KU LEUVEN

Measuring Herd Behavior[1]

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[1] Dhaene, J., Linders, D. & Schoutens, W. (2014) 'Model-free measurement of implied herd behavior', Research report, AFI, FEB KU Leuven.

Outline

Introduction

The financial market

- The interest rate r is deterministic and constant.
- n individual stocks:
  - Price of stock i at time T is denoted by $X_i(T) \equiv X_i$ and $X_i > 0$.
  - Vanilla options with strike K and maturity T : $C_i\left[K\right]$ and $P_i\left[K\right]$.
  - Traded strikes $K_{i,j}$:

$$0 = K_{i,0} < K_{i,1} < \ldots < K_{i,m_i} < K_{i,m_i+1} = F_{X_i}^{-1}(1) < \infty.$$

- Stock market index:
  - $S = w_1 X_1 + \cdots + w_n X_n$.
  - Index options with strike K and maturity T : $C\left[K\right]$ and $P\left[K\right]$.
  - Traded strikes K_i:

$$K_{-l} < K_{-l+1} < \ldots < K_{-1} < K_0 \leq \mathbb{E}[S] < K_1 < \ldots < K_{h-1} < K_h.$$

Introduction

Herd behavior in stock markets

- Our goal is to define an index which
  - reflects the perception of the market about future level of co-movement between stock prices;
  - is based on option prices;
  - is model-free and forward-looking.
- We call this index the **Herd Behavior Index** (HIX)[2]:
  - HIX $\in (0, 1]$
  - HIX $= 1$ implies perfect herd behavior;
  - a higher value of the HIX indicates a stronger co-movement.

[2] Dhaene, Linders, Schoutens & Vyncke (2012)

Introduction
Herd behavior in stock markets

- The HIX is closely related to the **Implied Correlation**.
  - Both indices may give information about the level of diversification that is possible when investing in an equity portfolio.
  - The implied correlation is not *model-free*.
  - References: Skintzi & Refenes (2005), CBOE (2009) and Linders & Schoutens (2013).
- The HIX is closely related to the **Volatility Index** (VIX)
  - The HIX and the VIX may give information about the current level of fear in the market.
  - The VIX reflects the market's perception about the future level of *volatility*.
  - References: Whaley (2000), CBOE (2005) and Carr & Wu (2006).

Convex order

- Definition <u>convex order</u>:

$$X \preceq_{cx} Y \Leftrightarrow \begin{cases} \mathbb{E}[X] = \mathbb{E}[Y], \\ \mathbb{E}\left[(X-K)_+\right] \ \leq \mathbb{E}\left[(Y-K)_+\right], \text{ for all } K \in \mathbb{R}. \end{cases} \tag{1}$$

- If $X \preceq_{cx} Y$, the r.v. Y is *more variable* than the r.v. X.
- Assume that: $\mathbb{E}[X] \neq \mathbb{E}[Y]$
  - Compare the standardized r.v.'s $\frac{X}{\mathbb{E}[X]}$ and $\frac{Y}{\mathbb{E}[Y]}$:

$$\frac{X}{\mathbb{E}[X]} \preceq_{cx} \frac{Y}{\mathbb{E}[Y]} \Leftrightarrow \frac{\mathbb{E}\left[(X-k\mathbb{E}\left[X\right])_+\right]}{\mathbb{E}\left[X\right]} \ \leq \frac{\mathbb{E}\left[(Y-k\mathbb{E}\left[Y\right])_+\right]}{\mathbb{E}\left[Y\right]}, \forall k > 0.$$

The financial market

Traded index options and assumptions

- The market is arbitrage-free and there exists a pricing measure $\mathbb{Q}$.
  - Index option prices:

