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Optimal Relativities and Transition Rules of a Bonus-Malus System

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Agenda

- Bonus-Malus System (BMS)
- Motivation
- ➢ 3 implications; 2 inadequacy scenarios
- Optimal relativities
- Effectiveness of transition rules
- Varying transition rules
- Numerical illustrations
- Future research

Bonus-Malus System (BMS)

- BMS levels
- Transition rules
  - no claim -> bonus ($\downarrow$ BMS levels)
  - claim -> malus ($\uparrow$ BMS levels)
- Relativities

  = premium adjustment coefficients

Motivation

Taylor (1997):

- justifiable BMS relativities need to recognize the differentiation of underlying claim frequencies by experience, but only to the extent that this differentiation has not been recognized within the base premiums.

Motivation

Implication 1:

deal with heterogeneity within each risk class but not the heterogeneity between different risk classes

- ➢ modeling of unobserved heterogeneity (see, e.g., Lemaire, 1995; Denuit et al., 2007)

Motivation

Implication 2:

a priori information should be incorporated into the determination of optimal relativity

- ➢ Taylor (1997): simulation approach
- ➢ Pitrebois et al. (2003): analytical formula

Motivation

Implication 3:

the average *a priori* expected claim frequencies of BMS levels should exhibit as little variations as possible

- ➢ not addressed in previous studies

Motivation

Inadequacy scenario 1:

Identical optimal relativities regardless of *a priori* expected claim frequencies

- Pitrebois et al. (2003): 2 sets of optimal relativities for urban/rural drivers
- the heterogeneity between different risk classes are dealt with separately, contradictory with implication 1

Motivation

Inadequacy scenario 2:

Identical transition rules regardless of current levels occupied

Optimal relativities

Θ = unobserved heterogeneity (true relative premium)
r_L = relativity for level L (actual relative premium)
Λ = unknown *a priori* expected claim frequency
$\bar{\lambda}$ = average expected claim frequency

Optimal relativities

Norberg (1976): without *a priori* classification

$$\min \mathbb{E}\left[\left(\bar{\lambda}\Theta - \bar{\lambda}r_L\right)^2\right] \equiv \min \mathbb{E}[(\Theta - r_L)^2]$$

Pitrebois et al. (2003): analytical formula

$$\min \mathbb{E}[(\Theta - r_L)^2]$$

- the minimization of expected squared difference between true relative premium and actual relative premium does not take into account the base premiums

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Optimal relativities

Our new objective function

$$\min \mathbb{E}[(\Lambda\Theta - \Lambda r_L)^2]$$

- by incorporating the amount of base premiums, the obtained solution would partially address inadequacy scenario 1.

Coene and Doray (1996):

financial equilibrium constraint $E(r_L) = 1$

Optimal relativities

Langrangian method:

$$\mathcal{L}(r,\alpha) = \mathbb{E}\left[(\Lambda\Theta - \Lambda r_L)^2\right] + \alpha(E(r_L) - 1)$$

Solution:

$$\alpha = \frac{\left(\sum_{\ell=1}^{j} \frac{\mathbb{E}\left[\Lambda^2\Theta \middle| L=\ell\right] \Pr(L=\ell)}{\mathbb{E}\left[\Lambda^2 \middle| L=\ell\right]}\right) - 1}{\sum_{\ell=1}^{j} \frac{Pr(L=\ell)}{2\mathbb{E}\left[\Lambda^2 \middle| L=\ell\right]}}$$

$$r_\ell = \frac{\mathbb{E}\left[\Lambda^2\Theta \middle| L=\ell\right]}{\mathbb{E}\left[\Lambda^2 \middle| L=\ell\right]} - \frac{\alpha}{2\mathbb{E}\left[\Lambda^2 \middle| L=\ell\right]}$$

Effectiveness of transition rules

Pitrebois et al. (2003): interaction of *a priori* and *a posteriori* ratings

$$\mathbb{E}[\Lambda|L=\ell]$$

Our contribution:

$$\mathbb{V}[\Lambda]=\mathbb{E}\big[\mathbb{V}[\Lambda|L]\big]+\mathbb{V}\big[\mathbb{E}[\Lambda|L]\big]$$

Implication 3 seek to minimize the 2nd component / maximize the 1st component

$$\text{effectiveness}, \tau_{\text{rule}}=\frac{\mathbb{E}\big[\mathbb{V}[\Lambda|L]\big]}{\mathbb{V}[\Lambda]}$$

Varying transition rules

To address inadequacy scenario 2, introduce varying rules dependent on the current level occupied.

Effective level transition for drivers staying in level ℓ and make k claims in current year = $t_{\ell,k}$

malus transition:

$$t_{\ell,k} \geq 0, t_{\ell_2,k} \leq t_{\ell_1,k} \text{ for } k \geq 1, \ell_2 \geq \ell_1$$

bonus transition:

$$t_{\ell,0} \leq 0, \left|t_{\ell_2,k}\right| \geq \left|t_{\ell_1,k}\right| \text{ for} \ell_2 \geq \ell_1$$

Numerical illustrations

$$t_{\ell,0} = \begin{cases} 0, & \text{for } \ell = 1 \\ -1, & \text{for } 2 \leq \ell \leq \left\lceil \frac{j}{2} \right\rceil + 1 \\ -2, & \text{for } \ell > \left\lceil \frac{j}{2} \right\rceil + 1 \end{cases}$$

$$t_{\ell,k} = \begin{cases} \min\left[j - \ell, \max\left[k, \left\lceil \frac{j-\ell}{p} \times k \right\rceil \right] \right], & \text{for } k \geq 1, \ell < j \\ 0, & \text{for } k \geq 1, \ell = j \end{cases}$$

