

**Progress Report PhD-Year 1:  
“Towards Quantifying Liquidity Premia on  
Corporate Bonds”**

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## Abstract

The Liquidity Premium on corporate bonds and their potential significance as an additional discount factor on long-dated insurance liabilities has been a high priority for Solvency regulators. The robust decomposition of the credit spread into credit- and liquidity components has been the principal objective of this report. Methodologies used in previous academic work, most notably Webber (2007), are reviewed and the proposed modeling approach addresses issues that made previous work unsuitable for industry wide applications.

Using the iBoxx GBP investment grade bond index a measure of relative liquidity is calculated. The Relative Bid-Ask Spread (RBAS) is a relative liquidity measure that allows liquidity comparisons between bonds of different characteristics such as rating, duration and age. A series of daily regressions model the Credit Spread using readily available bond characteristics such as duration, age, coupon, rating and RBAS. It also takes into account five levels of seniority of debt, collateralization and classifications such as financial/non-financial.

Previous literature, especially those focusing on calibrating structural models, relied on data sources that are not easily available and calibrated models using carefully chosen samples of bonds. The proposed modeling approach uses data that is readily available or easily computed for all bonds on a daily basis, producing daily estimates of liquidity premia on an individual bond level. Results regarding the size of the illiquidity premium are, on aggregate, similar to those reported previously for investment grade corporate bonds. We find that for the average bond, between 30%-40% of the credit spread is due to liquidity effects, but there is variation over time. In the time period just prior to the financial crisis, the average liquidity premium was between 5%-10%.

Liquidity Premium estimates for individual bonds can substantially differ from reported average; bonds with above an average level of illiquidity (50%-80% quantile) command a premium of 35%-55% and bonds with very high levels of illiquidity command in the region of 55%-85%.

Bond specific Liquidity Premium estimates are crucial when considering the possibility of a discount factor for insurance liabilities. Only the actual assets held to maturity, used to replicate liabilities' cash flows, can potentially be used in to calculate a discount factor. Moreover, a frequent review of a portfolio's (relative) liquidity and subsequent liquidity premia is an essential part of good risk management practice. The proposed modeling approach allows for both the Liquidity Premium to be estimated for individual bonds as well as constant (daily) review of these estimates.

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## **1. Executive Summary**

In 2009 the Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS) set up a special task force to investigate the Illiquidity Premium. In March 2010 the task force published its findings (CEIOPS, 2010) and argued for the introduction of an illiquidity premium.

The argument made by the insurance industry is simple in principle. The cash flows associated with annuity contracts are predictable and easy to match with traded assets; gilts and corporate bonds are used to match liability cash flows and will be held until maturity. The yield on a corporate bond is higher than the yield on a “risk-free” gilt or swap, with the additional yield referred to as the risk premium. This risk premium compensates investors for excess risk the holder bears; for example, the issuer of the bond may default, the issuer of the bond may get downgraded and the bond may be difficult to convert to cash, i.e. it is illiquid. Whereas the default related risk is always present, the difficulty to turn a bond into cash is easily avoided by holding the bond until maturity. The argument goes that the bond holder can capture the value of that liquidity risk premium in their valuation.

The difficulty lies in calculating or even defining the liquidity of insurance liabilities, as it cannot be easily compared to the liquidity of assets. For some skeptics this fundamental issue is where the insurer’s argument breaks down. On the other hand, CEIOPS defines the liquidity of liabilities as the degree to which a liability’s cash flows are predictable, since predictable cash flows can be replicated using illiquid assets. This implies that the definition of “illiquid” liabilities excludes liabilities for which cash flows are not as predictable (because the policyholder can surrender their policy before the redemption date for example). Determining the eligibility of liabilities based the proposed definition of “predictability” is problematic since the above cash flow, can nonetheless be matched with a combination of bonds and derivatives that can be held to maturity.

In its first report in 2010, the CEIOPS task force states that “to determine the part of the spread attributable to liquidity risk, the challenge that has to be faced is the accurate breakdown of this spread into its components” (CEIOPS, 2010). In the same report it offered a simple formula that could be used as a proxy for the Liquidity Premium, based on a fixed percentage of the spread. The calibration of this one parameter would determine the liquidity premium and issues were raised immediately to the potential subjectivity and the frequency of calibration.

The technical issue of quantifying the Liquidity Premium on corporate bonds has been the focus of the PhD-project during its first nine months. In particular, we focus on a review of popular (mathematical) methods of corporate spread decompositions and assess their results and practical use. Previous work regarding spread decompositions had four major issues making them unacceptable for practical industry-wide use: outcomes relied heavily on chosen sample of bonds and time period, modelling techniques required input data that is simply unavailable for a large part of the relevant bond universe with self-selection bias as a result, estimates of liquidity premium are aggregated and do not account for differences in for example rating, sector, duration or seniority and lastly, all estimates are at very low frequencies (monthly or quarterly).

Using the iBoxx dataset, covering Sterling denoted investment grade bonds, model the liquidity premium using readily available (observable) data at the individual bond level. Using daily data, our analysis is robust in the sense that it can capture changes in the market and update liquidity premium estimates on a daily basis. We believe any implementation of an additional discount factor based on illiquidity should take into account the ALM situation of the insurer in order to rule out arbitrage opportunities. Being able to robustly estimate the illiquidity premium on any particular set of assets is vital and is possible using our modelling approach. In particular we perform two modelling exercises in order to extract the Liquidity Premium: the first evolves around the modelling of the Bid-Ask Spread and the second analysis evolves around modelling the Benchmark Spread, also referred to as the Credit Spread, Option Adjusted Spread, Asset Swap Margin or Z-Spread; all of which differ slightly in the way they are calculated.

Future should focus on the refinement of the analyses; capturing sector specific information sufficiently, capturing more complex non-linearities in the data, the identification of potential outliers, a more elaborate investigation into the volatility of liquidity and the possibility to sub-divide rating classes. Additionally, since the Liquidity Premium is modelled directly, the decomposition of the Credit Spread into expected defaults, default risk and liquidity components, needs attention. As an aside to these models one could think of investigating whether liquidity clientele effects are empirically sound or one could think of looking into trading strategies using (relative) liquidity as a starting point. Research at a later stage in the project should focus on the application of an estimated illiquidity premium on corporate bonds. This work could focus on the theoretical underpinning of Market Consistent Valuation, the real-world matching of assets and liabilities and the predictability of cash flows. Empirical work could focus on scenario analysis and stress testing.

## **2. Literature Review**

### **2.1 Asset Liquidity**

#### **2.1.1 Liquidity in Financial Markets**

Liquidity in financial markets has many aspects and defining a liquid market can heavily depend on what group of market participants is asked. Before delving into definitions of liquidity, it is important to distinguish between two “types” of liquidity. Liquidity in financial markets can generally refer to either liquidity in funding, or liquidity in trading. Following Brunnermeier and Pederson (2009), we can define these two aspects of liquidity as follows;

- Funding Liquidity is the ease with which market participants can obtain funding.
- Market Liquidity is the ease with which an asset can be traded.

Even though we are primarily concerned with so-called market liquidity, the two cannot be seen in total isolation, especially during periods of (extreme) market distress when funding and market liquidity can lead to a spiral of illiquidity (Brunnermeier and Pederson, 2009). Brunnermeier and Pederson (2009) argue that their mutually reinforcing effect during crises periods causes well-known “liquidity phenomena” such as flight to quality.

When traders (speculators, hedge funds, investment bank, all mark-to-market) buy a security, it requires the trader to use some of his own capital (difference between security’s price and its collateral value) to finance the trade. Similarly, short selling requires a margin on all positions. Traders are less willing to put on trades if funding is sparse, especially in “capital-intensive” securities, which has direct consequences for liquidity across the entire market.

#### **2.1.2 Defining Liquidity**

Market liquidity is difficult to define, especially since different market participants (for example, day traders and pension funds) and different sides of the market (buy or sell), might have different requirements for a liquid market. We look at various definitions offered by recent literature and extract all aspects of liquidity that might need to be considered when evaluating the liquidity of a market or individual security.

In its Stability Report in April 2007 (Bank of England, 2007) argues that liquidity risk, the harmful consequences of illiquidity, is present when “one cannot easily offset or eliminate a position without significantly affecting the market price”. Linking liquidity



directly to movement in market price is likely to describe one aspect of liquidity as the existing body of empirical research often considers the price impact of trades when evaluating liquidity (often proxied using the Amihud-measure, (Amihud, 2002)). However, on its own this definition is rather narrow in scope and many other elements of liquidity need to be considered.

Like many authors we use a market microstructure theory approach to study market liquidity in the first instance. Market microstructure theory is defined by O'Hara (O'Hara, 1995) as "the study of the process and outcomes of exchanging assets under a specific set of rules. While much of economics originates from the mechanics of trading, microstructure theory focuses on how specific trading mechanisms affect the price formation process." For a comprehensive survey of the field, see Madhavan (2000). Kyle (1985) identifies three characteristics of a market that describe its liquidity:

- Tightness: size of the spread between bid-ask prices
- Depth: maximum trade size / volume that does not affect current prices, reciprocal of equilibrium price to trade volume
- Resilience: speed with which the price impact of trades disappear

Even though these three elements of market liquidity are unlikely to describe all aspects of liquidity, they are well-established. These dimensions form the basis of several factors that Amihud et al. (2005) identify as the main elements affecting the microstructure of a market and the market liquidity of traded assets: exogenous transaction costs, private information, inventory risk and search friction.

Exogenous transaction costs are costs incurred by both the buyer and seller of a security every time the security is traded. These costs can include transaction taxes, order processing costs and broker fees. Dealers will adjust their quote spread to protect themselves, on average, against counterparties with superior knowledge from which a trading loss will occur. Inventory risk refers to the wider spread dealers will quote for holding an inventory in a security that deviates from their desired inventory and search friction describes how an investor might be making concessions on price if he cannot find a counterparty for his trade; specifically, it refers to the opportunity cost of an investor between an immediate transaction (price concession) and waiting for willing counterparty in the market.

Since the bid-ask spread is at the centre of our study more specific microstructure theory, concerned with the decomposition of bid-ask spreads, is of interest to us. All of the above-mentioned factors are considered when decomposing the bid-ask spread. Early work on

decomposing the bid-ask spread focussed on explaining quoted spreads cross-sectionally using market variables such as trading volume and security risk (Benston & Hagerman, 1974 and Demsetz, 1968). More recent work decomposes the spread into adverse information dealer profit components (Glosten & Harris, (1988) and Stoll, 1989), where the dealer profit represents compensation for inventory holding and order processing costs.

Estimates by Stoll (1989) and Madhavan and Smidt (1991) indicate that inventory costs are relatively small for liquid asset classes but increase substantially for illiquid assets and, hence, the remainder of the spread is mostly determined by order processing costs and adverse information costs. Copeland and Stoll (1990) conclude that order costs, the clerical costs of carrying out the transaction, are fixed irrespective of trade size. Therefore the average cost of order processing per unit decreases with trade size. The effect of informed counterparties has been studied at length (see for example Glosten & Milgrom (1985)) and models describe how dealers / market makers demand compensation for losses incurred from counterparties with superior knowledge. Specifically, Lin et al. (1995) empirically verify a model developed by Easley and O'Hara (1987) in which well-informed traders prefer to trade larger amounts.

With the above in mind, especially considering the factor “search friction” as identified by Amihud et al. (2005), we can define another aspect of liquidity as “immediacy”; the ability to execute a trade contemporaneously. Of course, immediacy is highly dependent on the trade size; at any given point in time, the ability to execute a trade with immediacy will differ substantially for small and large quantities.

Even though liquidity of a market as a whole, or as a sub-section of the market is of interest to us, we ultimately want to define and measure the level of liquidity of any individual corporate bond, contemporaneously.

### **2.1.3 Liquidity Proxies**

All empirical work involving liquidity and asset prices uses variables that are inherited from microstructure literature and are associated with different levels of market liquidity. We attempt to give an overview of “liquidity proxies” that are extensively used; by no means is our overview exhaustive in the proxies it lists.

The most intuitive liquidity proxy, and central in our study, is the Bid-Ask spread, as it is often regarded as an “aggregate” measure of liquidity (Hasbrouck et al., 2001). Depending on data source and asset class, this direct measure might not be readily available. An indirect, implicit approximation of the Bid-Ask spread introduced by Roll (1984), continues to be an

important proxy in recent empirical work (Dick-Nielsen et al., 2012 and Bao et al., 2011). Roll's measure (1984) constructs implicit Bid-Ask spreads based on prices alone. Recognising that negative serial dependence exists between observed price changes when dealing with a market maker, as first pointed out by Niederhoffer and Osborne (1966), Roll computes the spread as twice the negative covariance between subsequent price changes. To see why this negative dependence exists, Roll considers the following. Assuming for simplicity that all transactions are with a market maker with constant spread and no new information about the security arrives, we can assume that successive transactions are equally likely to be a purchase or sale by the market maker since trades arrive randomly at both sides, exogenously. Therefore, the joint probability of successive price changes ( $\Delta p_t = p_t - p_{t-1}$ ) depends on whether the last transactions was at the bid- or ask side. As the transaction at time  $t - 1$  is equally likely to be at the bid- or ask side, the joint distribution of successive changes in price can be written down using the table below (Table 1).

		$\Delta p_t$		
		-s	0	+s
$\Delta p_{t-1}$	-s	0	.125	.125
	0	.125	.25	.125
	+s	.125	.125	0

**Table 1: Joint Distribution of Successive Price Changes (conditional on no new information).**

Given that means of both  $\Delta p_t$  and  $\Delta p_{t-1}$  are zero and can be ignored, the covariance is simply

$$Cov(\Delta p_t, \Delta p_{t+1}) = \frac{1}{8}(-s^2 - s^2) = \frac{-s^2}{4}$$

**Equation 1**

In addition to the Roll measure, other measures of (implicit) Bid-Ask spread have been used in literature with varying results (Fong et al., 2009), but Roll remains the most widely used implicit spread measure to date. Other (implicit) spread measures include Holden's (2009) effective spread measure based on observed price clustering and the LOT effective spread measure based on the assumption of informed trading on non-zero-return days and the absence of informed trading on zero-return days developed by Lesmond et al. (1999). Another popular measure of transaction costs, inferred from transaction data is the Unique

Roundtrip Cost (URC) measure, which matches up trades of similar volume within a short time period, assuming the trades occurred at different sides of the bid-ask. Contrary to literature that uses transaction data (Goldstein et al., 2007), primarily in the equity markets, the side of a particular trade as well as the type of agent executing the trade is unknown for Dick-Nielsen et al. (2012), using the TRACE (Trade Reporting and Compliance Engine) database for US corporate bonds.

A second class of liquidity proxies directly relates to market depth, one of Kyle's (1985) market liquidity characteristics, and it assesses the price impact of trades. A liquid asset should be able to trade at substantial quantities without moving price and, conversely, price movements will reflect the depth of an asset's market. By far the most used liquidity proxy in this class is the Amihud-measure (Amihud, 2002), which captures the "daily price response associated with one currency unit of trading volume", serving as a rough measure of price impact. The defined ILLIQ measure (Amihud, 2002) defined here as the average ratio of the daily absolute return to the (dollar) trading volume on that day,  $|R_{iyd}| / VOLD_{iyd}$ .  $R_{iyd}$  is the return on stock  $i$  on day  $d$  of year  $y$  and  $VOLD_{iyd}$  is the respective daily volume in dollars. Defining ILLIQ annually

$$ILLIQ_{iy} = \frac{1}{D_{iy}} \sum_{t=1}^{D_{iy}} \frac{|R_{iyd}|}{VOLD_{iyd}}$$

Equation 2

where  $D_{iy}$  is the number of days for which data are available for stock  $i$  in year  $y$ .

The use of the Amihud-measure as a proxy for liquidity (price impact) is widespread in academic literature; from Dick-Nielsen et al (2012) who use the measure in various regression analyses on the US corporate bonds market to Hasbrouck (2009) who links trading costs and returns in US equity markets.

Its popularity and widespread use also caused the Amihud-measure to be subjected to further study, criticism and refinement. Theoretical work by Brennan et al. (2012) questioning the symmetric microstructure framework suggested by Kyle (1985) finds that equilibrium rates of return are sensitive to changes between seller-initiated trades and returns, but not sensitive to buyer-initiated trades. Whereas the Amihud-measure treats positive and negative returns the same, Brennan et al. (2013) decomposes the traditional Amihud-measure into components that correspond to up-days and down-days, pointing towards research by for

example Brunnermeier and Pederson (2009) arguing liquidity in a down market may be different from liquidity in an up market. They find that for US equity markets, the down-day component of the Amihud-measure is associated with a return premium whereas the up-day component is not significantly priced.

A third class of liquidity proxies can be referred to as trading intensity variables, which frequently covers both measures based on turnover and zero-trading-days. Turnover is intuitively defined as the ratio of the trading volume in a given period and the amount outstanding. The inverse of the turnover measure can be interpreted as the average holding period. Alternatively, zero-trading-days is a measure of trading intensity as it simply calculates the percentage of trading days with no trades in a given time period. In addition to a strict asset specific measure, Dick-Nielsen (2012), analysing corporate bond liquidity, develop a firm specific zero-trading-days measure, the number of days in a given time period where none of the bonds issued by a particular firm trade. At any time, this measure tries to capture the fact issuers will have bonds of varying maturities outstanding and a shorter waiting time between trades within a firm indicate there is relatively frequent new information about the firm.

In addition to liquidity proxies on their own, authors also aggregate various proxies into one index or one component for their analyses. Kerry (2008) builds an index by averaging nine different proxies for liquidity including six microstructure variables evenly split between various bid-ask spread approximations and price impact (all return-to-volume). Dick-Nielsen (2012) provide a comprehensive review of many liquidity proxies, all of which they subject to Principal Component Analyses to both assess communality between individual proxies and create “new” aggregate liquidity measures. They conclude that the Amihud-measure and the Unique Roundtrip Cost measure are most consistent and statistically significant in explaining bond spreads, results that hold across bond of different quality and market conditions (pre sub-prime crisis and during sub-prime crisis).

## **2.2 Liquidity Premium Estimation Techniques**

The separation of the various components of the Credit Spread is not straightforward as none of the components can be observed direct. Two (categories of) methodologies are commonly employed in academic literature, which will be discussed in the following sections. In summary, the first methodology uses CDS premia to model the credit risk born by bonds and the second methodology is based Merton’s model to price risky debt. Each of

these modelling approaches come with their own set of merits and imperfections, which will be discussed in much more detail below.

## **2.2.1 Negative CDS Basis Approach**

### **2.2.1.1 Credit Derivatives**

Credit derivatives cover a broad class of securitized derivatives whereby the credit risk of the underlying loan is transferred to an entity other than the lender (Satyajit, 2005). Credit derivatives can be divided into two groups; unfunded credit derivatives, where two counterparties sign a bilateral contract, and funded derivatives where the seller of protection puts up initial capital to settle possible credit events. Examples of unfunded credit derivatives include Total Return Swaps and Credit Default Swaps (CDS), the latter accounting for almost forty per cent of the total credit derivative market (Mengle, 2012). Examples of funded credit derivatives include (synthetic) Collateralized Debt Obligations (CDOs).

The market for CDS was non-existent before the 1990s and the creation of CDS as we know them now is often credited to JP Morgan & Co in 1994 (Stulz, 2010), selling on the credit risk from the line of credit it had extended to Exxon. The market growth of CDS since its inception is extraordinary, peaking at the end of 2007 when the notional value of the CDS market reached \$62 trillion USD (ISDA, 2010). Even when one acknowledges that the cash flow generated by the market is only a fraction of its notional value (historically only 0.2% of investment grade companies default in a year (Moody's, 2011)), it has quickly grown to become an important financial market for all investors.

The structure of a single-name CDS contract is simple and works as follows. Two parties are involved in the contract, the protection buyer who is looking to insure against the possibility of default on a particular bond and the protection seller, who is willing to bear the risk. The company that issued the bond is referred to as the reference entity, the bond itself being the reference issue. In case of a credit event (default, failure to pay or other "trigger") the protection seller agrees to buy the reference issue at face value and in return receives a default swap premium, a periodic (quarterly) fee. The contract simply expires at maturity date in case no credit event happens during its lifetime and in case there is a credit event, the protection seller buys the reference issue at face value and the periodic payments are discontinued.

### 2.2.1.2 Estimation Methodology

The CDS premium, the periodic fee at which the protection seller is willing to take on the risk of a credit event on the reference issue, is central to the Negative CDS Basis approach. Using arbitrage, Duffie (1999) shows that the spread of a corporate floating rate note (FRN) over a default free FRN should equal the CDS premium. Even though this is an approximation when applied to ordinary fixed coupon bonds, it is generally accepted. In reality, the difference between the CDS premium and the spread on the bond can be observed to be negative, implying that other factors contribute to the entirety of the bond's spread. Longstaff et al. (2005) in their direct approach, interpret this negative basis as the difference in yield between an illiquid corporate bond (synthetically free of expected defaults and credit risk) and the yield on a liquid credit risk free bond. The residual yield is then interpreted as a direct quantification of the discounted yield associated with liquidity. This model free approach allows for the, in principal, easy computation of the liquidity premium using a simple equation:

$$\text{Liquidity Premium} = -\text{CDS basis} = \text{Corporate Bond Spread} - \text{CDS premium}$$

Equation 3

The extreme simplicity of the method comes with both a set of assumptions that might not be very realistic (under some market conditions) and data / computation restrictions. The key methodological assumption, based on Duffie (1999), is that the CDS premium is only compensation for bearing the credit risk of bond. In reality CDS contracts bear other risks, one of which is counterparty (credit) risk. One can argue that during periods of calm or benign financial markets, counterparty risk can be neglected, but in the aftermath of the 2008 global crisis, counterparty risk is high on the agenda. The extreme jumps in CDS spreads for troubled financials during the second half of 2008 seem to suggest that other factors, on top of mere credit risk, might contribute to the spread.

### 2.2.1.3 Empirical Results and Real-World Considerations

Using a unique proprietary dataset with quotes and trades from fourteen CDS dealers selling protection on the same set of reference firms, Arora et al. (2012) investigate how counterparty credit risk affects CDS pricing. They find that despite the significant relation between dealer credit risk and cost of credit protection, the effect on CDS premia is small.

Specifically, they estimate that a 645 basis point increase in dealer's credit spread translates into only a one basis point decline in selling the credit protection. Theoretical work exploring the magnitude of counterparty credit risk on CDS pricing generally estimates the price effect to be in the range of 7 basis points (Kraft and Steffensen, 2007) to 20 basis points (Hull and White, 2001), implying an effect many times the empirical estimates. However, it is crucial to recognize that the theoretical literature focuses on CDS contracts in which the liabilities are not collateralized. Standard market practice during the sample period of the studies was full collateralization by both parties to the contract. Full collateralization would seem to imply that counterparty credit risk is not priced in CDS contracts. Reality is however, as became clear after the Lehmann bankruptcy, that firms posing collateral in excess of their liabilities, often required by large Wall Street dealers (Arora et al., 2012), are at risk of becoming unsecured creditors of a defaulting counterparty. Arora et al. ultimately conclude that counterparty risk is most definitely priced in CDS contracts, but estimation cannot be seen without the context of collateralization, industry sector and chosen sample period.

Another factor that is likely to contribute to the CDS spread, beyond pure credit risk, is the illiquidity of CDS contracts themselves. Empirical work on the liquidity of credit derivatives is limited; Tang and Yan (2010) use regression analyses and capture the impact of expected liquidity and liquidity risk in CDS spread, Chen et al. (2005) use the term structure of CDS spreads to find both an expected liquidity premium and liquidity risk premium is earned and Bongaerts et al. (2011) infer liquidity risk premia on CDS prices from expected excess returns. In other derivative markets, examples include Deuskar et al. (2011) who conclude that interest rate options with low levels of liquidity trade at higher prices than liquid equivalents and Cetin et al. (2006) who incorporate illiquidity into a standard Black-Scholes framework.

Bongaerts et al. (2011) apply a theoretical asset pricing model that incorporates liquidity and allows for short selling to the credit default swap market. Estimating a non-linear asset pricing model by applying Generalized Method of Moments to quotes over a sample period between 2004 and 2008, they find significant and robust enough evidence of liquidity premia to conclude that CDS spreads cannot be used as a pure measure of credit / default risk. Their results hold for the last two quarters of 2008, when both CDS spreads and bid-ask spreads increased dramatically, arguably due to counterparty risk and deleveraging.

In addition to methodological imperfections the Negative CDS Basis approach also suffers from impracticality, despite its easy-to-compute nature. Finding equivalent CDS contracts and corporate bonds by issuer and maturity is not straightforward. Using an index to



approximate, introduces further practicalities: CDS indices are not widely available across different economies, do not cover high yield bonds as the market for these CDS contracts is very thin and using an index can result in a mismatch having substantial impact on the result. In case a (set of) bond(s) can be perfectly matched with a (pool of) CDS, ignoring the above mentioned methodological issues and assuming the CDS spread is a pure measure of credit risk, the estimated liquidity premium should not be generalized to a wider set of bonds (of similar duration, notional amount or sector). The availability of CDS contracts is self-selecting in nature, the more illiquid bonds are unlikely to have an active CDS market and any estimated liquidity premium is likely to understate the liquidity premium on bonds for which no active CDS market exists. Given the self-selecting nature the liquidity premia derived from a Negative CDS Basis approach that matches bonds and CDS directly, they can only ever be applied to the same set of bonds.

## **2.2.2 Structural Model Approach**

The second method to decompose the Credit Spread is often referred to as the “Merton”- or “Structural”-approach after Merton (1974), who developed structural models for pricing risky debt.

### **2.2.2.1 Merton’s Model**

Merton’s model is of the structural kind, meaning it attempts to describe the explicit relationship between default probabilities and capital structure. Merton’s model makes clever use of option pricing theory by treating a company’s equity as a call option on its assets. Merton’s (1974) first model of default is intuitive and simple; it assumes a very simple debt structure of one zero coupon bond maturing at time T and the underlying value of the firm follows a Geometric Brownian Motion

$$dV = \mu V dt + \sigma V dz$$

where  $V$  is the total value of the firm,  $\mu$  is the asset drift,  $\sigma V$  is the volatility of assets and  $dz$  is a standard Wiener process.