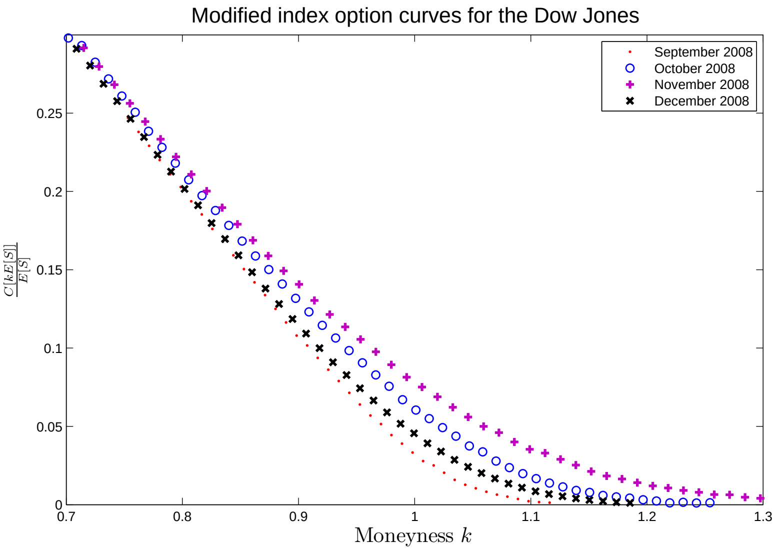
$$C[K] \quad = \quad \mathrm{e}^{-rT}\mathbb{E}_{\mathbb{Q}}\left[(S-K)_{+}\right].$$

  - The option curve C gives information about the risk-neutral index volatility.
- The forward deflated index price $= \frac{S}{\mathbb{E}[S]}$:

$$\mathrm{e}^{-rT}\mathbb{E}\left[\left(\frac{S}{\mathbb{E}[S]}-k\right)_{+}\right]=\frac{C[k\mathbb{E}[S]]}{\mathbb{E}[S]}, \quad \text{for } k \geq 0. \tag{2}$$

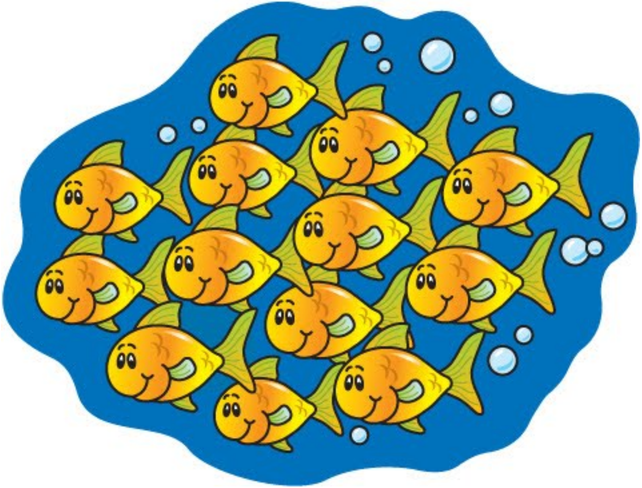
- The option curve of $\frac{S}{\mathbb{E}[S]}$ gives information about the variability of the index around its forward price.
  - k can be interpreted as the *forward moneyness* of the option.

Illustration: Convex order and index options



Comonotonicity

Perfect herd behavior = comonotonicity



Perfect herd behavior

- Perfect herd behavior = comonotonicity[3]:

$$(X_1, \ldots, X_n) \stackrel{\mathrm{d}}{=} \left(F_{X_1}^{-1}(U), \ldots, F_{X_n}^{-1}(U)\right)$$

  - U is a uniform $(0,1)$ r.v.
- Comonotonicity under equivalent measures:
  - $\mathbb{P}$ - world comonotonicity $\Leftrightarrow$ $\mathbb{Q}$ - world comonotonicity.
- Comonotonic stock prices are driven by a single factor.
  - Extreme positive dependence structure.

