



# Markov Chain Modeling of Policy Holder Behavior in Life Insurance and Pension

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# Outline

- Approaches to policy holder behavior
- Risk and behavior models
- Reserves
- Example
- Conclusion and recommendations



# Approaches to policy holder behavior

- Policy holder behavior
  - Surrender option
    - Early termination of the contract
  - Free policy option (paid-up policy option)
    - Stop premium payment and reduce benefit



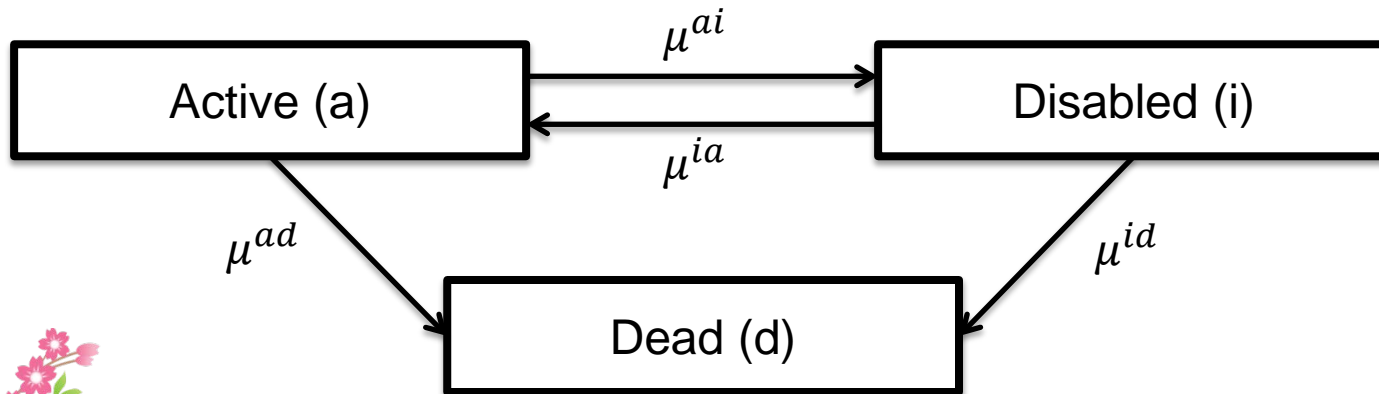
# Approaches to policy holder behavior

- Modeling by worst-case behavior (not here)
- Modeling by expected behavior
  - Intensity modeling by Markov chains
  - Dependence between policy risk and behavior by contract design