If, at time  $T$ , the value of the company's assets are lower than the amount of debt (interest and) principal ( $D$ ) due to be repaid, it is (in theory) rational for the company to default, leaving an equity value of zero. In case  $V_T > D$ , the company should make the repayment, leaving an equity value of  $D - VT$ . Therefore, from option pricing theory (Black & Scholes, 1973), the value of the company's equity ( $E$ ) at time  $T$  is given by:

$$E_T = \max(V_T - D, 0)$$

This resembles the payoff structure of a call option on the value of the assets with a strike price equal to the required repayment on the debt. Using the standard Black-Scholes (1973) formula for the value of the equity today yields:

$$E_0 = V_0 N(d_1) - D e^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln \frac{V_0}{D} + \left( r + \frac{\sigma_V^2}{2} \right) T}{\sigma_V \sqrt{T}}$$

and

$$d_2 = d_1 - \sigma_V \sqrt{T}$$

with constant risk-free rate  $r$ , and volatility of assets  $\sigma V$ . The risk-neutral default probability, using a drift rate of  $r$ , of the debt is given by  $N(-d_2)$ . In order to calculate the value of the equity we require  $V_0$  and  $\sigma V$ , both unobservable quantities in the marketplace. We can however observe  $E_0$  and estimate equity volatility  $\sigma E$  from Ito's Lemma

$$\sigma_E E_0 = \frac{\partial E}{\partial V} \sigma_V V_0 \quad \text{or} \quad \sigma_E E_0 = N(d_1) \sigma_V V_0$$

This gives us a pair of simultaneous equations that can be solved numerically for values of  $V_0$  and  $\sigma V$ .

### 2.2.2.2 Extensions to Merton Model

Empirical applications of Merton's model show that there is a discrepancy between the default probabilities produced by Merton and those observed historically. The fair credit spreads estimated by Merton's model are reported to underestimate the observed credit spread in the market, as found by Jones et al. (1984) who are one of the first to calibrate Merton's model to real-world data. Bohn (2000), in his survey of risky debt valuation using option pricing, similarly found model spreads to be consistently lower than observed credit spreads. Merton's model has been criticised for making a set of strict assumptions that may not be reflecting the real-world accurately enough to produce realistic outcomes. As a result, Merton's model has been subject to many extensions over several decades, each extension addressing some of the simplifying assumptions made by Merton. The list of extensions below is by no means exhaustive, but includes well documented extensions relevant to the structural models applied empirically.

1. Default in Merton's model could only occur at the debt's maturity date. The model can be modified to allow for early default by introducing a threshold level so that default occurs as soon as  $V_t$  falls below this level. In some models this can be the result of shareholders' strategy to maximize equity value (Fan and Sundaresan, 2000). Models with a default barrier were pioneered by Black and Cox (1976) and are often referred to as First Passage models. Within the group of models that can be referred to as First Passage models, an important distinction needs to be made; those specifying an exogenous default boundary and those specifying an endogenous boundary. A typical application of a structural model with an exogenous default barrier can be found in Longstaff and Schwartz (1995). They extend Black and Cox (1976) by not having the barrier to be a (discounted) constant but to have it evolve according to its own stochastic process. A typical, straightforward example of a model that incorporates an endogenous default barrier, that is, the default barrier is derived from various other parameters, is the model by Leland (1994). The bankruptcy point in this model is the value of the firm such that the market price of equity drops to zero.
2. The firm's assets are modelled by a Geometric Brownian Motion, implying log-normal distribution of assets. Critics have argued that a simple diffusion model might not be sufficient to capture the real world dynamics of the firm. Firms that are not currently in financial distress are, under the diffusion assumption, never in danger of defaulting on (very) short term debt obligations; i.e. firms cannot default

unexpectedly. If the diffusion dynamics were accurate, we would observe zero credit spreads on short term debt, which is strongly rejected. Credit spreads on bonds with short maturities are non-zero, positive and often substantial. Both Fons (1984) and Sarig & Waga (1989) report that near-zero credit spreads on short term debt does not agree with observed credit spreads, and that the yield spread curves of certain bonds are not upward sloping as implied by the diffusion model, but are flat or even downward sloping. To allow for unexpected defaults and subsequently increase the default probability on short term debt, a jump-diffusion model for the firm's assets can be used in the Merton framework. Zhou (2001) observes that incorporating a jump risk in the default process the model matches the size of the credit spreads more closely and the yield curve can take various shapes (upward, downward and humped), even if the firm is not in financial distress. Mason & Bhattachary (1981) were the first to notice the importance of including jump processes in the valuation of risky debt. Zhou (2001) claims that his model, based on a continuous diffusion process and a discontinuous jump process is more realistic compared to their model in which the evolution on the firm follows a pure jump process with jump amplitude following a binomial distribution, for reasons of flexibility and generality.

3. The assumption of a constant risk free interest rate is not realistic and a stochastic interest rate model can be incorporated into Merton's model or any of its extensions. This also allows for the stochastic element of the evolution of the firm to be correlated with the interest rate process, if this is desired. One example of a model where interest rate is modelled by a stochastic process is a study by Shimko et al. (1993), who estimate the effect of asset and interest rate correlation on credit spread. They incorporate the short term nominal Vasicek interest rate model (1977) into Merton's model and report that for a correlation of -0.25 between interest rate and asset dynamics, the estimated credit spread is between 5 and 7 basis point below the credit reported when using a non-stochastic interest rate. Both Kim et al. (1993) and Longstaff and Schwartz (1985) show and argue that introducing a stochastic interest rate might be conceptually right, but it has a relatively small effect on credit spreads and substantially increases the complexity of the analysis. They argue that the cost of added complexity does not weigh up against introducing stochastic rates for conceptual reasons. Leland (1994), also working with a non-stochastic interest rate, notes that stochastic rates do not only unnecessarily complicate calculations, even from a real-world perspective they add very little. Referring to the criticism by Jones

et al. (1984) that contingent claim approaching to valuing risky debt produce too small credit spreads, he argues that stochastic interest rates in the Merton framework lower spreads (assuming negative correlation between the asset and interest rate processes) and therefore does not solve the problem of small spreads.

4. Assuming a single zero-coupon bond as the total debt structure of the firm, or trying to map all existing debts into a single zero-coupon bond is clearly restrictive and not always feasible, respectively. The first structural credit risk model that allows for multiple debts with different characteristics was the Geske Compound Option model developed by Geske (1977). More recent work has tackled the oversimplified capital structure of the firm and several models allow for more complex capital structures. Extensions have been made to include coupon paying debt, see for example Nielsen et al. (1993) who extend the basic Merton model to include an exogenous stochastic default boundary that is triggered when cash flows are unable to meet interest payments. In addition to models that include coupon bearing debt, more complex capital structure can also be modelled. Leland (1994) assumes a straightforward model where debt is perpetual and pays a continuous coupon stream, and Fan and Sundaresan (2000) build on this model by also assuming single-layered perpetual debt but including negotiations between creditors and shareholders in case of distress. In order to avoid inefficient liquidation, the model allows for shareholders to service debt strategically, with bargaining power  $\eta$ . For  $\eta > 0$ , the default barrier is lower than its counterpart in Leland (1994), the model defaulting to the Leland (1994) model when  $\eta$  is zero. Leland and Toft (1996) move away from the perpetual capital structure by assuming the firm continuously issues debt of maturity  $Y$ ; implying the firm is also redeeming debt issued many years ago. Therefore, at any given moment in time, the firm has various debt obligations outstanding of various durations, which are all to receive coupon. Shorter maturities place a greater burden on the firm's cash flow because of the debt that needs to be redeemed and as a result the endogenous default barrier is much higher in the Leland and Toft (1996) model than in Leland (1994), especially for shorter specified maturities. For  $Y \rightarrow \infty$ , Leland and Toft (1996) converges to Leland (1994).
5. Almost all extended models include a continuous payout as part of the evolution of the firm's assets. Adjusting the drift rate to account for negative cash flow can be thought of as dividend payments to its shareholders. This simple extension was

offered by Merton himself in his first publication and is analogous to the pedagogical extension of the Black-Scholes model with continuous dividend payments. The value of the firm's assets then evolves as a Geometric Brownian Motion under the real world risk measure:

$$dV = (\mu - \delta)Vdt + \sigma Vdz$$

where  $\mu$  is  $(r + \lambda)$ , with  $r$  the risk free interest rate and  $\lambda$  the risk premium and  $\delta$  is the payout ratio.

6. In addition and similar to the inclusion of a payout ratio in the evolution of the firm's assets, many extensions (see for example, Leland (1994) or Fan & Sundaresan(2000)), include tax advantages in their model. This ties in directly with model extensions regarding the firm's capital structure since the tax advantage is applied to coupon payments and is added to the evolution of the firm's assets as a tax shield.
7. Further refinement of a more complex / realistic debt structure is needed as debt does not only vary in maturity but can have various seniority / priority structures too. Several extensions of Merton's model allow for debt to be issued with various levels of seniority (see for example, Benos & Papanastasopoulos (2007)). Debt of lower seniority is valued at a higher credit spread due to a decreased recovery rate in the event of default. Recovery rates represent the percentage of the face value of the debt that is received by debt holders in case of bankruptcy of the firm. These recovery rates are exogenous in structural models but are thought of to vary on a firm-to-firm basis in the real world, depending on the outstanding debt across seniority classes. Recovery rates are calibrated to fit historical averages across the market. Models that take into account seniority of debt assume that absolute-priority rules are fully adhered to in case of default so that outstanding debt is paid off in strict order of seniority. Empirical evidence suggests that absolute priority rules are violated in reality; Franks and Torous (1994) investigate firms that have either entered bankruptcy through Chapter 11 of the Bankruptcy Reform Act 1978 or have informally completed a distressed exchange of traded debt. Firms entering bankruptcy under Chapter 11 only do so after attempting to resolve financial issues informally, but these firms often benefit from specific provisions in Chapter 11 because they are less solvent or liquid. As a result, the average expected recovery rate for creditors' claims under a Chapter 11 bankruptcy are approximately 30% lower than recovery

rates under more informal proceedings. In addition, Eberhart et al. (1990) examined bankruptcy proceedings under the same Act and measured the amount paid to shareholders in excess of what they should have received under strict absolute priority rules. Eberhart et al. (1990) report this percentage to be 7.6% and also found evidence that equity markets expected deviations from the absolute priority rule as common share values reflect part of the value ultimately received by a violation of the strict rules.

### **2.2.2.3 Empirical Results Structural Models**

Structural models of default attempt to mathematically describe how default occurs, and what dynamics exist between the various factors playing a role in default. Most of the structural models that relax one or more of the simplifying assumptions are theoretical in nature and describe dynamics of the firm and the dynamics of the firm's debt. No structural model tries to relax all of the simplifying assumptions nor do any of the structural models claim to be an accurate representation of the real-world dynamics of the firm. Instead, structural models allow us to study the dynamics one (or several) specific factors and probability of default / credit spread. Examples the effect on credit spread of stochastic interest rates and a correlation between firm's asset process and interest rate process or the effect of a jump-diffusion process for the firm's assets on the default probability of debt with short maturities. The models themselves are theoretical in nature, the credit spreads (or default probabilities) produced by these models require values for all the parameters in the model; calibration.

The standard reference for the discrepancy between credit spreads observed historically and those produced by (early) structural models after calibration is Jones et al. (1984), who reported the yield spreads derived from structural models to underestimate historical spreads. They report that on average, predicted spreads underestimate observed spreads by 4.5%, the errors being the largest for speculative grade bonds. In addition they find that pricing errors are significantly related to maturity, equity variance and leverage. Ogden (1987), in a similar study of bond prices between 1977-1981, finds that the Merton model underestimates spreads by 104 basis points on average. Both studies emphasized the lack of stochastic interest rates in their conclusions. Given the time period under study, this is no surprise since US Treasury rates were extremely volatile as a result of the Federal

Reserve's money supply target during 1979-1982, when inflation reached double-digits. Few implementations of structural models using individual bond prices do not appear until Lyden and Saraniti (2000), who use a sample of non-callable bonds to calibrate and fit both Merton's original model (modified to treat coupon bonds as if they were a portfolio of zero-coupon bonds, each of which can be priced using the standard zero-coupon version of the model) and the model derived by Longstaff and Schwartz (1995), in which the firm issues a constant amount of new coupon paying debt with a fixed maturity and in which equity holders have to the option to issue new equity to service the debt or default. They find that both models underestimate yield spreads and pricing errors are correlated with coupon rate and maturity.

More recently, Eom et al. (2004) have taken five structural models, calibrated the required parameters of all models to the same sample of 182 bonds during the period 1986-1997 and compared spread predictions across models and with observed historical spreads. Using the Fixed Income Database they choose a very particular set of bonds to include in their study, meeting strict requirements; only non-financial firms are included to ensure the leverage ratio is comparable across the sample, utilities are excluded since return on equity and revenues (and thereby probability of default) are dependent on regulatory influences, only fixed-rate coupon bearing bonds that are not convertible and only bonds with a simple capital structure (firms with a maximum of two publicly traded bonds and exclusion of subordinated debt) are included. In addition, the chosen firms must have publicly trading stock in order to qualify for a structural approach in the first place. Eom et al. (2004) implement five structural models (Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001)) and, contrary to previous empirical literature, fail to conclude structural models and incapable of producing sufficiently high yield spreads. They do agree that the five structural models cannot accurately price corporate debt, but the difficulties are far from limited to an underestimation of spreads. Whereas Merton (1974) and Geske (1977) are reported to consistently underestimate yield spreads, as previous work indicated, Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001), on average, produce yield spreads that are too high. Both the Longstaff and Schwartz (1995) model and the Collin-Dufresne and Goldstein (2001) model have an incredible dispersion of predicted spreads; often they are either very small or extremely large. That across the sample of bonds this averages out (slightly) higher on average is rather irrelevant as prediction error on a bond-to-bond basis is often a magnitude several times the average prediction error. The Leland and



Toft (1996) model is different in the sense that it appears to consistently produce yield spreads that are too high, which Eom et al. (2004) attribute to the simplifying assumptions about coupons.

#### **2.2.2.4 Estimation Methodology**

Structural models have found limited use in the estimation of liquidity premia on corporate bonds. The underlying idea behind its use stems from the assumptions that the fair credit spread produced by structural models is obtained by viewing equity as a call option on its assets with the strike price equal to the face value of its debt. Therefore, the model exclusively describes credit risk and excludes any part of the spread associated with liquidity. The liquidity premium can then simply be interpreted as the residual between the estimated yield spread and the observed yield spread. The fact that there is ample empirical evidence that structural models fitted to historical data underestimate the observed yield spread (Jones et al. ,1984), seems to confirm this rationale. Confusing and contradictory seems to be the academic literature both criticising structural models, arguably accounting for credit risk only, and their incapacity to accurately match historically observed yield spreads while on the other there is ample theoretical and empirical evidence for the existence of a liquidity premium. A liquidity premium would imply that, given structural models only value credit risk, yield spreads produced by structural models are expected to be lower than observed spreads. Despite the evidence produced by liquidity premia literature, structural models are continuously refined and are, at least in part, judged by their ability to match historical yield spreads.

In the United Kingdom, the Bank of England used Leland and Toft's (1996) structural model to derive economy-aggregated liquidity premia, the results of which will be discussed shortly. In their implementation, the structural model derives the fair credit spread of a representative bond (either investment grade or high yield) and inputs in the model are therefore derived from average across the economy rather than specific to an issuer (or individual bond). The decomposition of the credit spread into the three typical components requires the model to be calibrated and run twice, with different risk preferences for the aggregate investor. The fair credit spread (compensation for expected default losses and compensation for unexpected losses added together) is obtained by setting the risk premium  $\lambda > 0$ , in the equation describing the evolution of the firm's assets.

$$dV = (r + \lambda)Vdt + \sigma Vdz$$

The compensation demanded by investors for the expected losses can be isolated by assuming that investors still demand a risk-bearing rate of return, but discount cash flows at the risk-free rate ( $\lambda = 0$  in the above). In short, this can be referred to as computing the fair credit spread under the real world probability measure and the risk neutral probability measure respectively. The component for unexpected default losses is then computed as the difference between the real-world yield spread and the risk-neutral yield spread. The size of what is labelled the non-credit related spread which represents, arguably among other things, the liquidity component, is obtained by subtracting the estimated fair credit spread and the observed aggregate (market) credit spread.

#### **2.2.2.5 Empirical Results and Real-World Considerations**

Using structural (Merton-style) models to estimate the liquidity premium on corporates bonds comes with a set of restrictions (on the data side), assumptions (on the model side) and arguably concerns over its validity altogether.

The simplifying assumptions made by Merton's original model, as well as the many extensions that either relax some assumptions or introduce other refinements, have been discussed in detail. As far as assumptions are concerned when implementing the model on a bond-to-bond basis, it is crucial to note that these models are not designed to be calibrated to, for example, complex real world debt structures of a firm. This is easily illustrated by the implementation of several structural models by Eom et al. (2004) who were very strict in selecting only those firms with a simple capital structure. They also excluded all financial firms as they have very different leverage ratios to non-financials; leverage ratios that are common for financials, are considered extremely dangerous for non-financials and would imply extreme spreads on those debts. Even the more subtle exclusions of convertible bonds, exclusions of utilities or the exclusion of with call options bonds, illustrate that structural models are not designed to capture the real world effect of the many aspects of a firm's or the market's structure.

The reported inaccuracy of various implemented structural models (Eom et al., 2004), rather than the (in)ability to predict spreads on average, is important since it illustrates that even in the hypothetical scenario where all data is perfectly and instantly available, current

structural models cannot be used to calculate yield spreads (or liquidity premia) on a bond-to-bond basis. The rationale followed by Webber (2007) to use a structural model to infer liquidity premia on an aggregate level, rather than infer on a bond-to-bond basis and aggregate the results subsequently to represent the state of the economy, is no surprise.

Even accepting that the estimation of liquidity premia using structural models cannot be done on a bond-to-bond basis and one decides to take an aggregates approach similar to the one followed by Webber (2007), there is ample choice between the structural models. Model choice obviously depends on data availability and reliability, but Eom et al. (2004) show that a range of structural models can be fitted using the same data. They also show that while structural models on the one hand produce similar outcomes, confirmed by Huang & Huang (2002), there are also substantial differences in the outcomes. Working from the assumption that all models produce similar results, at least in terms of producing similar yield spreads on average, one still needs to choose one structural model from a growing body of models. The Bank of England (Webber, 2007) chose to work with the Leland and Toft (1996) model and justified its choice for the model: “it provides a simple solution for coupon-paying non-callable bonds with a fixed maturity, which are the focus of this study”.

Aside from concerns over model implementation and assumptions, even the simplest structural models need a considerable number of parameters, whereas sophisticated extensions often require even more. This becomes an issue when using structural models for real-world purposes. For bond-to-bond applications, but also for aggregate applications similar to Webber (2007), the availability of input parameters becomes a concern. Deriving parameters for individual bonds might simply be impossible whereas for aggregate applications, it introduces a high degree of parameter uncertainty.

### **3. iBoxx Dataset**

The dataset used during the course of the PhD project comes from Markit, a financial information services company providing independent data, valuations and trade processing of assets. Markit aims to enhance transparency in financial markets and improve operational efficiency for its clients, predominantly institutional investors.

The iBoxx indices provide fixed income bond data on a daily basis, essential for many market participants in structured products, fixed income research, asset allocation and performance evaluation. iBoxx indices cover Euro, Sterling, Asian, US Dollar denominated markets, both investment grade and high yield. In addition to daily consolidated prices, a range of analytical values are provided for all the bonds in the Markit bond universe. For the PhD project we make use of the iBoxx GBP Investment Grade Index (historical); the majority of information presented below comes from the Markit iBoxx GBP Benchmark Guide (Markit, 2012a).

#### **3.1 Markit Bond Universe**

Several selection criteria are used to determine eligibility for index inclusion. The obvious restrictions are that only bonds, as apposed to any other money market instruments, are eligible; the issuer's domicile is not relevant. As far as bonds with certain characteristics are concerned, the following bond types are specifically excluded: bonds with American call options, floating-rate notes and other fixed to floater bonds, optionally and mandatory convertible bonds, subordinated bank or insurance debt with mandatory contingent conversion features, Collateralized Debt Obligations or bonds collateralized by CDOs. In addition, retail bonds and private placements are reviewed by iBoxx' technical Committee on an individual basis and excluded if deemed unsuitable. On the other hand, bonds with the following characteristics are included: "plain vanilla" fixed coupon bonds, zero coupon bonds, amortizing bonds, set-ups and step-up callable bonds with European options, callable/puttable and extendable bonds with European options, event-driven bonds such as rating- or tax-driven bonds or soft bullet bonds.

In addition to restrictions on the bond type, all bonds in the Markit iBoxx GBP universe must have a Markit iBoxx Rating of investment grade (Markit, 2012b). The average rating of Fitch Ratings, Moody's Investors Service and Standard & Poor's Rating Services determines the index rating. Investment grade is defined as BBB- or higher from Fitch and

Standard & Poor's and Baa3 or higher from Moody's. Ratings from the rating agencies are converted to numerical scores. The numerical score is averaged and consolidated to the nearest rating grade, the iBoxx Rating system does not use tranches.

Eligibility for inclusion is also conditional on the amount outstanding, where the issue needs to be of a minimum size. Gilts need to have an outstanding amount of at least GBP 2bn, whereas the minimum amount for non-Gilts is set to 250m.

### **3.2 Bond Classification**

All bonds are classified based on the principal activities of the issuer and the main sources of the cash flows used to pay coupons and redemptions. In addition, a bond's specific collateral type or legal provisions are evaluated. Hence, it is possible that bonds issued from different subsidiaries of the same issuer carry different classifications. The issuer classification is reviewed regularly based on updated information received by Markit, and status changes are included in the indices at the next rebalancing if necessary. This implies that even though several analytical values or variables in the dataset are considered "static", there is the possibility of them changing. The main sector classifications within the Markit iBoxx GBP Index are the following:

- Gilts. Bonds issued by the UK central government denominated in Sterling.
- Sovereigns. Bonds issued by a central government other than the UK and denominated in Sterling.
- Sub-Sovereigns. Bonds issued by entities with explicit or implicit government backing due to legal provision or the public service nature of their business.
  - o Agencies: Bonds issued by entities whose major business is to fulfil a government-sponsored role to provide public, non-competitive. Often, such business scope is defined by a specific law, or the issuer is explicitly backed by the government.
  - o Supranationals: Bonds issued by supranational entities, i.e. entities that are owned by more than one central government (e.g. World Bank, EIB).
  - o Public Banks: Bonds issued by publicly owned and backed banks that provide regular commercial banking services.
  - o Regions: Bonds issued by local governments (e.g. Isle of Man).
- Collateralized.

- Covered Bonds. Bonds which are secured by a general pool of assets in case the issuer becomes insolvent.
- Securitized Bonds. Asset Backed Securities (ABS), Housing Associations, Mortgage Backed Securities (MBS).
- Corporates. Bonds issued by public or private corporations. Bonds secured by a “floating charge” over some or all assets of the issuer are considered corporate bonds. Corporate bonds are further classified into Financials and Non-Financials bonds and then into their multiple-level economic sectors, according to the issuer’s business scope. The category insurance-wrapped is added under Financials for corporate bonds whose coupon and principal payments are guaranteed by a special monoline insurer. For a full overview of Economic-, Market- and Market Sub-Sector, please see Appendix I, Table 1.

In addition to a classification based on the issuer’s business scope and activities, corporate debt is further classified into senior and subordinated debt. Subordinated debt is mostly issued by financials, but other corporate issuers might be forced to if indentures on earlier issues mandate their status as senior bonds. Subordinated debt can be especially risk-sensitive since the bond holders only have claims on an issuer’s assets after other bond holders without the upside potential that shareholders enjoy.

Capital in the form of debt instruments is always sub-ordinated because senior debt does not count towards bank capital. From a regulator’s point of view the bank’s capital serves as protection of depositors, a safety net that can absorb unexpected losses to guarantee depositors. A financial institution’s debt can be categorized as one of the following:

- Tier 1: Shareholder’s equity and retained earnings are commonly referred to as a bank’s “core” (Tier 1) capital for regulatory purposes. The Tier 1 capital, as issued debt, consists of other securities qualifying as Tier 1.
- Upper Tier 2: From a regulatory perspective of a bank’s capital, Tier 2 debt comprises undisclosed reserves, revaluation reserves, general provisions, hybrid instruments (preferred) and subordinated term debt
- Lower Tier 2: From a regulatory perspective of a bank’s capital, only 25% of a bank’s total capital can be Lower Tier 2 debt; it is easy and cheap to issue. In order to ensure that a bank’s capital from subordinated debt issues does not fall substantially after and

issue matures, the regulator demands that Tier 2 capital debt amortises on a straight line basis from maturity minus five years.

The market information on the tier of subordination for insurance capital is often less standardized than the equivalent issues by banks. In these cases, the classification is based on the maturity, coupon payment and deferral provisions of the bond from the offering circulars of the bonds (Markit, 2012a).

Bonds with option-like characteristics (embedded options) are included in the dataset, as detailed in section 3.1. These option-like features are part of the bond rather than separately traded securities, the option-like characteristics are not necessarily mutually exclusive; one bond may have multiple option features embedded. Embedded bonds can include but are not limited to:

- Callable Bonds: Bonds that give the issuer the option of buying back the bond at a predetermined price at some point in the future. The lockout period refers to the initial time period in which the bond cannot be redeemed by the issuer.
- Puttable Bonds: bonds that give the bond holder the option to demand early redemption at a predetermined price at some point in the future.
- Convertible Bonds: Bonds that give the bond holder the option to demand conversion of bonds into stocks at a predetermined price at some point in the future.

Depending on the type of option feature(s) embedded in a bond, the Credit Spread can either be higher or lower than the Credit Spread for an equivalent bond without option features. No information about a bond's optionality is included in the dataset explicitly.