[3] Dhaene, Denuit, Goovaerts, Kaas & Vyncke (2002a,b)

Perfect herd behavior

The comonotonic market index

- Consider the vector $(X_1, \ldots, X_n)$ with risk-neutral cdf's F_{X_i}.
- The stock market index:

$$S = \sum_{i=1}^{n} w_i X_i$$

- The comonotonic stock market index:

$$S^c = \sum_{i=1}^{n} w_i F_{X_i}^{-1}(U)$$

- Comonotonic option prices:

$$\begin{aligned} C^c[K] &= \mathrm{e}^{-rT}\mathbb{E}\left[(S^c - K)_+\right], \\ P^c[K] &= \mathrm{e}^{-rT}\mathbb{E}\left[(K - S^c)_+\right]. \end{aligned}$$

- The risk-neutral cdf of the comonotonic stock market index can be determined if the marginal risk-neutral cdf's F_{X_i} are known.

Risk-neutral stock price distributions

- Vanilla option prices:

$$C_i[K] \quad = \quad \mathrm{e}^{-rT}\mathbb{E}_{\mathbb{Q}}\left[(X_i - K)_+\right].$$

- Option curves vs. cdf

$$F_{X_i}(x) = 1 + e^{rT} C_i'[x+].$$

- The curve C_i is only partially known, so F_{X_i} cannot be specified.
- Approximate the unknown option curve $C_i[K]$ of each stock i (dashed line) by the piecewise linear curve $\overline{C}_i[K]$ connecting the traded strikes (solid line).
- The approximation $\overline{F}_{X_i}$ of F_{X_i} follows from:

$$\overline{F}_{X_i}(x) = 1 + e^{rT} \overline{C}_i'[x+]$$

The comonotonic stock market index

- The comonotonic stock market index:

$$S^c = \sum_{i=1}^{n} w_i F_{X_i}^{-1}(U)$$

  - Cdf of S^c is unknown.

- The (approximated) comonotonic stock market index[4]:

$$\overline{S}^c = \sum_{i=1}^{n} w_i \overline{F}_{X_i}^{-1}(U)$$

  - Cdf of $\overline{S}^c$ can be determined:
    - from observed stock option prices,
    - in a model-free way.

[4] Hobson, Laurence & Wang (2005).

Comonotonic index option prices

- Comonotonic index option prices:
  - Definition:

$$\overline{C}^c[K] = e^{-rT}\mathbb{E}\left[\left(\overline{S}^c - K\right)_+\right],$$
$$\overline{P}^c[K] = e^{-rT}\mathbb{E}\left[\left(K - \overline{S}^c\right)_+\right].$$

- $\overline{C}^c[K]$ and $\overline{P}^c[K]$ can be determined from observed stock option prices[5]:

$$C[K] \leq \overline{C}^c[K] = \sum_{i=1}^{n} w_i \overline{C}_i[K_i^*],$$
$$P[K] \leq \overline{P}^c[K] = \sum_{i=1}^{n} w_i \overline{P}_i[K_i^*].$$

for appropriately chosen strikes K_i^*.

[5] Dhaene, Wang, Young, Goovaerts (2000); Chen, Deelstra, Dhaene, Vanmaele (2008).

Characterizing perfect herd behavior

Theorem (Dhaene, Linders, Schoutens & Vyncke (2012))

The following statements are equivalent:

1. $$\underline{X} \text{ is comonotonic.}$$
2. $$C[K] = C^c[K], \text{ for all } K \geq 0.$$
3. $$P[K] = P^c[K], \text{ for all } K \geq 0.$$

- Cheung (2010) proves this result when the marginals have finite mean.
- Cheung, Dhaene, Kukush & Linders (2013) show that for non-negative r.v. 's, no additional constraint on the first moment has to be imposed.