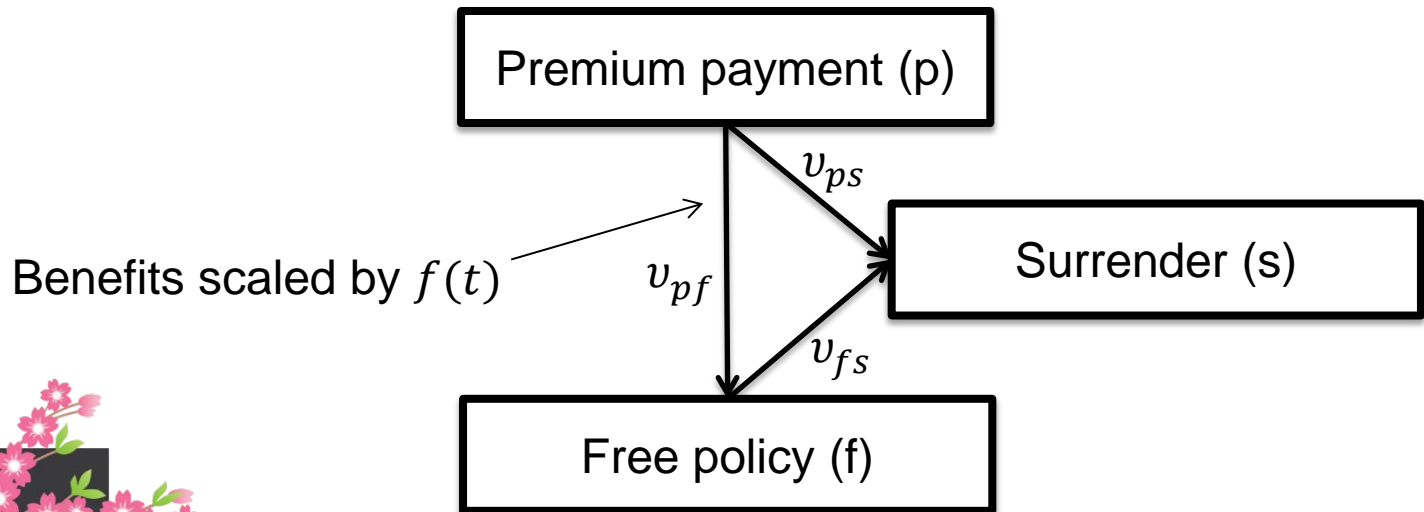
# Risk and behavior models

- Disability risk model
  - Deterministic intensities



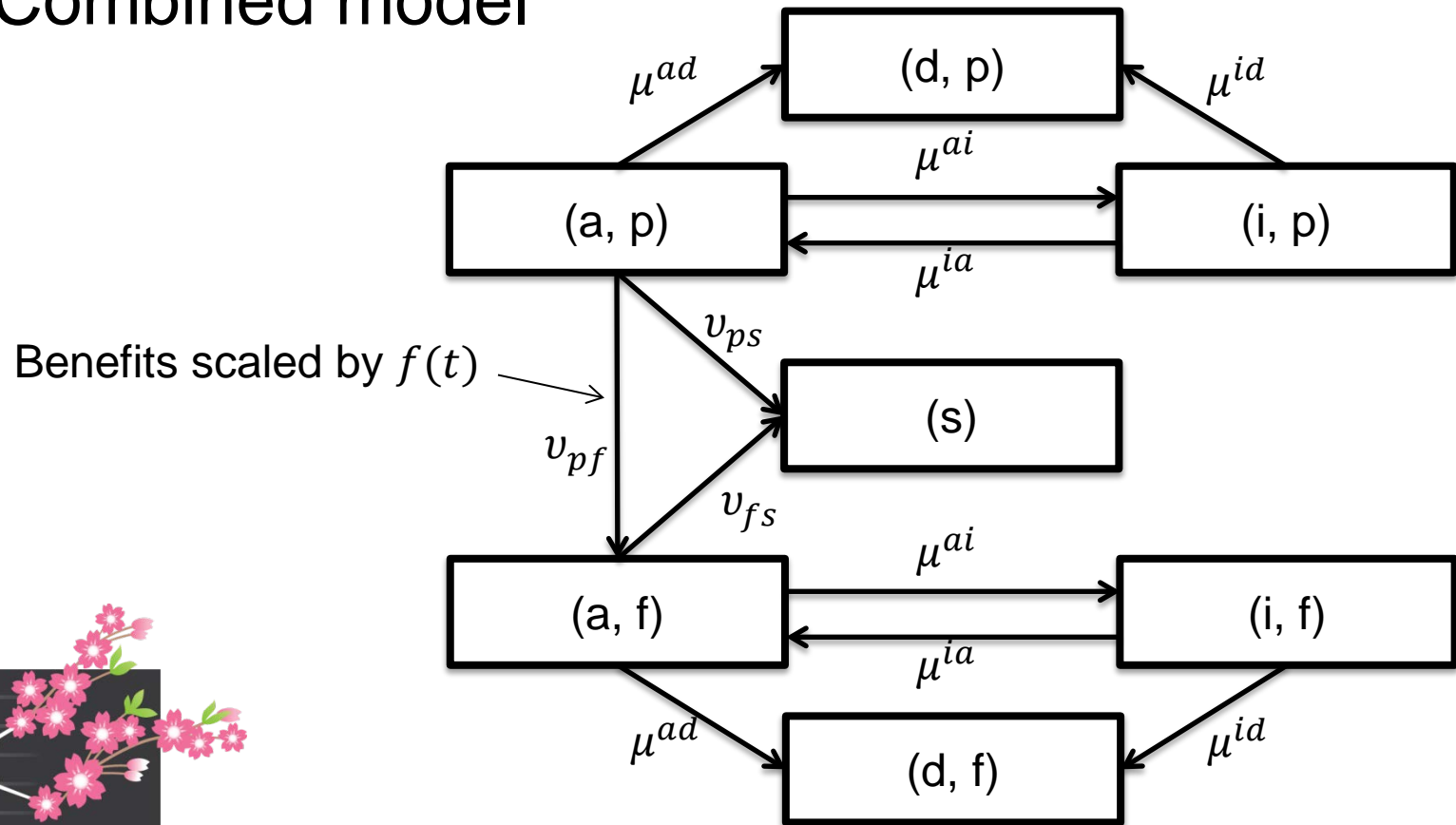
# Risk and behavior models

- Behavior model
  - Contract design: Only policy holder options when active



# Risk and behavior models

- Combined model



# Reserves

- In general complex expressions for reserves
  - Explicitness depends on reactivation
  - Relatively easy to handle with system of ordinary differential equations (Thiele)