### **3.3 Bond Price Consolidation**

Markit iBoxx index calculations are based on multi-sourced pricing which, depending on the structure of each market, takes into account a variety of data inputs such as transaction data, quotes from market makers and other observable data points. For indices calculated in real-time, for example EUR and GBP, the source of data is quotes from market makers. Currently ten market makers submit prices, including Barclays Capital, Goldman Sachs, HSBC, Deutsche Bank and JP Morgan. All submitted prices and quotes have to pass through a three-step consolidation process (Markit, 2013).

The first step is to pre-filter, to technically validate the quote format and timing of the contribution from the market maker. Secondly the dispersion of the quotes that pass the technical validation is checked. If the contributions from market contributors pass the Maximum Distance Test (the distance between the highest and lowest quote is within a specified limit), all contributions are eligible for consolidation. If contributions fail this test, they are subjected to a potential two further checks (Inner Distance Test and Markit Control Price) to determine eligibility. The last step of the process is the consolidation of contributions into bid- and ask-quotes. If less than two quotes are valid, no consolidated quotes can be generated. If two or three quotes are received, the consolidated quote is determined as the arithmetical mean of all eligible quotes. Lastly, if four or more quotes are received, the highest and lowest quotes are eliminated. Thereafter the arithmetic average of the remaining eligible quotes is calculated to determine the consolidated quote. For more detailed information about the price consolidation process, please see Markit's Pricing Rules (Markit, 2013).

### **3.4 Analytical Values**

In addition to a large amount of descriptive classification about a bond, a range of analytical values is available on a daily basis. This section aims to give an overview of the most important of these analytical values for the PhD project to date, focussing on Spread measures in particular. The analytical values that are available on a daily basis include key measures such as Years to Maturity, (Modified) Duration, (Annual) Yield, (Annual) Convexity and Coupon. In addition to these key measures many more are reported on a daily basis including Daily Returns, Month-Date>Returns, Number of Contributors, Excess Return over Sovereigns, Duration weighted exposure and next call date. Yield calculations use the bid price of the bond. Please refer to Appendix I for detailed formulae for several key measures.

### **3.5 Credit Spreads**

During the course of the sample period of the study (Oct. 2003 – May 2013) there have been more than a handful of changes to the availability of certain analytical values. Most importantly, over the years the available pieces of information of every bond on any given day has increased from 40 in October 2003 to 82 in May 2013. For this reason, to maintain



backwards compatibility in order to include the entire sampling period, many of the newer bits of information need to be excluded. This is not a problem for the majority of information as they add very little value (for example, 7 buckets of Expected Remaining Life), but when it comes to Spread calculations it is important to note what changes have occurred during the sample period. For the last day included in the sample, four different spread measures are reported on a daily basis.

### 3.5.1 Asset Swap Margin (ASW)

The Markit SWAP curve ( $z_t$ ), constructed from Libor rates and ICAP swap rates, is central to the process of asset swap spreads calculation. As soon as the curve is defined the present value of fixed and floating payoffs is calculated, and the asset swap spread is determined. From the perspective of the asset swap seller, assuming all payments are annual and made on the same day, the present value is

$$P = 100 + C \sum_{i=1}^{N_{\text{fixed}}} z(t_i) - \sum_{i=1}^{N_{\text{floating}}} \Delta_i(L_i + A)z(t_i)$$

Equation 4

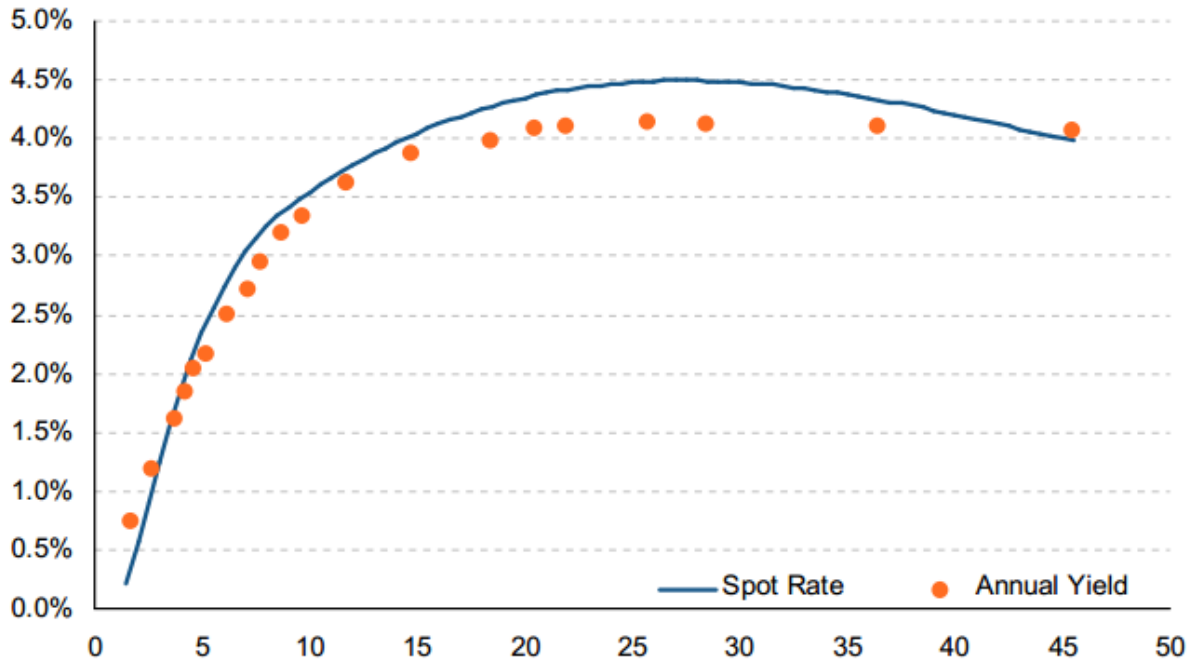
where  $Q$  is the annual coupon payment,  $L_i$  is the LIBOR set at time  $t_{i-1}$  and paid at time  $t_i$ ,  $\Delta_i$  is the accrual factor in the corresponding basis. The equation is iteratively solved for  $A$ , the asset swap margin.

### 3.5.2 Z-Spread

The Z-spread is a measure of the spread the investor would realize over the entire benchmark zero coupon curve if the bond is held to maturity and does not default. The Z-spread is calculated as the spread that will make the present value of the cash flows of respective bond equal to the market dirty price, when discounted at the benchmark spot rate plus the spread.

Benchmark zero curve  $z_t(L)$ , also known as spot rate curve, is calculated from dirty prices of defined benchmark bonds. The curve is constructed using natural splines with the annual yield of the benchmark bonds as defined knots. The Markit documentation uses the

following figure (Figure 1) to illustrate its construction of the benchmark zero curve using natural splines (Markit, 2012a).



**Figure 1: Construction of Benchmark Zero Curve.** On any given day 20-25 Gilts are classified Benchmark bonds. The 25 chosen bonds are used as determined knots and a curve is derived using natural splines

If the (dirty) price of bond ( $b$ ) is  $P_b$ , then the Z-spread ( $zsp$ ) is the value such that

$$P_b = \sum_{i=1}^N \frac{CF}{(1 + z_t(L_i) + zsp)^i} + \frac{FV}{(1 + z_t(L_N) + zsp)^N}$$

Equation 5

### 3.5.3 Option Adjusted Spread (OAS)

Similar to Z-spread, the OAS is the spread over the benchmark zero coupon curve realized if the bond is held until maturity. The major difference is the interest rate volatility assumption used in OAS. Due to the fact that interest rate changes can affect the cash flows of the security with embedded option the following relationship can be highlighted:

$$ZSpread = OAS + Option Cost$$

The OAS is calculated as the spread that will make the present value of the cash flows of the bond equal to the dirty price when discounted at the benchmark spot rate plus the OAS spread. Given spot rates and empirical volatility, a binomial interest rate tree is derived using the iterative search method. The theoretical price of the bond is found by conventional backward induction of future cash flows. The OAS is found iteratively by using the Newton method, such that adding the spread to every node of the interest rate tree would make the present value of the cash-flows equal to the market dirty price of the bond.

Intuitively, the relationship between the ZSpread and the OAS is straightforward. For example, for callable bonds, the option benefits the issuer (it allows him to buy back the bonds if rates go down, i.e. bond prices go up), and  $Option\ Cost > 0$  hence  $OAS < ZSpread$ . For putable bonds the opposite is true and for options without embedded features the spread measures are equal.

### 3.5.4 Annual Benchmark Spread

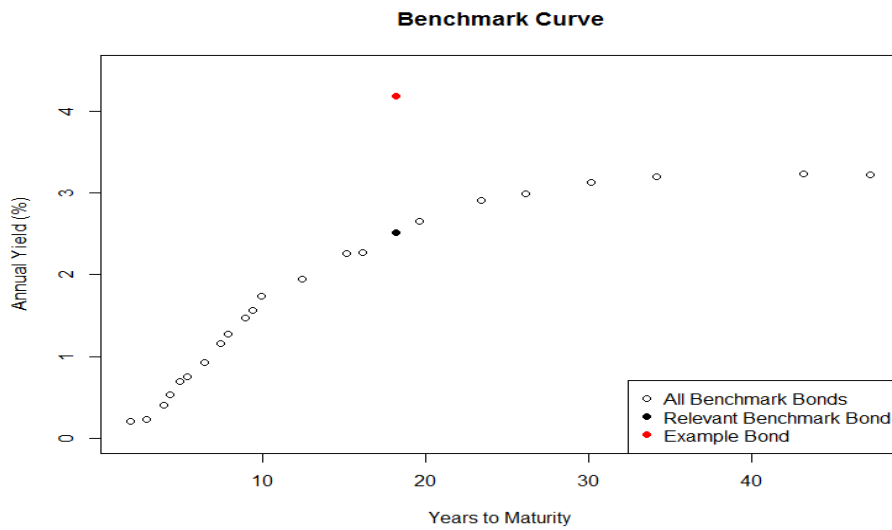
The benchmark spread can be defined as a premium above the yield on a default-free bond necessary to compensate for additional risk associated with holding the bond. The default-free yield to maturity is found by a linear interpolation of two benchmark bonds with maturities being just above and just below the time to maturity of a bond:

- Government bond is selected as an approximation of a “default-free” bond
- The difference between maturities of a bond and the benchmark bond is the smallest in comparison to other alternatives

The annual benchmark spread of a bond  $i$  at time  $t$  is:

$$BMS_{i,t}^a \begin{cases} 0 \\ Y_{i,t}^a - Y_{BM(i),t}^a \end{cases}$$

where  $Y_{i,t}^a$  is the annualized yield of bond  $i$  at time  $t$  and  $Y_{BM(i),t}^a$  is the annualized yield of benchmark bond  $i$  at time  $t$ . Note that for benchmark bonds  $BMS_{i,t}^a = 0$ . On any given day 20-25 Gilts are classified benchmark bonds. For example, on 9<sup>th</sup> of October 2012 using the iBoxx dataset, 23 Gilts are classified benchmark bonds used to calculate the annual benchmark Spread. Figure 2 below plots those 23 bonds (years to maturity versus annual yield), and shows how the spread is calculated.



**Figure 2: Calculating the Annual Benchmark Spread.** Rather than using a benchmark curve, as derived in section 3.5.2, the Benchmark Spread is based on the difference in annual yield between a bond and the corresponding benchmark bond. A benchmark bond is chosen as to minimise the difference in maturity between a bond and its reference.

Throughout the study we specifically refer to the Annual Benchmark Spread when mentioning to the “Credit Spread”. The choice for the Annual Benchmark Spread over the other spread measures is twofold. Firstly, the OAS and the Z-Spread are only available for days after June 2009 when the value was first reported as part of the index. Backwards compatibility is important to ensure consistency throughout the entire time period. Secondly, the spread is intuitively very easy and the reference point (effectively Gilts) is a readily tradable asset in its own right.

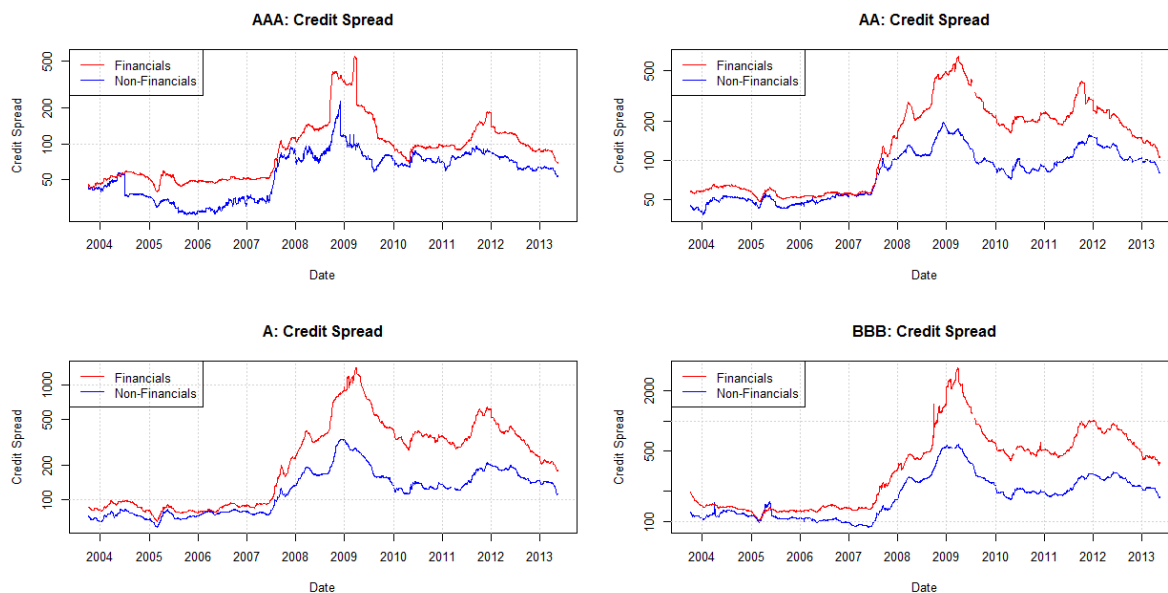
Since the Benchmark Spread is fitted to actual bonds in the database rather than a Benchmark Curve (linear interpolations) jumps in a bond’s benchmark spread can occur when its changes benchmark (bond). This should be documented.

Unfortunately not all spread measures are available for the entire time period under study. To maintain backwards compatibility, the Annual Benchmark Spread is the preferred measure of Spread. Future work should confirm the stability of regression coefficients and results when using a different measure of spread (on a subset of the data). This will add to the validity and robustness of the proposed modelling approach.

## 4. Exploring Credit Spreads

The Markit iBoxx dataset provides Annual Benchmark Spreads, for all bonds on every trading day which makes computations readily available. This allows us to monitor Credit Spreads through time for all individual bonds, but also allows us to monitor Credit Spreads at more aggregate levels; one can think for example, of Spreads aggregated by Rating, by Financials and Non-Financials, by Senior and Subordinate debt, or by a combination of several of the above. When examining data by means of plots, we will often split the data in eight categories or groups; split by Rating (AAA, AA, A and BBB) and Financials / Non-Financials. The split into eight categories is granular enough to get a good idea of what different segments of the market were doing at any one point, yet sufficiently aggregate not to get lost in details.

To accurately describe the distribution of Credit Spreads for eight of the eight categories on any given day during the sample period (Oct. 2003 – May 2013), we plot the Credit Spread time series using the mean Credit Spread as well as the interval between which 90% of all observed Credit Spreads lie. Since the plot is of considerable size it is moved to Appendix II Figure 1, and a smaller summary plot without daily intervals is included below (Figure 1, Credit Spread on a logged Y-axis).



**Figure 3: Summary of Credit Spreads (log scale) by Rating and (Non-)Financials.** Credit Spreads move together across Rating class (AAA, AA, A and BBB); this hold for both Financials and Non-Financials. Before 2007, Credit Spreads for Financials and Non-Financials are very similar but diverge in late 2007. Not only are Credit Spreads currently following a downward trend, Financials and Non-Financials have similar Spreads again; this does not hold for BBB-rated bonds.

Please refer to Figure 1 in Appendix II for a thorough investigation of Credit Spread, as Figure 1 above does not show daily intervals. Several, perhaps obvious, observations can be made from the time series plots:

- 1.) Credit Spreads move together, regardless of Rating and Financial Status. Credit Spreads across all eight groups start to increase mid-2007, peak early 2009 and decline rapidly in the following year.
- 2.) Credit Spreads for Financials and Non-Financials seem to be very similar (within a given Rating category) in the years prior to the “credit crunch”. During the crisis years Spreads of Financials rise substantially more than Spreads on Non-Financials; which is no surprise given the nature of the crisis.
- 3.) The distribution of Credit Spreads on any given day is narrow for the years leading up to the “credit crunch”, widens substantially during the crisis and remains wide in more recent years too.
- 4.) Credit Spreads increased in 2011 as a result of the debt crisis in Europe. The Credit Spreads peaked early 2012 and have since been declining. The trend continues downwards to this day.

#### **4.1 Timeline / Event**

Rather than treating “the credit crunch” and the subsequent “sovereign crisis” as single events or single periods of time, major financial events can be looked at on a (near) individual basis. This allows us to subjectively quantify the impact certain events had on Credit Spreads of the aforementioned groups and perhaps whether the market responded prior to the event. Fourteen major financial events have been selected:

A: 05-05-2005

Tony Blair is re-elected as Prime Minister of the United Kingdom by popular vote, with 35.2% the lowest majority government in British history

B: 09-08-2007

BNP Paribas indicate they cannot value the complex assets (CDOs) for three of their funds, for which trading freezes. It is the first major bank to acknowledge the risk of exposure to sub-prime mortgage markets. Northern Rock's chief executive later reflects saying that it was "the day the world changed"

C: 14-09-2007

Northern Rock has borrowed large sums of money to fund mortgages for customers, and needs to pay off its debt by reselling (or "securitising") those mortgages in the capital markets. Due to fallen demand, Northern Rock faces a liquidity crisis and it needs a loan from the British government. This sparks fears that the bank will shortly go bankrupt; Britain's first bank run since 150 as a result.

D: 17-02-2008

The government nationalises the troubled mortgage lender Northern Rock as a result of prolonged liquidity problems.

E: 14-03-2008

The investment bank Bear Stearns is bought out by JP Morgan. This makes it the biggest player fallen in the crisis.

F: 07-09-2008

The US government bails out Fannie Mae and Freddie Mac – two huge firms that had guaranteed thousands of sub-prime mortgages.

G: 15-09-2008

Lehman Brothers, deeply involved in the sub-prime mortgage markets, files for bankruptcy causing worldwide financial panic.

H: 17-09-2008

The UK's largest mortgage lender, HBOS, is rescued by Lloyds TSB after a huge drop in its share price.

I: 08-10-2008

Iceland's three biggest commercial banks – Glitnir, Kaupthing, and Landsbanki – collapse. To protect the deposits of their many British customers, Gordon Brown uses anti-terror legislation to freeze the assets of the banks' UK subsidiaries.

J: 13-10-2008

The British government bails out several banks, including the Royal Bank of Scotland, Lloyds TSB, and HBOS.

K: 02-04-2009

The G20 agrees on a global stimulus package worth \$5tn.

L: 02-05-2010

Signalling the start of the Eurozone crisis, Greece is bailed out for the first time, after Eurozone finance ministers agree loans worth €110bn. This intensifies the austerity programme in the country, and sends hundreds of thousands of protesters to the streets.

M: 28-11-2010

European ministers agree a bailout for Ireland worth €85bn

N: 05-05-2011

The European Central Bank bails out Portugal

O: 21-07-2011

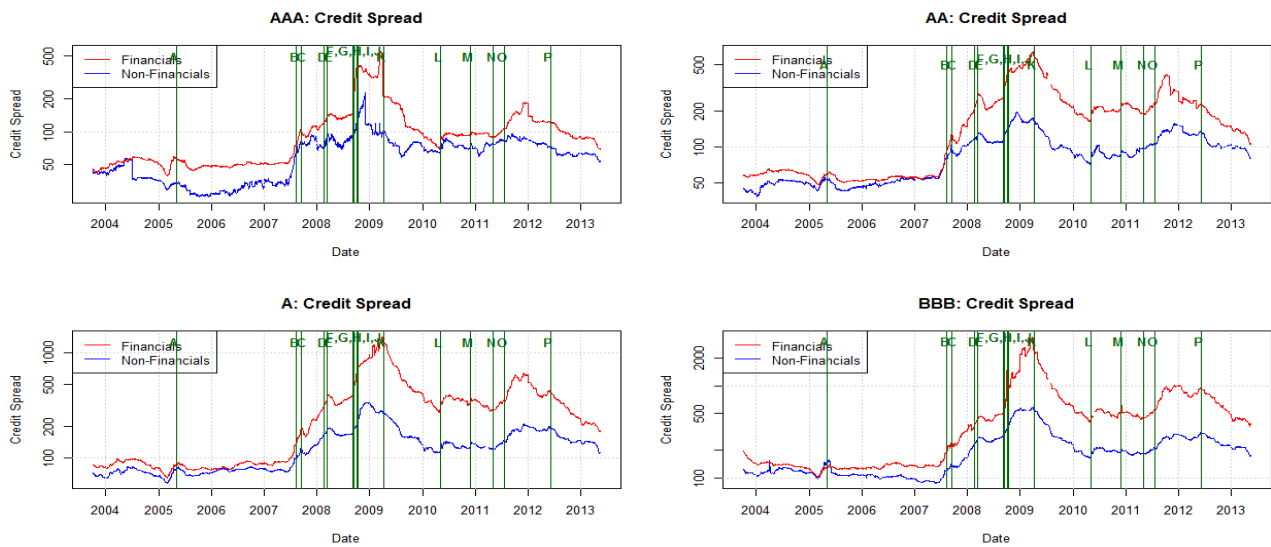
Greece is bailed out for a second time, after it failed to get all its affairs in order

P: 06-06-2012

The level of Spanish borrowing reaches a record high, indicating the Euro-crisis is still ongoing and recovery is a slow process



The events from the timeline have been added to the Credit Spread time series plot (Figure 4).



**Figure 4: Timeline overlay; Credit Spreads by Rating and (Non -) Financials.** Many events from the Financial world, and those associated with the Credit Crunch and European Sovereign crisis in particular, can be linked to (sudden) movements in the Credit Spreads.

Prior to the May 2005 parliamentary elections (event A) Credit Spreads increased for some time. Elections were announced on April 4<sup>th</sup>, a month before the elections on May 5<sup>th</sup>, but the date of the general election had been covered in the media for weeks. Uncertainty over the outcome may have been the increased risk causing higher spreads. The reason why the upcoming elections would cause an increase in spreads is unknown and might very well be spurious. The subsequent two events, B and C, are often regarded as the events that signify the start of the “financial crisis” and coincide with the start of the increase in Credit Spreads throughout the crisis. Looking at the time series closely we can see that the increase in Spreads precedes the actual events, indicating the market seemed to foresee these and later events. Looking very closely at B and C we can actually see that immediately after Northern Rock’s loan from the UK Government (event C), the Credit Spreads appear to drop, or at least move sideways for a brief period of time. The unrest in September 2008 later appeared not be foreseen by the market as the large jump in spreads exactly coincides with the events. One could argue that for financials markets this might have been the true start of the crisis. Spreads continue to increase until the UK government bailout of the Royal Bank of Scotland, Lloyds TBS and HBOS on 13 October 2008. Whereas the bailout coincides with the peak we see for Financials, it appears as if the non-Financials peaked a few months earlier, and were on the way to recover before Financials. The bailout seems to start a steady downward trend which continues well into 2011, only to be interrupted for a very brief period of time due to

the bailing out of Greece (event L) and Ireland (event M). The bailing out of Portugal followed by the second bailing out of Greece two months later (events N and O) cause another steady increase in Spreads until 2012 when again Spreads are slowly declining, a trend that is slow and continues to this day.

## 5. Bid-Ask Spread Analysis

### 5.1 Introduction

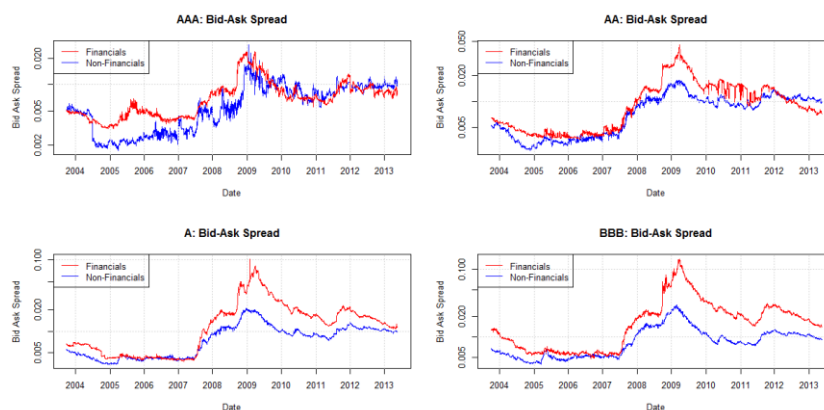
Using the daily bid- and ask prices from the data, the Bid-Ask spread is easily computed. On any given day  $d$ , the Bid-Ask Spread of bond  $i$  is defined as

$$BAS_{d,i} = \frac{Ask\ Price_{d,i} - Bid\ Price_{d,i}}{Bid\ Price_{d,i}}$$

Equation 6

Since the Bid-Ask spread represents the costs associated with a purchase and instantaneous sale of an asset, it is referred to as the Unique Roundtrip Cost by some. Dick-Nielsen et al. (2012) for example, estimate the Unique Roundtrip Cost from transaction data by identifying likely Bid- and Ask prices from a series of transactions.

