Stress scenario generation for solvency and risk management

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Outline

- Worst case scenario
- Relation to VaR and Solvency II
- Finding the worst case scenario
- Examples
- Conclusion

Worst case scenario

- Worst case scenario

  $\tilde{\mu} = \operatorname{argmax}_{m \in M}\{V_{j_0}(t_0; m)\}$
- $V_{j_0}(t_0; m)$ is the reserve in state j_0 at time t_0
- The $m \in M$ are the possible future biometrical scenarios (of transition intensities $t \to m_{jk}$(t))
- $V_{j_0}(t_0; \tilde{\mu}) = \sup_{m \in M}\{V_{j_0}(t_0; m)\}$

  if $\tilde{\mu}$ is a worst case scenario

Applications of worst case scenarios

- Solvency II standard formula for life (assuming assets unaffected)

  $SCR_{life} = \mathrm{VaR}_{0.995}\left(V_{j_0}(t_0;\mu) - V_{j_0}(t_0;\mu^{BE})\right)$

  μ: Stochastic transition intensities

  μ^{BE}: Deterministic best estimates transition intensities

- $SCR_{life} \leq \sup_{m\in M}\left\{V_{j_0}(t_0;m)\right\} - V_{j_0}(t_0;\mu^{BE})$

  if $P(\mu \in M) \geq 0.995$

- Worst case scenarios can also be used for premium settlement of traditional with-profits life insurance products to ensure sufficiently high premiums

Finding the worst case scenario

- **Theorem:** $\tilde{\mu}$ is a worst case scenario if $\forall t \in [t_0, n]$:

$$\tilde{\mu}(t) = \text{argmax}_{(m_{jk})_{j \neq k} \in M(t)} \left\{ \sum_{j \neq k} \tilde{p}_{j_0 j}(t_0, t) \left(b_{jk}(t) + \tilde{V}_k(t) - \tilde{V}_j(t) \right) m_{jk} \right\}$$

Thiele

$$\frac{\partial}{\partial t}\tilde{V}_j(t) = r(t)\tilde{V}_j(t) - b_j(t) - \sum_{k:k \neq j} \left(b_{jk}(t) + \tilde{V}_k(t) - \tilde{V}_j(t) \right) \tilde{\mu}_{jk}(t) \text{ and } \tilde{V}_j(n) = 0$$

Kolmogorov

$$\tilde{p}_{j_0 j}(t_0, t_0) = 1_{(j_0 = j)} \text{ and } \frac{\partial}{\partial s}\tilde{p}_{j_0 j}(t_0, s) = \sum_{l:l \neq j} \left(\tilde{p}_{j_0 l}(t_0, s)\tilde{\mu}_{lj}(s) - \tilde{p}_{j_0 j}(t_0, s)\tilde{\mu}_{jl}(s) \right)$$

- Numerically challenging due to initial condition at both t_0 and n
- General approach exists

  Christiansen and Steffensen (2013a)

  Christiansen, Henriksen, Schomacker and Steffensen

Cases where calculations simplifies

- Transition intensities independent

  $M(t) = \times_{\{(j,k)|j \neq k\}} M_{jk}(t)$

  $\tilde{\mu}_{jk}(t) = \text{argmax}_{m_{jk} \in M_{jk}(t)} \left\{ \left(b_{jk}(t) + \tilde{V}_k(t) - \tilde{V}_j(t) \right) m_{jk} \right\}$

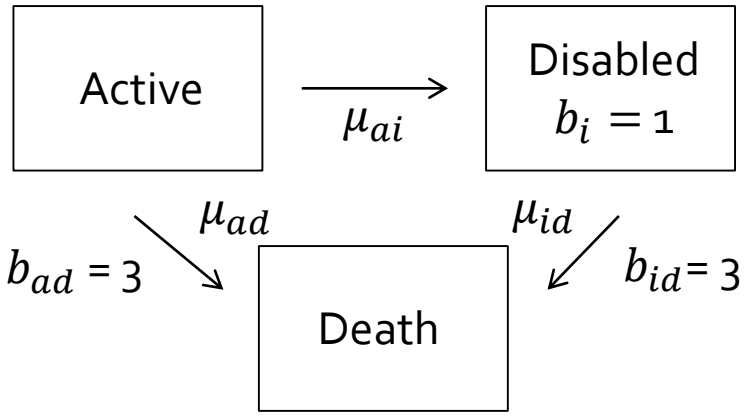
  Christiansen (2010)

