of hydropower reservoirs. Vestnik National University, p.30-33. (ISSN 2074-1847).
- [44]. Yunusi M., Ganiev Ch., Odinaeva S.A. Mathematical problems in estimating population abundance. Vestnik of the Tajik National University. 1/3 (85), Ser. naturally. Sciences, Dushanbe, Sino, 2012. p. 3-19. (ISSN 2074-1847).
- [45].Yunusi M., R. Odinaev. Investigations mathematical models of plant protection. Dushanbe, L.t.d., "Sarmad company", 2013, - 110c. (monograph).
- [46]. Yunusi M., S. Odinaeva. Investigations in mathematical modelling problems
- of protection and population estimates of ecological systems. Dushanbe, L.t.d.., "Sarmad company", 2013, - 136c. (monograph).