The Markit iBoxx dataset provides Bid- and Ask prices for all bonds on every trading day which makes Bid-Ask Spreads readily available. This allows us to monitor Bid-Ask Spreads through time for all individual bonds, but also allows us to monitor Bid-Ask Spreads at more aggregate levels; one can think, for example, of Spreads aggregated by Rating, by Financials and Non-Financials, by Senior and Subordinate debt, or by a combination of several of the above. To accurately describe the evolution of the (distribution of) the Bid-Ask Spread, both the mean as well as the interval between which 90% of all observed Bid-Ask Spreads fall are recorded on a daily basis. Since this plot is very large, it is moved to Appendix III (Figure 1, Appendix III); thorough inspection is highly recommended. Figure 5 below shows the average Bid-Ask Spreads split by Rating (AAA, AA, A and BBB) and Financials / Non-Financials.



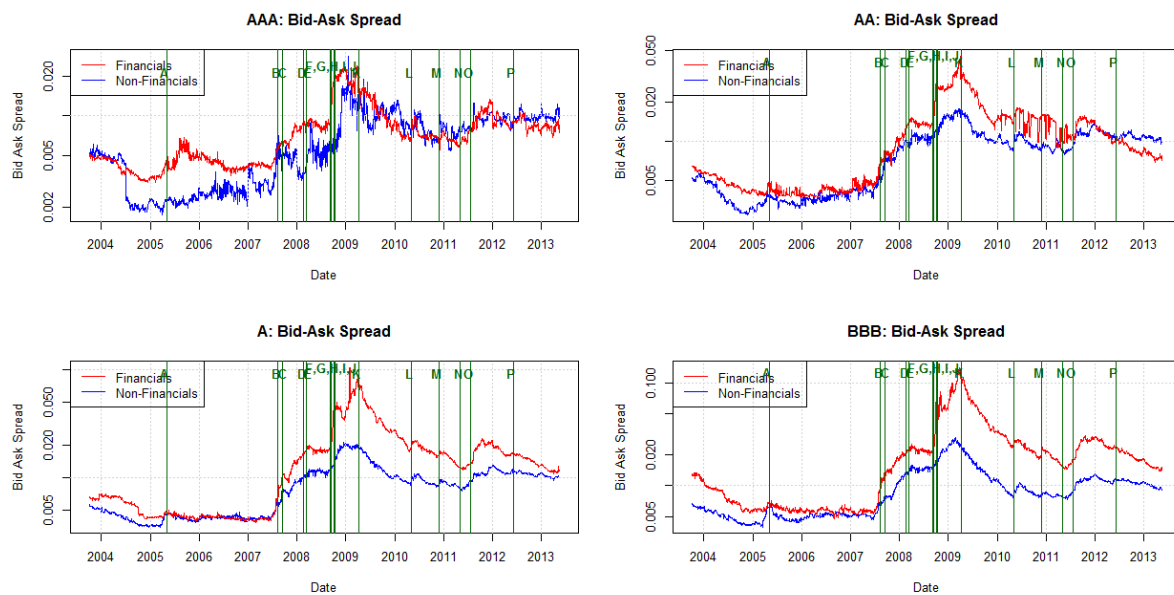
**Figure 5: Summary Bid-Ask Spread (log scale) by Rating and (Non -) Financials.** Similar to Credit Spreads, Bid-Ask Spreads move together across Rating class and across Financial/Non-Financial status. Noticeably, the onset of the financial

*crisis (mid-2007) caused the Bid-Ask Spreads to increase, but the Bid-Ask Spread of Financials increased substantially more than the Spreads for Non-Financials, something that is intuitive given the nature of the crisis. The divergence of Spreads between Financials and Non-Financials cannot be seen for bond of high Rating quality (AAA and AA).*

From Figure 5 above we can see that Bid-Ask Spreads across ratings and Financials / Non-Financials appear to move together, with all spreads substantially increasing during the credit crunch and starting to return stable levels in more recent years. On the other hand, the level at which the Bid-Ask Spread “stabilizes” after the years of distress is considerably higher than the stable levels in the years 2003-2007. It is too early to say whether markets are still experiencing a slow but persistent decline. Another observation, from Figure 1 in Appendix III in particular, is that the increase in Bid-Ask Spreads as well as the widening of the interval of observed spreads is far greater, proportionally, for Financials than for Non-Financials; this seems to make intuitive sense given the nature of the crisis during 2007-2010. The widened interval of observed Spreads does, however, persist in the years after the financial distress and does not return to the narrow interval observed during 2003-2007.

## **5.2 Timeline / Event Analysis**

Rather than treating “the credit crunch” and the subsequent “sovereign crisis” as single events or single periods of time, the Figure below (Figure 6) aggregates Bid-Ask Spreads by Rating and Financials / Non-Financials and overlays this plot with the timeline of major events discussed in Chapter 4. This allows us to subjectively qualify the impact certain events had on the Bid-Ask Spread of the eight groups.



**Figure 6: Timeline overlay; Bid-Ask Spreads by Rating and (Non -) Financials.** *Many of the event included in the timeline presented in Chapter 4, and in particular those associated with the global Credit Crunch and the European Credit crisis, can be linked to movement in the Bid-Ask Spread.*

We can see from the overlay that many of the major financial events included in the timeline coincide with substantial moves in aggregate Bid-Ask Spread, across the market. Since Figure 4 might be difficult to read by eye due to the logged scaling and rather small print due to image size, the impact of the major events is described qualitatively. Please note that no mathematical techniques are used for the estimates of spread increases; they are a mere reflection of “before-after” observations.

The very first of events (A) is the only non-financial event included in the timeline. The elections in May 2005 seem to coincide with a small peak in Bid-Ask Spreads across, predominantly, Non-Financials; which could very well be coincidental and spurious.

The subsequent two events, B and C, are often regarded as the events that signify the start of the “financial crisis” and coincide with the start of the increase in Bid-Ask Spreads throughout the crisis. Following the unrest in September 2008 (event F, G, H and I) an even larger jump in spreads is clearly visible across all Rating qualities. Spreads continue to increase until the UK government bailout of the Royal Bank of Scotland, Lloyds TBS and HBOS on 13 October 2008. The bailout seems to start a steady downward trend which continues well into 2011, only to be interrupted for a very brief period of time (event L and M). The bailing out of Portugal followed by the second bailing out of Greece two months

later (events N, O) cause another steady increase in spreads until 2012 when again spreads are slowly declining, a trend that is slow and continues to this day.

### 5.3 Modelling Bid-Ask Spread

To understand the dynamics of the Bid-Ask Spread through time, two modelling approaches are taken: modelling Bid-Ask Spread as an AR(I)MA process and regressing Bid-Ask Spreads on various analytical and classification variables in the dataset. The latter approach is believed to provide practitioners with actionable insight, is of great importance for the development of subsequent models and as a result is discussed in great detail.

#### 5.3.1 Time Series Modelling

##### 5.3.1.1 Fitting AR(I)MA process

The Box-Jenkins methodology can be described as using Autoregressive and Moving Average processes to create a time series that best fits past values of the time series, projecting the time series into the future (Box and Jenkins, 1970). Not concerned with projecting Bid-Ask Spreads into the future at this point, the focus of this modelling approach is fitting a mean level processes to Bid-Ask Spreads for each of the eight categories defined previously (by Rating and Financials / Non-Financials) and to subsequently estimate market (segment) wide Bid-Ask Spread volatility. The ARMA(p,q) model is given by:

$$\left(1 - \sum_{i=1}^p \alpha_i L^i\right) \ln(BAS_t) = \left(1 + \sum_{i=1}^q \theta_i L^i\right) \epsilon_t$$

Equation 7

where  $L$  is the lag operator,  $\alpha_i$  are the autoregressive components,  $\theta_i$  the moving average components and  $\epsilon_i$  the error terms, assumed to be  $N(0,h)$  i.i.d. Specifying a GARCH model the variance of the error is time dependent making the  $\epsilon_i$  term  $N(0,h_t)$  i.i.d.

The time series to which an ARMA(p,q) is fitted can be the original or, if the time series is non-stationary, can be fitted to differenced series to remove the non-stationarity. From simply looking at time series plots for each of eight groups one might conclude that the series does not seem to be stationary, something that could be confirmed by for example, an

Augmented Dickey-Fuller unit root test (Box & Jenkins, 1970). On the other hand one might argue that the Bid-Ask Spreads, especially on the aggregated levels under consideration, will always revert to a constant mean level in the long run and should be considered stationary as a result. From Bid-Ask Spread time series plots (Figure 5) one may conclude that after the substantial increase in Bid-Ask Spreads during the credit crunch, levels are reverting back to what they were before. This however, does not take away that the time series was trending during 2007-2011 and should be modelled as a differenced series, as indicated by the ADF test.

In addition to choosing the order of differencing, the orders of the autoregressive ( $p$ ) and moving average ( $q$ ) components need to be determined. Rather than using (the shape of) the autocorrelation and partial autocorrelation function to determine the order of the components, we perform a grid search over  $p$  and  $q$ . Since the interpretation of the (partial) autocorrelation functions can be rather subjective in nature, a grid search over plausible components and choosing the orders that minimise a fit criterion (AIC) is a superior choice given the volume of models to be fitted. Aikaike's Information Criterion (AIC) is a measure of relative quality of a model and is often used for model selection. Given several candidate models, ARMA models of various orders in  $p$  and  $q$ , the criterion deals with the trade-off between model complexity (additional parameters) and goodness of fit by penalising parameters:

$$AIC = 2k - 2\ln(L)$$

where  $k$  is the number of model parameters and  $L$  the maximum likelihood of the model.

Table 2 summarizes the grid search for each of the eight groups, identified by Rating and Financial (F) or Non-Financial (NF) status.

Series	Order											
	(0,0,0)	(1,0,0)	(0,0,1)	(0,1,0)	(1,1,0)	(0,1,1)	(1,1,1)	(2,1,0)	(0,1,2)	(2,1,1)	(1,1,2)	(2,1,2)
AAA-F	3161	-7902	184	-7900	-8176	<b>-8224</b>	-8222	-8215	-8222	-8220	-8220	-8218
AAA-NF	4611	-5468	1723	-5463	-5666	-5730	<b>-5754</b>	-5708	-5747	<b>-5755</b>	-5754	-5753
AA-F	4747	-7504	1641	-7505	-7729	-7805	<b>-7816</b>	-7786	<b>-7816</b>	-7814	-7814	-7812
AA-NF	3984	-9853	803	-9855	-10076	-10121	<b>-10125</b>	-10109	-10124	<b>-10126</b>	-10125	-10124
A-F	5881	-10174	2572	-10179	-10199	-10212	-10240	<b>-10282</b>	-10277	-10280	-10281	-10280
A-NF	3578	-11971	342	-11974	-12232	-12240	<b>-12242</b>	-12241	<b>-12242</b>	-12240	-12240	-12238
BBB-F	6001	-5927	2936	-5928	-6252	-6383	<b>-6392</b>	-6363	<b>-6392</b>	-6391	-6390	-6289
BBB-NF	3428	-11839	212	-11842	-12036	-12048	<b>-12049</b>	<b>-12049</b>	-12048	-12048	-12046	-12045

Table 2: AIC Grid Search, AR(D)MA Models

The grid search shows that for many of the eight groups there are several candidate models that score extremely similar on the AIC, shown in bold (Table 2). Model selection

purely based on AIC values is rather arbitrary in this case and the estimated model parameters need to be investigated before deciding on the appropriate order. In model selection parsimonious models are preferred (hence AIC) and simple models are to be preferred over their more complex counterparts in case of comparable relative fit (Burnham & Anderson, 2002). Keeping this in mind, the estimated model parameters are examined to see whether the additional parameters estimated by the more complex candidate models are statistically significant. Table 3 below summarizes the estimated parameters for the candidate models AA-NF (1,1,1) and AA-NF (2,1,1).

	AR (1)		AR (2)		MA (1)	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
AA-NF(1,1,1)	0.149	0.059	-	-	-0.488	0.052
AA-NF(2,1,1)	0.267	0.086	0.057	0.038	-0.608	0.083

**Table 3: AR(I)MA Candidate Model, AA-NF example**

The additional AR(2) parameter has a high standard error estimate relative to the coefficient and the effect of the coefficient is not statistically different from zero as a result ( $\alpha = 0.01$ ). Therefore, the more complex (2,1,1) model is rejected in favour of the (1,1,1) “mixed” model. For most models, the (1,1,1) specification is preferred. Table 1 in Appendix III summarizes the model parameters for the selected models.

Even though parameters cannot be easily be compared across different model specifications, parameter estimates are similar across all models (1,1,1) with several exceptions. All MA(1) estimates are of the same direction and similar in magnitude, except for A-NF, for which the estimated effect is far smaller. The A-NF model is also the only (1,1,1) model for which the first autoregressive dependency (AR(1)) is negative, something seen in both of the estimated higher order models (A-F and BBB-NF). Another observation from the above is that series of good Rating quality (AAA or AA) seem to be driven by the same process, with the exception of no autoregressive dependency for AAA-F. Series of lower Rating quality are not only different to their higher Rating quality counterparts, but are also incoherent as a sub group with higher order processes and both positive and negative autoregressive components.



### 5.3.1.2 Fitting GARCH(1,1)

A Ljung-Box test (Box & Ljung, 1978) is used to confirm that the error terms of the fitted processes are independently distributed, i.e. no autocorrelation remains (Ljung-Box test, p-values > .1). The error terms do on the other hand, exhibit non-constant variance. In particular, one can observe clusters of volatility. Figure 2, in Appendix III shows a time series plot of the squared residuals of the ARMA process fitted to the (differenced) Bid-Ask Spread of group AA-NF. All other groups show similar behaviour, clusters of volatility.

In order to model the non-constant variance of the time series, and to estimate its variance (i.e. volatility) at any point in time, a GARCH (1,1) model is used (Bollerslev, 1986):

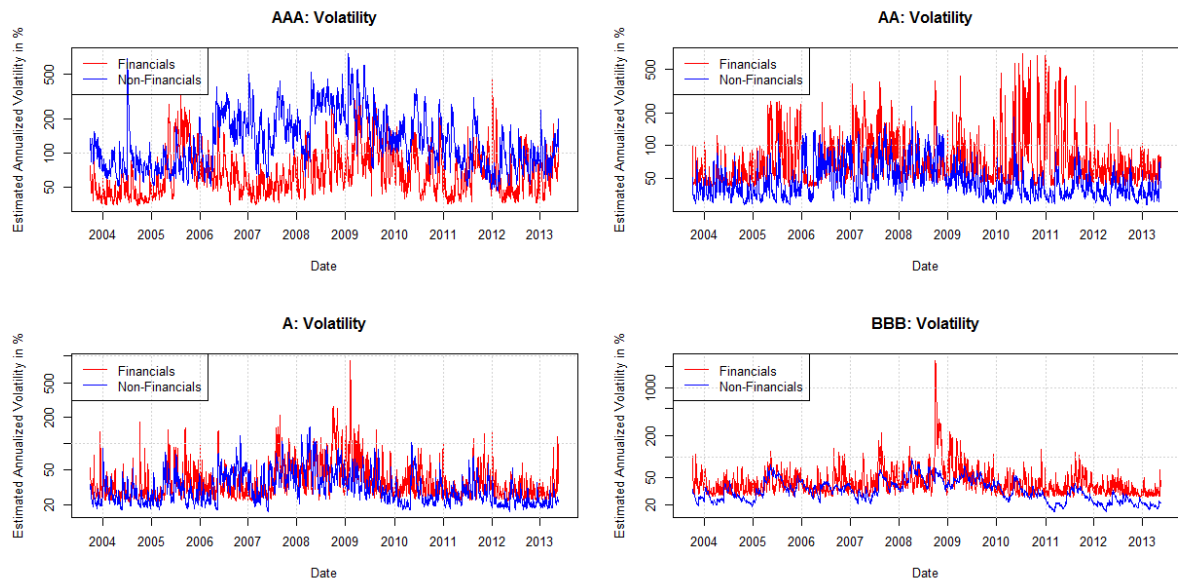
$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

Equation 8

where  $h_t$  is today's conditional variance of  $\varepsilon_t$ ,  $\omega$  is the weighted long run variance.

GARCH(1,1) models are fitted with both Gaussian and Student's T distributed error terms and fit subsequently examined using AIC. Despite being able to accommodate fatter tails and therefore being the preferred model, residuals of the fitted models still exhibit non-normality (Jarque-Bera, p-value < 0.01) when using a Student's T distribution. For a comparison between the relative fit of models with different error distributions, including tests for normality and autocorrelation, please see Table 2, Appendix III.

The daily computed conditional variance can be turned into an annualized volatility by straightforward manipulation of  $g_t = h_t \sqrt{252}$ , where  $g_t$  is the estimated annualized volatility at time  $t$ . Figure 7 below shows the estimated volatility for each of the eight market segments.



**Figure 7: Bid-Ask Spread Volatility (by Rating and (Non-) Financials).** *Volatility estimates for each of the eight categories are high, but most importantly appear to be very volatile and noisy.*

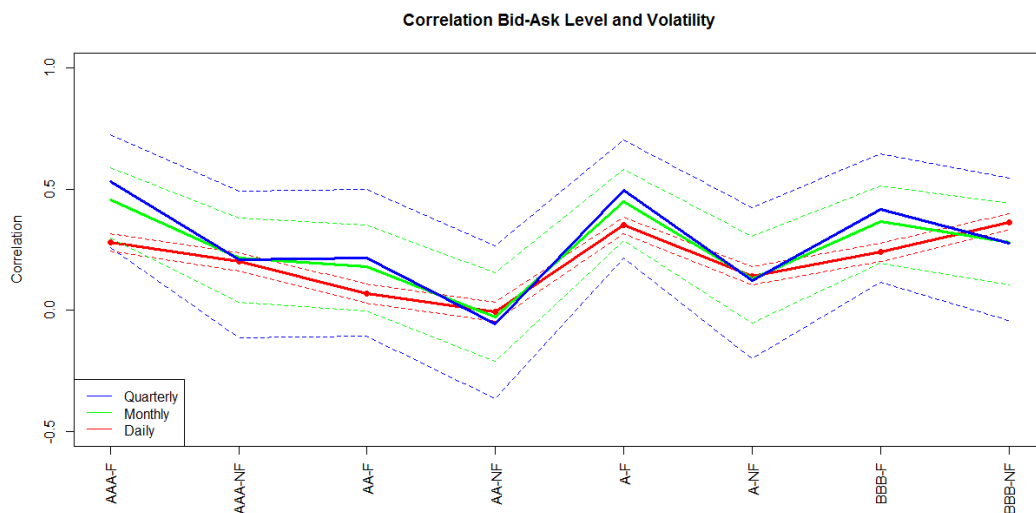
Several observations can be made from the above plots:

- Estimated levels of Bid-Ask volatility are high across all Rating and Financial status, the possible exception being BBB-NF, which is the only market segment for which volatility never exceeds 100% (annualized). For all other market segments, volatilities above 100% are regularly observed.
- Estimated levels of Bid-Ask volatility are similar across Rating, but some differences can definitely be observed. The volatility of Non-Financials seems to be slightly lower when compared to Financials, except for those rated AAA, where Non-Financials seem to have higher volatility.
- Estimated levels of Bid-Ask volatility are very volatile themselves, the possible exception being BBB-NF. This begs the question whether the updating function is calibrated appropriately. Further investigation into this issue is desired.

### 5.3.1.3 Bid-Ask Spread and Volatility

Dick-Nielsen et al. (2012) find that there is a positive correlation between their liquidity proxy's risk (uncertainty) and the level of their liquidity proxy. Their analysis aggregates the level of liquidity by quarter (liquidity risk being the standard deviation in any given quarter). In order to test whether a similar effect is observed here, correlations between Bid-Ask

Spread en Bid-Ask volatility are computed for each of the eight groups over the entire time period. Apart from group AA-NF, all correlation coefficients are significantly different from zero ( $\alpha=0.05$ ). In terms of magnitude the above results are slightly different to those reported by Dick-Nielsen et al. (2012). They report a correlation of 83% between Unique Roundtrip Cost (a liquidity proxy similar to Bid-Ask Spread) and its risk. For liquidity proxy for the price impact of a trade, they report a correlation of 56%. Those results are consistent with Acharya and Pedersen (2005) who likewise find a high correlation between liquidity and liquidity risk. The correlations reported here are not to be ignored, but cannot be considered “high”. Dick-Nielsen et al. (2012) even report multicollinearity concerns when including both terms in several of regression equations. It is important to note that both studies used monthly or quarterly aggregated data and as our daily estimates for Bid-Ask volatility vary substantially on a daily basis, rolling up daily estimates into monthly and quarterly figures gives higher correlations coefficients. Figure 8 below shows the estimated correlations by group for daily, monthly and quarterly estimates.



**Figure 8: Correlation Bid-Ask Spread and Bid-Ask Spread Volatility (by Rating and (Non -) Financials).** *The correlation between level of liquidity and liquidity risk (volatility) is well-documented. Using the Bid-Ask Spread and its volatility as a proxy for liquidity we are unable to replicate the high correlation found by others. The noisy volatility estimates are likely to cause some disruption in the estimates.*

In addition to the expected widening of the confidence interval due to a decrease in number of observations (for example, daily  $N=2430$  whereas quarterly  $N=8$ ), we can see that estimated correlations are lower when using daily estimates of volatility and Spread. The increased correlations estimates are promising and demonstrate that the results of our daily estimates are similar to other studies, especially given that those high correlations reported in other studies concerned individual bonds rather than market segments. We can also see that

the correlations estimates for monthly aggregated data and quarterly aggregated data are extremely similar. A last observation would be that within a given rating class, correlation estimates for Financials are higher than those for Non-Financials. This seems to tie in with general market expectations regarding Financials and volatility.

### 5.3.2 Bid-Ask Spread Regression Modelling

Rather than modelling Bid-Ask Spread as univariate, aggregate time series, the regression approach is used to model the Bid-Ask Spread of individual bonds as being determined by a set of explanatory variables from the iBoxx dataset. Using this cross-sectional approach the relationship between a bond's many characteristics and attributes can be quantified. This section will discuss a purely cross-sectional approach, a cross sectional approach with temporal (daily) components and the construction of the Relative Bid Ask Spread (RBAS).

#### 5.3.2.1 Cross-Sectional Modelling

For the regression analysis the dataset is divided into four categories according to Rating quality (AAA-BBB) and separate analyses are run for each. The specified regression model is therefore of the general form:

$$\log(\text{Bid} - \text{Ask Spread}_{i,j}) = c_j + \beta_{j,1}x_{i,j,1} + \beta_{j,2}x_{i,j,2} + \dots + \beta_{j,k}x_{i,j,k} + \varepsilon_{i,j}$$

Equation 9

where  $i$  refers to one particular observation of a bond within group  $j$  (AAA, AA, A or BBB) and the random errors  $\varepsilon_{i,j}$  are independent normally distributed random variables with zero mean and constant variance  $\sigma_j^2$ . More specifically, substituting in the independent variables the following regression is fitted:

$$\begin{aligned}
& \log(\text{Bid} - \text{Ask Spread}_{i,j}) \\
&= c_j + \beta_{j,1}\sqrt{\text{Age}}_{i,j} + \beta_{j,2}\log(\text{Notional Amount})_{i,j} \\
&+ \beta_{j,3}\log(\text{Duration})_{i,j} + \beta_{j,4}\log(\text{Duration} * \text{Non} - \text{Financial})_{i,j} \\
&+ \beta_{j,5}\log(\text{Duration} * \text{Sovereign})_{i,j} + \beta_{j,6}(\text{Non} - \text{Financial})_{i,j} \\
&+ \beta_{j,7}(\text{Sovereign})_{i,j} + \beta_{j,8}(\text{Senior})_{i,j} + \beta_{j,9}(\text{Collateralized})_{i,j} + \varepsilon_{i,j}
\end{aligned}$$

Equation 10

Explanatory variables can either be static (e.g. Level of Seniority) or dynamic (e.g. Duration), where static variables remain the same during a bond's lifetime and the dynamic variables (can) change on a daily basis. Dummy or indicator variables are used for mutually exclusive categorical variables. The variables Non-Financial, Sovereign, Senior and Collateralized are examples of indicator variables. The interaction effects of duration and the Non-Financial / Sovereign indicator result in an adjustment of the  $\beta_{j,3}$  parameter (effect of Duration) for those bond classifications. Table 4 below shows a summary of the regression results for the four specified regressions.

	AAA		AA		A		BBB	
	Beta	t-value	Beta	t-value	Beta	t-value	Beta	t-value
Constant	-3.455	-147.3	-7.481	-195.0	-7.671	-238.6	-5.87	-153.0
Age sqrt()	0.126	128.6	0.153	124.3	0.205	200.5	0.165	151.9
Notional Amount log()	-0.153	142.8	0.076	38.3	0.105	64.6	0.092	47.1
Duration log()	0.612	173.3	0.581	275.3	0.429	226.9	-0.031	-10.2
Non-Financial	-0.203	-19.9	-0.466	-64.6	-1.118	-206.7	-2.110	-311.3
Sovereign	-0.409	-52.5	-0.151	-15.6				
Senior			-0.149	-51.1	0.028	14.3	0.209	90.3
Collateralized	-0.204	-30.3	-0.182	-39.5	-0.251	-74.1	-0.479	-137.8
Duration*Non-Financial	0.009	2.2	0.130	36.7	0.378	137.1	0.744	205.6
Duration*Sovereign	0.037	13.3	-0.06	-12.17				
R-Squared	0.521		.354		.245		0.316	
% in-sample  10%	10.9%		9.7%		11.2%		11.6%	
% in-sample  30%	33.2%		30.4%		32.1%		34.9%	
N	534240		475714		888464		596456	

Table 4: Summary Results: Cross-Sectional Bid-Ask Model

Whereas the pure cross-sectional modelling of Bid-Ask Spreads is a great tool to identify possible factors that determine the Spread, in light of the financial turmoil that the markets have seen during the sampled time period, it is unlikely that relationships have stayed constant for over a decade. With this in mind, a purely cross-sectional model is of little value, even though one can make clear observations for some of the coefficients. For

example, the Financial indicator appears to play a much greater role when it comes to Bid-Ask Spreads at the lower end of the IG Rating scale. Not only the Financial indicator itself is most important, but the difference in the effect of duration is also the biggest in the lowest Rating class. Apart from several key differences when it comes to coefficients, there is a certain degree of consistency between the results, at least when aggregated over a decade; Age and Duration have similar effects across Rating, with an effect slightly decreasing going down the Rating ladder. The model above is therefore also not on the level of granularity that it could be; it would allow to include far more specific bond classifications rather than the Financials/Non-Financial breakdown. The number of observations for several (sub-) market sections is more than sufficient to fit stable regression coefficients. The danger is fitting to bonds or issuers specifically rather than the economic sector as a group. Even though this extra layer of granularity is something to explore going forward (for example, Utilities and Retail are both likely to respond differently to financial distress as are Banks and Insurance firms), the temporal component of the relationship is first explored.