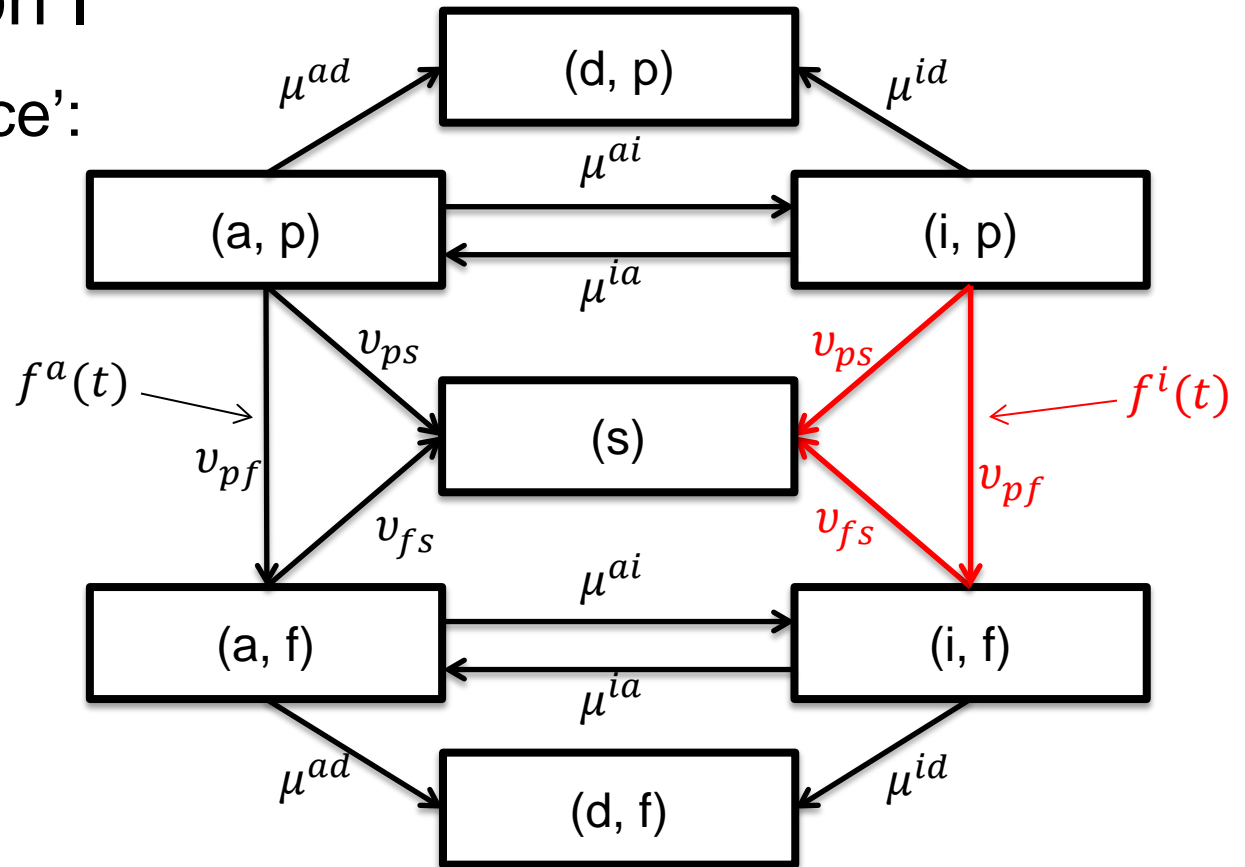




# Reserves

- Simplification I

'Independence':



# Reserves

- Simplification II

‘Same  $f(t)$ ’:

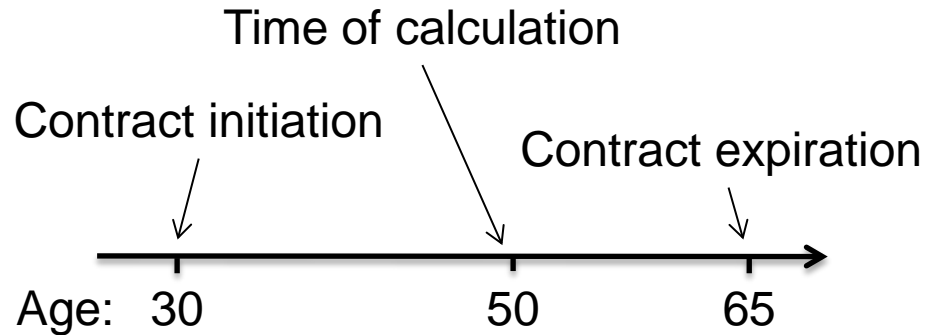
$$V^a(t) = \int_t^n e^{-\int_t^s r} p_{pp}(t, s) c(t, s) ds + \int_t^n e^{-\int_t^s r} W(t, s) c^+(t, s) ds$$

- $W(t, s) = \int_t^s p_{pp}(t, \tau) v_{pf}(\tau) f(\tau) p_{ff}(\tau, s) d\tau$
- $c(t, s)$  cash flow in case of no free policy intervention
- $c^+(t, s)$  cash flow in case of free policy intervention