Measuring herd behavior

The comonotonicity gap

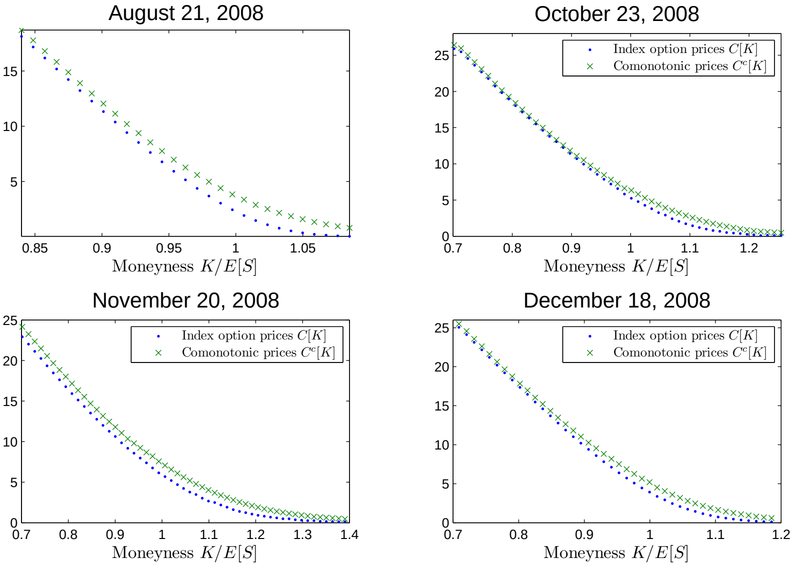
- The market is comonotonic:
  - if, and only if, the observed index option curve coincides with the comonotonic one.
  - In general, the market will never be comonotonic
    - ★ The observed prices are always constrained from above by their comonotonic modifications:

$$C[K] \leq C^c[K].$$

- Comonotonicity gap
  - Distance between the comonotonic case and the observed situation.
  - Small gap = sign for a high degree of herd behavior.
    - ★ Index options are priced as if the stocks will move almost perfectly together in the future.
  - The at-the-money comonotonicity gap[6]:

$$\text{ATM comonotonicity gap} = \frac{C\left[K_0\right]}{\overline{C}^c\left[K_0\right]}.$$

[6] Laurence (2008)



August = 63.73%, October = 82.9%, November = 80.38% and December = 75.58%.

Measuring herd behavior

Capturing the market implied comonotonicity gap in a real number

- Our goal is to capture the comonotonicity gap in a real number π:
  - $\pi \in (0, 1]$.
  - $\pi = 1$ represents a market with perfect herd behavior[7]
- Two approaches:
  - Characterize the index option surfaces by the swap rates $\mathbb{E}\left[\left(\frac{S}{\mathbb{E}[S]}\right)\right]$ and $\mathbb{E}\left[\left(\frac{S^c}{\mathbb{E}[S]}\right)\right]$
  - Characterize the index option surfaces by the distorted expectations $\rho_g[S]$ and $\rho_g[S^c]$

[7] Cheung, Dhaene, Kukush & Linders (2013).

Swap contracts on the index

Model-free expression for implied swap rate

- Consider the swap contract, underwritten at time 0:
  - floating leg pays $u\left(\frac{S}{\mathbb{E}[S]}\right)$ at maturity T;
  - the fixed leg pays P at maturity T.
- The time-0 price of the forward contract is 0 :

$$P = \mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right]$$

- Model-free expressions for the swap rate[8]:

$$\mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right] - u(1) = \frac{\mathrm{e}^{rT}}{\mathbb{E}[S]^2}\left(\int_0^{\mathbb{E}[S]} u''\left(\frac{K}{\mathbb{E}[S]}\right) P[K]\,\mathrm{d}K + \int_{\mathbb{E}[S]}^{+\infty} u''\left(\frac{K}{\mathbb{E}[S]}\right) C[K]\,\mathrm{d}K\right).$$

[8] u has an absolutely continuous derivative u'.

Swap rates and convex order

Measuring implied volatility

- The swap rate gives information about the variability of the index around its forward rate.
  - Consider $\underline{X}$ and $\underline{Y}$ and their respective sums S_X and S_Y.
  - If u is convex:

$$\frac{S_X}{\mathbb{E}[S_X]} \preceq_{cx} \frac{S_Y}{\mathbb{E}[S_Y]} \Longrightarrow \mathbb{E}\left[u\left(\frac{S_X}{\mathbb{E}[S_X]}\right)\right] \leq \mathbb{E}\left[u\left(\frac{S_X}{\mathbb{E}[S_X]}\right)\right].$$

- For a convex function u, the swap rate $\mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right]$
  - measures the *implied index volatility*;
  - captures the *expectation of the market* about future volatility.
- Each convex function u results in a corresponding volatility index.