### 5.3.2.2 Temporal Cross-Sectional Modelling

The temporal component of this model comes from the fact that a regression model, very similar to the one presented before, can be fitted not on the entire sample period but on several subsections of the data. More specifically, a regression is run for every day in the dataset. Given ~1200 data points on any given day, we have sufficient data to fit daily regression models. Using the regression model from equation 9, on a daily basis yields the following specified model:

$$\begin{aligned}
& \log(\text{Bid} - \text{Ask Spread}_{i,j,d}) \\
&= c_{j,d} + \beta_{j,d,1} \sqrt{\text{Age}_{i,j,d}} + \beta_{j,d,2} \log(\text{Notional Amount})_{i,j,d} \\
&+ \beta_{j,d,3} \log(\text{Duration})_{i,j,d} \\
&+ \beta_{j,d,4} \log(\text{Duration} * \text{Non} - \text{Financial})_{i,j,d} \\
&+ \beta_{j,d,5} \log(\text{Duration} * \text{Sovereign})_{i,j,d} + \beta_{j,d,6} (\text{Non} - \text{Financial})_{i,j,d} \\
&+ \beta_{j,d,7} (\text{Sovereign})_{i,j,d} + \beta_{j,d,8} (\text{Senior})_{i,j,d} \\
&+ \beta_{j,d,9} (\text{Collateralized})_{i,j,d} + \varepsilon_{i,j,d}
\end{aligned}$$

Equation 11

where  $d$  refers to days, and as before,  $i$  refers to one particular observation of a bond within group  $j$  (AAA, AA, A or BBB).

In order to summarize the results and compare fit between the non-temporal regression model (Equation 9) and the temporal regression model (Equation 10), the same measures of fit are used to describe the models. Model (1) refers to the fit of the purely cross sectional model (Equation 9) and Model (2) is the same model (explanatory variables) using daily regressions.

R-Squared (also  $R^2$  or coefficient of determination) is a measure of fit is defined as

$$R^2 = \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

**Equation 12**

where  $\bar{y}$  is the mean of the dependent variable,  $y_i$  is the observed value of the dependent variable for observation  $i$ , and  $\hat{y}_i$  is the fitted value (regression) of the dependent variable for observation  $i$ .

Alternatively one can express the  $R^2$  as the squared correlation between observed and fitted values for the dependent variable

$$R^2 = [\text{corr}(y, \hat{y})]^2$$

**Equation 13**

If the  $R^2$  value is close to 1, most of the variability in the data is explained (captured) by the regression and the regression model fits the data well. Conversely, values close to 0 reflect a poor fit.

The Root Mean Square Error (RMSE) is another measure of fit, related to the  $R^2$ . Even though the RMSE dependent on the dependent variables' scaling, making it unsuitable to compare models across sampled populations, it can be used to compare fit between Models (1) and Models (2).

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_n - \hat{y}_n)^2}{n}}$$

Equation 14

A third measure of fit (Accuracy Interval) directly related to the magnitude of the error terms can be defined as the percentage of observations ( $n$ ) that are fitted to lie within a certain range ( $\alpha$ ) of the observed dependent variable ( $y$ )

$$Accuracy\ Interval = \frac{n \mid \left( \frac{|y_n - \hat{y}_n|}{y_n} < \alpha \right)}{n}$$

Equation 15

Model	AAA		AA		A		BBB	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Mean R-Squared	.521	.833	.354	.727	.245	.743	.316	.632
RMSE	.0045	.0032	.0093	.0065	.0169	0.0116	.0286	.0237
AI, $\alpha = 0.1$	10.9%	27.9%	9.7%	28.3%	11.2%	31.1%	11.6%	22.3%
AI, $\alpha = 0.3$	33.2%	69.1%	30.4%	68.1%	32.1%	71.7%	34.9%	59.3%
N	534240		475714		888464		596456	

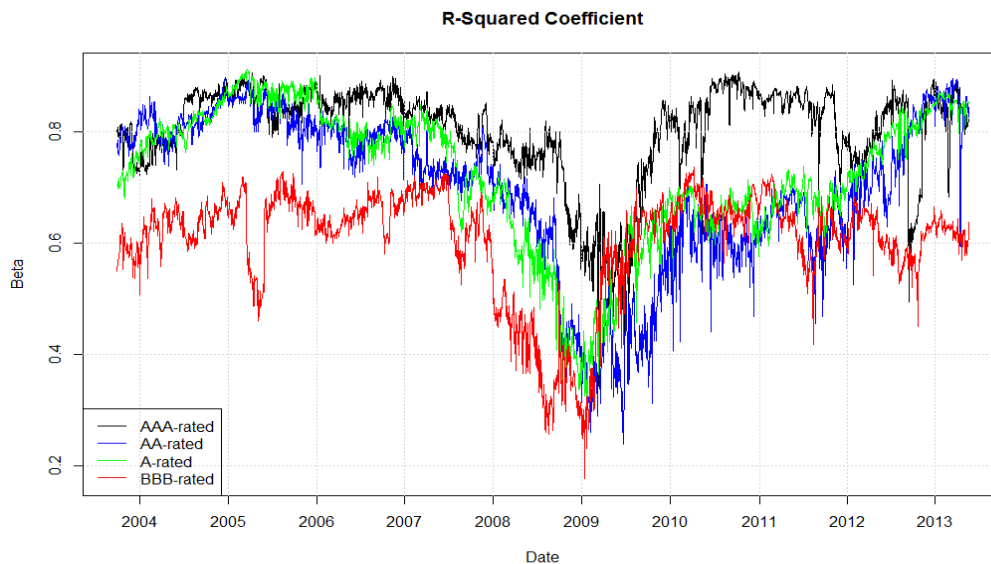
Table 5: Adding Temporal Component: Summary of Fit Measures

All fit measures have improved drastically across all models, but there is still a substantial part of the data that is not well captured, especially for lower rated bonds. Since explicit credit risk, as proxied by Rating and captured by the four constants in the model, increases substantially going down the Rating ladder, it is not surprising BBB does not fit as well. The broad (no tranches) Rating class BBB is a far more diverse universe of credit risk bearing instruments: those of actually decent quality and those about to be downgraded to junk status. Even with additional variables in the model, or a different functional form, we are unlikely to capture the basis credit risk associated with a category as diverse as BBB accurately with a single value (constant in regression model).

Using the temporal aspect of the model, we can look at the fit of the model through time, as measured by the models R-Squared coefficient. From Figure 9, we can see that the R-Squared is, generally, higher for models of a higher Rating quality. Importantly, the models seem to describe the data worse during the “credit crunch”, across Rating categories. This seems to suggest that factors are missing, factors that prove to be descriptive during



periods of financial distress, but not in other periods as suggested by high R-Squared values before and after the crisis. Another possible explanation is that a different functional form describes the relationships (during the “credit crunch”).



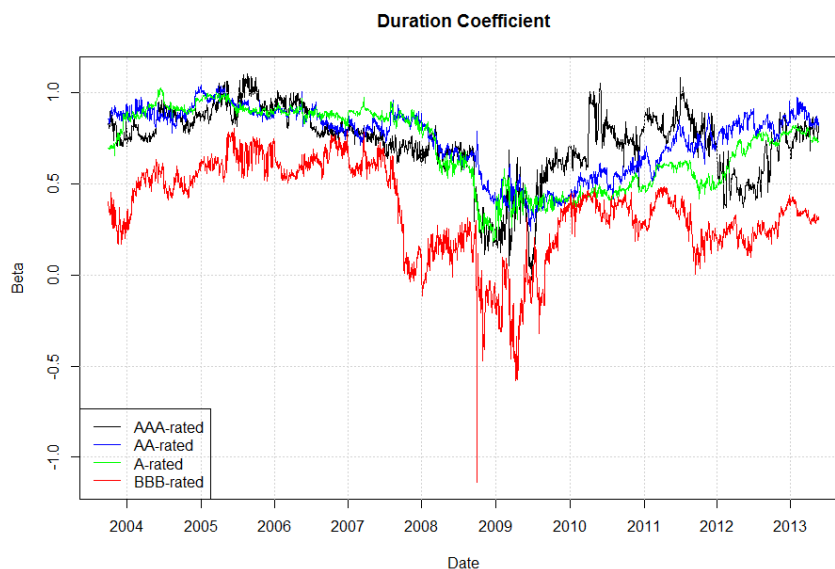
**Figure 9: Time-varying R-Squared in the Bid-Ask Model.** *The  $R^2$  of the temporal model differs between Rating classes, with BBB having the lowest  $R^2$ . Additionally, the fit of the model, across Rating class, declined during the Financial crisis, an indication that Bid-Ask Spread dynamics are difficult to capture using the current equation during times of extreme financial distress.*

Despite the comforting improved fit of the model, the real insight and value of the temporal aspect lies with the evolution of the regression coefficients through time and across Rating. In addition to plots included here, all plots can also be found in Appendix III (Figure 3, 4, 5 & 6), where estimates of regression coefficients are supplemented with their Confidence Intervals. These Confidence Intervals are important to determine the true “range” of estimated coefficients and to determine whether coefficients are different across Rating qualities, after accounting for their uncertainty.

### 5.3.2.2.1 Individual Betas: Duration

The relationship between Duration and Bid-Ask Spread is strong, positive and intuitive. Due to transformations in the regressions equation (log-log), the estimated beta here represents the elasticity; a coefficient of .5 means that a 1% increase in Duration increases the Bid-Ask Spread by .5%. From the plot we can make several important observations. The first observation is that the coefficients move together and that coefficients are of a similar magnitude, with the exception of the BBB coefficient, whose coefficient is estimated to be

substantially lower than other Ratings. The second observation is that, for all Rating categories except BBB, the coefficient is highly positive (generally 0.6 - 1.0) with decreased coefficients during the crisis where coefficients were below 0.5. The BBB duration coefficient is substantially lower (< 0.5) and actually turned negative during the height of the crisis period. It is crucial to mention at this point that for BBB-rated bonds the Duration effect appears to be very different for Financials and Non-Financials, especially during the crisis years (see Individual Betas: Other). Effectively, the Duration coefficient for Non-Financials turns out not to be negative at all during this period. Additionally, where it seems that the coefficient of Duration is substantially lower than for all other Ratings, this is effectively only true for Financials, Non-Financials have Duration coefficients very similar to those of a higher Rating quality. This will be discussed in more detail in Section 5.3.2.2.4

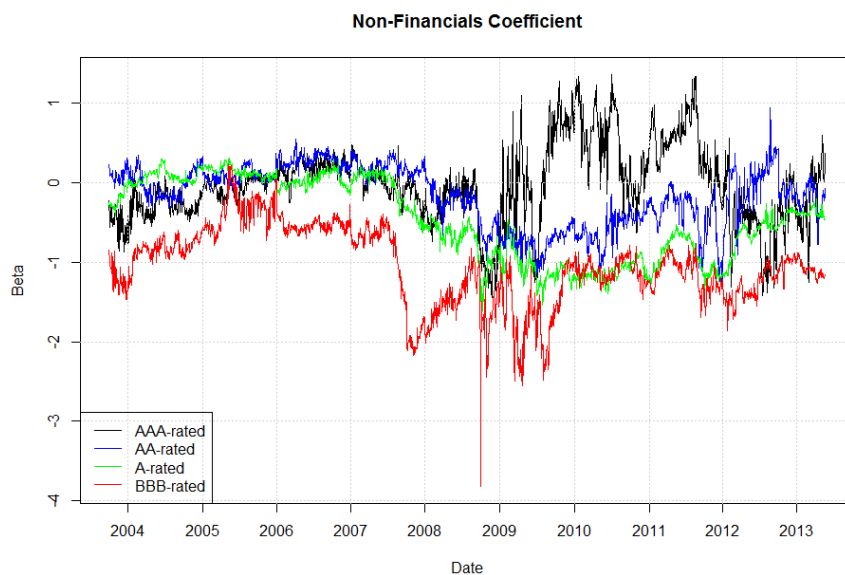


**Figure 10: Time-varying Duration coefficient in Bid-Ask Model.** *The Duration coefficient is rather stable throughout time, adding to the validity of the model. The decline of the coefficient, most noticeable for BBB-rated bonds, can be directly linked to Northern Rock and Lehman Brothers.*

The non-constant nature of the coefficients merits further analysis. For example, the sharp, sudden decline in the BBB-coefficients during the second half of 2007 can be directly linked to the unrest around Northern Rock. This event can also be marked as the start of a decline in the Duration coefficient across all Rating classes. Similarly the tremendous drop of the BBB-coefficient, on one particular day can be linked to Lehman Brothers. The Lehman Brothers event appeared to have another effect with respect to Duration coefficients.

### 5.3.2.2 Individual Betas: Non-Financials

The coefficient of Non-Financials (versus) Financials indicates, *ceteris paribus*, whether Non-Financials have a lower Bid-Ask Spread (negative beta). An estimated coefficient of zero indicates that both Financials and Non-Financials trade at the same Bid-Ask Spread. From Figure 9 we can see that this is indeed the case for Financials and Non-Financials before the credit crunch, with the exception of BBB-rated bonds. For these BBB-rated bonds the coefficient is negative, indicating that Non-Financials traded at a lower Bid-Ask Spread. Mid-2007 the coefficient dropped down for BBB and started a slower decline for all other Ratings, suggesting that Non-Financials were now trading at a considerable lower Bid-Ask Spread, with those BBB-rated being the most responsive. The AAA coefficient jumps around a lot, which is simply the result of there not being a lot of observations (AAA-rated Financials after the crisis), but a flight to quality (AAA and AA) is nevertheless apparent. Currently, Financials and Non-Financials seem to be trading for the same Bid-Ask Spreads (perhaps a slight premium for Financials) again for IG bonds of higher credit Rating than BBB.



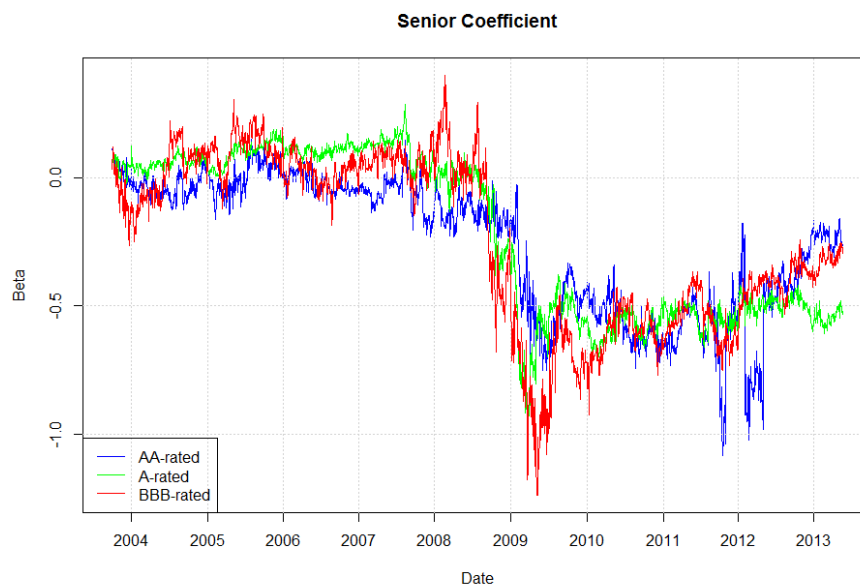
**Figure 11: Time-varying (Non -) Financials coefficient in Bid-Ask Model.** *The coefficient of the indicator variable is reasonably stable over time, with the exception the time periods around several key events (such as Northern Rock and Lehman Brothers).*

Similar to the Duration coefficient, we can clearly see the impact of the Northern Rock and Lehman Brothers events. The strong reaction of this coefficient indicates that Bid-Ask Spreads of Financials reacted far stronger to the event (widened) than the Bid-Ask Spread of Non-Financials. Looking at the AAA-coefficient we can also see the impact of the

well-documented “flight-to-quality” during financial distress. With a coefficient of around one or even slightly above one for the time period 2009-2012, this would indicate that Non-Financials were trading at the same or even at a wider Bid-Ask spread than Financial equivalents.

### 5.3.2.2.3 Individual Betas: Seniority

Similar to the coefficient for Non-Financials, the Seniority coefficient indicates whether, *ceteris paribus*, Senior and Subordinate bonds trade at different prices (spreads). An estimated coefficient of zero indicates that both trade at the same Bid-Ask Spread, which is exactly what we see across all Rating classes until the credit crunch. During the credit crunch, the coefficient drops sharply, but the coefficient remains very similar across all Rating classes during the entire sample period. Recovery of the coefficient in the direction of zero seems to start somewhere late 2011 and the upward trend continues to this day.



**Figure 12: Time-varying Seniority coefficient in Bid-Ask Model.** *The coefficient for Seniority is not stable over time and it appears as if a step-change has occurred during 2008. In the years prior to 2008, no effect of Senior bonds could be observed, but from 2009 onwards Senior bonds trade at tighter spreads. Currently, it seems the coefficient is slowly returning towards 0.*

### 5.3.2.2.4 Individual Betas: Others

For plots of all other estimated coefficients and their confidence intervals, please refer to Figures 3, 4, 5 & 6 in Appendix III. The coefficient for Notional Amount (log, or  $\beta_{j,d,2}$  in

Equation 10) is negative before the crisis years indicating that issues of a larger size have a lower Bid-Ask Spread; this is what we would intuitively expect. During the crisis years however, for all Rating qualities except AAA, the coefficient shoots up substantially to levels above zero, which indicates that larger issues trade at higher Bid-Ask Spreads. This is likely to be misleading in the sense it might not truly reflect the trading mechanisms. During the crisis years, Bid-Ask Spreads were at far higher levels across the board. Bid- and Ask Prices for (very) small issues might not have been updated by market makers as these instruments were not being actively traded during these times. As a result, the Bid-Ask Spread of larger issues is “adjusted” with the new financial climate, whereas smaller issues are not. After 2010 the coefficient started to move towards the levels seen before the crisis and even though the coefficient is currently still higher than in 2004-2007, if the current trend continues those levels will be reached in a year’s time. This seems to be an indication that smaller issues are being traded again, itself an indication of liquidity returning to the market from 2010 onwards.

The interaction term between the Non-Financial indicator and Duration ( $\beta_{j,d,6}$  in Equation 10) indicates whether the effect of Duration on Bid-Ask Spreads is different for Financials and Non-Financials, it adjusts the Duration coefficient described in section 5.3.2.2.1. A coefficient is zero, indicates no difference in Bid-Ask Spread. For the high quality ratings (AAA and AA) no effect is observed throughout the entire period. It is important to note that even though Financials and Non-Financials traded at different levels of Bid-Ask Spreads (see Non-Financials beta from section 5.3.2.2.2), the effect of Duration did not change. The difference in the effect of Duration is immediately visible for BBB-rated bonds and the Bid-Ask Spreads of A-rated bonds diverge slowly for Financials and Non-Financials. The positive coefficients indicate that the effect of Duration is stronger (positive) for Non-Financials. The results presented here need to be seen in conjunction with the estimated coefficient of Duration. Whereas at first it seems as if the Duration coefficient for BBB-rated bonds behaves very different from other Rating qualities, based on Figure 10, it turns out that after making the appropriate adjustments, it is only the Financials that behave differently.

### 5.3.2.3 Relative Bid-Ask Spread (RBAS)

The Relative Bid-Ask Spread (RBAS) is a measure of a bond's Bid-Ask Spread (liquidity) relative to bonds with the exact same characteristics. Mathematically, the RBAS is defined using fitted values of the temporal regression model (Equation 10):

$$\log(RBAS_{i,j,d}) = \epsilon_{i,j,d} = \log(Bid - Ask Spread_{i,j,d}) - f_{i,j,d}$$

Equation 16

where  $f_{i,j,d}$  are the fitted values of the temporal Bid-Ask Spread regression equation (Equation 10) for bond  $i$  of Rating quality  $j$  on day  $d$ .

The construction of the RBAS measure is an insightful measure in its own right. It scales the Bid-Ask spreads of very different bonds of different Rating qualities on the same scale. It therefore allows (potential) investors to compare asset liquidity against a fair benchmark. One can think of a scenario where a portfolio manager wants to include a bond from a certain market segment, for reasons of diversification. Given a considerable set of bonds with different maturities, issue size, seniority and age, the portfolio manager can now deliberately choose a bond that is relatively liquid (or illiquid depending on his preferences). If not the reason to choose certain bonds, it can at least be an additional factor that plays a role in the decision making process. Without the RBAS, no straightforward liquidity comparison was possible. Furthermore, the RBAS is a useful measure in subsequent analyses that estimate the liquidity premium on bonds. Stripped from the effects of various factors such as Duration, the unmitigated effect of those factors as well as bond liquidity is determined.

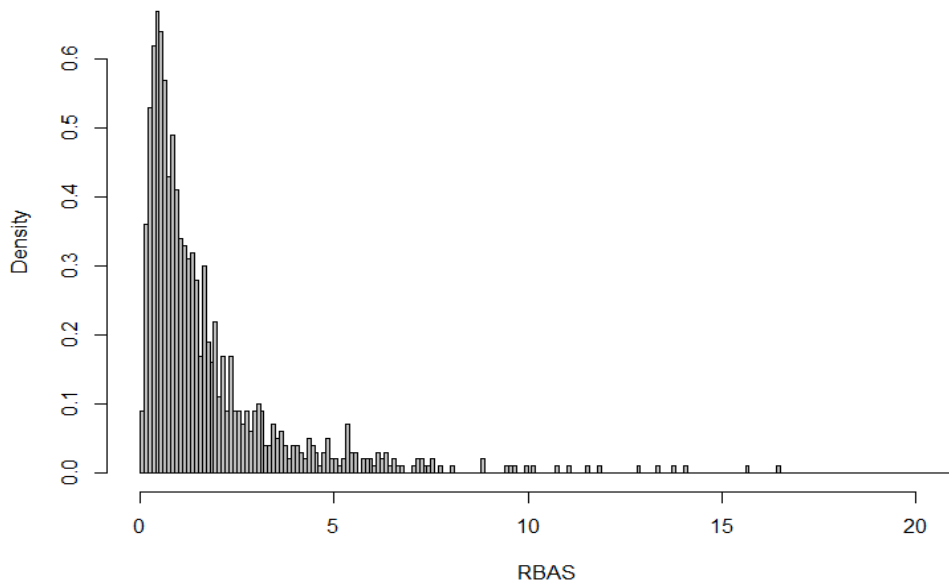
The typical distribution of the RBAS can be approximated using straightforward arithmetic since the  $\epsilon_{i,j,d}$  of Equation 10 are  $N(0, \sigma_{j,d}^2)$  iid. Therefore,

$$RBAS_{i,j,d} = e^{\epsilon_{i,j,d}}$$

Equation 17

Generating RBAS for 1000 bonds for  $N(0,1)$  iid residuals yields the distribution illustrated in Figure 13.

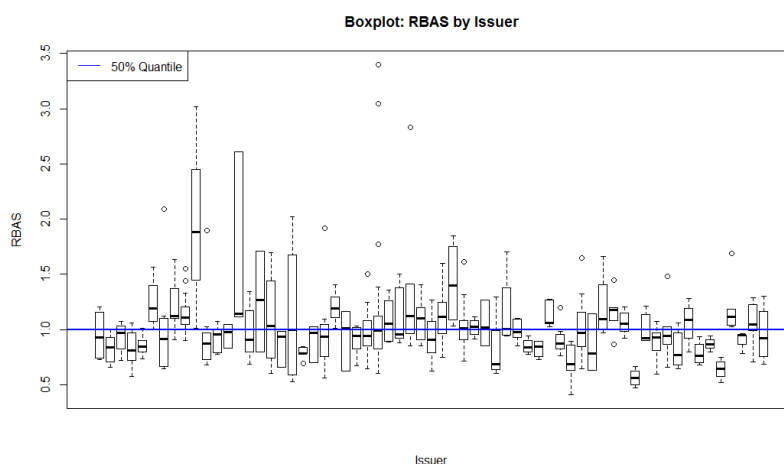
### Typical Distribution of RBAS



**Figure 13: Typical Distribution RBAS.** *The Relative Bid-Ask Spread (RBAS) on any given day, for any of the four Rating categories, is distributed with a long tail as pictured.*

#### 5.3.2.3.1 Issuer Specific RBAS

Since the regression equation accounts for differences in bond characteristics, the remaining RBAS allows us to freely compare liquidity across bonds. It is not unlikely that bonds from the same issuer have (on any given day) similar estimates of RBAS. The RBAS therefore not only includes purely bond specific liquidity information but also consists of issuer specific liquidity. To illustrate the clustering of RBAS for bonds with the same issuer we are looking at the 70 issuers with the most bonds outstanding on 26 June 2006, with the largest issuer having 24 bonds outstanding and the smallest issuer included in this list having 3 bonds outstanding. Using simple boxplots from Figure 11 we can see that there is most definitely some clustering of RBAS for bonds of the same issuer.



**Figure 14: Issuer specific Liquidity.** *To investigate whether the RBAS of bonds of the same issuer are similar, boxplots of the RBAS by Issuer reveal that RBAS is likely to contain an issuer-specific component*

#### 5.3.2.4 Future Work

The Bid-Ask Spread models using a temporal component provide an extra dimension to the analysis. The observation that the effect of many characteristics and analytical values of bonds have changed over the last decade, in particular due to the “credit crunch”, is key to understanding the dynamics of the Bid-Ask Spread. Inclusion of these time-varying coefficients improves every measure of fit, ensuring that Spreads are predicted accurately. The fact that the model’s predictions are not 100% accurate and for some Rating categories, during certain time periods, were no-where near 100%, does not invalidate the model in any way. The residual (Bid-Ask Spread) of the models simply refers to idiosyncratic or bond-specific Bid-Ask Spread. One can think of a lack of model fit (during certain time periods), as a high percentage of spreads being the results of bond or issuer specific (liquidity) factors, working off the assumption that the specified model is correct and captures all structural components effectively. On the other hand a combination of the two, a larger liquidity spectrum as well a model that underperforms during certain periods of time, is more plausible.

