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A Cautious Note on Natural Hedging of Longevity Risk

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Longevity risk

$\Downarrow$

Policyholders' future realized mortality rates

$\Downarrow$

Life insurers' liabilities

Approaches to protecting against longevity risk:

- Stochastic mortality forecasting models
- Externally $\longrightarrow$ Mortality-linked securities
- Internally $\longrightarrow$ natural hedging
  - life insurances $\leftrightarrow$ annuities

Literature:

- Cox and Lin (2007): Companies selling both life and annuity products charge cheaper prices $\Rightarrow$ evidence of natural hedging
- Wetzel and Zwiesler (2008): Portfolio composition significantly impacts longevity exposure
- Tsai et al. (2010): Optimal product mix to minimize *CVaR*

Introduction

Background & Literature Review

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Contributions

Underlying mortality forecasting models:

- Existing literature:
  - (Low-dimensional) factor models: Lee-Carter model (Lee and Carter (1992)), CBD model (Cairns et al. (2006))
  - ⇒ Error term σ_t affects time-t mortality rates at different ages simultaneously
  - ⇒ Cannot capture disparate shifts in mortality rates at different ages
  - Life insurances (working class) ⇔ annuities (retirees)
  - ? Positive conclusions of natural hedging
- This paper:
  - Parametric factor model & non-parametric mortality model
  - ⇒ Natural way to test natural hedging

Main findings:

- Using factor models helps to create a perfect hedge for mortality risk by utilizing natural hedging
- **BUT:** Different result from non-parametric mortality model

⇒ Natural hedging might not be as effective as we think

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Mortality Forecasting Models

Non-Parametric Model

Forward survival probabilities:

$$ {}_{\tau}p_x(t)\, \mathbf{1}_{\{\Upsilon_{x-t}>t\}} = \mathbb{E}^{\mathbb{P}}\left[\mathbf{1}_{\{\Upsilon_{x-t}>t+\tau\}} \middle| \mathcal{F}_t \vee \{\Upsilon_{x-t} > t\}\right],\ 0 \le T \le t \le T+\tau $$

Generational survival data ${}_{\tau}p_x(t_j)$: $j = 1, \ldots, N$

$$ F\left(t_j, t_{j+1}, (\tau, x)\right) = -\log\left\{ \frac{{}_{\tau+1}p_x(t_{j+1})}{{}_{\tau}p_x(t_{j+1})} \Big/ \frac{{}_{\tau+1+t_{j+1}-t_j}p_{x-t_{j+1}+t_j}(t_j)}{{}_{\tau+t_{j+1}-t_j}p_{x-t_{j+1}+t_j}(t_j)} \right\} $$

- $\bar{F}(t_j, t_{j+1}) = \left(F(t_j, t_{j+1}, (\tau, x))\right)_{(\tau,x)\in\tilde{\mathcal{C}}}$, $j = 1, 2, \ldots, N-1$

$\Rightarrow$ $\bar{F}(t_j, t_{j+1})$ are i.i.d. Gaussian distributed (Prop. 2.1, Zhu and Bauer (2013))

$\Rightarrow$ Simulate $\bar{F}(t_N, t_{N+1})$ based on sample mean and covariance matrix from $F\left(t_j, t_{j+1}, (\tau, x)\right)$, $j = 1, \ldots, N-1$

$\Rightarrow$ ${}_{\tau}p_x(t_{N+1})$

Parametric Factor Model

Forward force of mortality (easier to model/work with than ${}_\tau p_x(t)$):

$$\mu_t(\tau, x) = -\frac{\partial}{\partial \tau} \log \{{}_\tau p_x(t)\}$$

Consider **time-homogenous diffusion-driven** models (cf. Bauer et al. (2012))

$$d\mu_t = (A\,\mu_t + \alpha)\; dt + \sigma\, dW_t$$

- Drift condition (Cairns et al. (2006, ASTIN)): With W_t Brownian motion under $\mathbb{P}$,

$$\alpha(\tau, x) = \sigma(\tau, x) \times \int_0^\tau \sigma'(s, x)\, ds$$

- Bauer et al. (2012): μ_t allows for a Gaussian finite-dimensional realization (FDR) iff

$$\sigma(\tau, x) = C(x + \tau) \times \exp\{M\tau\} \times N$$

- Zhu and Bauer (2013):

$$\sigma(\tau, x) = (k + c\, e^{d(x+\tau)})\,(a + \tau)\, e^{-b\tau}$$

Economic Capital for a Stylized Insurer

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Economic Capital Calculation

- Newly founded life insurer selling traditional products (term-life, endowment, annuity); Equivalence Principle; risk-neutral w.r.t. mortality risk
- Available Capital at time zero: $AC_0 = E$
- Available Capital at time one: $AC_1 = \mathbb{E}^{\mathbb{Q}}[Assets|\mathcal{F}_1] - \mathbb{E}^{\mathbb{Q}}[Liabilities|\mathcal{F}_1]$
- One-year mark-to-market approach for calculating **Economic Capital**:

$$EC = \rho\left(\underbrace{AC_0 - AC_1\, p(0,1)}_{L}\right)$$

- ρ: monetary risk measure ($L^2(\Omega, \mathcal{F}, \mathbb{P}) \to \mathbb{R}$)
  - *Solvency Capital Requirement* (Solvency II):

$$\text{EC} = \text{SCR} = \text{VaR}_\alpha(L) = \arg\min_x\{\mathbb{P}(L > x) \leq 1 - \alpha\}$$