Swap contracts on the index

Approximating the implied swap rate

- Traded strikes: $K_{-l}, K_{-l+1}, \ldots, K_{-1}, K_0, K_1, \ldots, K_{h-1}, K_h$.
- Approximation for $\mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right]$: (composite trapezoidal rule)

$$\mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right] - u(1) \approx \frac{e^{rT}}{\mathbb{E}[S]^2}\sum_{i=-l}^{h} u''\left(\frac{K_i}{\mathbb{E}[S]}\right)\Delta K_i Q[K_i] - \frac{u''\left(\frac{K_0}{\mathbb{E}[S]}\right)}{2}\left(\frac{\mathbb{E}[S]-K_0}{\mathbb{E}[S]}\right)^2.$$

  - $\Delta K_i = \frac{K_{i+1}-K_{i-1}}{2}$ for $i = -l+1, \ldots, h-1$.
  - $\Delta K_{-l} = K_{-l+1} - K_{-l}$ and $\Delta K_h = K_h - K_{h-1}$.
  - $Q[K_i]$ defined by

$$Q[K_i] = \begin{cases} P[K_i], & \text{if } K_i < K_0 \\ \frac{C[K_i]+P[K_i]}{2} & \text{if } K_i = K_0 \\ C[K_i], & \text{if } K_i > K_0 \end{cases}$$

Measuring implied volatility

Example: The Volatility Index (VIX)

- Define the function $v : \mathbb{R}_0^+ \longrightarrow \mathbb{R}$ as

$$v(x) = -2 \ln x.$$

- Approximation for $\mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right]$ is called the Volatility Index
- The Volatility Index (VIX):

$$\text{VIX}[T] = 2\mathrm{e}^{rT} \sum_{i=-l}^{h} \frac{\Delta K_i}{K_i^2} Q[K_i] - \left(\frac{\mathbb{E}[S] - K_0}{K_0}\right)^2$$

  - The VIX is the leading barometer for the perceived level of volatility, based on near term S&P500 options.
    - The methodology can be applied for any index and maturity, as long as index options are available.

Perfect herd behavior and swap rates

- Assume that stock prices move perfectly together
  - the price vector is the comonotonic vector $(X_1^c, X_2^c, \ldots, X_n^c)$;
  - the index option curves are C^c and P^c.
- The (approximate) comonotonic swap rate $\mathbb{E}\left[u\left(\frac{\overline{S}^c}{\mathbb{E}[S]}\right)\right]$ can be determined in a model-free way from the index option curves $\overline{C}^c$ and $\overline{P}^c$.
- In general, the price vector $(X_1, X_2, \ldots, X_n)$ is not comonotonic
  - For a convex u: the swap rate $\mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right]$ is always constrained from above by the comonotonic swap rate:

$$u \text{ is convex } \Longrightarrow \mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right] \leq \mathbb{E}\left[u\left(\frac{S^c}{\mathbb{E}[S]}\right)\right]. \qquad (3)$$

Characterizing perfect herd behavior

Theorem

The strictly convex function u has an absolutely continuous derivative u' and $\mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right]$ is finite. Then the following statements are equivalent:

1. $$\underline{X} \text{ is comonotonic.}$$
2. $$\mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right] = \mathbb{E}\left[u\left(\frac{S^c}{\mathbb{E}[S]}\right)\right].$$
3. $$C\left[K\right] = C^c\left[K\right], \text{ for all } K \geq 0.$$
4. $$P\left[K\right] = P^c\left[K\right], \text{ for all } K \geq 0.$$

