

### Optimal Life Cycle Portfolio Choice with Variable Annuities offering Liquidity and Investment Downside Protection



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### VA's + GMWBs *(Guaranteed Minimum Withdrawal Benefits)*

Key features:

- Allows equity investment in the VA (separate account)
- Limited annual withdrawals possible
- At retirement: Can annuitize fund value for lifetime payouts or take lump sum
- Money back guarantee / downside protection
- Fund Account / Guarantee Account



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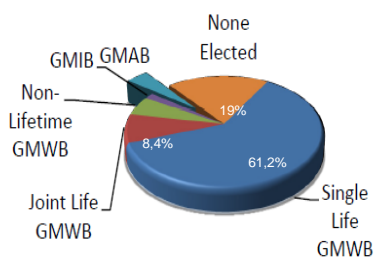
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### Motivation

Variable Annuity Sales Data for Q1 2010 (\$210 B)



National Association for Variable Annuities (NAVA)

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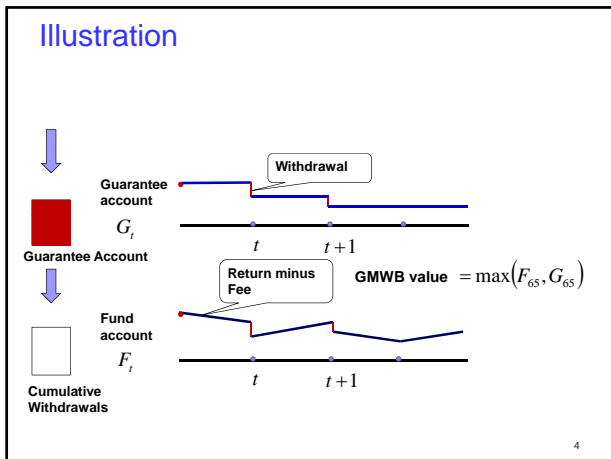
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### GMWBs: Mechanics

Fund Account:  $F_{t+1} = \max(F_t - E_t, 0) \cdot (1 - \varphi) \cdot R_{t+1}$

Guarantee Account:  $G_{t+1} = G_t - E_t$

Withdrawals until  $t < K$ :  $0 \leq E_t \leq \min(G_t, E^{\max})$  Max. annual withdrawal

Value at ret.  $t = K$ :  $L_K = \max(G_K, F_K)$

Final Lump Sum  $t = K$ :  $0 \leq LS_K \leq L_K$

Annuity Income  $t = K$ :  $PA_K = \frac{\max(F_K, G_K) - LS_K}{\ddot{a}_K}$

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### GMWBs: Insurer's Risk Exposure

- In Retirement: Mortality Risk:
  - ✓ Management through Pooling Independent Individual Risks
- Before Retirement: Capital Market + Mortality Risk
  - ✓ Fund Account < Guarantee Account
  - ✓ GMWB resembles complex American Put Option with Time-varying Strike Price
    - ✓ Risk Charge: Option Pricing + Actuarial Theory (Bauer/Kling/Russ 2008, Dai/Kwok/Zong 2008)
    - ✓ Risk Management: Dynamic Hedging (Using Derivatives)

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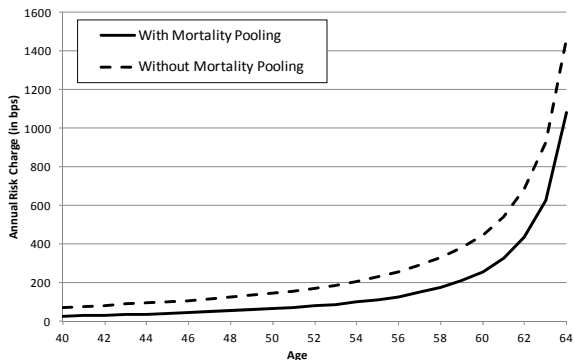
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### Annual GMWB Risk Charge




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### Life-Cycle Model



- Household
  - Time-additive CRRA utility function over the ages 40 to 100 (Base Case: RRA=5, DF=0.96, no bequest motive)
  - Single US female with high school degree
  - (Risky) labor income profile à la Love (2011); retirement age 65
  - Each year: **decide** optimal consumption/saving, liquid investments (stocks, bonds), purchases/withdrawals GMWB-VA
- Annuity & capital markets
  - Deferred variable annuity with GMWB (fix payments from age 65)
  - Withdrawals before retirement age 65 / Lump sum option at age 65
  - Stock LN ~ (6%, 18%); Bonds fix 2%
- Numerical solution by backward induction, (5 state variables: time, wealth; fund account, guarantee account, cost)

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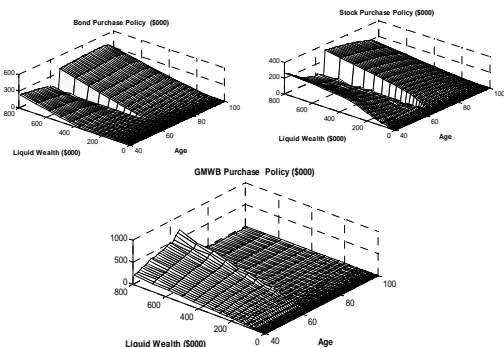
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### Output: Optimal Portfolios w/ GMWBs