- argmax constant with respect to $\tilde{p}_{j_0 j}(t_0, t)$

  Decouples Kolmogorov equations

  Christiansen and Steffensen (2013b)

Examples: Model

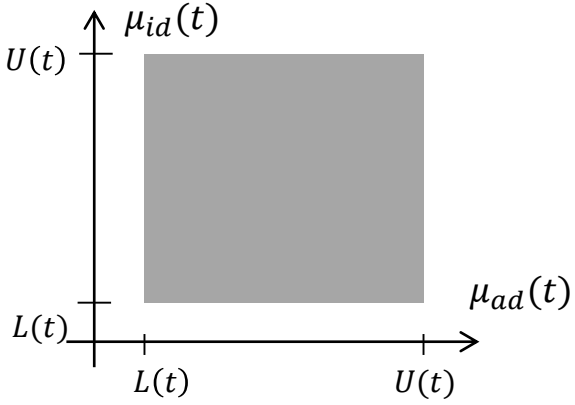


- Fixed standard Danish disability intensity
- Find worst case scenarios $\tilde{\mu}_{ad}$ and $\tilde{\mu}_{id}$
- Examples calculated using Actulus® Calculation Platform

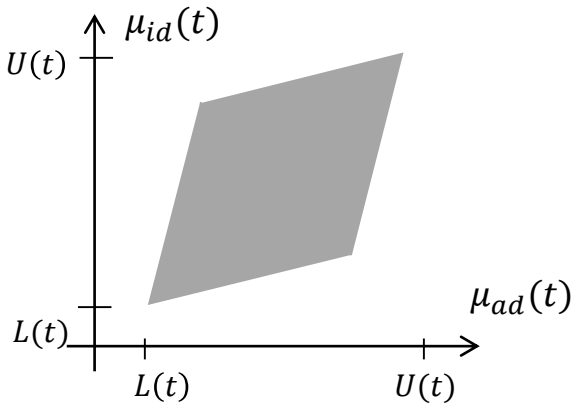
Examples: Possible transition intensity scenarios

- Best estimate death intensity
  $\mu^{BE}(t)$ standard Danish intensity
- Scenarios based on Solvency II mortality and longevity stress
  $L(t) := (1 - 20\%)\mu^{BE}(t)$
  $U(t) := (1 + 15\%)\mu^{BE}(t)$