# Example

- Contract



## Premium

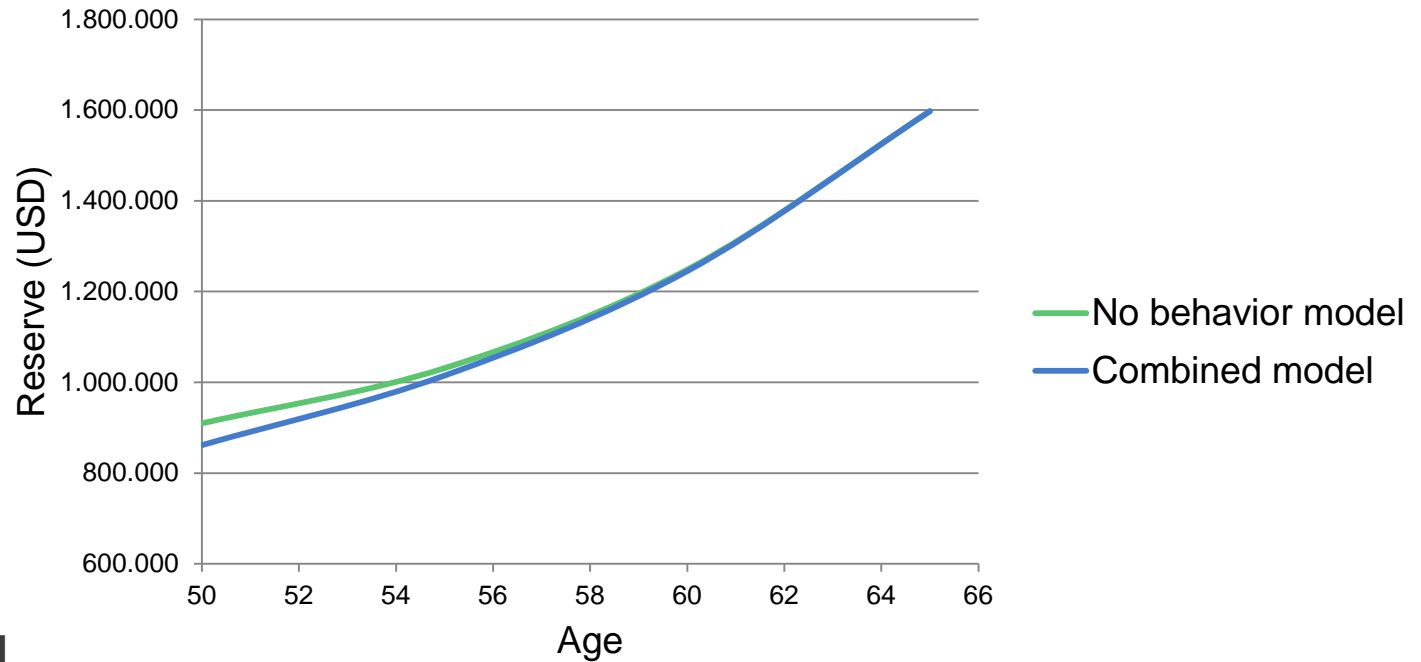
20,000 USD p.a. when active

Benefits	Amount
Disability annuity	100,000 USD p.a.
Term insurance	400,000 USD
Pure endowment at expiry	1,597,593 USD