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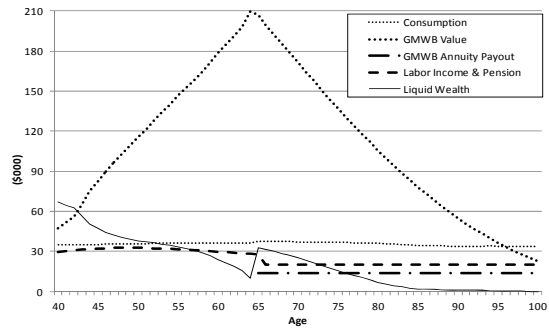
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### Expected Life-Cycle Profiles in GMWB Environment



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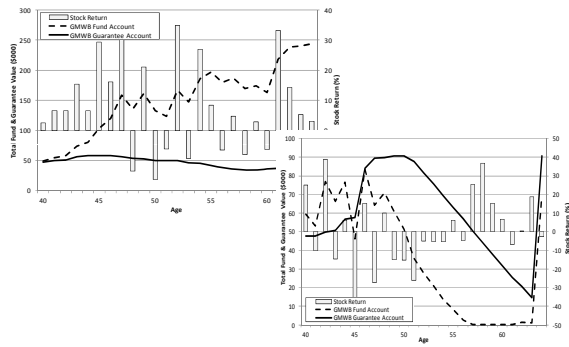
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### Results for Good and Poor Market Performance



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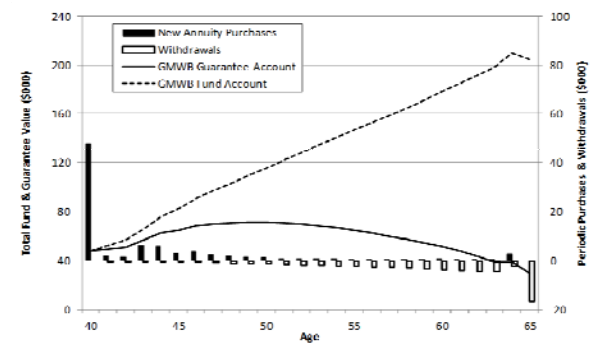
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### Expected Fund and Guarantee Profiles and GMWB Purchases and Withdrawals



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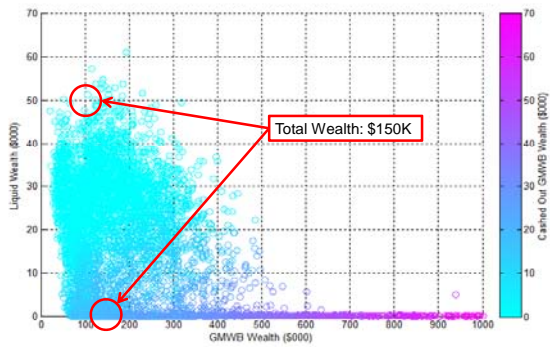
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### GMWB Cash Outs at Age 65 (10,000 Life-Cycles)



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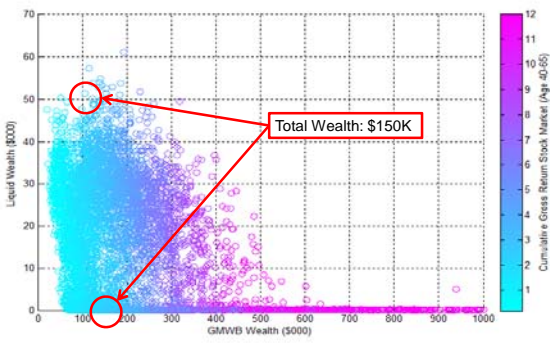
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### Cumulative Stock Market Performance Drives Results



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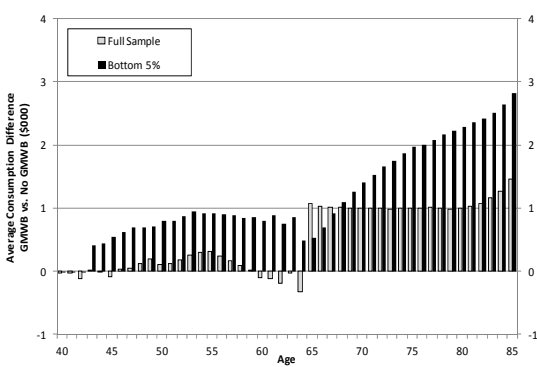
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### Excess Consumption From GMWBs



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So:

- GMWB combination of downside protection, liquidity, stock market participation, and annuitization is very appealing;
- GWMBs quite attractive for household portfolios, raising lifetime consumption and utility.



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Overall Conclusions:



- Payout VAs a *crucial* component of household LC portfolio.
  - Provide access to capital market and survival credit, enhancing lifetime wellbeing.
  - Guarantees, liquidity, participation all valuable but change pricing.
- Complex and often confusing to real-world consumers.
  - We show their contribution to welfare.

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