Comonotonic swap rates

- Comonotonic swap rates:

$$\mathbb{E}\left[u\left(\frac{S^c}{\mathbb{E}[S]}\right)\right]-u(1)$$
$$=\frac{\mathrm{e}^{rT}}{\mathbb{E}[S]^2}\left(\int_0^{\mathbb{E}[S]}u''\left(\frac{K}{\mathbb{E}[S]}\right)P^c[K]\,\mathrm{d}K+\int_{\mathbb{E}[S]}^{+\infty}u''\left(\frac{K}{\mathbb{E}[S]}\right)C^c[K]\,\mathrm{d}K\right).$$

- Approximation:

$$\mathbb{E}\left[u\left(\frac{\overline{S}^c}{\mathbb{E}[S]}\right)\right]-u(1)$$
$$\approx\frac{\mathrm{e}^{rT}}{\mathbb{E}[S]^2}\sum_{i=-l}^{h}u''\left(\frac{K_i}{\mathbb{E}[S]}\right)\Delta K_i\overline{Q}^c[K_i]-\frac{u''\left(\frac{K_0}{\mathbb{E}[S]}\right)}{2}\left(\frac{\mathbb{E}[S]-K_0}{\mathbb{E}[S]}\right)^2.$$

  - ΔK_i and $\overline{Q}^c[K_i]$ defined as above.

The implied degree of herd behavior

- Swap contract with pay-off $u\left(\frac{S}{\mathbb{E}[S]}\right)$ at time T.
  - Swap rate $= \mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right]$.
  - Swap rate is determined in a model-free way by the index option curve Q.
  - The swap rate depends on the marginals and the dependence structure.
- $\mathbb{E}\left[u\left(\frac{S^c}{\mathbb{E}[S]}\right)\right]$ is the swap rate, provided the stock prices are expected to move perfectly together.
- u is strictly convex.
  - Bounds for $\mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right]$:

$$u(1) \leq \mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right] \leq \mathbb{E}\left[u\left(\frac{S^c}{\mathbb{E}[S]}\right)\right].$$

The implied degree of herd behavior

- Measuring the degree of herd behavior: (u strictly convex)

$$\text{Degree of Herd Behavior}_u = \frac{\mathbb{E}\left[u\left(\frac{S}{\mathbb{E}[S]}\right)\right] - u\left(1\right)}{\mathbb{E}\left[u\left(\frac{S^c}{\mathbb{E}[S]}\right)\right] - u\left(1\right)}.$$

- Definition of the Herd Behavior Index[9]:

$$\text{HIX}_u\left[T\right] = \frac{e^{rT}\sum_{i=-l}^{h} u''\left(\frac{K_i}{\mathbb{E}[S]}\right)\Delta K_i Q\left[K_i\right] - \frac{u''\left(\frac{K_0}{\mathbb{E}[S]}\right)}{2}\left(\mathbb{E}\left[S\right] - K_0\right)^2}{e^{rT}\sum_{i=-l}^{h} u''\left(\frac{K_i}{\mathbb{E}[S]}\right)\Delta K_i \overline{Q}^c\left[K_i\right] - \frac{u''\left(\frac{K_0}{\mathbb{E}[S]}\right)}{2}\left(\mathbb{E}\left[S\right] - K_0\right)^2}$$

  - Nominator:
    - Captures real market situation.
    - Follows from observed **index option prices**.
  - Denominator:
    - Captures comonotonic market situation.
    - Follows from observed **stock option prices**.

[9] Linders, Dhaene & Schoutens (2013).