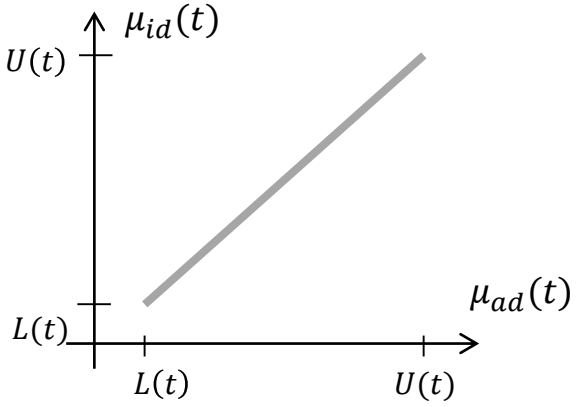
M(t): Independence



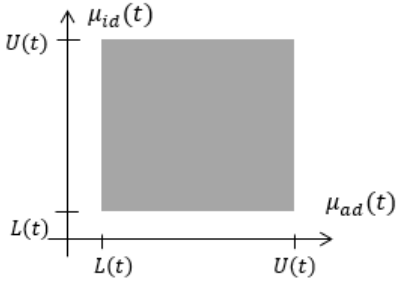
M(t): Dependence

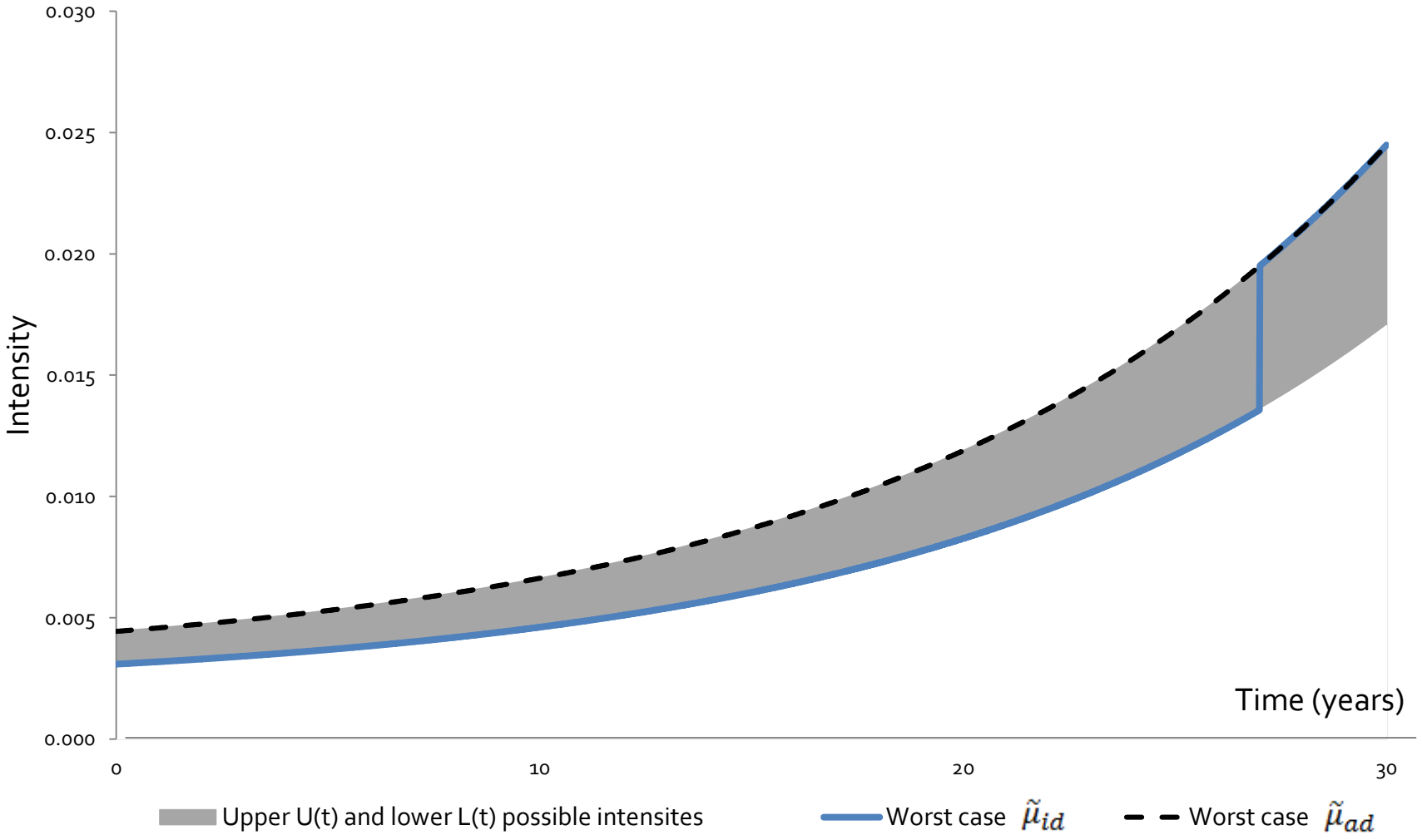


M(t): Linear dependence

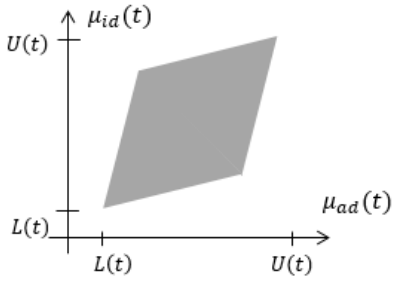


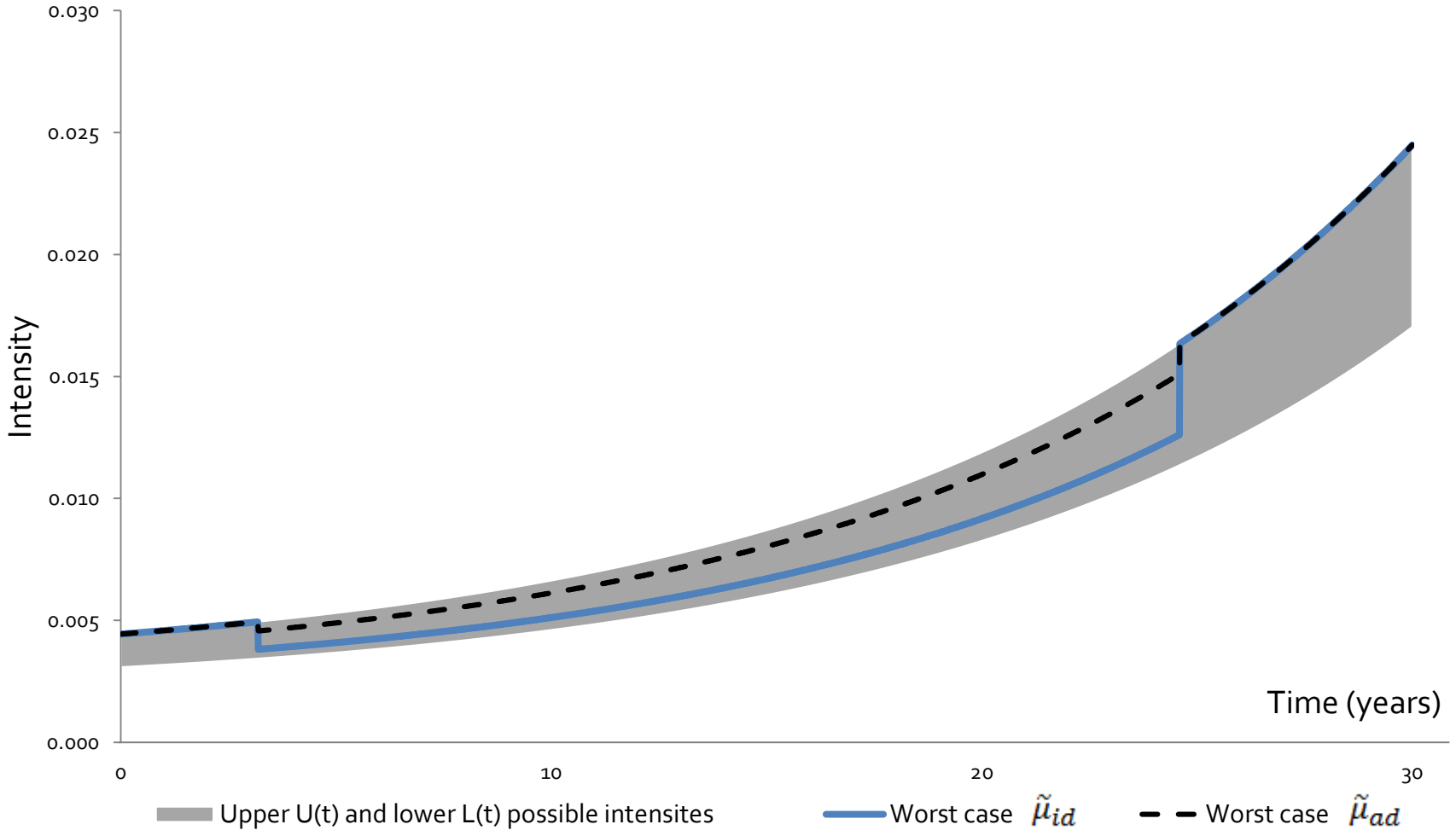
Independence