  - *Conditional Tail Expectation* (used within SST):

$$\text{EC} = \text{CTE}_\alpha = \mathbb{E}\left[L | L \geq \text{VaR}_\alpha(L)\right]$$

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Mortality estimation:

- U.S. female data (Human Mortality Database), year 1933-2007
- 46 generational life tables: 1963-2008, age: 0-100 $\mapsto {}_{\tau}p_x(t_j)$, $j = 1, \ldots, 46$
- Calibrate and forecast under:
  0. Deterministic mortality (Lee-Carter)
  1. Non-parametric model
  2. Parametric factor model

Financial market estimation:

- Financial portfolio: stock, 5-year, 10-year, and 20-year gov. bond
- Financial market model: Extended Black-Scholes model with stochastic interest rates (Vasicek model)
- Calibrated to U.S. data from 01-1982 to 07-2012 using Kalman filter

50,000 simulations of A_1 and $V_1 \Rightarrow AC_1 \Rightarrow EC$

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Implementation

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Economic Capital for a Stylized Insurer

Base Case

Duration match with financial portfolio; $E = \$20,000,000$

	x	i	$n_{x,i}^{\text{term/end/ann}}$	$B_{\text{term/end/ann}}$
Term Life				
	30	20	2,500	\$100,000
	35	15	2,500	\$100,000
	40	10	2,500	\$100,000
	45	5	2,500	\$100,000
Endowment				
	40	20	5,000	\$50,000
	45	15	5,000	\$50,000
	50	10	5,000	\$50,000
Annuities				
	60	(40)	2,500	\$18,000
	70	(30)	2,500	\$18,000

Economic capital:

	Deterministic Mortality	Factor Model	Non-parametric Model
95% VaR			
	\$60,797,835	\$61,585,667	\$62,802,167

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Financial Risk Hedging

Optimal static hedge:

- Minimizing economic capital by changing weights in bonds/stock

Economic capital:

95% VaR	Deterministic Mortality	Factor Model	Non-parametric Model
	$3,201,921	$9,871,987	$10,049,401

Optimal weights:

	Deterministic Mortality	Factor Model	Non-Parametric Model
Stock	0.2%	1.5%	0.9%
5-year Bond	2.5%	0.1%	0.5%
10-year Bond	87.3%	88.0%	90.8%
20-year Bond	10.0%	10.4%	7.8%

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Natural Hedging of Longevity Risk

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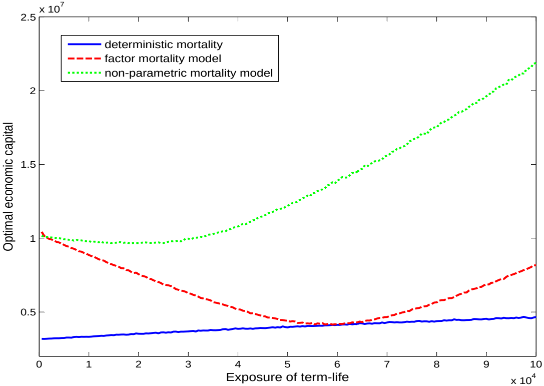
Optimal static hedge:

- Exposure in annuity/endowment ⇒ fixed
- Adjust exposure in term-life insurance n^{term}:
  - Minimize capital with optimizing financial risk
- Three cases: deterministic mortality vs. factor mortality model vs. non-parametric model



Natural Hedging of Longevity Risk

Observations

- Without systematic mortality, EC increases in n^{term}
- With factor mortality model, EC convex of n^{term} ($n^{term*} = 60,000$)
- BUT With non-parametric forecasting model, only very mild effect of natural hedging

Economic capital: ($n^{term} = 60,000$)

	Deterministic Mortality	Factor Model	Non-parametric Model
95% VaR	$4,128,345	$4,165,973	$13,872,739

- Using the factor mortality model, adding mortality risk increases the optimal economic capital slightly
  - ? (Almost) perfect hedge of mortality risk with natural hedging
- Using the non-parametric mortality model, adding mortality risk increases the optimal economic capital considerably
  - ⇒ Natural hedging does not work as well as we expect
  - ▶ Factor models too simplified

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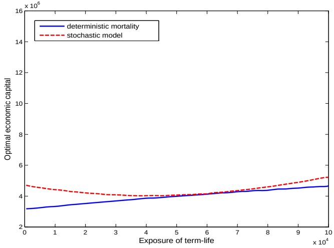
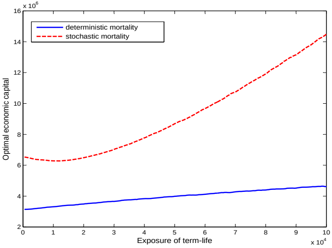
Alternative Mortality Models

- Repeat the calculations for alternative mortality models
  - Stochastic Lee-Carter model (one-factor model)
  - Non-parametric bootstrapping model from Li and Ng (2010, JRI)
- U-shape EC curve for the Lee-Carter model $\rightarrow$ highly effective natural hedging ($n^{term*} = 60,000$)
- Mild effect of natural hedging from the Li&Ng model



(a) Lee-Carter model



(b) Li&Ng model

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Natural hedging proposed to handle longevity risk

- Positive results from existing literature
  - Use factor mortality models
  - Neglect disparate mortality evolutions under different ages
  - Entail potential biases
- We compare results derived from both parametric factor and non-parametric stochastic mortality model
  - Concur the existing literature when the factor model used
  - With non-parametric model, natural hedging much less effective

How much should we trust model-based results?

- Advantages: simple, easy to use, etc.
- CAVEAT: important features might be stripped

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