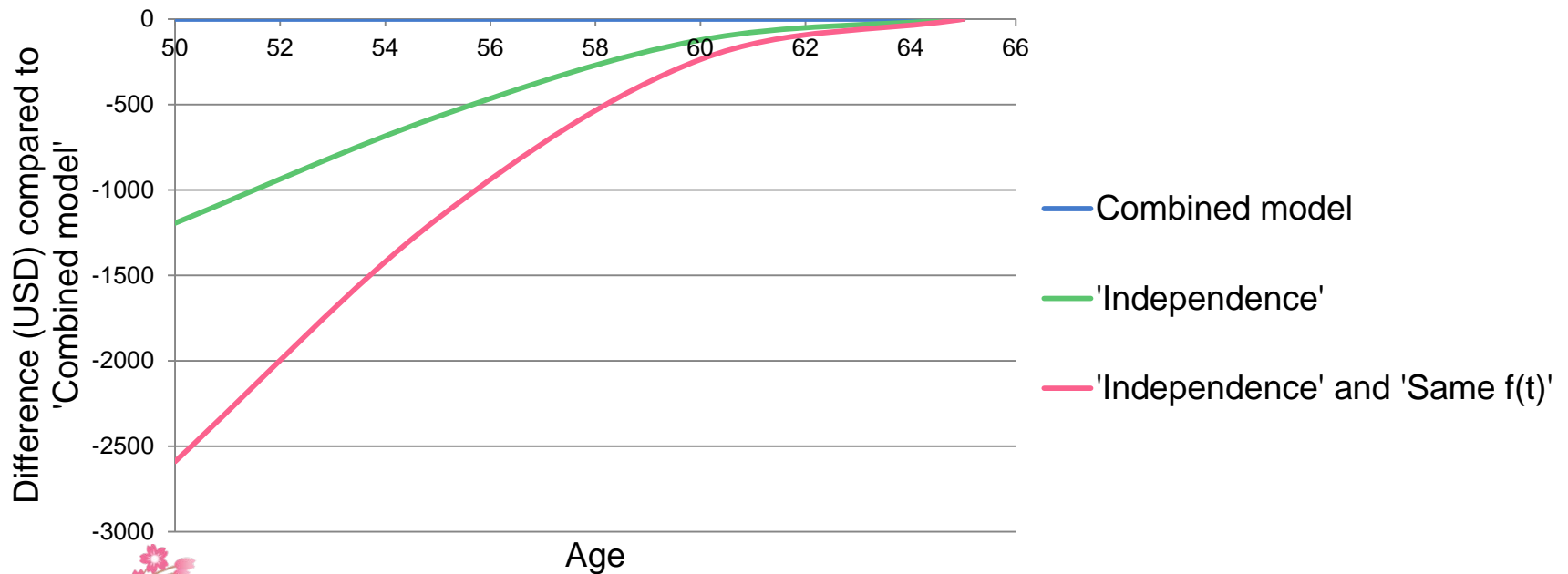
# Example

- Reserves



# Example

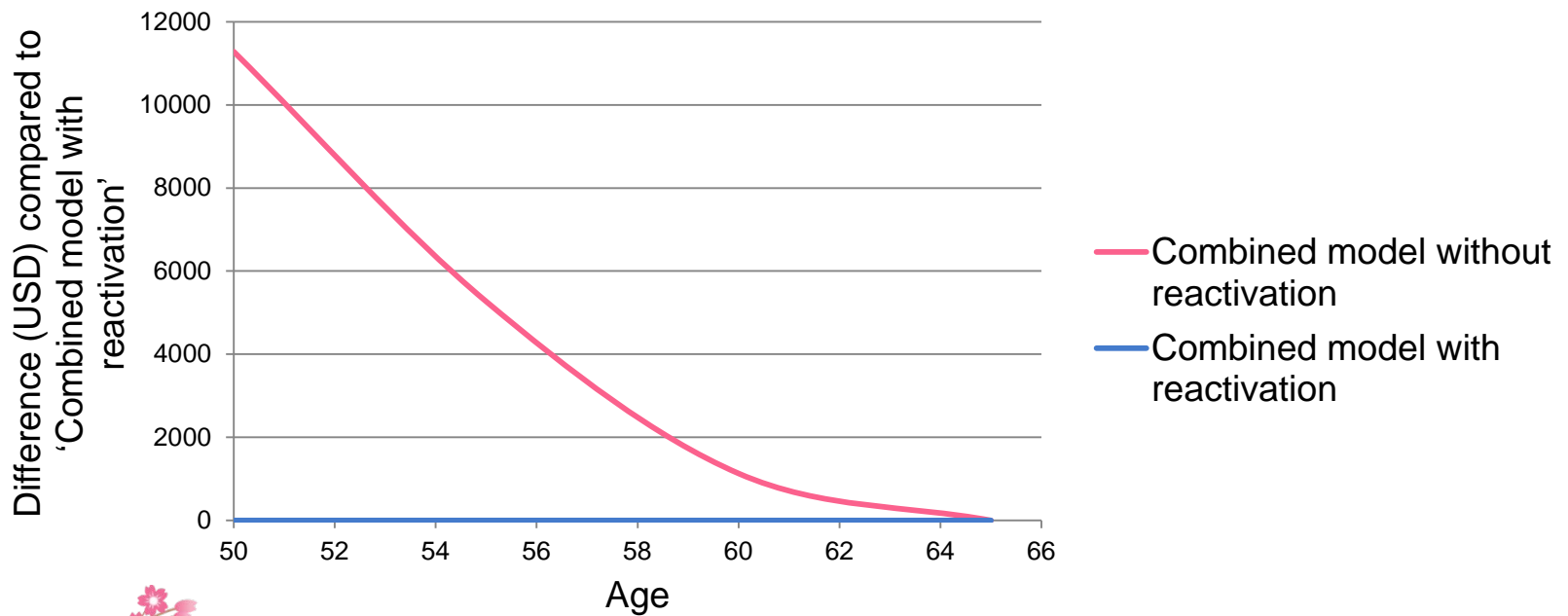
- Consequence of simplifications



- The sign and size of the error is contract dependent
- The error is systematic

# Example

- Consequence of modeling without reactivation



- The size of the error is contract dependent
- The error is systematic

# Conclusion and recommendations

- Without behavior modeling  
'right' = 'simple' – (5% to 6%)
- Simplified behavior modeling  
'right' = 'simple' + (0% to 1%)
- Without reactivation  
'right' = 'simple' – (1% to 2%)
- Contract and basis dependent





You don't know how 'wrong' is wrong  
until you know how 'right' is right