The Herd Behavior Index (HIX)

- Define the function $u : \mathbb{R}^+ \longrightarrow \mathbb{R}$ as:

$$u\left(x\right) = \left(x-1\right)^2.$$

- The degree of herd behavior is measured by:

$$\begin{aligned} \text{Degree of Herd Behavior} &= \frac{\mathsf{Var}\left(S\right)}{\mathsf{Var}\left(S^c\right)} \quad (4) \\ &= \frac{\int_0^{\mathbb{E}[S]} P\left[K\right] \mathrm{d}K + \int_{\mathbb{E}[S]}^{+\infty} C\left[K\right] \mathrm{d}K}{\int_0^{\mathbb{E}[S]} P^c\left[K\right] \mathrm{d}K + \int_{\mathbb{E}[S]}^{+\infty} C^c\left[K\right] \mathrm{d}K}. \end{aligned}$$

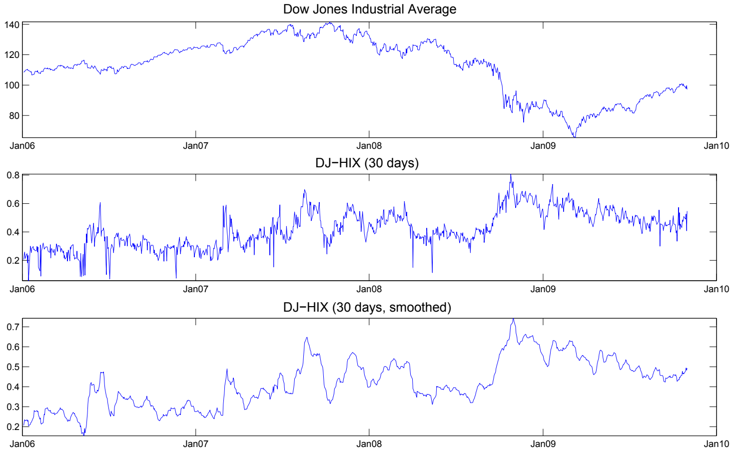
- The HIX:

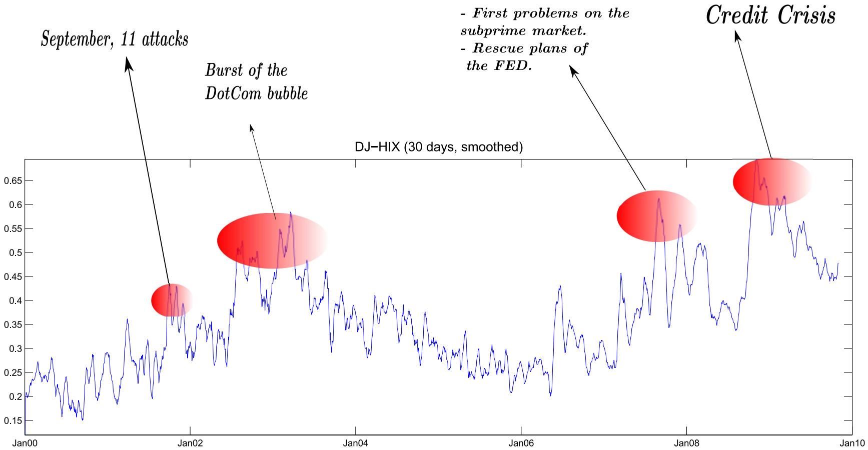
$$\mathsf{HIX}\left[T\right] = \frac{2\mathrm{e}^{rT}\sum_{i=-l}^{h} \Delta K_i Q\left[K_i\right] - \left(\mathbb{E}\left[S\right] - K_0\right)^2}{2\mathrm{e}^{rT}\sum_{i=-l}^{h} \Delta K_i \overline{Q}^c\left[K_i\right] - \left(\mathbb{E}\left[S\right] - K_0\right)^2}. \quad (5)$$

The DJ-HIX

Herd behavior over time

- Values of **DJ-HIX** for $T = 30$ calendar days.
- Time period: January, 2006 - October, 2009.





September, 11 attacks
Burst of the DotCom bubble
- First problems on the subprime market.
- Rescue plans of the FED.
Credit Crisis
DJ−HIX (30 days, smoothed)
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
Jan00
Jan02
Jan04
Jan06
Jan08
Jan10

Thank you for your attention!

Men, it has been well said, think in herds, it will be seen that they go mad in herds, while they only recover their senses slowly, and one by one.

Charles Mackay

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