TABLE 1

Transition rules for the simple rules of -1/+2 (varying rules with $p = 4$)

	Level occupied if				
Starting level	0	1	2	3	$\geq$4
	claims are reported				
9	8 (7)	9 (9)	9 (9)	9 (9)	9 (9)
8	7 (6)	9 (9)	9 (9)	9 (9)	9 (9)
7	6 (5)	9 (8)	9 (9)	9 (9)	9 (9)
6	5 (5)	8 (7)	9 (8)	9 (9)	9 (9)
5	4 (4)	7 (6)	9 (7)	9 (8)	9 (9)
4	3 (3)	6 (6)	8 (7)	9 (8)	9 (9)
3	2 (2)	5 (5)	7 (6)	9 (8)	9 (9)
2	1 (1)	4 (4)	6 (6)	8 (8)	9 (9)
1	1 (1)	3 (3)	5 (5)	7 (7)	9 (9)

TABLE 2

Transition rules for the simple rules of -1/+3 (varying rules with $p = 3$)

Starting level	Level occupied if 0 claims are reported	1	2	≥3
9	8 (7)	9 (9)	9 (9)	9 (9)
8	7 (6)	9 (9)	9 (9)	9 (9)
7	6 (5)	9 (8)	9 (9)	9 (9)
6	5 (5)	9 (7)	9 (8)	9 (9)
5	4 (4)	8 (7)	9 (8)	9 (9)
4	3 (3)	7 (6)	9 (8)	9 (9)
3	2 (2)	6 (5)	9 (7)	9 (9)
2	1 (1)	5 (5)	8 (7)	9 (9)
1	1 (1)	4 (4)	7 (7)	9 (9)

TABLE 3

Interaction between a priori and a posteriori ratemakings

	-1/+2		Varying $p = 4$		-1/+3		Varying $p = 3$	
level ℓ	$\Pr[L=\ell]$	$\mathbb{E}[\Lambda \vert L=\ell]$	$\Pr[L=\ell]$	$\mathbb{E}[\Lambda \vert L=\ell]$	$\Pr[L=\ell]$	$\mathbb{E}[\Lambda \vert L=\ell]$	$\Pr[L=\ell]$	$\mathbb{E}[\Lambda \vert L=\ell]$
9	2.87%	19.26%	0.70%	21.35%	5.06%	18.02%	1.19%	20.10%
8	2.35%	18.27%	0.54%	19.23%	4.09%	17.21%	1.12%	18.07%
7	2.31%	17.37%	1.26%	18.73%	4.17%	16.36%	2.83%	17.38%
6	2.31%	16.78%	2.34%	17.34%	3.94%	15.90%	2.14%	16.59%
5	3.20%	16.01%	4.24%	16.70%	3.72%	15.58%	5.51%	16.28%
4	3.37%	15.68%	4.17%	16.17%	7.29%	14.83%	9.18%	15.25%
3	8.09%	14.90%	8.97%	15.16%	6.20%	14.70%	7.57%	15.04%
2	6.84%	14.77%	7.44%	14.97%	5.33%	14.59%	6.34%	14.87%
1	68.66%	13.97%	70.34%	14.03%	60.21%	13.87%	64.13%	13.97%

TABLE 4

EFFECTIVENESS OF TRANSITION RULES

Transition rules	τ_{rule}
-1/+2	93.87%
varying $p = 4$	94.74%
-1/+3	94.56%
varying $p = 3$	95.18%

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TABLE 5

OPTIMAL RELATIVITIES FOR THE BMS

	-1/+2		Varying $p=4$		-1/+3		Varying $p=3$	
level ℓ	r_ℓ	$r_\ell^{\text{unconstrained}}$	r_ℓ	$r_\ell^{\text{unconstrained}}$	r_ℓ	$r_\ell^{\text{unconstrained}}$	r_ℓ	$r_\ell^{\text{unconstrained}}$
9	266.57%	261.07%	420.41%	419.11%	233.67%	227.73%	293.04%	288.85%
8	235.37%	229.27%	317.11%	315.62%	205.54%	199.03%	230.71%	225.56%
7	207.95%	201.22%	307.54%	305.95%	176.56%	169.34%	211.92%	206.36%
6	188.51%	181.29%	234.82%	233.11%	159.35%	151.70%	183.85%	177.74%
5	162.42%	154.47%	215.07%	213.20%	146.58%	138.59%	173.05%	166.69%
4	150.04%	141.74%	186.85%	184.91%	115.41%	106.55%	133.57%	126.30%
3	118.70%	109.49%	127.08%	125.08%	109.31%	100.29%	124.15%	116.67%
2	112.85%	103.46%	118.27%	116.22%	103.93%	94.75%	116.18%	108.51%
1	72.92%	62.35%	69.43%	67.10%	66.34%	56.11%	70.86%	62.08%
$\mathbb{E}[r_L]$	100%	90.20%	100%	97.79%	100%	90.68%	100%	91.90%

Future research

- how the varying extent of *a priori* classification affects the choice of sufficiently effective transition rules
- Gilde and Sundt (1989): linear relativities

Q&A

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