The indicator for Financials/ Non-Financials is extremely important since the dynamics and level of Bid-Ask spread differ greatly between the two. More of those differences could be investigated by including more indicator variables for specific economic market (segments). One can imagine that on any given day after the Credit Crunch, the



number of AAA-rated Banks is very small. In order to avoid fitting a “Banks” parameter to individual bonds or issuers, we refrain from using too specific bond classifications at this point. Important to mention that substantial improvement in the predictive power is likely to be achieved if certain (sub-) sectors were fitted in addition to Financials/Non-Financials. In the near future the improvement of the regression equation is high on the agenda, with the inclusion of cleverly chosen classifications as a priority. One can think of adding value to the model by separating certain economic sectors that are more or less prone to stress in financial markets beyond the Financial / Non-Financial classification. Banks / Insurance is one example from the Financial spectrum of bonds, Retail / Utilities serves as an example for the Non-Financial subset of the data.

An improvement in fit measures is also (easily) achieved if conventional (automated) outlier detection algorithms were used. Using influence measures on the regression equation to identify observations that do not meet certain criteria (Cook’s distance, hat matrix (Belsey, Kuh & Welch, 2005)), measures of fit will improve substantially. The removal of “outliers” is also likely to improve estimates of coefficients (smaller standard error). At this point the entire sample is used for estimating equations as well as fitting Bid-Ask Spreads. The simple reason is that all bonds in the dataset are true and correct data points, and exclusion of data points should be given ample consideration. However, it might be true that some bonds in the dataset simply follow a different set of rules (different equation) when it comes to Bid-Ask Spread. Therefore, we believe that data points should not be excluded from an individual regression point of view, but certain bonds (ISIN codes) might be excluded if, and only if, that same bond is identified as an outlier during a substantial (or all) period of time. The exclusion of certain bonds will improve the regression equations but the exclusions are interesting to investigate in their own right. The potential inclusion of issuer specific information could also alleviate this problem. Nevertheless, future research will focus on “outliers”.

The last direction of future work should look into the modelling approach. Even though the regression equation (using temporal component) seems to produce valid and reasonably accurate results, one can imagine that more complex non-linear relationships go unnoticed. A more complex investigation into the functional form of some of the variables included in the regression analysis should be desired. In particular the effect of a bond’s age needs to be investigated further, since this is easy to relate to real practitioner insight.

## 6. Credit Spread Analysis

Taking an approach very similar to the approach taken when modelling the Bid-Ask Spread, we use several analytical values as well as bond classifications to model Credit Spread. As mentioned before, the analytical value used as the Credit Spread is the Annual Benchmark Spread (please see Section 3) and the terms will be interchangeably. The modelling setup is very similar to the one seen before; four regression equations are set up that are run on a daily basis. The analytical values used to model Credit Spreads does not limit itself to merely those provided by iBoxx as we have created one already, the Relative Bid Ask Spread (RBAS) and will also include the volatility of Bid-Ask Spread.

### 6.1 Bid-Ask Spread Volatility

Considering that not only the level of Liquidity but also the volatility of liquidity can be an important determinant of Credit Spreads, estimates of volatility will be another analytical value that is considered in the regression equation. When modelling the Bid-Ask Spread as a univariate time series using ARIMA models, we already estimated the volatility of eight markets segments using a GARCH (1,1) model. The possibility of computing the Bid-Ask Spread volatility on an individual bond basis is explored and these estimates are considered when construction regression models.

To estimate the volatility of individual bonds we use the Exponentially Weighted Moving Average (EWMA) model that is widely used to estimate and forecast volatility (Roh, 2007) and is defined by

$$\sigma_i^2 = \omega + (1 - \lambda)r_i^2 + \lambda\sigma_{i-1}^2$$

where  $\sigma_i^2$  is the conditional variance of  $\log(BAS)$ , with  $\lambda$  the weight assigned to lagged variance and  $(1 - \lambda)$  the weight assigned to the daily changes in  $\log(BAS)$ .

One can see that the EWMA is a particular case of a GARCH model where  $\omega = 0$ ,  $\alpha = 1 - \lambda$  and  $\beta = \lambda$ . The parameter to control is  $\lambda$ , with higher values indicating a slower decay, whereas low values increase the responsiveness to daily changes in BAS. Hull (2012) says that the estimator tends to be above 0.90, but heavily depends on the modelled asset class and

time period. As an example he reports that the estimator is close to 0.9 for currency markets, but near 1.0 for money markets. The original value used by JP Morgans' RiskMetrics is 0.94 (RickMetrics, 1996) and is often used as a point of reference. In choosing the optimal  $\lambda$  parameter a series of possible volatility series are computed, with different values of  $\lambda$ . Using a simple, cross-Rating correlation analysis of Credit Spreads, the volatility series with the highest correlation coefficient is chosen.

$\lambda$	Pearson- $\rho$
.9	0.14
.91	0.13
.92	0.16
.93	0.18
.94	0.20
.95	0.21
.96	0.19
.97	0.15
.98	0.13

Table 6: EWMA, choosing an optimal  $\lambda$ .

From Table 6 above we can see that aggregated over the entire sample period and across all bonds, a  $\lambda$  of 0.95 has the highest correlation coefficient (0.21). The choice of optimal parameter might seem slightly arbitrary as the results for several values of  $\lambda$  around 0.95 are very similar.

## 6.2 Temporal Cross-Sectional Modelling

In keeping with the model for Bid-Ask Spreads we split the dataset in four subsets according to Rating classes and define regression analyses as follows:

$$\log(\text{Credit Spread}_{i,j,d}) = c_{j,d} + \beta_{j,d}x_{i,j,d,1} + \beta_{j,d}x_{i,j,d,2} + \dots + \beta_{j,d}x_{i,j,d,k} + \varepsilon_{i,j,d}$$

Equation 18

where  $i$  refers to one particular observation of a bond within group  $j$  (AAA, AA, A or BBB) on day  $d$  and the random errors  $\varepsilon_{i,j,d}$  are independent normally distributed random variables

with zero mean and constant variance  $\sigma_{j,d}^2$ . More specifically, substituting in the independent variables the following regression is fitted:

$$\begin{aligned}
& \log(\text{Credit Spread}_{i,j,d}) \\
&= c_{j,d} + \beta_{j,d,1}\sqrt{\text{Age}}_{i,j,d} + \beta_{j,d,2}\log(\text{Duration})_{i,j,d} \\
&+ \beta_{j,d,3}(\text{Coupon Rate})_{i,j,d} + \beta_{j,d,4}(\text{RBAS})_{i,j,d} \\
&+ \beta_{j,d,5}(\text{BAS Volatility})_{i,j,d} + \beta_{j,d,6}(\text{Non - Financial})_{i,j,d} \\
&+ \beta_{j,d,7}(\text{Sovereign})_{i,j,d} + \beta_{j,d,8}(\text{Senior})_{i,j,d} + \beta_{j,d,9}(\text{Tier 1 Debt})_{i,j,d} \\
&+ \beta_{j,d,10}(\text{Upper Tier 2 Debt})_{i,j,d} + \beta_{j,d,11}(\text{Lower Tier 2 Debt})_{i,j,d} \\
&+ \beta_{j,d,12}\log(\text{Duration} * \text{Non - Financial})_{i,j,d} \\
&+ \beta_{j,d,13}\log(\text{Duration} * \text{Sovereign})_{i,j,d} \\
&+ \beta_{j,d,14}(\text{RBOS} * \text{Non - Financial})_{i,j,d} + \beta_{j,d,15}(\text{RBOS} * \text{Age})_{i,j,d} \\
&+ \beta_{j,d,16}(\text{RBOS} * \text{Sovereign})_{i,j,d} + \varepsilon_{i,j,d}
\end{aligned}$$

**Equation 19**

Some restrictions apply as not all covariates are fitted for all groups in  $j$ ; variables with the Sovereign indicator are only fitted for  $j=\{\text{AAA}, \text{AA}\}$  and the indicator variables for the various Debt Tiers are excluded for the same groups.

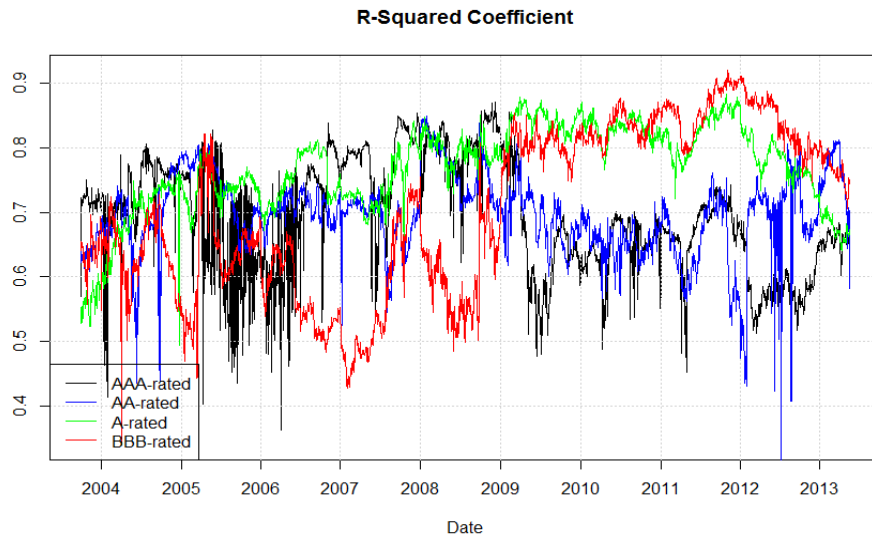
Using the same temporal component to modelling Credit Spreads, we allow for the effect of the independent variables to vary over time. These independent variables include Duration, Seniority, Coupon, Financial indicator, Sovereign indicator, Relative Bid-Ask Spread, volatility of Bid-Ask Spread, Age in Years and several interaction effects including Duration\*Financial, RBAS\*Financial and RBAS\*Age. Before delving into the estimated time-varying coefficients, the fit of the model (Equation 18) is examined in Table 7.

Model	AAA	AA	A	BBB
Mean R-Squared	.689	.701	.778	.706
AI, $\alpha = 0.1$	62.3%	69.0%	71.1%	67.9%
AI, $\alpha = 0.3$	83.3%	89.7%	90.1%	86.7%
N	534240	475714	888464	596456

**Table 7: Measures of fit; Credit Spread model**

Based on the measures of fit presented above, the model describes the data reasonably accurately. Based on the fitted Credit Spreads, there is only a small margin of error (on

average). The accuracy of the sub-models are likely to vary by time, the Bid-Ask Spreads model presented in Chapter 5 for example, did not fit the years 2007-2009 as well as it did other years. Figure 15 below describes the fit for the four sub-models through time.



**Figure 15: R-Squared through time; Credit Spread sub-models.** *The  $R^2$  coefficient for AAA-rated bonds is very noisy up to 2006, something not seen for other categories. The coefficients are high and reasonable stable over time with the exception of BBB-rated bonds.*

From Figure 15 we can see that this does not appear to be true for Credit Spreads, perhaps with the exception of BBB, which seems to have little explanatory power during 2007-2009, compared with other years as well as in comparison to other Rating qualities during the same time period. For all other Rating classes, and A in particular, R-Squared measure of fit is consistent through time. Higher Rating classes (AAA and AA) seem to have a lower fit during the years following the financial distress of 2007-2009.

### 6.3 Extracting Liquidity Premia

Whereas both the CDS and structural approach to extracting liquidity premia rely on the accurate estimation of expected defaults of and credit risk and imply a residual liquidity related premium, the above modelling technique explicitly models the liquidity components of the spread. The liquidity premium of any bond, on any given day is easily extracted in three steps:

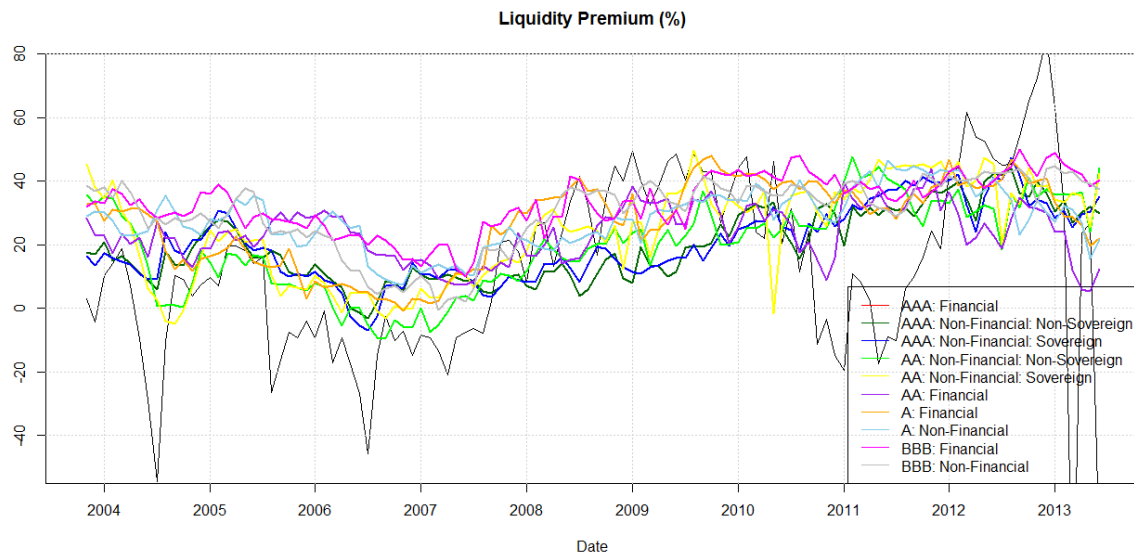
- Estimate the  $Credit\ Spread_{i,j,d}$  using the appropriate regression equation ( $Rating_j$  and  $Day_d$ ) and its independent variables. This is the model fitted value using the actual independent values; i.e. the Credit Spread given its liquidity (RBAS and volatility of Bid-Ask Spread).
- Estimate the  $Credit\ Spread_{i,j,d}$  for a bond identical in all aspects (independent variables), except liquidity. By setting the RBAS and volatility of Bid-Ask Spread to zero, we effectively create a synthetic bond, perfectly liquid.
- The difference in basis points between the estimated credit spread of the “real bond” and the estimated credit spread of the perfectly liquid equivalent is the Liquidity Premium in bps. Dividing the estimated Liquidity Premium by the estimated Credit Spread, gives an estimate of the % Spread attributed to Liquidity.

For example, please consider the following Nationwide 6.25% 2024 bond on the 18<sup>th</sup> of August 2011. The Credit Spread, measured by the observed Benchmark Spread provided by iBoxx is 672 bps. Using the temporal model (Equation 18), we estimate the Credit Spread to be 643, a reasonably accurate estimate using the RBAS of the bond on this day (RBAS = 1.6). Setting the RBAS (and Bid-Ask Spread volatility) to zero we create a synthetic, perfectly liquid equivalent which, using the same temporal model (Equation 18) is estimated to have a Credit Spread of 367bps. The difference between the two estimates (643 - 367 = 276) is the estimated Liquidity Premium. Quoting the Liquidity Premium as a percentage of total spread the 276bps translate into a proportion of 41.5%.

#### **6.4 Liquidity Premium Estimates**

It is important to express the estimated Liquidity Premia in both basis points and as a proportion of total Credit Spread. At least some of the recent discussions regarding the Liquidity Premia in the context of liability discounting evolve around the concept of applying a (fixed) percentage to the Credit Spread. This seems an intuitive way of dealing with a diverse universe of bonds trading at very different levels of spreads (bps). This simple method simply results in bonds with a higher spread having Liquidity Premium, but proportionally so. This seems restrictive as certain bond characteristics can intuitively be linked to higher liquidity premia. To investigate on an aggregate level, we have again divided the bonds by Rating, Financial status and, for good quality Ratings, Sovereign status. For a thorough overview, please see Figure 1 in Appendix IV; both the Liquidity Premium in basis

points and as a proportion of total Spread have been plotted in great detail (by Rating and Daily). It is highly recommended to study these Figures carefully. Figure 16 below summarizes the aforementioned plots from the Appendix by showing the proportion of Spread for each of the ten categories as a function of time.



**Figure 16: Liquidity Premium (%) over time by category aggregated by month.** *The estimates represent the average liquidity premium (%) for each of the ten categories in a given month. Even though the Liquidity Premium (as an aggregate) is reasonably stable over time, the exception being the brief time period before the financial crisis, some differences can be seen. For example, BBB seem to have, on average a slightly higher premium than AA bonds. However, for bonds of average liquidity, the premium is consistently between 20%-40%.*

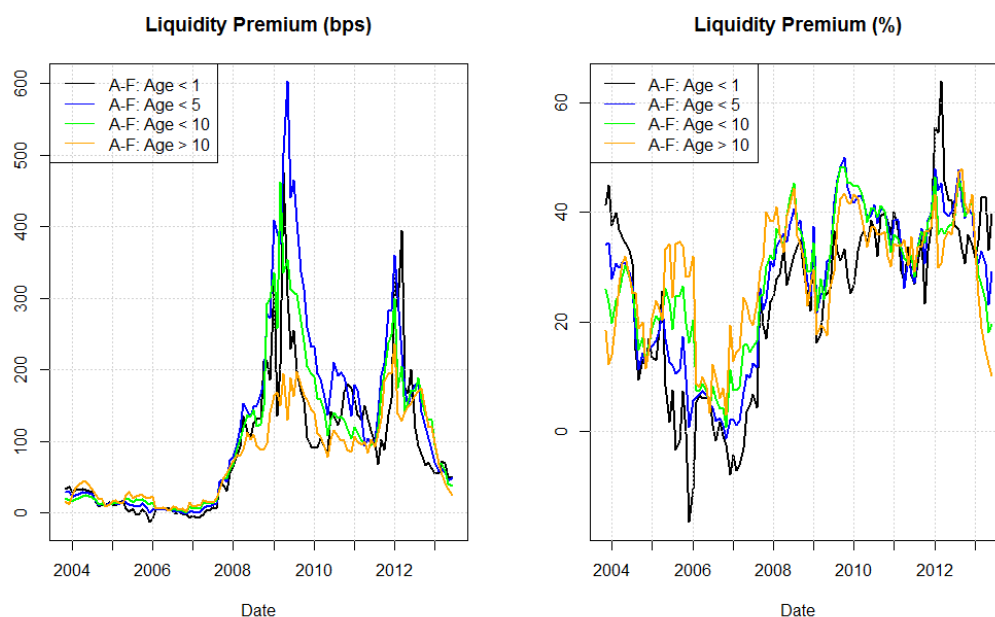
Looking at the above Figure (Figure 16), we can see that regardless of Rating class, Financial- or Sovereign status, Liquidity Premium Estimates move together throughout the time period. The only exception perhaps is AAA-Financials which seem rather volatile and out width the “normal” range of Liquidity Premia.

The observations we can draw from Figure 1 in Appendix IV are very similar; the differences in Liquidity Premium in basis points differs substantially between Rating classes and between bond classifications, the differences between bond classifications within a given Rating call are small and not significant. What does becomes clear from both the Figure above and the Figure in the Appendix (IV) is that the Liquidity Premium as a proportion of total Spread, does, even on an aggregate levels, change over time. We can see that the Liquidity Premium started to decline late in 2004 to reach a low during the first half of 2007, where Premia were low, close to zero or even negative on an aggregate level. Premia returned to “normal” levels (30%-40%) during 2008 and have stayed constant since. It is important to keep in mind that the results presented here are aggregate Liquidity Premia, insightful to get

an overview of the magnitude of Liquidity Premia of different segments of the market. Within each of these segments, there will be bonds with substantially higher, and with substantially lower Premium estimates than the averages presented here. Individual bond characteristics determine the Credit Spread and Liquidity Premium.

### 6.4.1 Liquidity Premium and Bond Age

As an example of a very simple way how to investigate the effect other bond characteristics may have on the Liquidity Premium, we are going to look at one category for different levels of several such characteristics, concluding with the Relative Bid-Ask Spread (RBAS). Firstly, we look at the average Liquidity Premium of A-Financials for different levels of Bond Age. We create four age brackets: 0-1 years, 1-5 years, 5-10 years and 10+ years and investigate the Liquidity Premium (in bps and %).



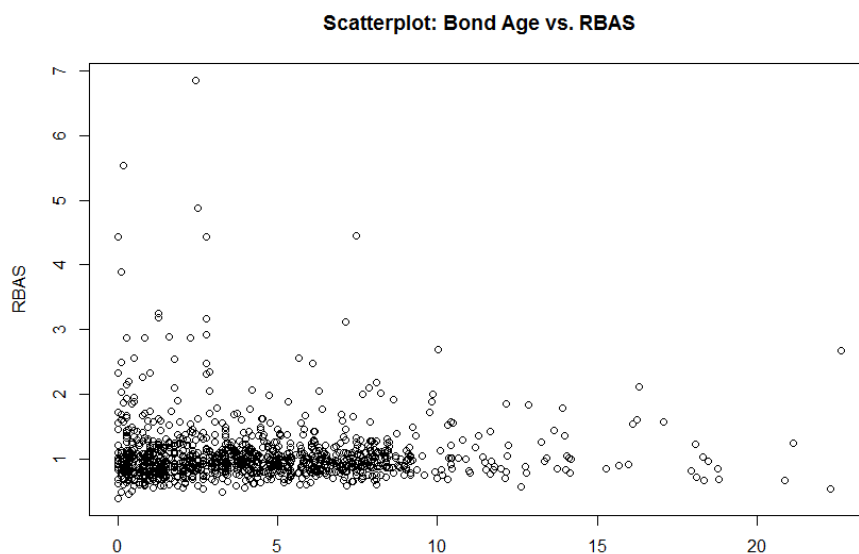
**Figure 17: Liquidity Premium as a function of Bond Age (A-rated Financials).** *Expressing the Liquidity Premium in bps, we can see (left) that older bonds reacted the least to the financial crisis. Expressing the Premium in % of total spread, we can only observe that during 2005-2007 there seemed to be a difference, where younger bonds had the smallest liquidity premium.*

The above Figure (Figure 17) shows that estimated Liquidity Premia across age categories differ, with the oldest bonds in the market commanding the lowest Premium. In terms of total spread there does not seem to be a substantial difference (Figure 17, right), except for the brief period prior to the financial crisis. During this period (2005-2007) Liquidity Premia were at their lowest, with the youngest bonds (Age < 1 year) commanding the lowest



Premium, approximately 0% of the total Spread on average. For other Age brackets this effect does not seem as pronounced.

The observed effect could simply be a function of a bond's specific liquidity, if and only if there were some relationship between a bond's age and its RBAS. Since Bond Age is explicitly included in the temporal regression that is used to determine RBAS (Equation 10) it is not correlated with Age at all. The scatterplot in Figure 18 of Bond Age and corresponding RBAS, for all bonds on 3rd April 2007, clearly illustrates the above.

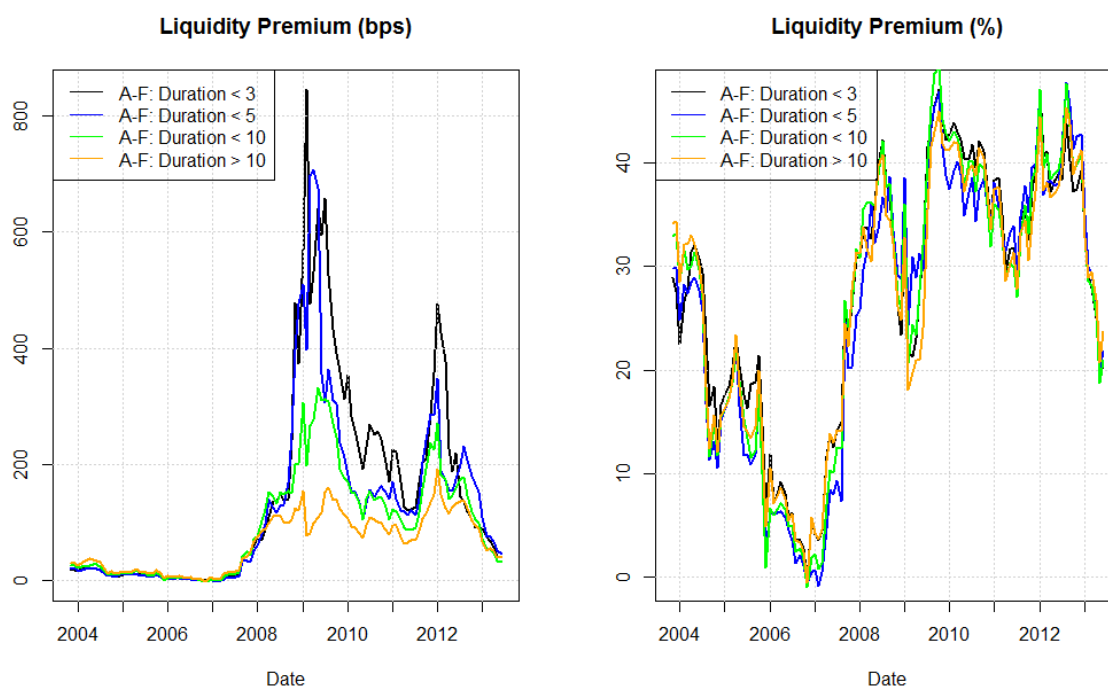


**Figure 18: Relationship Bond Age and RBAS.** *Since Bond Age is explicitly included in the construction of the RBAS, RBAS does not mitigate the effect of Age on the Liquidity Premium. The scatterplot confirms no relationship exists between Age and RBAS*

The reason for the discrepancy between the Age brackets stems from the estimated coefficient of the interaction term between Bond Age and RBAS ( $\beta_{j,d,15}$  in Equation XX). This term is rather insignificant over the decade under study with the exception of the few years prior to the crisis in which the effect of RBAS differed depending on Bond Age. During the time period just before the crisis the effect of RBAS was slightly negative, indicating that bonds traded at no liquidity premium and the most illiquid bonds even traded at a lower credit spreads. The effect of RBAS is adjusted by the interaction and this negates the negative effect for older bonds, leaving the younger bonds exposed to the negative RBAS coefficient, resulting in very low/negative Liquidity Premia. For an overview of the main coefficients, please see Appendix IV Figure 3. Confirmation from industry regarding the Age and Liquidity dynamics, during the brief time period prior to the financial, would be very helpful.