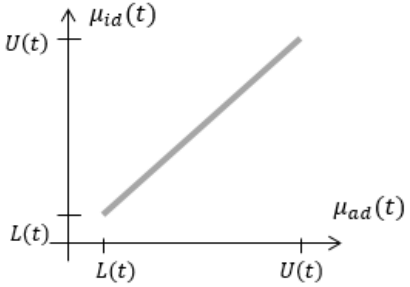


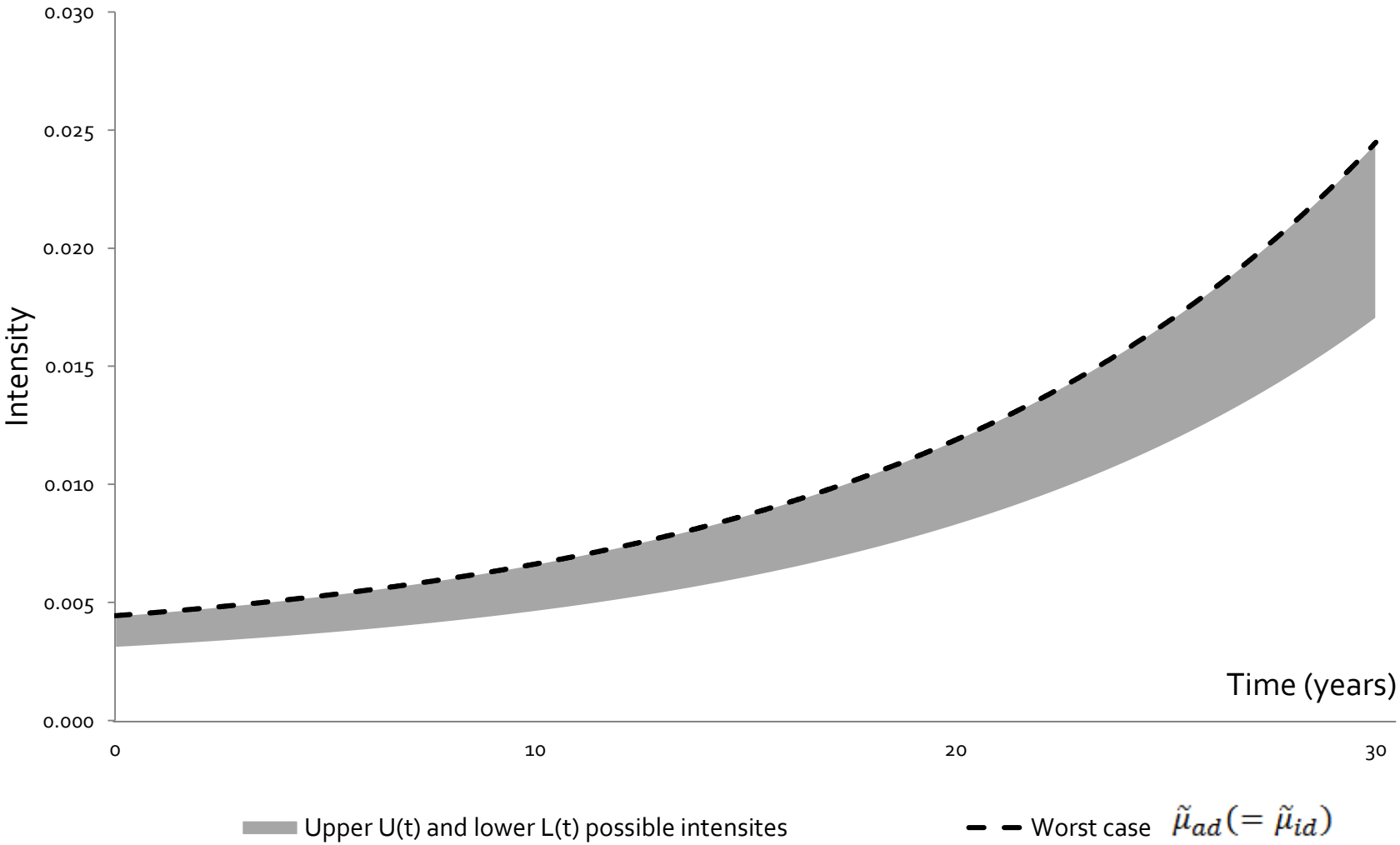
Dependence


$\mu_{id}(t)$
$U(t)$
$L(t)$
$\mu_{ad}(t)$
$L(t)$
$U(t)$


0.030
0.025
0.020
0.015
0.010
0.005
0.000
Intensity
Time (years)
0
10
20
30
Upper U(t) and lower L(t) possible intensites
Worst case $\tilde{\mu}_{id}$
Worst case $\tilde{\mu}_{ad}$

Linear dependence





Conclusion

- Worst case scenarios related to VaR and Solvency II
- Find the worst case scenario by iteration
- Worst case scenarios can include interest rate
- Extends to portfolio of policies

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