## 6.4.1 Liquidity Premium and Duration

Following the same steps outlined above, again using A-rated Financials as our (arbitrarily) chosen reference category, we investigate the effect of Duration. Four groups are formed and the Liquidity Premium investigated for each.



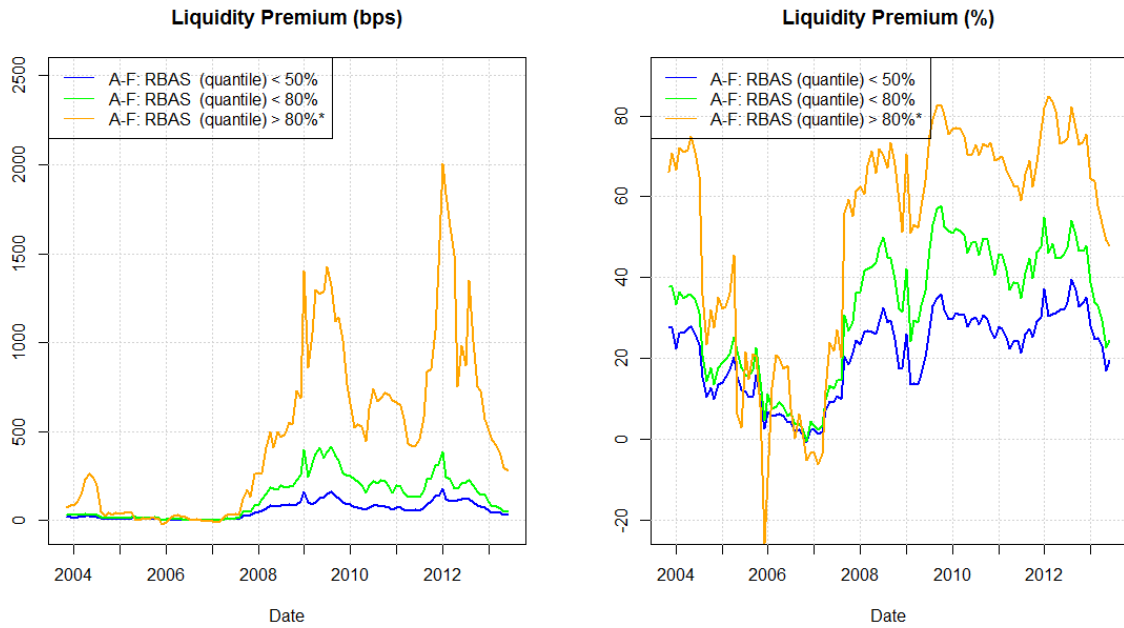
**Figure 19: Liquidity Premium as a function of Duration (A-rated Financials).** Bonds with the longest Duration responded the least to the financial distress during 2008-2010, in terms of Liquidity Premium.

The Liquidity Premium as a proportion of total Spread is the same across Duration buckets, but in terms of basis points there are substantial differences. This implies (the perhaps obvious observation) that Duration is an important driver of Credit Spread. Careful examination of Figure 19 (left) does tell us that the relationship between Duration and Credit Spread has changed during the decade we are studying. Prior to the financial crisis Duration has a positive effect on Credit Spreads (higher Duration meant higher Credit Spread, *ceteris paribus*). One might argue from the above plot that this effect is negligible as estimates in basis points are very similar compared to the differences seen during the financial crisis. However, it is important to take into account that all Credit Spreads were higher during this time period, the percentage difference between buckets is almost identical. During the crisis, longer Durations saw lower Liquidity Premia, implying a lower Credit Spread (as Liquidity

Premium as a proportion of total Spread is consistent between Duration buckets). The observed change in Duration effect can be seen clearly from the model coefficients (please refer to Figures 2, 3, 4 and 5 in Appendix IV for a selection of model coefficients), which drop from an elasticity of approximately 0.4 prior to late 2007 to -0.4 in a matter days. The coefficient seems to be recovering and on the most recent data point (end May 2013) the coefficient is very close to zero. This seems to correspond to Figure 19 (left), where the Liquidity Premium estimates are the same for the last data points.

#### **6.4.2 Liquidity Premium and Relative Bid-Ask Spread**

Perhaps most important is to how Liquidity Premium estimates differ between bonds of varying degrees of (relative) liquidity. Please recall that liquidity, measured by the Relative Bid-Ask Spread (RBAS), describes the liquidity relative to bonds with identical characteristics. Similar to our previous analysis we divide the RBAS scores in three buckets according to the RBAS quantiles: bucket one contains bonds of average liquidity (50% quantile), whereas bucket two contains those between the 50% and 80% quantile, the third bucket contains the bonds at the (very) illiquid end of the spectrum. The plotted estimates in Figure 20 are averages for bucket one and two whereas for bucket three the median is plotted. The third bucket contains several extreme estimates (very end of the spectrum) which would increase the estimator considerably. Please note that especially within the third bucket far more extreme estimates are observed.

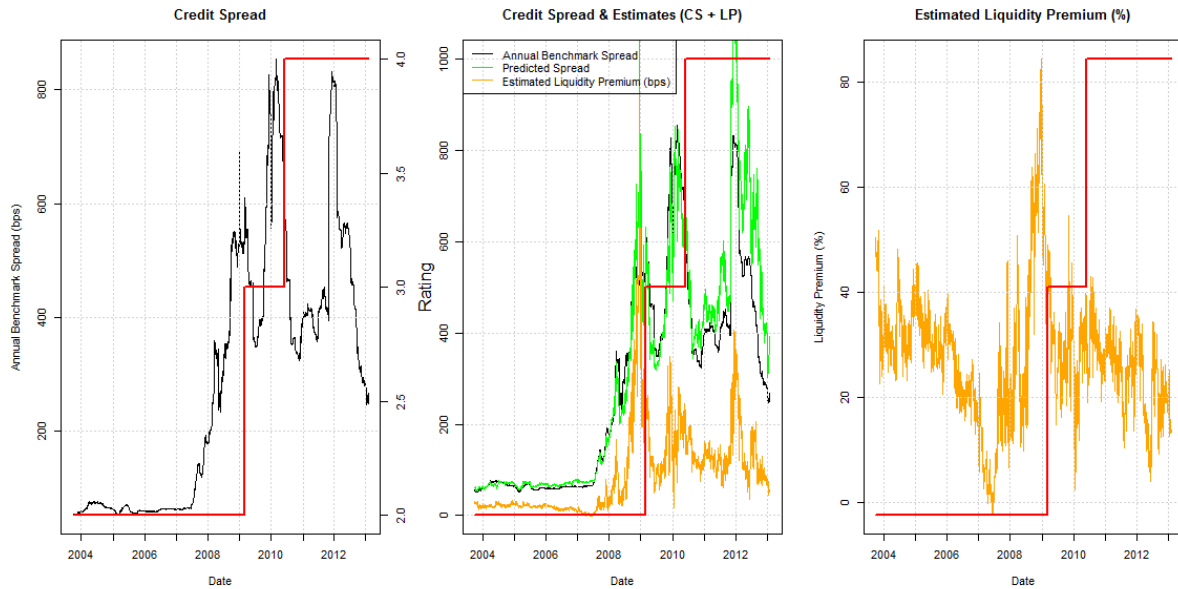


**Figure 20: Liquidity Premium as a function of RBAS (A-rated Financials).** *The RBAS is most directly related to the size of the Liquidity Premium, both in terms of basis points and proportion of total spread. The ability to discriminate between bonds on the basis of bond-specific liquidity levels is key in the modelling approach.*

The RBAS is a superior measure of liquidity since it is free from any factors that influence a bond's liquidity (see Section 5.3.2 for Bid-Ask Spread modelling). From Figure 17 we can clearly see that higher values of illiquidity result in a higher Liquidity Premium, both in basis point and as a proportion of total Spread. This is an obvious, yet important result that is often overlooked in other academic literature; the modelling of the Liquidity Premium on an individual basis allows us to differentiate between bonds of different liquidity.

## 6.5 Credit Spread around Rating Changes

It is well known that credit ratings are a lagged indicator of a firm's affairs, and the Markit iBoxx Rating used in this project is no exception of course. The question that naturally arises is whether our modelling approach is capable of capturing a Credit Spread that reacts to new market information quicker than Rating does. It seems, that the model is able to capture rising Credit Spreads prior to a downgrade, perhaps surprisingly so. Figure 16 below (left) shows the evolution of the Credit Spread of one particular bond (Halifax (HBOS) coupon paying Lower Tier 2 debt bond issued in 1988 and maturing 2014) during the sample period with Rating. Please note that at the start of the time period this bond is rated AA (2), is downgraded to A (3) and BBB (4) in 2009 and 2010 respectively.



**Figure 21: Case Study of Credit Spreads, Liquidity Premium and Rating Changes for the individual bond**

Before either downgrade the Credit Spread had been increasing, confirming that Rating is a lagged indicator of what is already incorporated in market prices. From Figure 16 (middle) however we can see that this does not appear to decrease the fit of the model in any way. The predicted Credit Spread remains a very good estimator of Credit Spread prior to the downgrade. Perhaps not so obvious is the fact that after the downgrade to BBB the model consistently overpredicts the Credit Spread. This seems to indicate that within the (very diverse) BBB-universe of bonds, this particular bond is of relative good credit. Also from Figure 16 (middle), the estimated Liquidity Premium estimates do not seem to “jump” as a result of a downgrade but move together with the correctly estimated Credit Spread prior to a downgrade. Figure 16 (right) shows us a plot with very familiar numbers and trends over time, none affected by the downgrading, merely by market wide events (especially several UK financials such as HBOS).

## 6.6 Future Work

Very similar in approach to modelling Bid-Ask spread, much of the anticipated future work is similar. A more granular bond classification will be explored, for which the expectations are a substantial improvement in model fit. Outliers will be dealt with using the same approach presented for dealing with outliers in Bid-Offer Spreads, and again one need to be cautious not to exclude perfectly valid observations. The modelling of the Credit Spread using a more

sophisticated modelling technique, able to capture complex non-linearities in the data, is important. Please see Section 5.3.2.4 for a more detailed discussion of future work in these capacities.

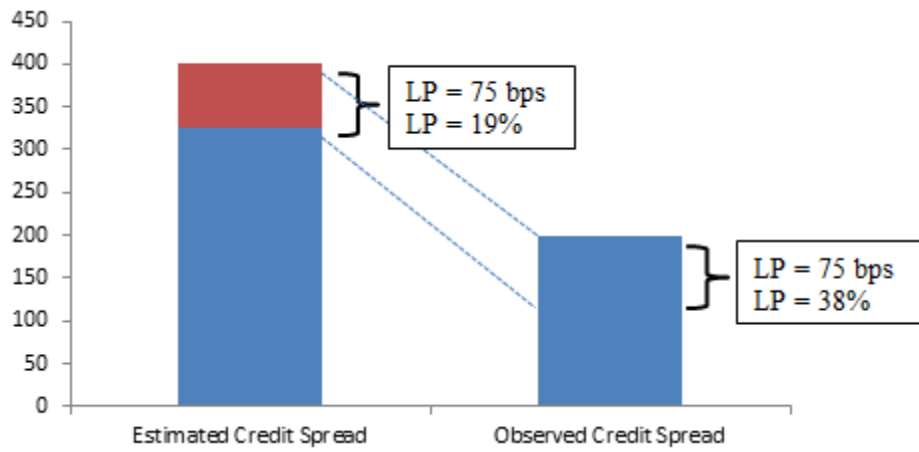
Further investigation of the volatility of the Bid-Ask Spread and/or the volatility of the Relative Bid-Ask Spread is necessary. Currently the volatility estimates are rather insignificant and add little to the analysis. A more thorough understanding of the volatility, including all possible specifications (volatility / variance of BAS or volatility / variance of RBAS) is a priority.

Within one given Rating quality, ignoring differences in bond classifications and analytical values, all bonds are essentially modelled as having the same credit risk. This is especially questionable for the lower rated bonds in the IG spectrum (BBB) as the group is very diverse. Ideally we would be able to incorporate Issuer (or Ticker) specific variables. Potential variables should be explored and could include for example, total amount of outstanding debt (ideally as ratio to firm size) or number of outstanding bonds. For a substantial subsection of the bond universe one could also think of equity market variables if the issuer is quoted on a stock exchange or balance sheet variables if available in quarterly reports. Whereas these variables are definitely not available for the entirety of the dataset, they may be an improvement on the existing model.

The fact that an improving or deteriorating credit position of the firm within a given Rating class cannot be accounted for becomes clear from the analysis in section 6.5, where we can see that keeping all variables constant, a downgrade can cause the estimated Spread to be consistently too high or too low, as it the case in Figure 14 after the downgrade to BBB. To account for this, in addition to adding more variables a two-stage fitting process could be considered. In this approach the estimated Credit Spread using the current methodology is compared to the actual observed Annual Benchmark Spread. Depending on the discrepancy between the two, the Spreads are estimated again using a different equation (Rating quality). The result are two estimates of Credit Spreads and depending on the discrepancy between observed and estimated spreads, a weighted average of the two can be taken. This “trick” could potentially account for spreads that are consistently estimated to be too high or low. A third and last possible way to account for differences within a given Rating category would be to fit the equation as is currently done, fix all beta coefficients and allow the constant of the model to vary on a bond by bond basis, minimizing the residual. This effectively allows us to create an infinite number “tranches”.

Important to note in the approach taken is that is model the Liquidity Premium explicitly, in contrast to for example any structural approach. Spread decompositions tend to include three components; Expected Defaults, Default Risk and Liquidity Premium. Using the approach taken here we are also able to identify three components but a different set: Liquidity Premium, Liquidity Risk Premium and the remainder of the spread, Credit Premium. This Credit Premium consists of both the expected defaults as well as default risk. The current model does not distinguish between the two. Even though the addition of a fourth component is a clear improvement, the separation of the two default related components is important too. Future work should explore the possibility to extract these and decompose the Credit Spread in four parts.

Currently the results have of Liquidity Premium estimates have been presented in both number of basis points as well as a proportion of the total Credit Spread. Critical to mention is that the point of reference for the Liquidity Premium as a calculated proportion is the estimated Credit Spread, produced by the model. For real-world, practitioner applications this might be problematic as illustrated using the following example (see also Figure 17). Suppose the estimated Credit Spread on Bond A is 400bps and the estimated Liquidity Premium is 75bps, which is 19% of the Spread. The practitioner could take away from this prediction the 75bps, apply this to the Credit Spreads observed in the market (200 bps) and will work with completely different Liquidity proportions. The danger therefore, at this point, is for model predictions that are quite different from the true spreads and applying the raw estimated liquidity premium. Even though model estimated are (very) close to observed Credit Spreads, discrepancies do and will persist to occur. A quick and simple fix would be to calculate the proportion of Liquidity Premium using the estimated Credit Spread and work out the Liquidity in bps from the actual market observed spread. Practical use and application of Liquidity Premium estimates should be given thought in the near future.



**Figure 22: Applying Estimated Liquidity Premium to real-world Spreads.** *Currently the Liquidity Premium is expressed in basis points and as a proportion of Spread, where Spread refers to the estimated Spread according to the regression model. When the discrepancy between the modelled Credit Spread and the observed market Credit Spread is large, working in basis points might present a bias for industry professionals.*



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## **8. Appendices**

### **8.1 Appendix I: Markit iBoxx**

### **8.2 Appendix II: Credit Spreads**

### **8.3 Appendix III: Bid-Ask Spread Modelling**

### **8.4 Appendix IV: Credit Spread Modelling**

## 8.1 Appendix I: Markit iBoxx

**Table 8: Detailed Bond Classification**

	Economic Sector	Market Sector	Market Sub-Sector
Financials	Financials	Banks	Banks
		Insurance	Life Insurance
			Nonlife Insurance
		Financial Services	General Financial
			Real Estate
		Insurance Wrapped	Insurance Wrapped
Non-Financials	Oil & Gas	Oil & Gas	Oil & Gas Producers
			Oil Equipment & Service
	Basic Materials	Chemicals	Chemicals
		Basic Resources	Industrial Metals
			Mining
			Forestry and Paper
	Industrials	Construction	Construction
		Industrial Goods	Aerospace & Defense
			Electronic & Electrical Equipment
			General Industrials
			Industrial Engineering
			Industrial Transport
			Support Services
	Consumer Goods	Automobiles	Automobiles
		Food & Beverage	Beverages
			Food Producers
		Personal Goods	Household Goods
			Personal Goods
			Tobacco
	Health Care	Health Care	Pharmaceuticals & Biotechnology
	Consumer Services	Retail	Food & Drug Retailers
			General Retailers
		Media	Media
		Travel & Leisure	Travel & Leisure
	Telecommunications	Telecommunications	Fixed Line Telecommunications
			Mobile Telecommunications
	Utilities	Utilities	Electricity
			Gas / Water & Multi-utilities
	Technology	Technology	Software & Computer Services
			Technology Hardware

### Key Measures: Detailed Formulae

The (average redemption) yield is a key metric for any bond and is calculated as follows:

$$(P_{i,t}A_{i,t})F_{i,t} = \sum_{j=1}^n CF_{i,j}(1 + y_{i,t})^{-L_{i,j}}$$

where,  $P_{i,t}$  is the clean bond price for bond  $i$  at time  $t$ ,  $A_{i,t}$  is the accrued interest,  $CF_{i,j}$  the cash-flow in the  $j$ th period,  $L_{i,j}$  the time difference in coupon periods between  $t$  and  $j$  and  $y_{i,t}$  the yield. The Newton iteration method is used to solve the equation for  $y_{i,t}$ .  $F_{i,t}$  is a redemption factor that is relevant for sinking funds, amortizing bonds and unscheduled full redemptions. For other bond types,  $F_{i,t}$  is always equals 1.

The duration of a bond is another key metric and also heavily features in this study:

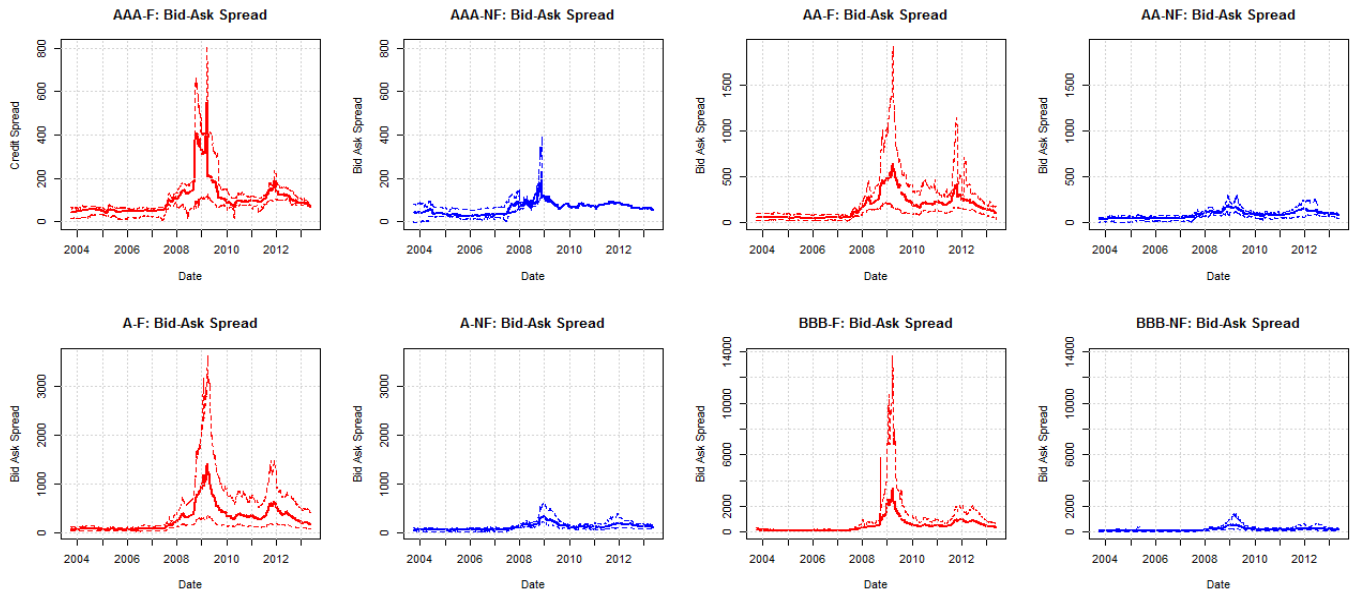
$$D_{i,t} = \frac{\sum_{j=1}^n CF_{i,j}L_{i,t,j}(1 + y_{i,t})^{-L_{i,t,j}}}{\sum_{j=1}^n (1 + y_{i,t})^{-L_{i,t,j}}} = \frac{1}{(P_{i,t} + A_{i,t})F_{i,t}m} \sum_{j=1}^n CF_{i,j}L_{i,t,j}(1 + y_{i,t})^{-L_{i,t,j}}$$

where,  $P_{i,t}$  is the clean bond price for bond  $i$  at time  $t$ ,  $A_{i,t}$  is the accrued interest,  $CF_{i,j}$  the cash-flow in the  $j$ th period,  $y_{i,t}$  is the bond's yield,  $m$  is the number of coupon payments per year,  $F_{i,t}$  the product of the redemption adjustment and the pay-in-kind adjustment factors for bond  $i$  at time  $t$



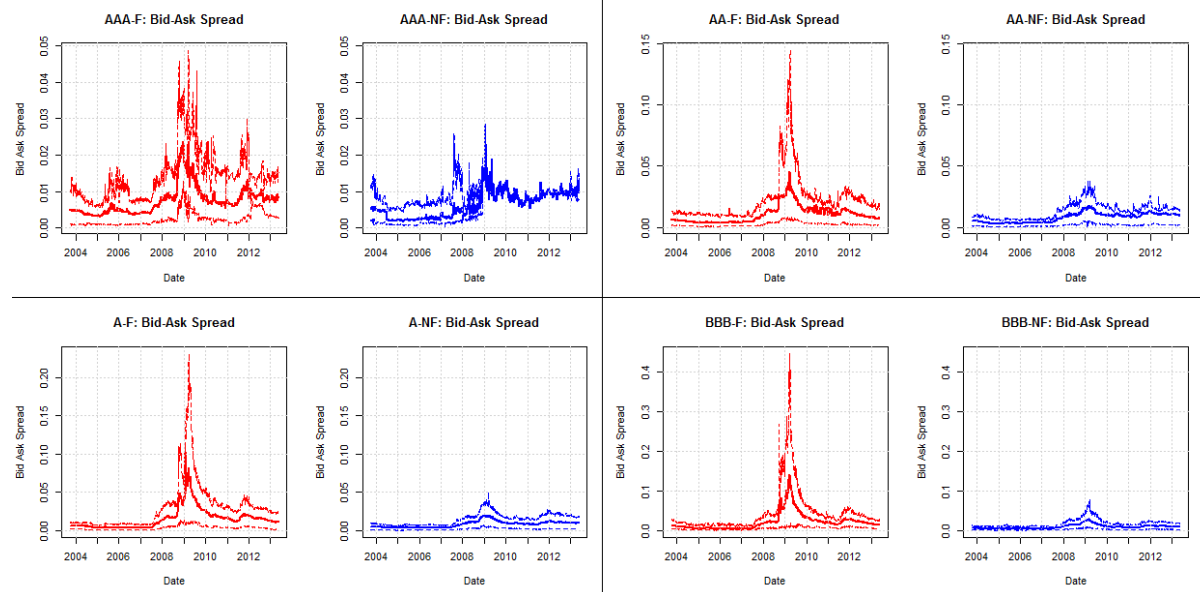
## 8.2 Appendix II: Credit Spreads

**Figure 1: Credit Spread and Quantiles (5% - 95%).** Detailed overview of Credit Spreads aggregated by 8 groups (Rating Class and Financial / Non-Financial status). The drawn quantiles show that the distribution of Credit Spreads changes over time; most significantly low rated Financials as a result of the Financial Crisis during 2007-2010



### 8.3 Appendix III: Bid-Ask Spread Modelling

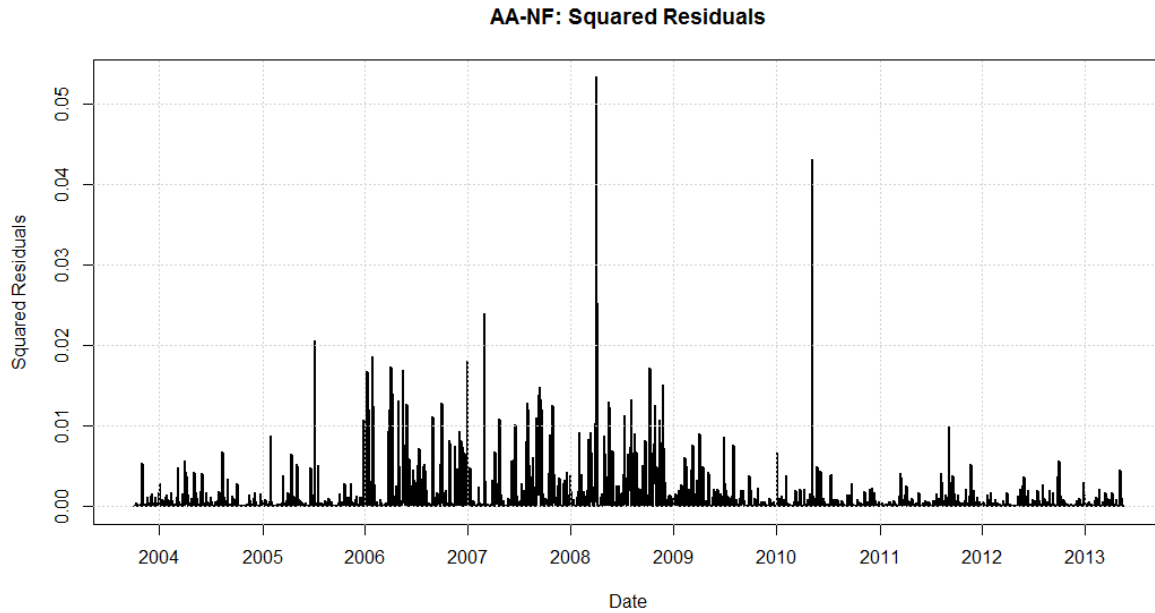
**Figure 1: Bid-Ask Spread and Quantiles (5% - 95%).** Detailed overview of Bid-Ask Spreads aggregated by 8 groups (Rating Class and Financial / Non-Financial status). The drawn quantiles show that the distribution of Bid-Ask Spreads changes over time; most significantly low rated Financials as a result of the Financial Crisis during 2007-2010



**Table 1: Coefficients chosen ARIMA:** Overview of the parameters of the Autoregressive and Moving Average components of the models chosen for each of the eight groups.

Model	AR(1)	AR(2)	MA(1)
AAA-F (0,1,1)			-.380 (.019)
AAA-NF (1,1,1)	.286 (.051)		-.627 (.043)
AA-F (1,1,1)	.187 (.049)		-.550 (.042)
AA-NF (1,1,1)	.149 (.059)		-.488 (.053)
A-F (2,1,0)	-.113 (.019)	-.184 (.018)	
A-NF (1,1,1)	-.117 (.061)		-.221 (.060)
BBB-F (1,1,1)	.153 (.043)		-.605 (.036)
BBB-NF(2,1,0)	-.298 (.020)	-.079 (.020)	

**Figure 2: Squared Residuals ARIMA process.** Time series plot of the squared residuals after fitting an ARIMA process to the Bid-Ask Spread (AA-NF). The non-constant variance and variance clustering in particular merit the use of conditional variance models.

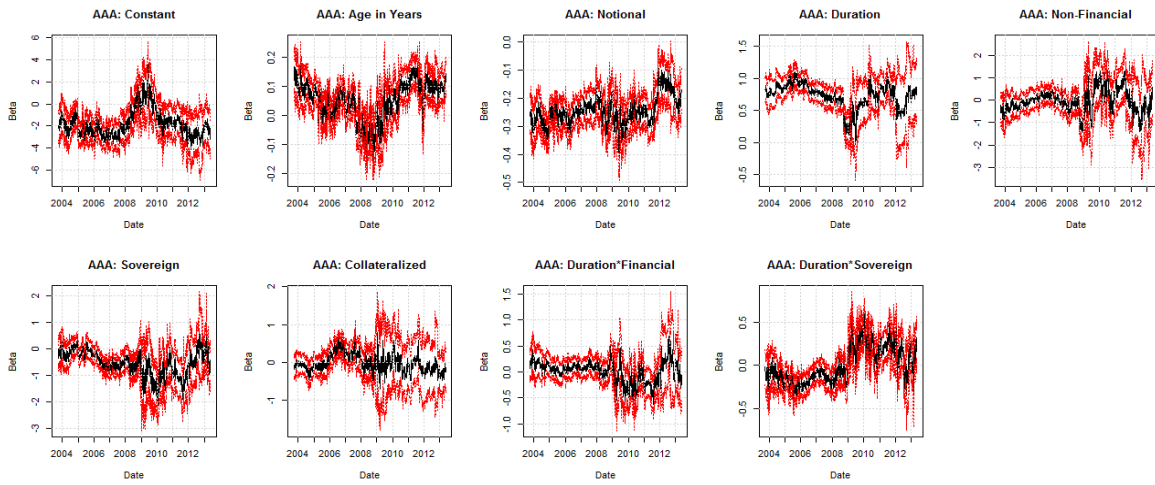


**Table 2: GARCH(1,1) Error Distribution.** *Fitting the GARCH(1,1) models using Gaussian errors causes the errors of the GARCH process to be non-normal. Fitting Student T errors to allow for fatter tails does result in an improved fit, as measured by the Log-Likelihood, it does not solve the non-normality of errors.*

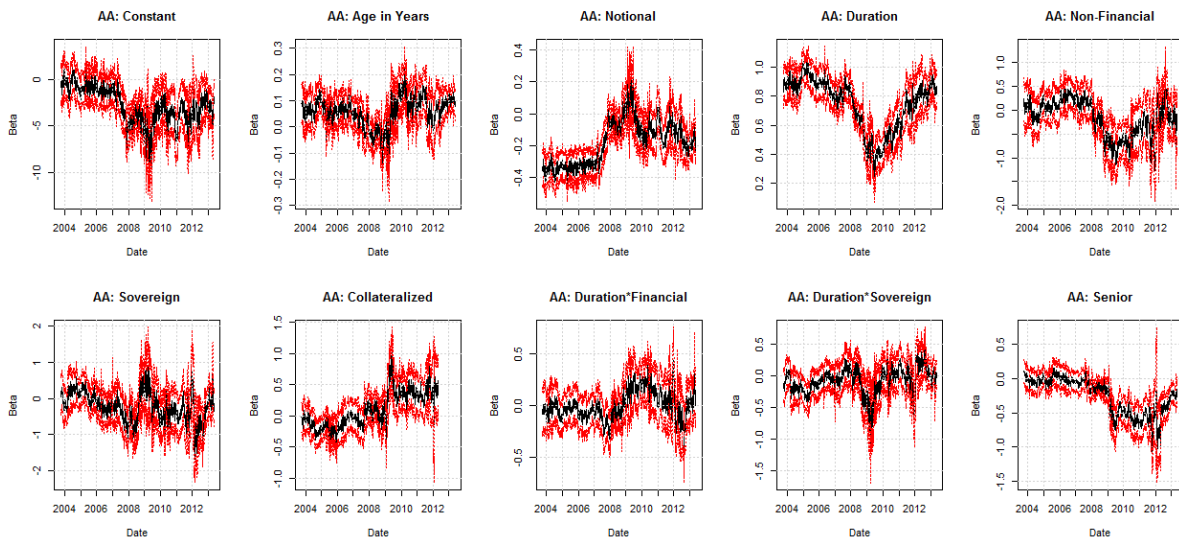
Model	Error	LogLik	Jarque –Bera	Ljung –Box (5)
AAA-F (0,1,1)	Gaussian	-4510	$p < 0.01$	$p = 0.91$
	Student T	<b>-4811</b>	$p < 0.01$	
AAA-NF (1,1,1)	Gaussian	-3273	$p < 0.01$	$p < 0.01$
	Student T	<b>-3736</b>	$p < 0.01$	
AA-F (1,1,1)	Gaussian	-4588	$p < 0.01$	$p = 0.25$
	Student T	<b>-5199</b>	$p < 0.01$	
AA-NF (1,1,1)	Gaussian	-5340	$p < 0.01$	$p = 0.26$
	Student T	<b>-5543</b>	$p < 0.01$	
A-F (2,1,0)	Gaussian	-6072	$p < 0.01$	$p = 0.11$
	Student T	<b>-6508</b>	$p < 0.01$	
A-NF (1,1,1)	Gaussian	-6475	$p < 0.01$	$p = 0.43$
	Student T	<b>-6730</b>	$p < 0.01$	
BBB-F (1,1,1)	Gaussian	-5523	$p < 0.01$	$p = 0.02$
	Student T	<b>-5549</b>	$p < 0.01$	
BBB-NF(2,1,0)	Gaussian	-6298	$p < 0.01$	$p = 0.16$
	Student T	<b>-6483</b>	$p < 0.01$	

**Figure 3: Temporal Bid-Ask Spread Modelling, AAA coefficients.** *Time series plot of the fitted coefficients and 95% Confidence Interval of the temporal model described by Equation 10 in the main text. Coefficients are stable over time, but there is some movement. For example, the coefficient*

of the interaction effect of Duration and Sovereign, changes considerably in light of the Sovereign Crisis; the Bid-Ask Spread becomes very Duration dependent.

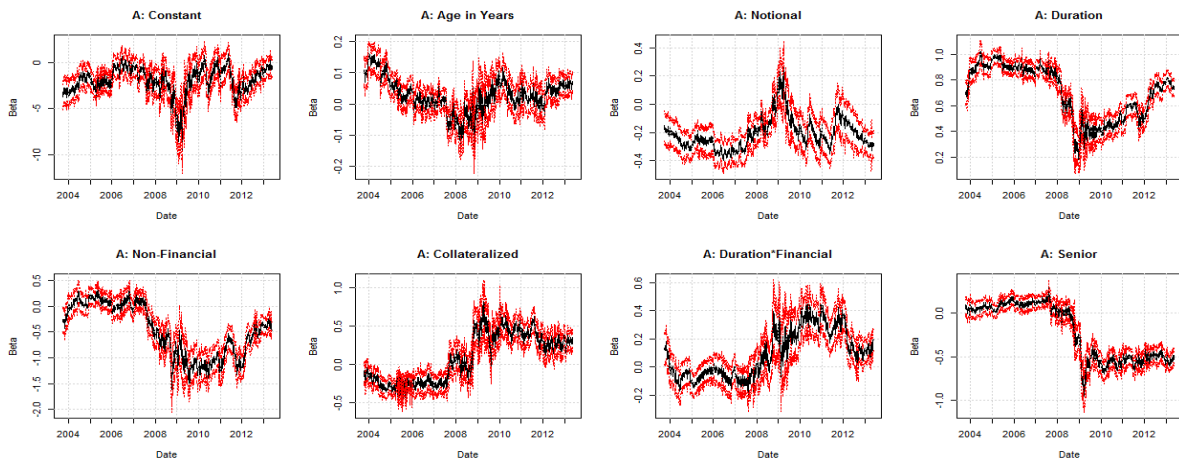


**Figure 4: Temporal Bid-Ask Spread Modelling, AA coefficients.** Time series plot of the fitted coefficients and 95% Confidence Interval of the temporal model described by Equation 10 in the main text. Coefficients are stable over time, but there is some movement. For example, the coefficient of the Duration changes drastically, at the same time the coefficient of Notional Amount changes drastically in the opposite direction. These changes can be rationalized by changing priorities of investors in the market during extreme distress.

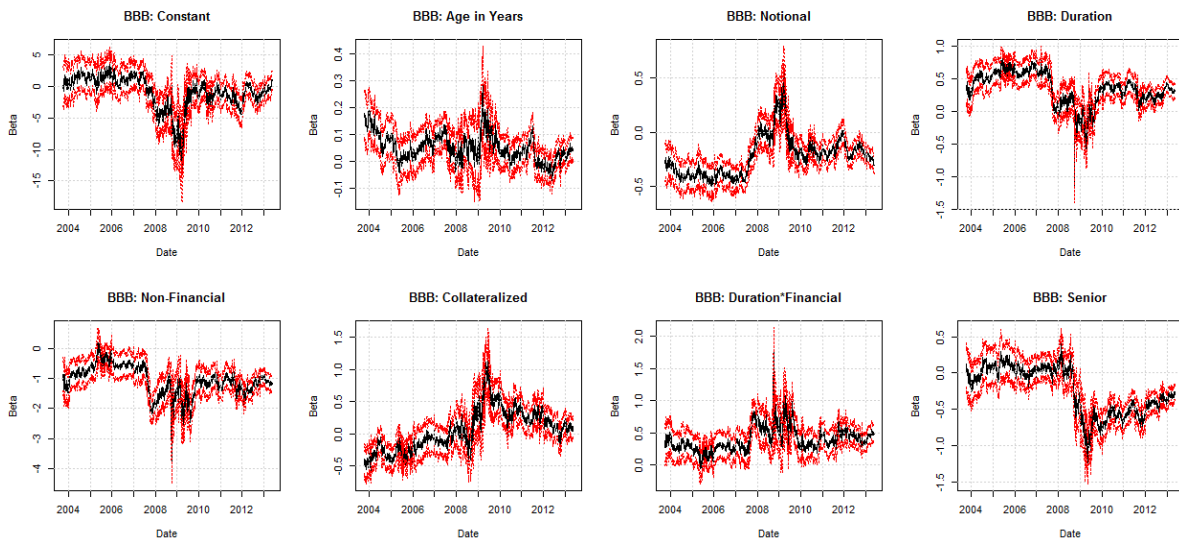


**Figure 5: Temporal Bid-Ask Spread Modelling, A coefficients.** Time series plot of the fitted coefficients and 95% Confidence Interval of the temporal model described by Equation 10 in the main text. Coefficients are stable over time, but there is some movement. For example, the coefficient of

Senior bonds, also discussed in the main text, drops sharply and has not started to show any signs of recovery towards old levels.

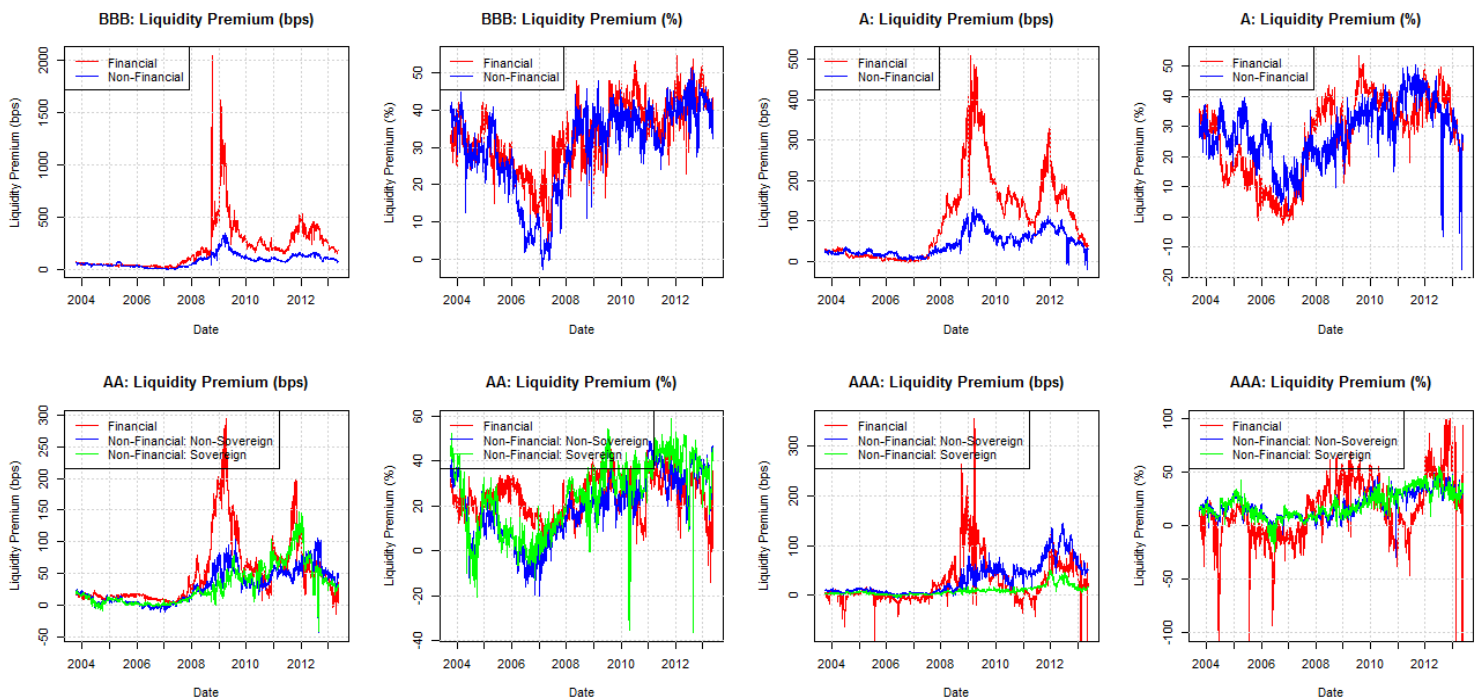


**Figure 6: Temporal Bid-Ask Spread Modelling, BBB coefficients.** Time series plot of the fitted coefficients and 95% Confidence Interval of the temporal model described by Equation 10 in the main text. Coefficients are stable over time, but there is some movement. For example, the coefficient of the interaction effect of Duration and Financials, changes considerably during the height of the crisis. The trading on price rather than yield is apparent for the now extremely risky (perceived) Financials of low Rating quality.

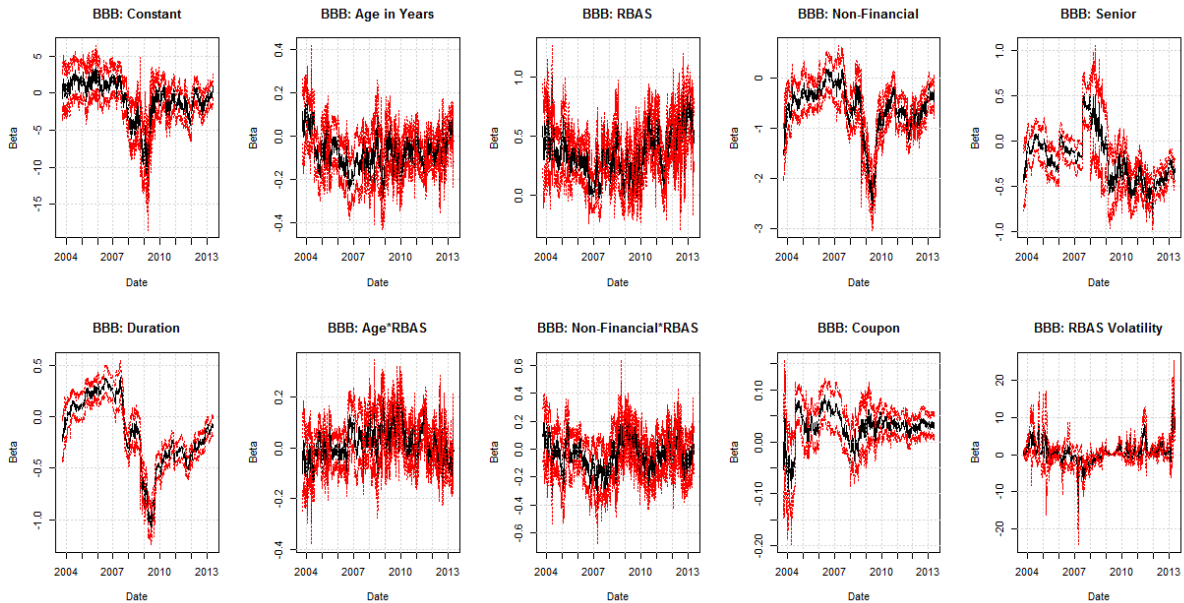


## 8.4 Appendix IV: Credit Spread Modelling

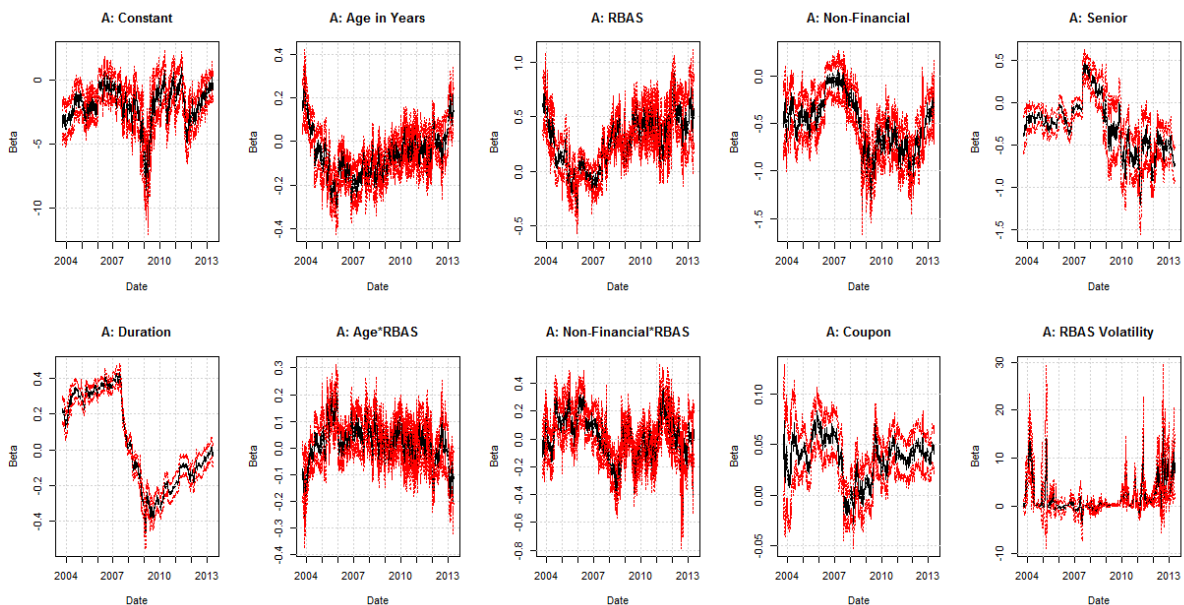
**Figure 1: Average Liquidity Premium Estimates.** For each of the ten categories (split by Rating class and status Financial/ Sovereign/ Non-Sovereign), the average Liquidity Premium in calculated on a daily basis, in both bps and as a proportion of the total Credit Spread. We can clearly see that even though the Liquidity Premium as a proportion of total Credit Spread does not differ between Financials/ Non-Financials that the estimates in basis points do differ, indicating that the Credit Spreads of Financials was magnitudes higher than the Credit Spread of Non-Financials. In all categories, most notably the AAA it appears as if some computation error causing sudden drops; this needs investigating



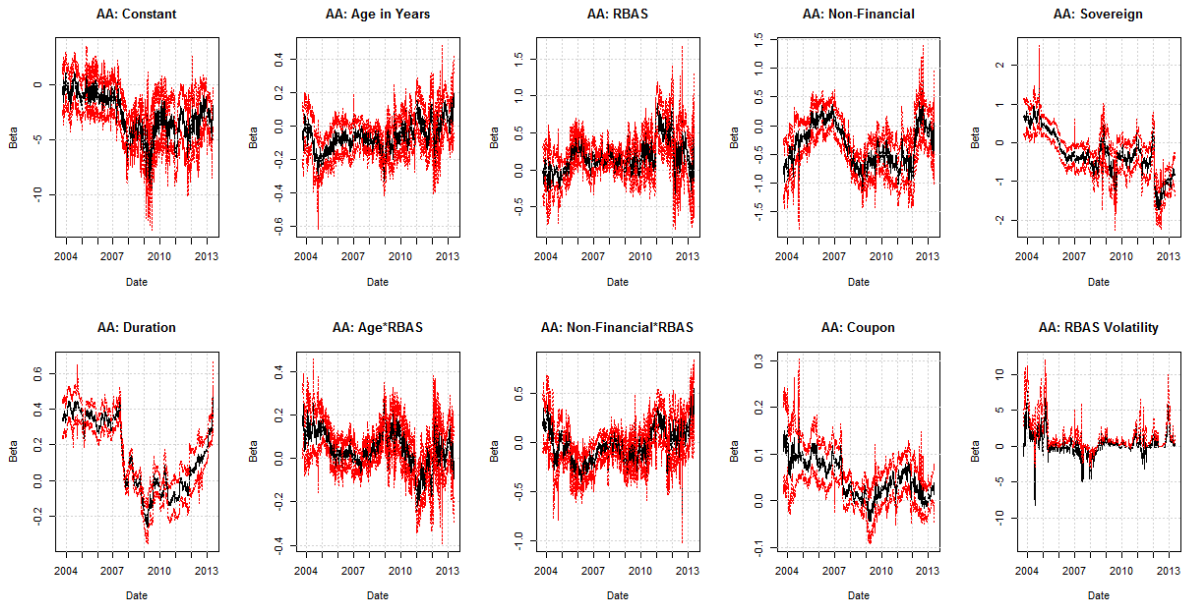
**Figure 2: Temporal Bid-Ask Spread Modelling, selection of BBB coefficients.** *Time series plot of a selection of the fitted coefficients and 95% Confidence Interval of the temporal model described by Equation 18 in the main text. Coefficients are stable over time, but there is some movement. Most notably the sharp drops in the Duration, Constant and Non-Financial coefficient. The RBAS coefficient is quite noisy, the coefficient of the volatility does not appear to have a consistent effect.*



**Figure 3: Temporal Bid-Ask Spread Modelling, selection of A coefficients.** *Time series plot of a selection of the fitted coefficients and 95% Confidence Interval of the temporal model described by Equation 18 in the main text. Coefficients are stable over time, but there is some movement. Most notably the sharp drops in the Duration, Constant and Non-Financial coefficient. The Non-Financial coefficient has fully recovered since the sharp drop in 2009. The RBAS coefficient is not quite noisy as for the BBB model and follows the same pattern adding to the validity of this coefficient. The coefficient of the volatility does not appear to have a consistent effect.*



**Figure 4: Temporal Bid-Ask Spread Modelling, selection of AA coefficients.** *Time series plot of a selection of the fitted coefficients and 95% Confidence Interval of the temporal model described by Equation 18 in the main text. Coefficients are stable over time, but there is some movement. Most notably the sharp drops in the Duration, Constant and Non-Financial coefficient. The temporary drop in the RBAS coefficient during 2005-2006 seen in the BBB and A model is not so apparent for the AA model. Interestingly, the coefficient of Coupon seems to follow a downward trend, now around 0, indicating little/no effect of coupon rate.*



**Figure 5: Temporal Bid-Ask Spread Modelling, selection of AA coefficients.** *Time series plot of a selection of the fitted coefficients and 95% Confidence Interval of the temporal model described by Equation 18 in the main text. For some of the coefficients confidence intervals are extremely wide and it seems as if computational problems appear. These are also the likely cause of the Liquidity Premia estimates that seemed unstable in Figure 1. The fact that very few bonds would have been AAA and Financials is the likely cause of the problem.*

